Stellarators and the path from ITER to DEMO

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A demonstration of fusion power, called DEMO, requires information on a number of physics issues that are addressed by non-axisymmetric shaping of the plasma. Stellarator experiments have shown that non-axisymmetric shaping provides the control needed to address the maintenance of the magnetic configuration, robustness against disruptions, and restrictive upper limits on the plasma density. Shaping is the primary means of control of a toroidal fusion plasma. The importance of axisymmetric shaping is recognized and exploited. The remaining freedom is in non-axsiymmetric shaping, which has physics benefits but technical challenges. The axisymmetric vision of DEMO as a self-organized plasma with weak external control is a design choice and not a requirement. DEMO cannot be perfectly axisymmetric, so non-axisymmetric magnetic fields must be controlled. The questions are at what level, of what type, and for what purpose.

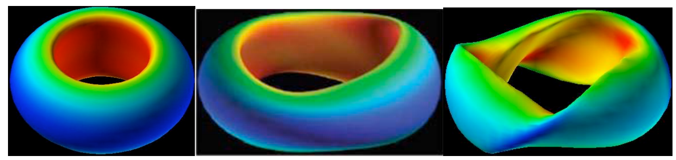
I. INTRODUCTION

Historically tokamaks and stellarators were considered distinct paths to the demonstration of fusion power, DEMO. However, non-axisymmetric shaping can be applied at any level from zero, an axisymmetric tokamak, to strong, a stellarator. Plasma shaping offers control, which is eliminated by the design choice of strict axisymmetry. Tokamak experiments have shown that non-axisymmetric magnetic fields allow the control of Edge Localized Modes (ELMs) and that the control of the effects of magnetic field errors is far easier than the elimination of asymmetries. Stellarator experiments have shown that shaping can maintain the magnetic configuration, provide robustness against disruptions, and remove restrictive upper limits on the plasma density.

The success of the axisymmetric tokamak has defined a path to DEMO, but the plasma is self-organized and weakly controlled. Acceptable power multiplication in DEMO limits the externally driven current to no more than 20%, so the fusion power and the plasma profiles are essentially self-determined [1]. If 80% of the poloidal magnetic field can be self produced, relatively weak non-axisymmetric shaping could eliminate the need for technically challenging current drive systems. Though with non-axisymmetric shaping, the externally controlled part of the poloidal field can be set at whatever level is optimal.

The non-axisymmetric shaping that produces a poloidal magnetic field also forms a cage around the plasma, which strongly centers it in the chamber. Empirically this makes the plasma robust against disruptions and resilient to ideal magnetohydrodynamic (MHD) instabilities.

Why is shaping uniquely effective in providing plasma control in DEMO? A plasma equilibrium, $\vec{\nabla}p = \vec{j} \times \vec{B}$, is determined by the profiles of the plasma pressure and current plus the shape of the outermost plasma surface [2]. In DEMO, the pressure profile will be largely self-determined through a balance between fusion energy production and microturbulent transport. The plasma current must be 80% bootstrap current, which is also determined by the pressure and temperature profiles. Shape is left as the primary means of control. The importance of axisymmetric shaping is recognized and exploited in all modern tokamaks. The remaining freedom is in non-axisymmetric shaping. Non-axisymmetric shaping has an order of magnitude more shaping parameters than the well-known axisymmetric parameters: aspect ratio, ellipticity, triangularity, and squareness.



Tokamak ι_{vac}=0 ARIES-RS but ι_{vac}/ι=20% NCSX ι_{vac}/ι=75%

FIG. 1: No clear demarcation exists between a tokamak, an ARIES-RS tokamak [4] modified by L.-P. Ku so 20% of its transform is due to non-axisymmetric shaping, and NCSX with 75% of its transform from non-axisymmetric shaping. The drift orbits in all are similar due to quasi-axisymmetry.

Plasma control using shaping can utilize either static or dynamic magnetic fields. Control using static fields is intrinsically simpler and is the usual view of plasma control in stellarators. For example, the location of the plasma in the vacuum chamber can be controlled in two ways: (1) statically using the centering effect of non-axisymmetric fields or (2) dynamically using feedback and axisymmetric fields. Dynamic magnetic fields are used in tokamaks on the resistive time scale of the chamber walls to control vertical and resistive wall mode instabilities. For these examples of dynamic control, the plasma accurately obeys $\vec{\nabla}p = \vec{j} \times \vec{B}$, so the changes in the fields effect the plasma through its shape.

No clear demarcation exists between tokamaks and quasi-axisymmetric stellarators. Particle trajectories act as if the device is axisymmetric as long as the magnetic field strength satisfies $B(\ell) = B(\ell + L_0)$, where ℓ is the distance along the magnetic field lines and L_0 is a constant for each field line [2]. This condition for quasi-axisymmetry can be accurately imposed at any level of non-axisymmetric shaping from zero to strong, as in the NCSX stellarator, which was being contructed at Princeton [3], Figure (1).

Non-axisymmetric shaping can be imposed in ways that differ more fundamentally from a tokamak than quasi-axisymmetric shaping, and this can give even greater control. The best known example is the quasi-isodynamic shaping of the W7-X stellarator under construction at Greifswald, Germany [5]. Quasi-helical shaping is also possible and being studied [6] on the HSX stellarator at the University of Wisconsin.

If an axisymmetric DEMO can be built, so can a DEMO with some level of non-axisymmetric shaping. Many stellarators, including the world's largest LHD [7], have been constructed without major problems or delays. Nevertheless, the construction delays of W7-X and NCSX have focused attention on the technical challenges of non-axisymmetric shaping. Both the physics benefits and the technical challenges can be addressed by theory and design. Rapid progress in the methods was made as part of the design efforts for W7-AS [8], W7-X [5], NCSX [3], and QPS [9], but much remains to be done.

The level of control in DEMO is a design choice, but only weak control is consistent with axisymmetry. ITER may clarify the importance of stronger control, but the information needed to define the design choices must come from research performed in parallel to ITER.

II. ISSUES ADDRESSED BY STRONG SHAPING

A. Magnetic configuration maintenance

The fundamental topological property of the magnetic field confining a toroidal plasma is the profile of the rotational transform $\iota(\psi_t)$. The transform is the average number of poloidal circuits a field line makes per toroidal circuit on a surface that encloses a toroidal magnetic flux ψ_t. The reciprocal of the transform is the safety factor, $q(\psi_t) = 1/\iota(\psi_t)$.

The rotational transform has three contributions,

$$\iota = \iota_{vac} + \iota_{drive} + \iota_{boot}. \tag{1}$$

The transform that would be present even without a plasma, ι_{vac}, requires a non-axisymmetric variation δa in the radius a of the magnetic surfaces, $\iota_{vac} \propto (\delta a/a)^2$. The transform due to currents driven either inductively or through steady-state current drive is ι_{drive}. The transform due to the bootstrap current, which means a net steady-state current produced by the plasma density and temperature gradients, is ι_{boot}.

