Sherwood 2000 (International Fusion/Plasma Theory Conference)
March 27–29, 2000, Los Angeles, California

Electron Cyclotron Current Drive Efficiency in Finite Collisionality Regime

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Reduction of rf current drive efficiency by trapped electrons in tokamaks is a widely discussed subject in the literature. Nevertheless, most quantitative investigations of the effect invoked the bounce average approximation, which assumes that effective collision frequency is much smaller than bounce frequency for trapped electrons at all energies. The zero-collisionality assumption neglects the de-trapping process of electrons due to collision and underestimates current drive efficiency. Collisionality enhancement of current drive efficiency might be small in high temperature reactor-grade tokamak plasmas, but the situation is less clear for the present-day experiments, especially in the case of off axis electron cyclotron current drive (ECCD). In this work, we use Green's function formulation to calculate ECCD efficiency in the regime of finite collision frequency. The Coulomb collision operator is simplified by considering pitch-angle scattering only, *i.e.*, Lorentz gas model. The numerical problem involved is reduced to solving a two-dimensional finite difference equation. Collisionality corrections of ECCD efficiency are evaluated using DIII–D experimental parameters and impact on Advanced Tokamak (AT) operational scenarios will be discussed.

This is a report of work supported by U.S. Department of Energy Grant DE-FG03-95ER54309.

INTRODUCTION

- Electron Cyclotron Current Drive (ECCD) is a leading candidate for current profile control in Advanced Tokamak (AT) operation

- Localized off-axis ECCD, in both directions of the plasma current, was clearly demonstrated in recent proof-of-principle experiments on DIII–D

- The measured ECCD efficiency agrees with the coupled ray-tracing and bounce averaged Fokker-Planck calculations for the cases near magnetic axis; but it meets and exceeds the predicted value at larger radius.

- The bounce averaged calculations assume that bounce frequency is much larger than collision frequency for trapped electrons at all energies, hence, underestimate current drive efficiency

- Preliminary finite-collisionality calculations using a velocity-space interpolation formula give modest increase in ECCD efficiency

- Non-bounce averaged calculations of ECCD efficiency for the Lorentz gas model (pitch-angle scattering only) are performed in the present work to gain better understanding of collisionality effects

