THEORY OF THE POLOIDAL SPIN-UP PRECURSOR TO INTERNAL TRANSPORT BARRIER FORMATION

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Theory of the Poloidal Spin-up Precursor to Internal Transport Barrier Formation

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The phenomenon of a sudden change in the poloidal flow prior to the reduction in transport and the steepening of temperature and density profiles has been observed both at the edge (high-modes) and in the core enhanced reversed shear (ERS) modes of tokamaks. The poloidal spin-up precursor is narrowly localized in the (radial) direction across magnetic flux surfaces. Although the reduction of turbulent transport is consistent with the theory of ExB flow shear suppression, the localized poloidal spin-up precursor has not been explained by the theory until now. It will be shown that the observed flow pattern is well described by a new class of bifurcation to the momentum balance equations. The new physics follows from extending the standard neoclassical theory of poloidal flow damping to include the turbulent viscous stress. The new bifurcation results from balancing the non-linear turbulent viscous stress with the linear poloidal flow damping due to the neoclassical parallel viscous stress. The new bifurcation results in a mono-polar ExB flow structure (with a large poloidal component) which is narrowly localized in the radial direction. This flow pattern will be referred to as a jet. The equations, which result in the jet bifurcation, are dual to the usual Ginzburg-Landau phase transition theory. The non-linearity appears in the gradient of the field (kinetic energy) rather than in the field (potential energy). The jet solution is a type of topological soliton. Whereas the Ginzburg-Landau equations have the property that once a phase transition is favorable a perturbation of the new phase will expand to fill all space, the jet bifurcation remains localized, saturating at finite width. An analytic model of the perpendicular turbulent viscous stress is constructed based on the properties of drift wave instabilities. This model is then solved for the jet bifurcation and compared with data* from an ERS transition. It is found that the peak flow velocity and the width of the observed flow pattern can be fit with the model using realistic values of the drift-wave linear growth rate, the effective diffusivity due to the drift-wave turbulence and the neoclassical poloidal damping rate. The observed experimental phenomenology of poloidal spin-up precursors is well explained by the jet bifurcation. The jet forms at the neoclassical poloidal flow damping rate which is faster than the energy or particle transport rate. The peak in the jet flow is shown to reduce and finally disappear as the diamagnetic velocity shear increases. A seed perturbation is required to initiate the jet bifurcation. The spin-up precursor is most likely to occur in a plasma with balanced neutral beam heating and is unlikely to occur when there is a strong toroidal momentum source. A jet is not required for a transport barrier to form. A new feature predicted by the theory, but not yet observed, is the existence of a toroidal flow excursion in the same location as the poloidal jet. This feature is due to the strongly off-diagonal nature of the turbulent viscous stress tensor predicted from drift-wave theory.

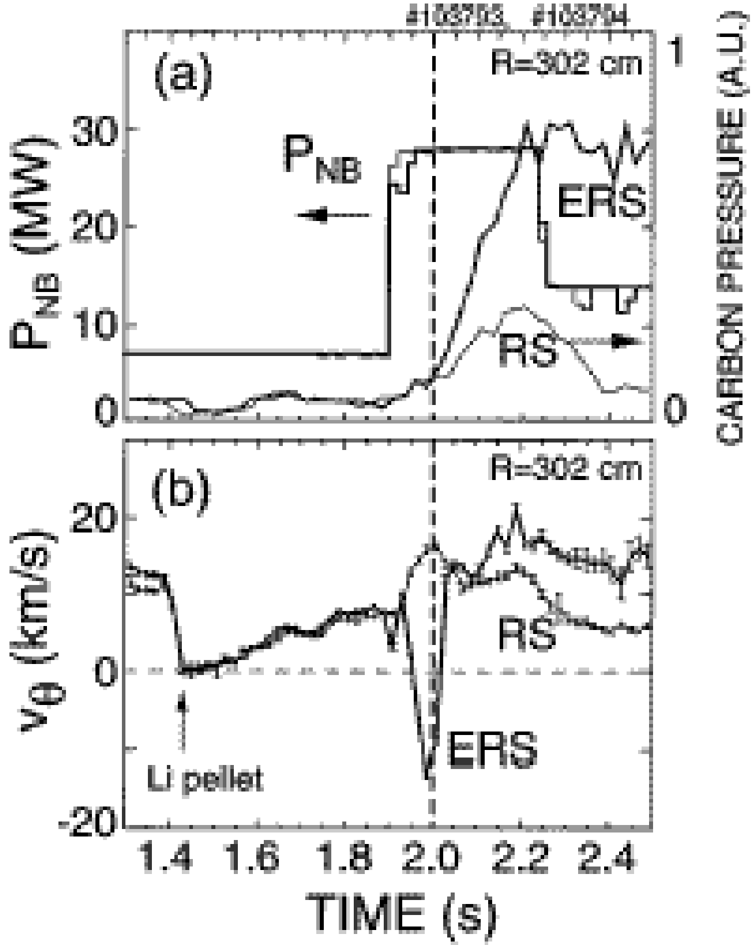
This work was supported by U.S. Department of Energy Grant DE-FG03-95ER54309.