An acceptable energy multiplication in a fusion system places a strict limit on ι_{drive}. DEMO [1] requires $\iota/\iota_{drive} > 5$. A driven current implies non-Maxwellian particles, and a high power is required to sustain these particles against collisional relaxation [10].

The magnitude and profile of the bootstrap current, and hence $\iota_{boot}(\psi_t)$, are given by the plasma density and temperature profiles. In DEMO, these profiles also determine the fusion power, which is balanced by the microturbulent transport to determine the profiles. The profiles and the magnetic configuration also directly influence the nature of the microturbulent transport. An empirical validation of this self-organized, microturbulent state of an axisymmetric burning plasma has been a major argument for the burning plasma experiment ITER. Nevertheless, a large extrapolation will be required to go from $\iota_{boot}/\iota_{drive} \approx 1.0$, which is expected in ITER, to $\iota_{boot}/\iota_{drive} > 4.0$, which is required for an axisymmetric steady-state DEMO.

Since the bootstrap current is determined by the microturbulent plasma state, the external control of the magnetic configuration is measured by the ratio $(\iota_{vac} + \iota_{drive})/\iota_{boot}$. In an axisymmetric tokamak the measure of control, $\iota_{drive}/\iota_{boot}$ must be extrapolated from an expected value of unity in steady-state ITER to less than a quarter in a steady-state DEMO. With non-axisymmetric shaping the ratio ι_{vac}/ι_{boot} is unconstrained by fundamental issues, such as energy multiplication. The ratio can be made small to minimize the non-axisymmetric shaping or large to optimize physics or to reduce the extrapolation risk to DEMO. In the NCSX design $\iota_{boot}/\iota_{vac} \approx 0.3$, and in other stellarator geometries, such as that of the W7-X design, ι_{boot}/ι_{vac} can be essentially zero.

B. Robustness of plasma equilibria

The shaping that gives the vacuum rotational transform ι_{vac} centers the plasma in the vacuum chamber and acts as a cage around a tokamak.

The location of an axisymmetric plasma within a vacuum chamber is a balance between the hoop stress of the plasma current, which is sensitive to the plasma state, and the force due to external vertical field. No natural tendency exists to center the plasma in the chamber. An external rotational transform ι_{vac} defines a central location for the plasma with a centering force that increases rapidly as the plasma approaches the coils. The external transform, ι_{vac}, not only gives greater control it also enhances the robustness of the plasma. The effect is profound.

The sudden termination of a plasma, called a disruption, poses an increasing risk to the survival of the confinement device itself the larger the plasma current. A disrupting plasma is associated with forces, heat loads, and relativistic electron avalanches that are capable of producing catastrophic damage. Disruption free operation of axisymmetric tokamaks requires staying sufficiently far below certain operational limits, which involve the plasma current, the density, and the pressure

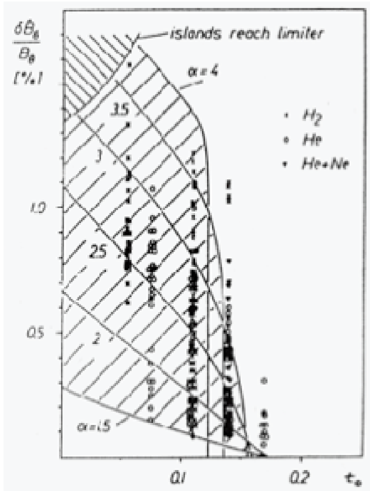


FIG. 2: MHD activity, which often led to disruptions, dropped to essentially zero in the W7-A stellarator, Fig. (7) of Ref. [12], when the vacuum rotational transform, $t_o = \iota_{vac}$, was greater than about 0.15.

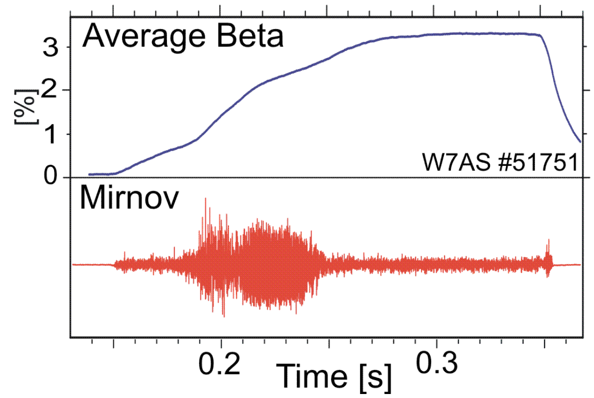


FIG. 3: Although magnetic activity was seen in W7-AS as beta crossed the threshold for ideal MHD stability, the effect on confinement was small and the magnetic activity went away for higher beta, Fig. (6) of Ref. [15].

[11]. Disruptions can also be induced by the drag on the plasma rotation, called mode locking, associated with even a small magnetic field error. To avoid mode locking in ITER, $\delta B/B < 5 \times 10^{-5}$ was suggested in the 2007 discussion of the ITER physics basis [11].

Although disruptions define the operational limits of axisymmetric tokamaks, this is not the case in stellarators. How small can ι_{vac} be and avoid disruptions? Early stellarators had a strong plasma current for heating that produced most of the rotational transform; disruptions ceased [12], Figure (2) when $\iota_{vac} > 0.15$. Despite the robustness of stellarators, tearing modes can occur and can strongly degrade the confinement. An $m = 2/n = 1$ island due to a tearing mode was seen in W7-AS as $\iota_{edge} \to 1/2$ when about 25% of its edge and 65% of its central transform was due to plasma current [13].

The robustness of stellarator equilibria is also illustrated by two other empirical results: (1) the softness of beta limits, $\beta \equiv 2\mu_0 p/B^2$ and (2) beta maintenance.

In W7-AS it was found [14] the pressure driven magnetohydrodynamic (MHD) modes limit neither the overall plasma pressure, the pressure gradient, nor the energy confinement. An ideal MHD stability analysis does predict unstable modes at certain values of beta, and enhanced magnetic activity is seen in the vicinity of these beta values. However, this activity does not provide a barrier to access to much higher values of beta, Figure (3). Since MHD stability does not provide an apparent empirical limit on the achievable values of beta, it remains controversial what will.

The relevant plasma parameters for DEMO are those that can be held in steady state. The W7-AS and LHD stellarators have found [16], Figure (4), that once a value of beta is obtained it can be held for over a hundred energy confinement times, while in tokamaks the maximum beta value that can be maintained for the longest periods of plasma maintenance, about fifty energy confinement times, is only about 60% of the maximum achievable beta.