RADIAL AND POLOIDAL SCANS HAVE BEEN OBTAINED TO TEST THE EFFECTS OF TRAPPED PARTICLES

P_{ECH} = 0.95–1.14 MW

$\bar{n}$ = 1.66–1.85 10^{13} cm^{-3}

q_{95} = 5.95–6.33

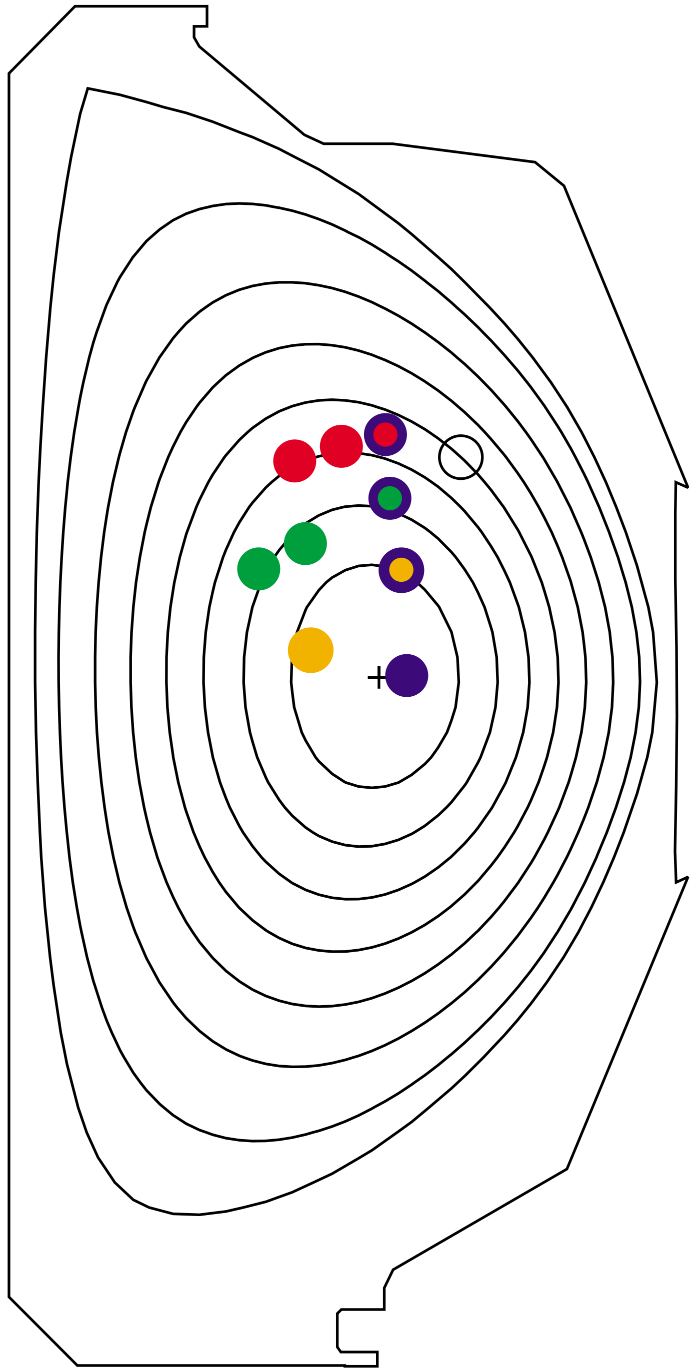
Radial Scan

Poloidal Scan ρ = 0.2

Poloidal Scan ρ = 0.34

Poloidal Scan ρ = 0.47

ECCD experiments were also conducted in the counter-current campaign



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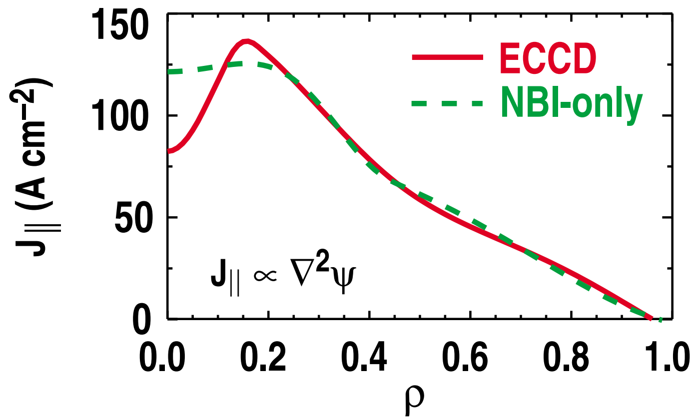
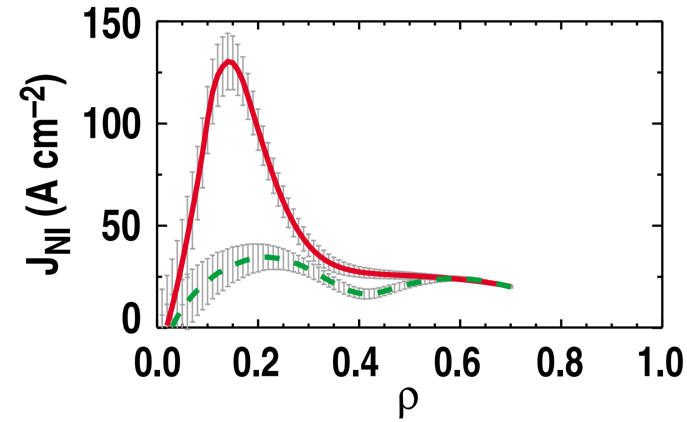
LOCALIZED CURRENT DRIVE IS CLEARLY MEASURED