*R.E. Bell, F.M. Levinton, S.H. Batha, E.J. Synakowski and M.C. Zarnstorff, Phys. Rev. Lett. **81** (1998) 1429.

Prefer Oral

THE POLOIDAL SPIN-UP PRECURSOR IS TRANSIENT

R.E. Bell, F. M. Levinton, S. H. Batha, E. J. Synakowski and M. C. Zarnstorff, Phys. Rev. Lett. 81 (1998) 1429.



The poloidal velocity of carbon ions has been observed to spin-up as a precursor to Type I ERS transitions on TFTR.

The spin-up is in the perpendicular velocity contribution to the ExB velocity not the diamagnetic velocity.

The poloidal velocity spins-down as the pressure profile steepens during barrier formation

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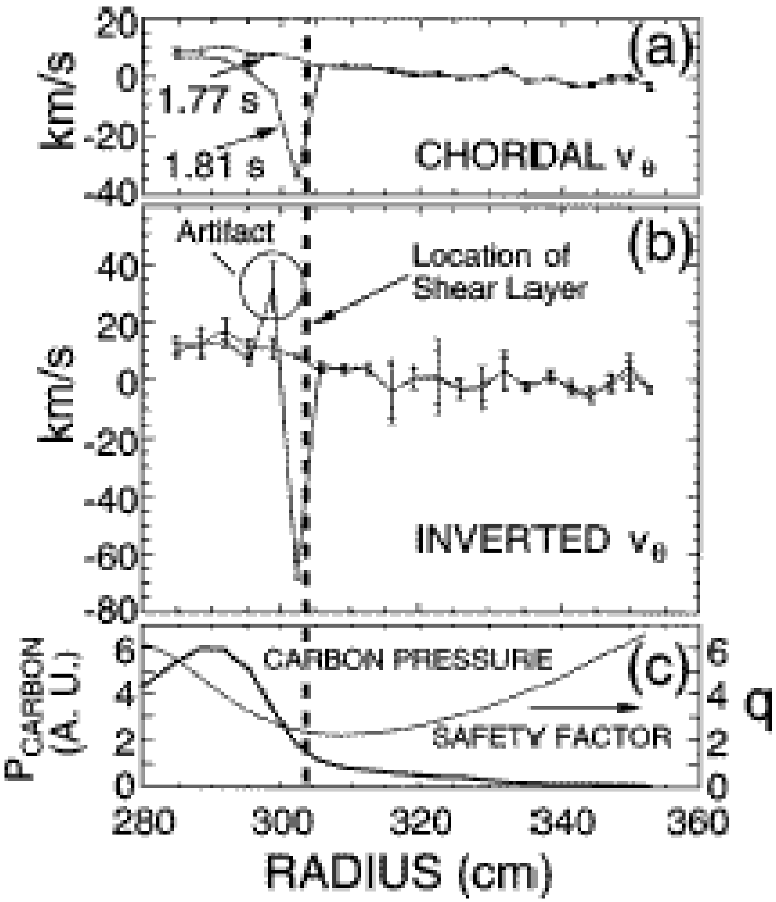
THE POLOIDAL SPIN-UP PRECURSOR IS LOCALLIZED

R.E. Bell, F. M. Levinton, S. H. Batha, E. J. Synakowski and M. C. Zarnstorff, Phys. Rev. Lett. 81 (1998) 1429.