Non-axisymmetric shaping also addresses two other stability issues that require non-trivial control systems [11] on ITER: Neoclassical Tearing Modes (NTMs) and Resistive Wall Modes (RWM). The NTM is an enhanced drive for the opening of a magnetic island due to the elimination of the bootstrap current through the flattening of the density and temperature gradients by the island itself. The enhanced drive only occurs when $d\iota/d\psi_t < 0$ as in a normal-shear, $dq/d\psi_t > 0$, tokamak. The natural sense of shear in a system with strong non-axisymmetric shaping is reversed, $d\iota/d\psi_t > 0$; for this sign of shear, the NTM effect tends to heal islands. Resistive wall modes arise when a strong bootstrap current is required, $\iota_{boot}/\iota_{drive}$ large, at a high plasma beta, $\beta \equiv 2\mu_0 p/B^2$. The strength of the bootstrap current at a given plasma beta is a design choice in a system with non-axisymmetric shaping.

An important question is how much non-axisymmetric shaping or vacuum transform is needed to make a toroidal plasma sufficiently robust to disruptions. An axisymmetric DEMO is premised on zero being sufficient. But if not zero, how much is required?

C. Upper limit on plasma density

Two effects place an upper limit on the density in an axisymmetric DEMO: (1) the efficiency of current drive, which scales approximately inversely with density, and (2) the Greenwald limit [17], which has the form $n < I_{pl}/\pi a^2$. Exceeding the Greenwald limit in an axisymmetric tokamak leads to the termination of the plasma through a disruption.

The density limit of axisymmetry has important implications for DEMO. (1) The plasma temperature must be

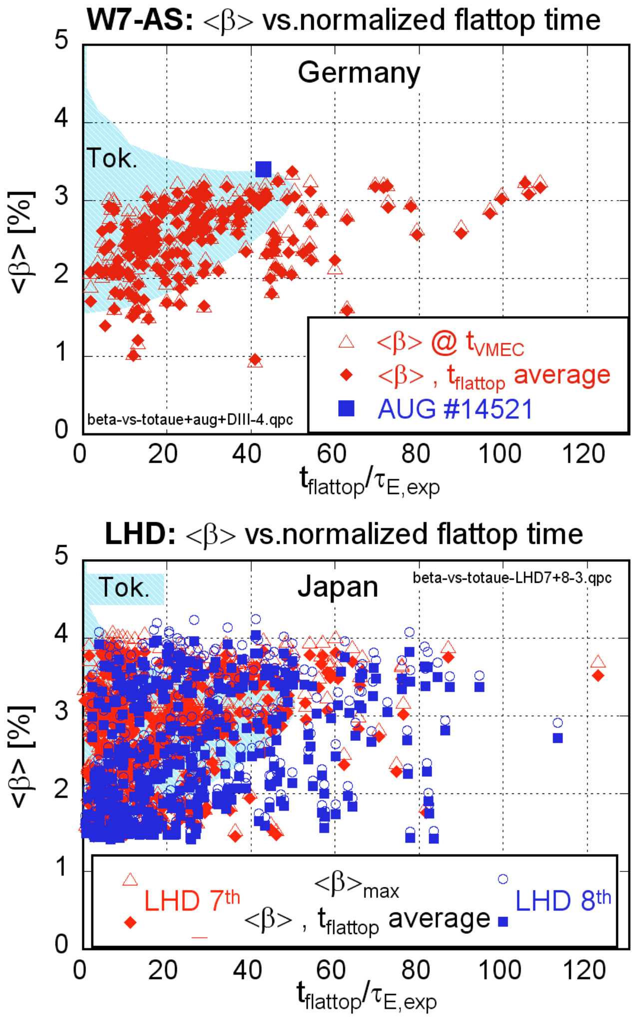


FIG. 4: Both W7-AS and LHD have found, Fig. (2) of Ref. [16], that once a value of beta is obtained it can be held for over a hundred energy confinement times, while in tokamaks the maximum beta value that can be maintained for the longest periods of plasma maintenance, about fifty energy confinement times, is only about 60% of the maximum achievable beta. Even higher values of beta than those illustrated have been achieved for short periods in tokamaks and stellarators.

very high to achieve the required fusion power density. The high temperature and low collisionality make the energy density of non-thermal alpha particles sufficiently high to provide a strong drive for Alfvénic instabilities, which can cause a rapid loss of the fusion alphas. (2) The density of the divertor plasma must be low, which makes handling the power flow in the divertor difficult.

In stellarators with strong non-axisymmetric shaping the plasma density can greatly exceed the Greenwald limit when the plasma current is replaced by the rotational transform ι. Indeed, the best plasma performance is often at a high density [18], Figure (5).

High density divertor plasmas have been studied on

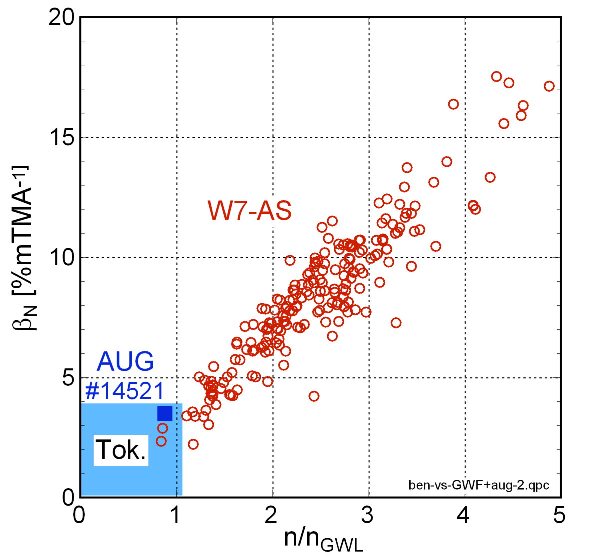


FIG. 5: The performance of W7-AS continues to improve as the density is raised above the Greenwald density, Fig. (4) of Ref. [18].

both LHD and W7-AS. Both machines could operate [19] so the plasma recombined before reaching material surfaces, which is called detachment. The power is then lost by radiation, which eases the issue of divertor power handling. Without a divertor, density control in W7-AS was difficult but with a divertor a steady improvement in confinement was found with higher density, the High Density H-Mode (HDH). Stable detached divertor conditions could be maintained with essentially 100% of the power radiated [14].

An important question is how much non-axisymmetric shaping is needed to remove the restrictions of the Greenwald density limit.

D. Microturbulent transport modification

The empirical level of transport in stellarators has a scaling similar to that of tokamaks [20], Figure (6). The empirical scalings of both stellarators and H-mode tokamaks are close to gyro-Bohm.