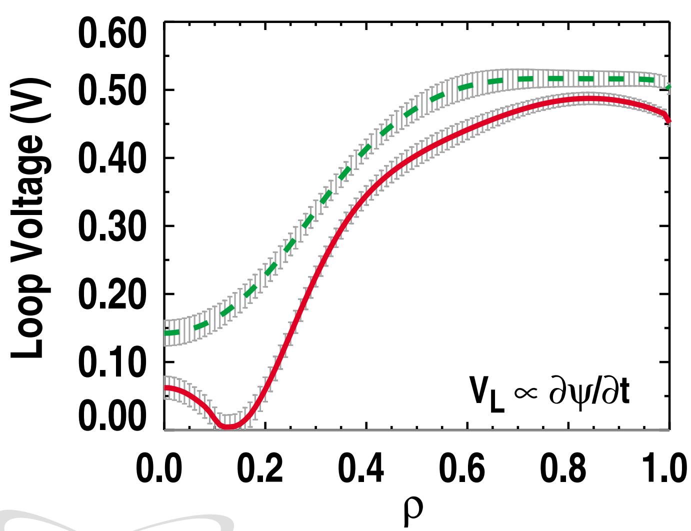


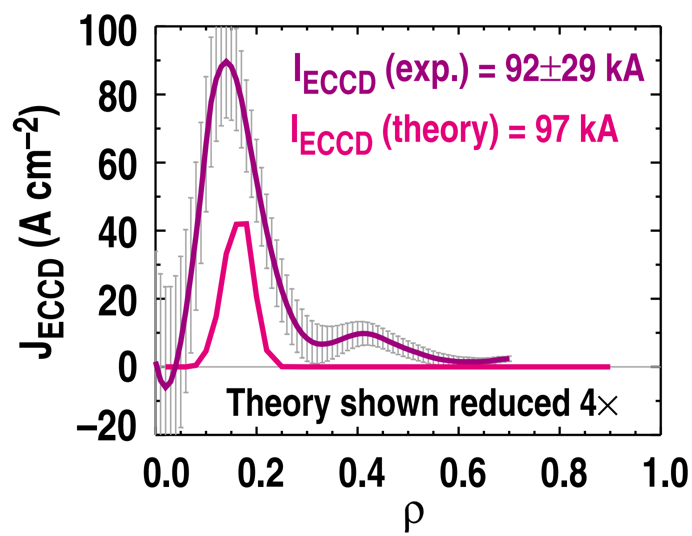
$J_{||}$ and loop voltage obtained from magnetic reconstructions with high resolution motional Stark effect spectroscopy (MSE)



Assumption of neoclassical resistivity gives $J_{NI} \equiv J_{||} - \sigma E_{||}$

Comparison of ECCD case with NBI-only fiducial separates ECCD from bootstrap and NBCD





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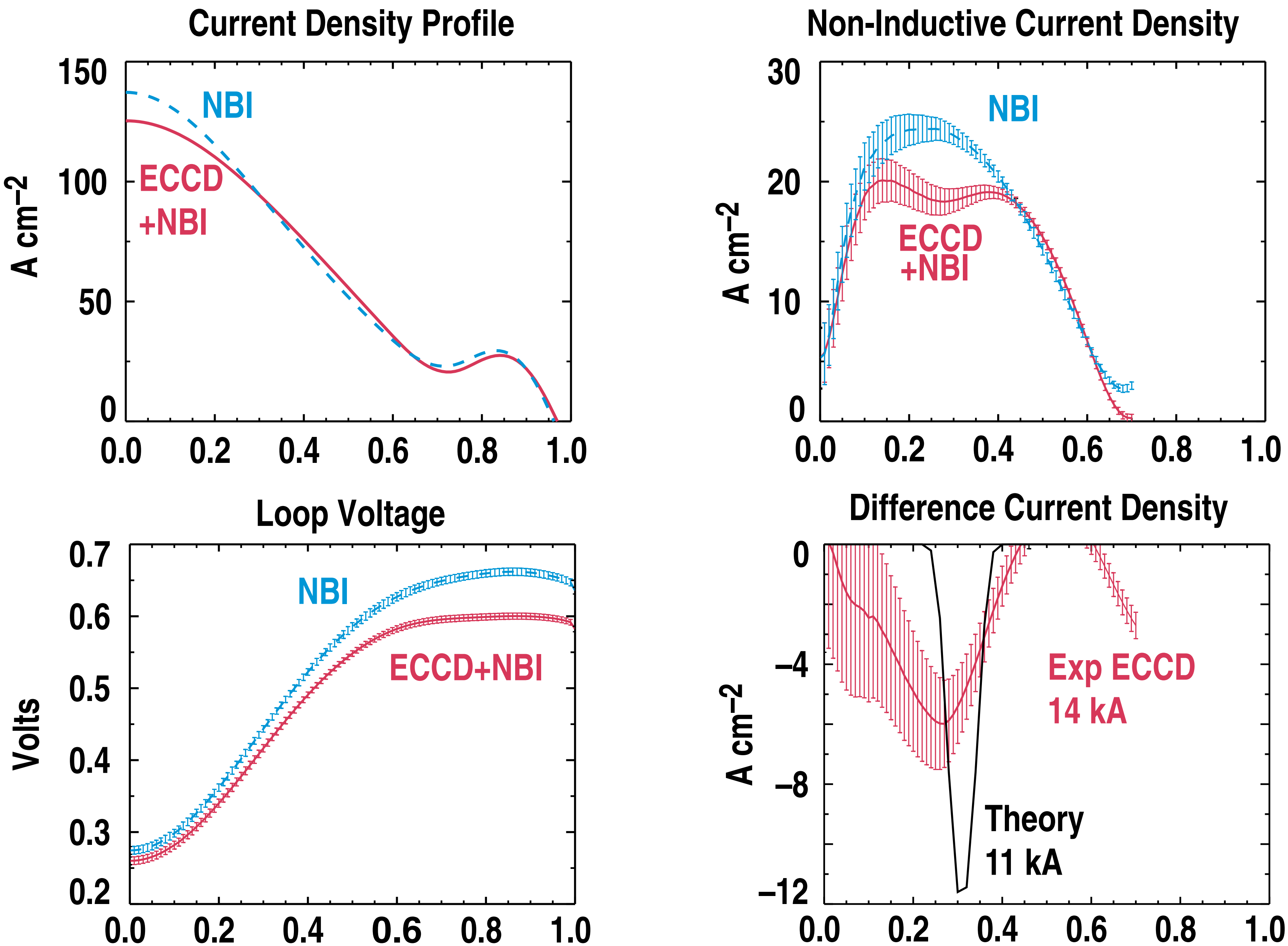
OFF-AXIS COUNTER ECCD WAS MEASURED