The precursor is localized to a very narrow layer.

The precursor is a monopolar excursion of the ExB velocity.

Transport barriers without poloidal spin-up precursors are also observed (Type II ERS, DIII-D NCS, and slow H-modes)

It will be shown that all of these properties are reproduced by a new class of bifurcated solution of the momentum balance equations, which will be called a jet .

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WHAT IS A JET?

The jet solution is :

A new solution of the momentum balance equations which balances the neoclassical poloidal damping against the viscous stress due to driftwave turbulence.

An excursion of the ExB velocity from its neoclassical value.

A precursor to an internal transport barrier for balanced NBI.

A transient which grows up fast ($\sim\nu$) and decays with the build-up of diamagnetic velocity shear.

Naturally localized to a thin layer of width $\sim (\mu/\nu)^{1/2}$

A new type of topological soliton.

A self-generated monopolar velocity excursion without the need of external torques.

The poloidal spin-up precursor to H-mode transitions may also be a jet.

INCLUSION ON THE VISCOUS STRESS DUE TO TURBULENCE IS REQUIRED

The source free steady state momentum balance equations are:

(x,y,z) = (radial, perpendicular, parallel)

Toroidal:

$$d\Pi_{x\varphi}/dx = 0, \quad \text{where} \qquad \Pi_{x\varphi} = (B_\varphi\, \Pi_{xz} - B_\theta\, \Pi_{xy})/B$$

Perpendicular:

$$d\Pi_{xy}/dx + mn\nu\,(U_{ExB} - U^{nc}) = 0, \quad \text{where}$$

$$U_{ExB} = -cE_x/B_z = U_y - (dP/dx)/(enB)$$

A thin slab approximation has been used which is appropriate for a localized velocity excursion.

The velocity and electric field gradient lengths are assumed to be smaller than those of the other fields (density, temperature, magnetic field). The steep velocity gradients are required in order for the turbulent viscous stress to compete with the neoclassical damping term (ν).

THE VISCOUS STRESS DUE TO DRIFTWAVES IS OF SIMILAR MAGNITUDE IN BOTH PARALLEL AND PERPENDICULAR DIRECTIONS

Quasilinear calculations in a sheared slab magnetic field geometry have shown that the viscous stress tensor due to drift waves (ITG,TEM) has the general form:

[R. R. Dominguez and G. M. Staebler, Phys. Fluids B5 (1993) 3876]

$$\Pi_{xy} = \eta_{yy}\,\gamma_{ExB} + \eta_{yz}\,\gamma_z \qquad \text{where} \qquad \gamma_z = -du_z/dx$$

$$\Pi_{xz} = \eta_{zz}\,\gamma_z + \eta_{zy}\,\gamma_{ExB} \qquad \text{where} \qquad \gamma_{ExB} = -du_{ExB}/dx$$

The steady-state toroidal momentum balance equation ($\Pi_{x\varphi} = 0$) has the non-trivial solution:

$$\gamma_z = C_z\,\gamma_{ExB}\,, \text{ where } \quad C_z = (B_\theta\eta_{yy} - B_\phi\eta_{zy})/(B_\phi\eta_{zz} - B_\theta\,\eta_{yz})$$

Elimination of the parallel velocity shear leaves the perpendicular viscous stress an implicit function of only the ExB velocity shear.

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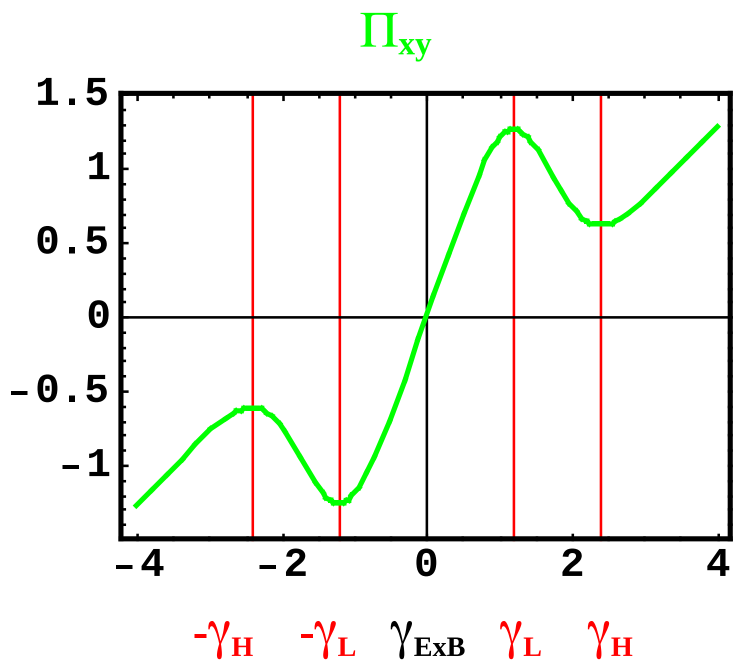
THE VISCOUS STRESS DUE TO DRIFTWAVES IS MULTIVALUED