The reason for similar empirical confinement in tokamaks and stellarators is not well understood theoretically. Microturbulence is influenced by the magnetic geometry through the magnetic field line curvature, local shear, and distance scale along the magnetic field lines over which the curvature and shear change. The theoretical expectation is, therefore, that microturbulent transport could be modified by the use of shaping, and this is seen in computer simulations of microturbulent transport.

Pavlos Xanthopoulos and Frank Jenko's group have used their nonlinear microtubulence code GENE to simulate Ion Temperature Gradient (ITG) turbulence [21] in W7-X [22], in NCSX, and in tokamaks. The GENE

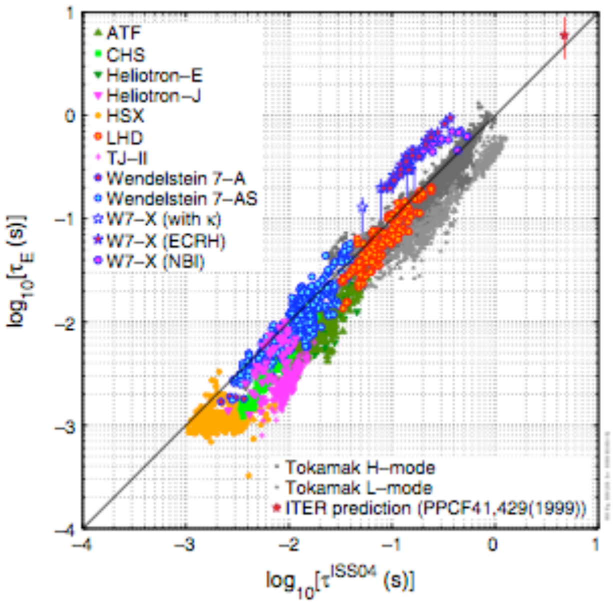


FIG. 6: The empirical scaling of transport in stellarators is similar to that of H-mode tokamaks. The energy confinement times of various stellarator and tokamak experiments is shown, as well as projections for W7-X and ITER, versus a stellarator confinement scaling law, Fig. (7) of Ref. [20].

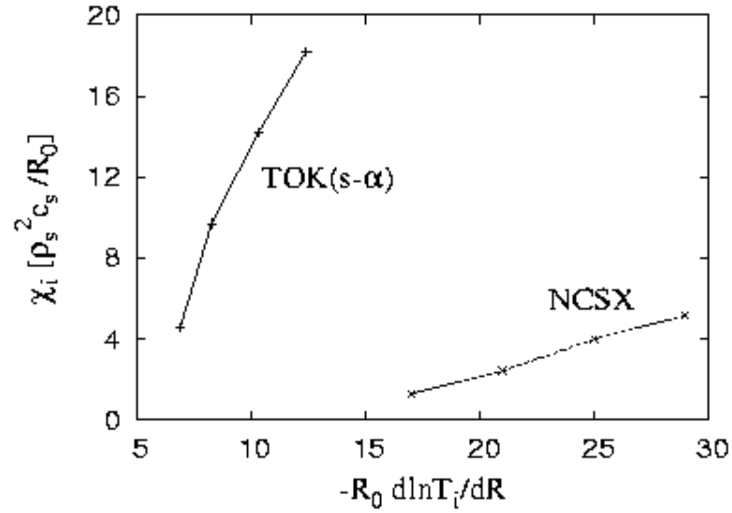


FIG. 7: The GENE code [21] finds that above the stability threshold for Ion Temperature Gradient (ITG) turbulence the ion heat transport increases rapidly in the tokamak, but the increase is much weaker in NCSX. The temperature gradient is defined using the major radius at the location of the largest bad curvature of the magnetic field lines

code finds, as do a number of other codes, that in tokamaks, zonal flows cause both a large reduction in the strength of the collisionless ITG turbulence and a shift of the marginally stable gradient to larger values. However, GENE finds that zonal flows often tend to have a weaker impact on ITG turbulence for stellarators, which may be due to a difference in the magnetic shear. Moreover, above marginal stability, the ion heat transport rapidly increases in the tokamak, but the increase tends to be weaker in W7-X and NCSX, Figure (7). The rapid increase in ITG transport seen in the tokamak simulations will force the plasma towards the critical gradient. When this occurs, the overall plasma confinement is determined by the plasma edge. The effect of shaping on electron confinement is also important but has not yet been studied using GENE.

An important question is whether non-axisymmetric shaping can be used to make the plasma performance sufficiently independent of the heating profile to allow burning plasma performance to be assessed by experiments in non-burning plasmas.

III. IMPORTANCE OF EVEN WEAK SHAPING

A. Mitigation of magnetic field errors

The use of non-axisymmetric shaping to mitigate magnetic field errors in tokamaks sounds peculiar. Why not just eliminate the magnetic asymmetries? The problem is that certain very small asymmetries $\delta B/B \sim 10^{-4}$ can cause disruptions. Tokamaks can neither be built with the required accuracy nor can practical correction coils reduce all asymmetries to $\delta B/B < 10^{-4}$. However, a simple coil set can control the magnetic asymmetries that cause disruptions, though often by making $\delta B/B$ larger. Error field mitigation does not mean error field elimination or even error field reduction.

Recently a capability was developed for quickly and accurately solving the equilibrium equation, $\vec{\nabla}p = \vec{j}\times\vec{B}$, for tokamaks that are slightly perturbed from axisymmetry, the Ideal Perturbed Equilibrium Code (IPEC). When IPEC is used [23] to analyze empirical error field correction experiments on DIII-D and NSTX, several paradoxes were resolved, which arose from common theory approximations being so inaccurate as to be misleading.

Empirically tokamaks are found to stop rotating, which is called mode locking, and often disrupt when the density is lowered below a critical value. This locking density is found empirically to be proportional to the external asymmetric magnetic field. Empirical error field correction means the currents in a set of control coils are adjusted to minimize the locking density.

The expected drive for magnetic islands found by an IPEC analysis [23] of the DIII-D error field correction experiments is given in Figure (8) along with an analysis carried out by the conventional approximation of superimposing the external asymmetric field on the axisymmetric equilibrium field. The plasma response, due to the helical distortion of the equilibrium currents by the perturbation, greatly amplifies the perturbation. The IPEC results show the expected approximate linear dependence

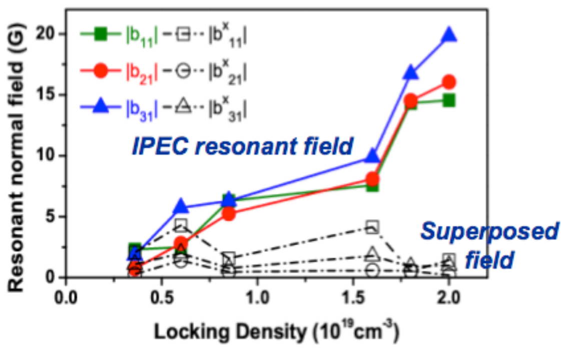


FIG. 8: The colored curves give the IPEC calculation [23] of the magnetic field that is driving islands at the $q = 1, 2$, and 3 rational surfaces in various configurations of the DIII-D control coils versus the plasma density at which plasma rotation locks to zero. This density is known to be proportional to the magnitude of the current in a fixed external current distribution. The black dashed lines give the resonant field for driving islands at the $q = 1, 2$, and 3 rational surfaces in a model in which the external perturbation is superposed on the unperturbed equilibrium field.