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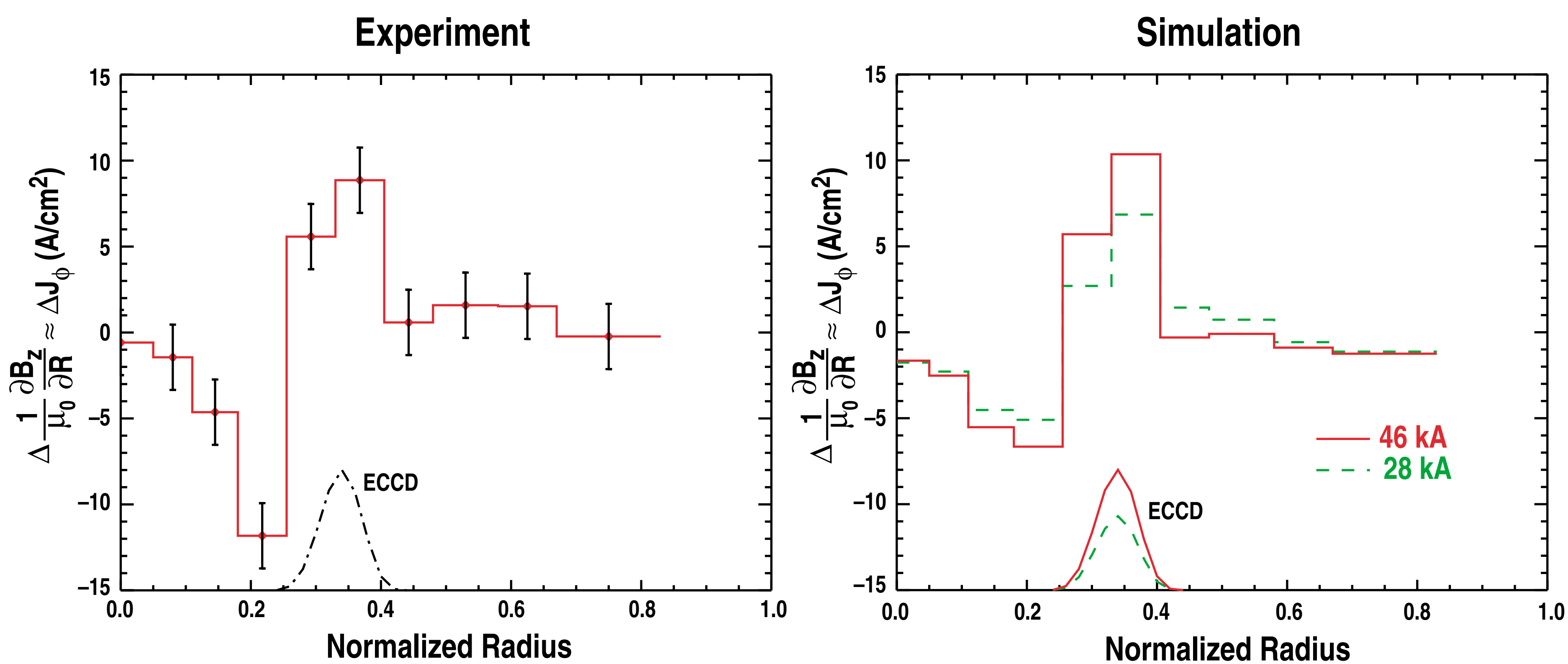
MSE SIMULATION TECHNIQUE IS USED TO DETERMINE PROFILE AND MAGNITUDE OF ECCD