The viscous stress due to drift waves has the general property that for low ExB velocity shear, $|\gamma_{ExB}| < \gamma_L$, the effective viscosity is large due to ion temperature gradient (ITG) modes.

For large ExB shear $|\gamma_{ExB}| > \gamma_H$ the effective viscosity is smaller since the ITG modes are quenched by the ExB shear.

A transport bifurcation based on these properties was first proposed in G.M. Staebler and R. R. Dominguez, Nucl. Fusion 33, 77 (1993)

THE JET SOLUTION BALANCES THE TURBULENT VISCOUS STRESS WITH THE NEOCLASSICAL POLOIDAL VELOCITY DAMPING

Setting: $U_Z = C_Z\, U_{ExB}$ and assuming: C_z, n, ν, are constant.

Gives the reduced 1-D perpendicular momentum balance equation:

$$d\Pi_{xy}/dx + mn\nu\,(U_{ExB} - U^{nc}) = 0,$$ **where** $\Pi_{xy} = \Pi_{xy}(\gamma_{ExB})$

and $U^{nc} = \{K_\theta dT/dx - (1/n)dP/dx\}/(eB + eB_\theta C_Z)$

Using:

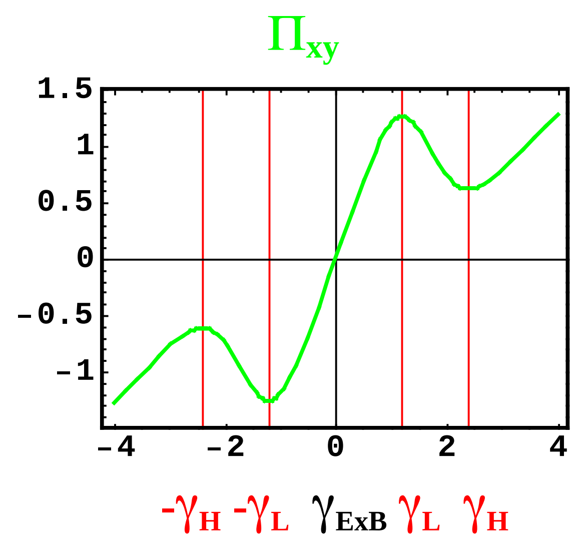
$$d\Pi_{xy}/dx = (d\Pi_{xy}/d\gamma_{ExB})\, d\gamma_{ExB}/dx$$

$$\eta_{inc} = d\Pi_{xy}/d\gamma_{ExB}$$

The perpendicular momentum balance equation reads:

$$\eta_{inc}\, d^2U_{ExB}/dx^2 = mn\nu\,(U_{ExB} - U^{nc})$$

$$\eta_{inc} < 0 \text{ for } \gamma_L < |\gamma_{ExB}| < \gamma_H$$



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GENERALIZED PHASE TRANSITION MODEL

The perpendicular momentum balance equation is a generalized phase transition model, which is dual to the usual Ginzburg-Landau model. The multivaluedness is in the field gradient rather than the field.

Defining: $\gamma^{nc} = -dU^{nc}/dx$

and assuming $d\gamma^{nc}/dx = 0.$

A first integral of the perpendicular momentum balance equation can be performed.