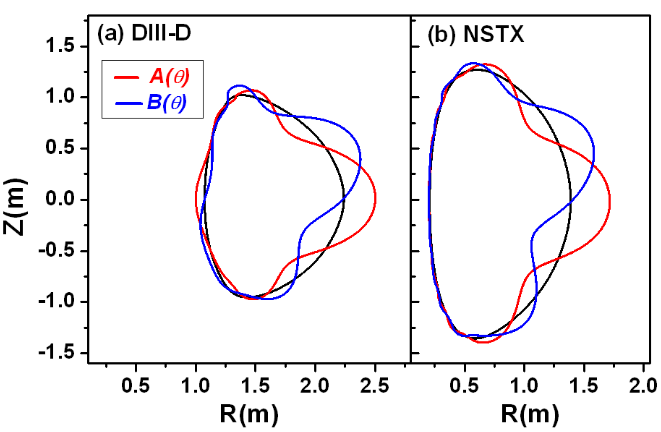


FIG. 9: The black curve is the plasma boundary. The distance between the red and the blue curves and the plasma boundary gives the magnitude of the external magnetic perturbation $\delta\vec{B}_x \cdot \hat{n} = A(\theta)\cos\varphi + B(\theta)\sin\varphi$ to which the plasma is most sensitive, Fig. (7) of Ref. [23].

of the driving field for the magnetic islands on the locking density while the superposition results show no correlation. The IPEC results also show the drive at three different rational surfaces $1/\iota = q = 1, 2, 3$ is highly correlated, which implies a single external perturbation dominates the response at all three rational surfaces. This external perturbation, which is illustrated in Figure (9), is dominant by an order of magnitude and is essentially the same for DIII-D and NSTX. Indeed one finds a similar perturbation dominates the error field response of ITER over its broad range of operational scenarios [24].

Error field mitigation on a nominally axisymmetric tokamak requires that a coil system be available to efficiently control a particular distribution of non-axisymmetic magnetic field.

Some level of non-axisymmetric shaping control is required for a practical tokamak to perform as if it were axisymmetric. The design choice for this control is an important issue for the success of ITER and DEMO.

B. Control of ELMs

The transport in the edge pedestal of an H-mode tokamak is too small. The plasma gradients steepen in that region until an instability occurs, which is called an Edge Localized Mode (ELM). The large pulses of energy associated with ELMs degrade divertor components and can unacceptably shorten their life. Small asymmetric perturbations have been used on the DIII-D tokamak [25] to control of ELMs while maintaining the enhanced confinement of the H-mode. The mechanism by which the asymmetric fields enhance the transport remains controversial: magnetic stochasticity, enhanced ion diffusion due to effects of drifts, or as suggested by R. Goldston enhanced damping of the zonal flows increasing the microturbulent transport.

Whatever the mechanism by which the asymmetric fields enhance the transport in the pedestal, the asymmetric fields cannot be allowed to unacceptably degrade the central plasma. The simplest way to enforce this condition is to use asymmetric perturbations with a high toroidal mode number since such perturbations have a rapid decay through space. Unfortunately this rapid spatial decay implies the driving coils must be close to the plasma surface, which is challenging in ITER and even more difficult in DEMO. An alternative is to optimize the asymmetric field, so the plasma edge is affected far more than the center. Even a simple optimization produces a major improvement. Figure (10) shows an IPEC optimization for ITER carried out by J.-K. Park with coil sets both on and off the outboard midplane. If more freedom were allowed in the external magnetic field, approximate quasi-axisymmetry could be imposed on the perturbing field, so the amplitude of the perturbation in the plasma interior would be irrelevant. This would allow ELM control coils to be placed much further from the plasma but would probably require some perturbation to the axisymmetry of the magnetic field on the inboard side.

On a nominally axisymmetric tokamak DEMO, an understanding the physics and the control of non-axisymmetric fields is important for the control of ELMs.

IV. CONSTRAINTS ON NON-AXISYMMETRIC SHAPING

A. Good magnetic surfaces

The magnetic field lines in rational magnetic surfaces close on themselves after m toroidal transits and

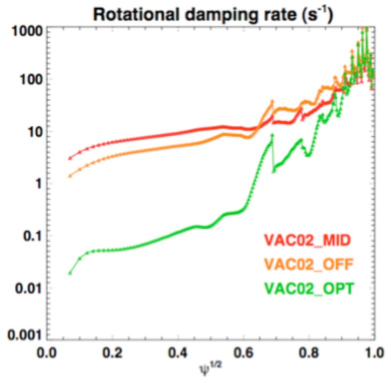


FIG. 10: Toroidal rotation damping can be greatly reduced in the plasma core while maintaining strong asymmetric effects near the plasma edge. The figure shows the damping rate in ITER as a function of the normalized poloidal flux for three configurations of ELM control coils: (1) a midplane coil set only (red), (2) a set of coils off the midplane only (orange), and (3) both coils sets with the relative currents optimized to minimize rotation damping in the plasma core while retaining strong effects at the plasma edge (green). Note the logarithmic scale for damping rates. Figure provided by J.-K. Park.

n poloidal transits. In other words, the rotational transform is a rational number $\iota = n/m$ on a rational surface. In an axisymmetric tokamak, the transform is a smooth function of the toroidal flux $\iota(\psi_t)$, so rational surfaces are unavoidable.

An important feature of rational surfaces is that a magnetic perturbation that has the (m, n) Fourier $e^{i(n\varphi - m\theta)}$ harmonic $b_{mn} = (\delta\vec{B}\cdot\vec{\nabla}\psi_t/\vec{B}\cdot\vec{\nabla}\varphi)_{mn}$ can split the ι_{mn} rational magnetic surface and form a magnetic island with a width that scales as $\sqrt{|b_{mn}|}$, Section (III A) of Ref. [2]. When islands from different rational surfaces overlap, a single field line can ergodically cover a region that includes these surfaces, which destroys the plasma confinement in that region.