$E = S - 0.5\ mn\nu(U_{ExB}-U^{nc})^2$

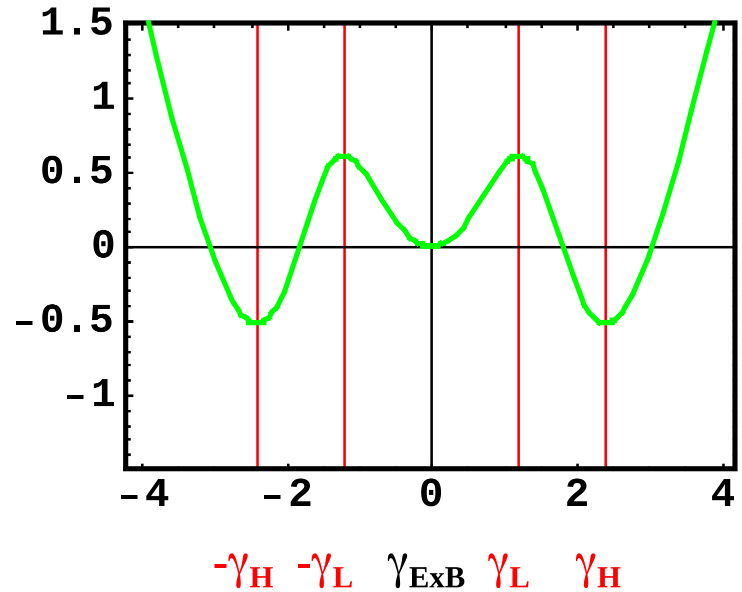
where

$S = (\gamma_{ExB} - \gamma^{nc})\Pi_{xy} - F$

and

$\Pi_{xy} = dF/d\gamma_{ExB}$

$S(\gamma_{ExB})$ for $\gamma^{nc} = 0$



Example : $F = 0.5\ (\mu_H + \mu_L /(1 + (\gamma_{ExB} /2)^4))\ \gamma_{ExB}{}^2$

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THE JET SOLUTION HAS BIFURCATIONS

Defining:

$U = U_{ExB} - U^{nc}$
$\eta = (d\Pi_{xy}/d\gamma_{ExB})/nm\nu$

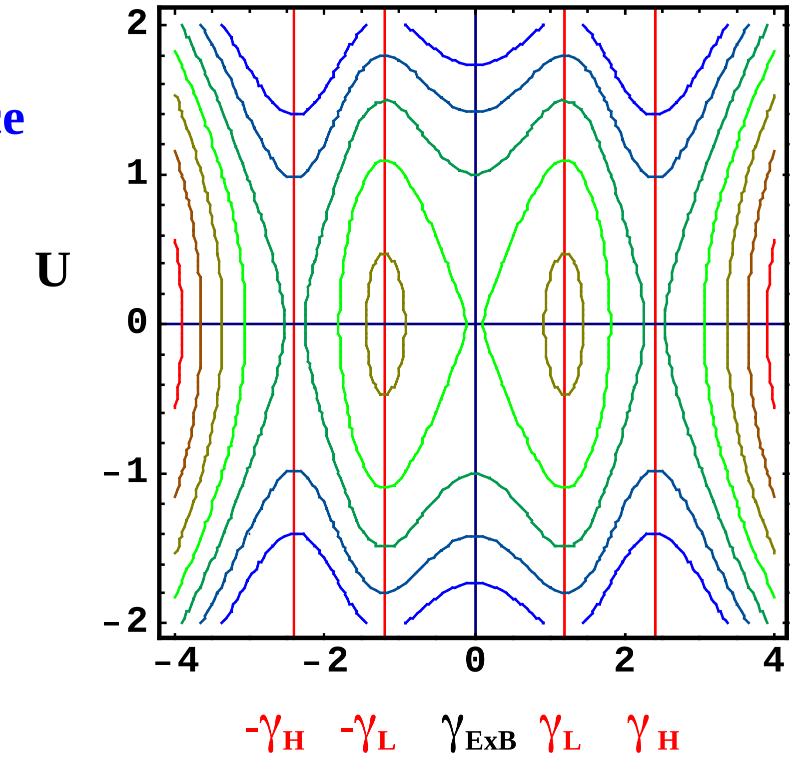
The perpendicular momentum balance equation can be written:

$\eta \, d\gamma_{ExB} = -U \, dx$
$dU = -\gamma_{ExB} \, dx$

In the (U, γ_{ExB}) phase plane the jet solution connects the two x-points at $\mp\gamma_H$, U=0.

The x-trajectories cannot cross the singular lines where $\eta = 0$ with continuous γ_{ExB}.

E-contours for $\gamma^{nc} = 0$



THE CENTAL BIFURCATION CONNECTS NON-DEGENERATE GROUND STATES

For $\gamma^{nc} > 0$ the two x-points at $\mp\gamma_H$, U=0, are non-degenerate. A jet solution can still be found:

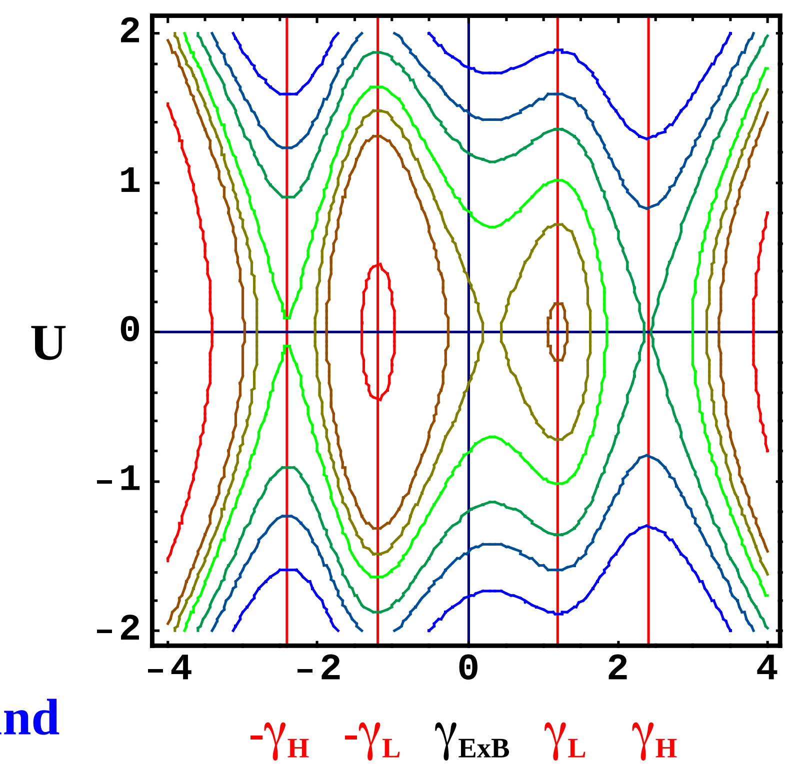
$$0.5mn\nu(U_{ExB}-U^{nc})^2 =$$
$$S - S(-\gamma_H) \quad \text{for } -\gamma_H < \gamma_{ExB} < -\gamma_C$$
$$S - S(\gamma_H) \quad \text{for } \gamma_L < \gamma_{ExB} < \gamma_H$$

Where γ_C is determined from continuity of the velocity.

$$S(-\gamma_C)-S(\gamma_L) = S(-\gamma_H)-S(\gamma_H)$$

The bifurcation in the middle compensates for the different ground state values of S.

E-contours for $\gamma^{nc} = 0.3\gamma_L$



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THE JET IS A LOCALIZED MONOPOLAR VELOCITY EXCURSION

The width of the jet scales like:

$\lambda = (\alpha/\nu)^{1/2}$

where

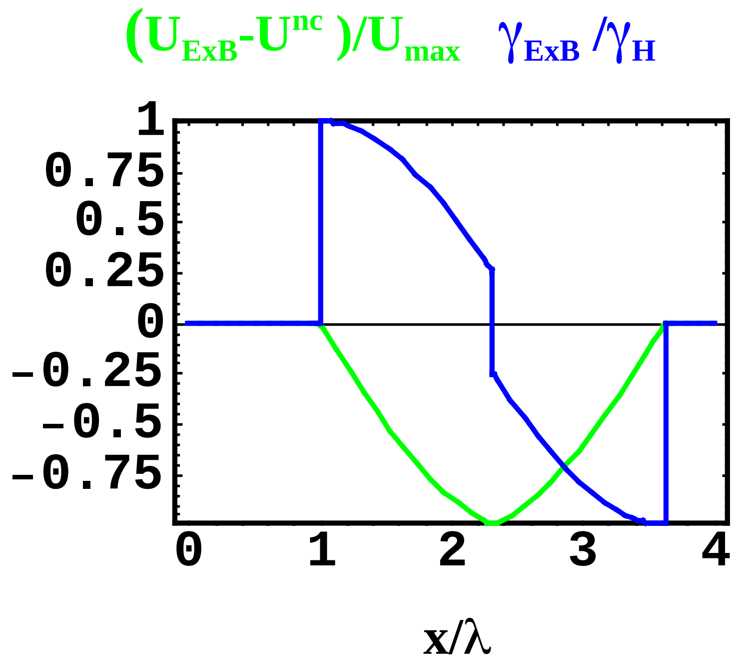
$\alpha = (\mu_L\gamma_L - \mu_H \gamma_H)/(\gamma_H - \gamma_L) = -\eta_{inc}/mn$

The maximum velocity excursion ($U_{ExB}-U^{nc}$) is:

$U_{max} = \lambda\, [(\gamma_H - \gamma^{nc})^2 - (\gamma_L - \gamma^{nc})^2]^{1/2}$

Shown is a case with

$\gamma_L = 0.25\, \gamma_H$, $\gamma^{nc} = 0$

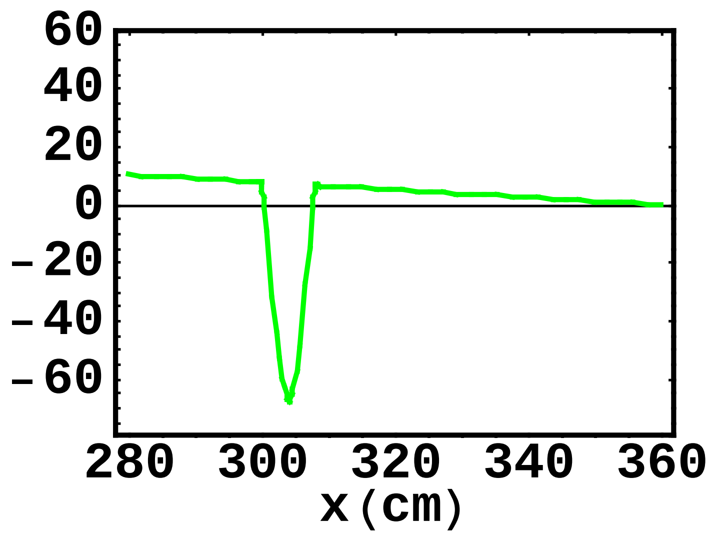


THE JET FITS THE POLOIDAL SPIN-UP PRECURSOR

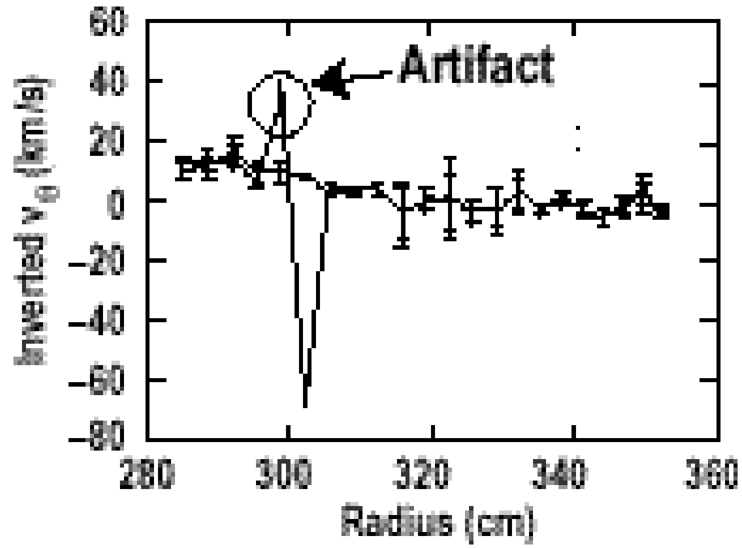
A good fit to the TFTR data is obtained for the model parameters:

$\gamma_L = 3x10^5/s$, $\gamma_H = 9\gamma_L$, $\mu_L = 2.8 m^2/s$, $\mu_H = \mu_L/40$, $\nu = 346/s$, $\gamma^{nc} = 1.3x10^4/s$

U_{ExB} (km/s)



TFTR



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THE VELOCITY EXCURSION SHRINKS AND DISSAPEARS AS THE DIAMAGNETIC VELOCITY SHEAR INCREASES

For $|\gamma^{nc}| < \gamma_L$:

A jet requires a seed perturbation since $U_{ExB} = U^{nc}$, $\gamma_{ExB} = \gamma^{nc}$ is stable.