Even when islands do not overlap, the presence of an island modifies the plasma rotation and enhances the transport. Plasmas can flow along, but not across a magnetic island. An island that is locked to an external magnetic field stops one component of the plasma flow at its rational surface, Section (V B 3) of Ref. [2]. The wobble of the magnetic field lines through a distance proportional to $\sqrt{|b_{mn}|}$ also breaks the condition $B(\ell) = B(\ell + L_0)$ that is required for quasi-axisymmetry (or axisymmetry).

An axisymmetric tokamak can have no islands since only $n = 0$ Fourier harmonics can be present, so locked magnetic islands are due to magnetic field errors. However, systems with non-axisymmetric shaping can have intrinsic islands, which means islands that have a toroidal mode number n equal to an integer times the number of toroidal periods N_p of the shaping.

Intrinsic islands, which are given by the non-axisymmetric shaping itself, can only be controlled by careful design. The intrinsic islands can be made exponentially small by using non-axisymmetric fields that have a large number of toroidal periods N_p. This follows from the theorem that the Fourier transform of an analytic function converges to zero exponentially with high mode number. Indeed, many stellarators are designed with a large number of periods. For example, the largest stellarator, LHD in Japan [7], has $N_p = 10$. However fundamental geometry limits the plasma aspect ratio per period $R/N_p a$, where R is the major and a is the minor radius of the torus. The desire to build fusion systems with a small total power output limits the aspect ratio R/a, and makes it highly desirable to have as few periods as possible. For example, the NCSX stellarator had $N_p = 3$. Although physics considerations, such as beta limits, force axisymmetric tokamaks to have a small aspect ratio R/a, desirable physics properties can be enforced easier with non-axisymmetric shaping the larger R/a. It is only the total power output and the cost of individual devices, which scale linearly with R/a, that drives stellarator designs towards a small aspect ratio. The minor radius a is essentially fixed by having a scale consistent with the shield thickness and by considerations of wall loading.

The theorem on exponential convergence of a Fourier transformation implies intrinsic islands are of concern only for low order rational surfaces $\iota = n/m$. The standard code [26] for non-axisymmetric equilibria, VMEC, does not give a reliable measure of the intrinsic islands. During the design of NCSX, the PIES code was used to ensure the intrinsic islands were sufficiently narrow [27]. Recently, a fast algorithm was developed [28], [29], which is based on Carolin Nührenberg's CAS3D code, to perturbatively correct the VMEC equilibria to find the intrinsic islands.

B. Particle trajectory confinement

Two types of restrictions on the particle trajectories arise in a fusion plasma. First, the fusion alphas must be retained in the plasma to provide heating. A far more restrictive, and somewhat controversial, requirement on the alpha confinement arises if the loss of even a few percent of the alphas at their birth energy would result in unacceptable damage to the chamber walls. Second, the thermal ions and electrons must remain sufficiently close to a magnetic surface between collisions. For ions $\delta\psi_t/\psi_t < 10^{-1}$ and for electrons $\delta\psi_t/\psi_t < 10^{-2}$, where $\delta\psi_t$ is the deviation from a magnetic surface.

This condition on the thermal ions follows from fundamental kinetic theory and thermodynamics. The deviation δf of either the electron or ion distribution function, $f = f_M \exp(\delta f)$, from a Maxwellian must satisfy $\delta f \sim 1/\sqrt{\nu_c \tau_E}$, where ν_c is a characteristic collision frequency and τ_E is the energy confinement time [30], [2].

The deviation of the distribution function δf is determined by the deviation of the particle trajectories from the magnetic surfaces. The kinetic energy of a particle is $H - q\Phi(\psi_t) = mv^2/2$, where $\Phi(\psi_t)$ is the electric potential. If the maximum field strength along a field line $B_{max} < (H - q\Phi)/\mu$, where $\mu = mv_\perp^2/2B$ is the adiabatically conserved magnetic moment of the particle, then the particle is passing. A passing particle covers a whole magnetic surface, and stays close to it. Consequenly the deviation of the trapped particles, $B_{max} > (H - q\Phi)/\mu$, from a magnetic surface, $\delta\psi_t$, must be sufficiently small if δf is to satisfy $\delta f \sim 1/\sqrt{\nu_c\tau_E}$. For a reactor relevant energy confinement time τ_E, the ions must satisfy $\delta\psi_t/\psi_t < 10^{-1}$ and the electrons $\delta\psi_t/\psi_t < 10^{-2}$.

The deviation of the trapped particle trajectories from a magnetic surface can be determined using the constancy of the action $J = \oint v_{||}d\ell$, which can be written as

$$J = \sqrt{2\frac{\mu}{m}} \oint \sqrt{B_{turn} - B(\ell)}d\ell, \qquad (2)$$

where magnetic field strength at the turning point of the particle is $B_{turn} \equiv (H - q\Phi(\psi_t))/\mu$ and ℓ is the distance along a field line.

In strict quasi-symmetry, the magnetic field strength along each field line has a periodicity length $L_0(\psi_t)$, so $B(\ell) = B(\ell + L_0)$. When $B(\ell)$ is strictly quasi-symmetric, the action J is a constant on a magnetic, or ψ_t, surface for a particle of given energy and magnetic moment, Figure (11). The conservation of action implies the turning points of the particle will remain on the same magnetic surface, so the only deviation the particle can have from its home magnetic surface is its deviation between turning points, known as the banana orbit width, which is proportional to the gyroradius. In practice the banana orbit is sufficiently narrow that the critical condition for trajectory confinement is the constancy of the action J on a magnetic surface for particles of given energy and magnetic moment.

Strict quasi-symmetry is not possible except when the axisymmetry is perfect, though it can be well approximated. To the extent, quasi-symmetry is broken, Figure (11), the action of a particle of given energy and magnetic moment depends on its location in the surface, $J = J_0(\psi_t) + \delta J_s$. The conservation of action implies the particle must move radially a distance $\delta\psi_t = -\delta J_s/(dJ_0/d\psi_t)$ as it precesses around the torus. A more general principle for achieving trajectory confinement is to minimize δJ_s.

A minimization of δJ_s, which is the variation within a magnetic surface of the action of a particle of given energy and magnetic moment, is said to be toward a system that is isodynamic. The minimization of δJ_s was used in the design of W7-X. The only way a magnetic field can be exactly isodynamic is if it is exactly quasi-symmetric $B(\ell) = B(\ell + L_0)$. However, when δJ_s is non-zero an important distinction exists between approximations to being quasi-symmetric and isodynamic. An isodynamic, unlike a quasi-symmetric, optimization al-

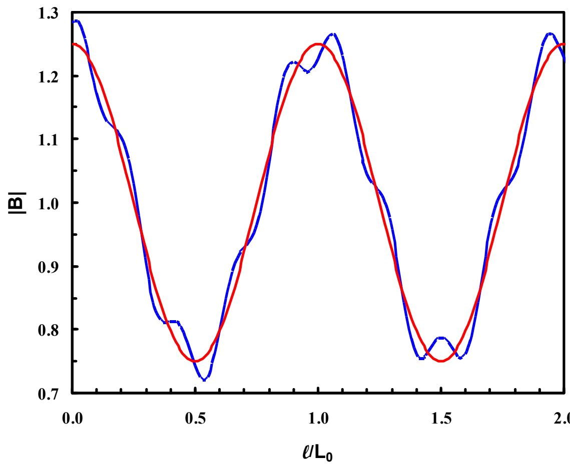


FIG. 11: The magnetic field strength in quasi-symmetry has a periodic variation along each magnetic field line, the red curve. When that periodicity is broken, the blue curve, the action integral, Eq. (2), of a particle of given energy and magnetic moment depends on the location, ℓ_t, of its turning point along the field line, $B(\ell_t) = B_{turn}$. The turning point will then move off the magnetic surface to conserve the action as a particle precesses around the torus.

lows a minimization of the parallel current, $j_{||} \equiv \vec{j} \cdot \vec{B}/B$, both the Pfirsch-Schlüter current, which is required to ensure $\vec{\nabla} \cdot \vec{j} = 0$, and the bootstrap current. If $j_{||}$ were zero, the shape of the plasma surfaces and the rotational profile profile would be independent of the plasma pressure. The zero $j_{||}$ condition is approximated by W7-X, which gives strong external control of the plasma.

The dependence of the particle trajectories on the variation in the field strength along the fields lines, not the variation in space, accounts for much of the sensitivity of tokamaks to a small breaking of the toroidal symmetry. Magnetic perturbations that are locally near resonance, $n/m \approx \iota(\psi_t)$, cause a large distortion of the magnetic surface, which creates a large asymmetry in the equilibrium current and can greatly amplify the perturbation. The distortion of the magnetic surfaces causes the magnetic field lines to move in and out in the intrinsic curvature, or $1/R$, variation in the field strength, which produces a large change in $B(\ell)$ and a large δJ_s.

When quasi-symmetry $B(\ell) = B(\ell + L_0)$ is broken, the radial diffusion coefficients for ions and electrons are generally unequal. The preservation of quasineutrality requires the electric potential to relax to an ambipolar potential $\Phi_A(\psi_t)$, so the radial transport of the poorer and better confined species become equal. A relaxation of the plasma potential to the ambipolar potental changes the $E \times B$ rotation, so it involves a torque. In practice, a number of factors can affect the actual electric potential $\Phi(\psi_t)$, for example a competition between cross field viscosity and the non-ambipolar transport associated with a breaking of quasi-symmetry. In such situations, the actual potential does not equal the ambipolar potential for a particular term in the breaking of quasi-symmetry and toroidal angular momentum must be transmitted by the associated asymmetric magnetic field perturbation to the coil that produces the asymmetry. However, this torque has an upper limit, ultimately set by the Maxwell stress tensor $\overleftrightarrow{T} = (\vec{B}\vec{B} - B^2\overleftrightarrow{1}/2)/\mu_0$, where $\vec{\nabla} \cdot \overleftrightarrow{T} = \vec{j} \times \vec{B}$.

If a small asymmetry causes too much radial transport, the current associated with the torque must shield the symmetry-breaking perturbation to prevent a violation of the torque limit associated with the Maxwell stress tensor. Indeed apparent shielding of this type has been seen [31] on NSTX. The implication is that if a quasi-symmetry is well enough approximated, the plasma will tend to complete the symmetrization itself.

C. Practical coils and structures

The externally produced magnetic field has two purposes: (1) to produce the toroidal magnetic flux and (2) to enforce the condition $\vec{B} \cdot \hat{n} = 0$ at the plasma boundary. The second condition follows from the equilibrium, $\vec{\nabla}p = \vec{j} \times \vec{B}$, constraint that $\vec{B} \cdot \vec{\nabla}p = 0$. In a tokamak the toroidal field coils provide the toroidal flux and the poloidal field coils enforce $\vec{B} \cdot \hat{n} = 0$ at the plasma boundary.

Given profiles of pressure $p(\psi_t)$ and transform $\iota(\psi_t)$, one can find the external magnetic field normal to the plasma boundary $\vec{B}_{ext} \cdot \hat{n}$ that would be required to support a plasma of given shape. A given $\vec{B}_{ext} \cdot \hat{n}$ on the plasma boundary requires a larger field at the location of the coils to produce it. The first practicality constraint of coil design is to ensure that all distributions of $\vec{B}_{ext} \cdot \hat{n}$ that increase too rapidly between the plasma and the coils are eliminated by modifications to the plasma shape.

Curl-free magnetic fields increase through space as $\exp(kx)$, where k is the wavenumber of the variation in the field, so only external fields with a sufficiently small k can be used. This constraint on the variation of the external field limits axisymmetric shaping to four shape parameters: aspect ratio, ellipticity, triangularity, and squareness. The same constraint is consistent with an order of magnitude more non-axisymmetric shape parameters.

The elimination of fields that increase rapidly through space can simplify the shape of the vacuum vessel and the support structures. If the coils can be more distant from the plasma, the vacuum vessel can be designed to have a simpler shape than the plasma.

Since the shape of the plasma determines the physics properties of the magnetic configuration, the coils and the plasma must to a certain extent be designed together. Modern methods for optimizing stellarators were developed as part of the W7-AS [8] design and improved during the design of W7-X [5], NCSX [3], and QPS [9]. The STELLOPT/COILOPT code [32] was developed to perform the joint plasma-coil optimization. The basic ideas of stellarator optimization will be explained, not as they were applied in any of these four designs, but in a way that illustrates the possibilities: (1) for improvements to a given stellarator configuration, say a particular NCSX equilibrium, or (2) for the design of a set of control coils for a specific purpose. An example is the design of coils to control ELMs by enhancing the transport near the edge but not in the body of the plasma.