For $0.5(\gamma_H+\gamma_L) > |\gamma^{nc}| > \gamma_L$:

No seed is required for a jet.

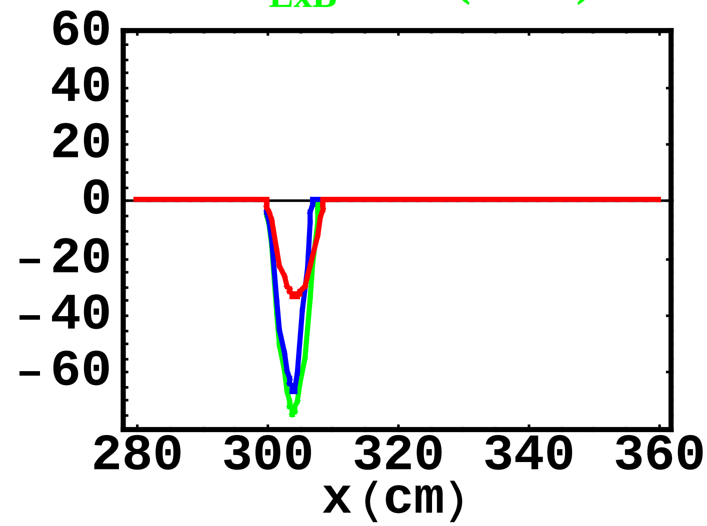
$U_{ExB} = U^{nc}$, $\gamma_{ExB} = \gamma^{nc}$ is unstable.

For $|\gamma^{nc}| > 0.5(\gamma_H+\gamma_L)$:

A jet solution does not exist.

From $U_{ExB} = U^{nc}$, $\gamma_{ExB} = \gamma_H$, no E-contour crosses the singular line $\gamma_{ExB} = \gamma_L$.

$U_{ExB}-U^{nc}$ (km/s)



$\gamma^{nc} = 0.04\gamma_L$, $\gamma^{nc} = \gamma_L$, $\gamma^{nc} = 0.5(\gamma_L + \gamma_H)$

THE JET IS A GENERALIZED TOPOLOGICAL SOLITON

The jet solution has a topological invariant similar to the "kink number" of a topological soliton.

$$\Delta \Pi_{xy} = \int_{x_1}^{x_2} dx \, d\Pi_{xy}/dx = \Pi_{xy}(\gamma_L) - \Pi_{xy}(\gamma_H) + \Pi_{xy}(-\gamma_H) - \Pi_{xy}(-\gamma_C)$$

From the perpendicular momentum balance equation it follows that:

$$\Delta \Pi_{xy} = - \int_{x_1}^{x_2} dx \, mn\nu(U_{ExB} - U^{nc})$$

Thus, the monopolar velocity excursion is only possible because the continuity of Π_{xy} is violated by the bifurcations in γ_{ExB} at the ends and the center of the jet.

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CONCLUSION

- **The jet soliton does indeed reproduce the properties of the observed poloidal spin-up precursor:**

  The jet grows up faster (ν) than the pressure gradient (χ/L^2).
  The jet decays as the barrier develops due to increasing diamagnetic velocity shear.

  The strong radial localization is natural. $\lambda \sim (\mu/\nu)^{1/2}$

  The monopolar character is explained without external torques.

- **An analytic model for the jet can be fit to TFTR data with realistic values for the critical ExB shear rate, and the effective momentum diffusivity.**

- **The theory predicts the existence of a parallel velocity component within the ExB jet due to the off-diagonal nature of the viscous stress.**

- **Triggering jets with localized perpendicular momentum sources (e.g. IBW) could dramatically lower the power required for a transport barrier.**