The explanation of stellarator optimization, which is given in this paragraph, clarifies the freedom that exists to impose particular controls on the plasma but is not required for an overall understanding of the section. The change in the physics quality of a plasma equilibrium by an external magnetic perturbation $\delta\vec{B}_{ext} \cdot \hat{n}$ can be measured by a set of I_q parameters. Examples of these parameters are the driving field for magnetic islands b_{mn} at a given rational surface, the change in the energy, or δW, of the least stable mode in an ideal MHD analysis, or the change in the action deviation δJ_s in a magnetic surface. The $(I_q \times I_b)$ matrix $\overleftrightarrow{Q}$ that relates I_q quality parameters to changes in an arbitrarily large number, I_b distributions of external field $\vec{B}_{ext} \cdot \hat{n}$ can be determined using Carolin Nührenberg's perturbed equilibrium code CAS3D. Using Peter Merkel's magnetic codes [33], an $I_b \times I_c$ efficiency matrix $\overleftrightarrow{E}$ can be found that relates I_c normal magnetic field distributions at a surface that characterizes the location of the coils to the I_b external magnetic field distributions on the plasma surface. With the choices $I_c >> I_b >> I_q$, a Singular Value Decomposition (SVD) analysis of the matrix $\overleftrightarrow{Q} \cdot \overleftrightarrow{E}$ gives the magnetic field distributions at the coils that most efficiently control any collection of quality parameters of the plasma. If I_I intrinsic islands are to be constrained to have negligible width, an SVD analysis of an $(I_I \times I_c)$ submatrix of $\overleftrightarrow{Q} \cdot \overleftrightarrow{E}$ determines the magnetic field distributions on the coil surface that most efficiently control those widths. These field distributions can be adjusted to null the intrinsic islands. A very large number of the magnetic field distributions at the coils, at least $I_c - I_q$, have no effect in linear order on any of the plasma quality measures and can be chosen to simplify the coils. If $\overleftrightarrow{W}$ is a diagonal matrix that gives an appropriate weight to each quality parameter and has a positive or negative weight depending on whether the quality parameter improves or degrades the physics, then the overall quality of a configuration is determined by the $(I_c \times I_c)$ target matrix $\overleftrightarrow{T} \equiv \overleftrightarrow{E}^\dagger \cdot \overleftrightarrow{Q}^\dagger \cdot \overleftrightarrow{W}^\dagger \cdot \overleftrightarrow{W} \cdot \overleftrightarrow{Q} \cdot \overleftrightarrow{E}$. A diagonalization of $\overleftrightarrow{T}$ determines which magnetic field distributions at the coil surface improve the overall physics, the ones with positive eigenvalues, or degrade the overall physics, the ones with negative eigenvalues. In total $\overleftrightarrow{T}$ can give at most I_q field distributions that affect the plasma quality parameters while the other field distributions, which are the null space of $\overleftrightarrow{T}$, give the field distributions that can be modified to simplify the coils.

The difficulty of constructing non-axisymmetric devices can be eased by a study of the required construction tolerances. The sensitivity of plasma equilibria to magnetic field errors can be determined in stellarators using Carolin Nührenberg's CAS3D perturbed equilibrium code [28], [29] just as the error field sensitivity of tokamks is determined by IPEC [23]. The sensitivity of stellarators to field errors is no greater than tokamaks. Trim coil systems can be designed to control the most sensitive external perturbations. Construction tolerances should be set by the optimal engineering trade-off between the strength and complexity of the trim coil set and the costs of additional construction accuracy.

A necessary condition for attractive coils is that they efficiently support the plasma configuration. Given that, a number of relatively unexplored choices in coil design remain. Coils have three topological types: (1) modular, which means shaped toroidal field coils, (2) helical, or (3) saddle coils, which means coils that do not encircle the plasma. Coils sets that are hybrids of these types could ease fabrication. Many choices also exist for the design and fabrication of the vacuum vessel and supporting structures, which could make the construction of non-axisymmetric systems easier.

Non-axisymmetric shaping can have important applications to DEMO over a broad range of levels and of purposes. Design studies are needed to clarify the choices. For example, ELM control coils could be located far from the plasma if they were designed to obey a quasi-axisymmetric constraint.

V. DISCUSSION

The standard vision of DEMO as having a plasma in a self-organized microturbulent state with little effective external control is a design choice not a requirement. Research towards DEMO has two foci with a relative importance set by the perception of non-axisymmetric shaping: (1) Demonstration that a weakly controlled plasma indeed self-organizes into an attractive state for fusion power. (2) Determination of what type and level of control reduces the risk of DEMO and optimizes its attractiveness as a power plant.

Different research foci naturally lead to different research programs. A far broader spectrum of research contributes to the focus on control than on demonstration. Experiments as small as the Compact Toroidal Hybrid (CTH) stellarator at Auburn can give important information on the level of vacuum transform required to avoid disruptions. But, experiments at a far larger scale are required to determine the effects of microturbulent transport, the effect of high beta on disruptions, or what sets the practical beta limit in a system with non-axisymmetric shaping. The LHD is providing important information and W7-X will. However, the cancellation of NCSX means no experiment is focused on the use of non-axisymmetric shaping to enhance the attractiveness of tokamaks for fusion.

The possibility of experiments of a moderate scale, which can be carried out in parallel to ITER, changing fundamental assumptions is one of the attractions of a research program based on understanding plasma control. Research on error field, resistive wall mode, and ELM control on nominally axisymmetric tokamaks illustrate this benefit of non-axisymmetric shaping.

A change in the prevailing paradigm of theory is also required to assess the optimal type and level of plasma control on DEMO. In no other area could scientific discovery through computing with more certainty have a major impact on the cost, schedule, and plans for the development of magnetic fusion energy. Theory could both assess the physics benefits and ease the technical challenges. A broader drive is required than just advanced computing since many areas of highest potential can be addressed by existing codes and computers.

What are the major questions for research? Can non-axisymmetric shaping be used to make the plasma performance sufficiently independent of the heating profile to allow burning plasma performance to be assessed by experiments in non-burning plasmas. A positive answer appears likely with profound implications on the cost for the development of fusion. The more technically difficult non-axisymmetric systems are perceived to be, the more important are the questions: How much non-axisymmetric shaping or vacuum transform is needed to make a toroidal plasma sufficiently robust to disruptions? How much non-axisymmetric shaping is needed to remove the restrictions of the Greenwald density limit? Clearly if an axisymmetic DEMO can be built, so can a DEMO with some level of non-axisymmetric shaping. But, a question remains. What are the limitations of engineering feasibility imposed on non-axisymmetric shaping?

Whatever decision is made on the optimal level of control, the fusion program will be ready to build DEMO only when sufficient flexibility can be included in the design to acceptably compensate for the uncertainties in the science. The greater the flexibility, the more uncertainty in the science that can be tolerated, and the greater the plasma control, the less the uncertainty in the science.

If indeed plasmas stably self-organize so $\iota_{boot}/\iota_{drive} > 4$ gives an acceptable DEMO, then relatively little non-axisymmetric shaping would be required to replace the driven current with vacuum transform. Nevertheless, needless lack of control implies needless risk, so the benefits and challenges of additional control must be carefully assessed.

DEMO cannot be perfectly axisymmetric, so non-axisymmetric magnetic fields must be controlled. The questions are at what level, of what type, and for what purpose.

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