- Evolution of magnetic equilibrium simulated using ONETWO transport code
  - Plasma boundary is fixed to experimental shape
  - Time history of measured density and temperature profile is included
  - Magnetic equilibrium determined from Grad-Shafranov equation
  - Current and loop voltage evolution determined by Faraday's and Ohm's law
  - ECCD profile given by TORAY ray tracing code (magnitude treated as free parameter)

MSE MEASUREMENT SHOWS THAT THE INCREASE IN CURRENT DENSITY FROM ECCD IS AT LEAST AS LOCALIZED AS THE THEORETICAL PREDICTIONS



Change in MSE between ECH + NBI and NBI-only plasmas at t = 1.43 s is shown

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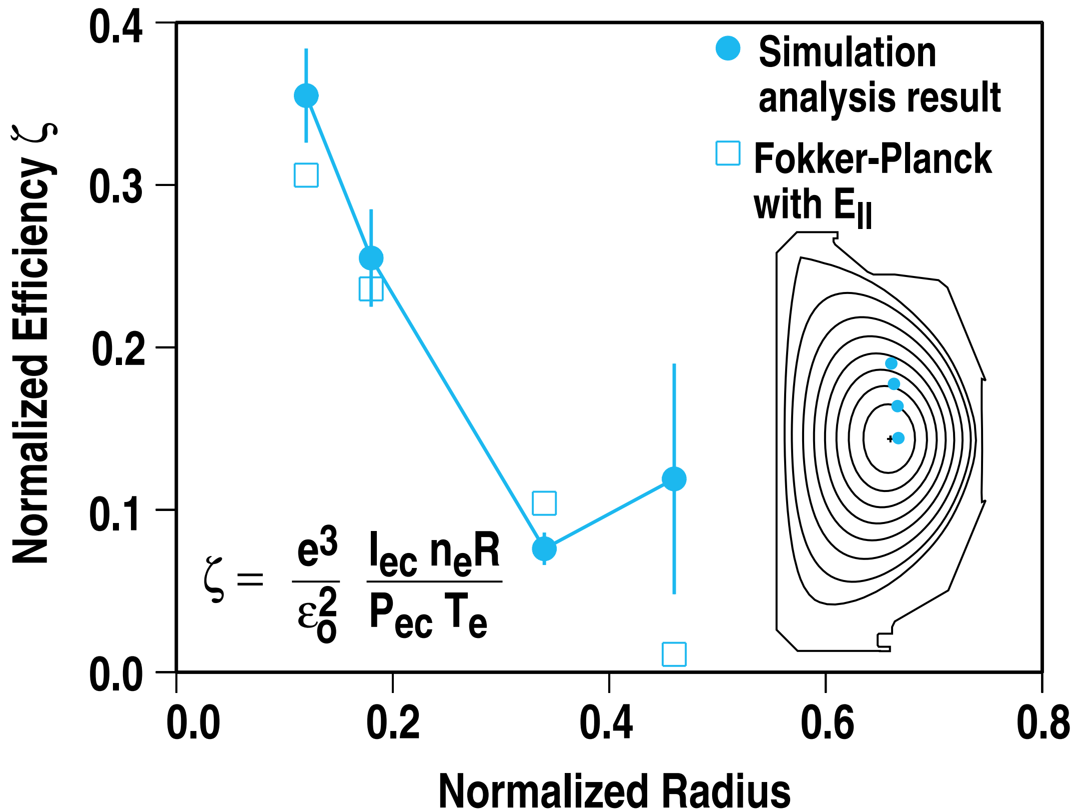
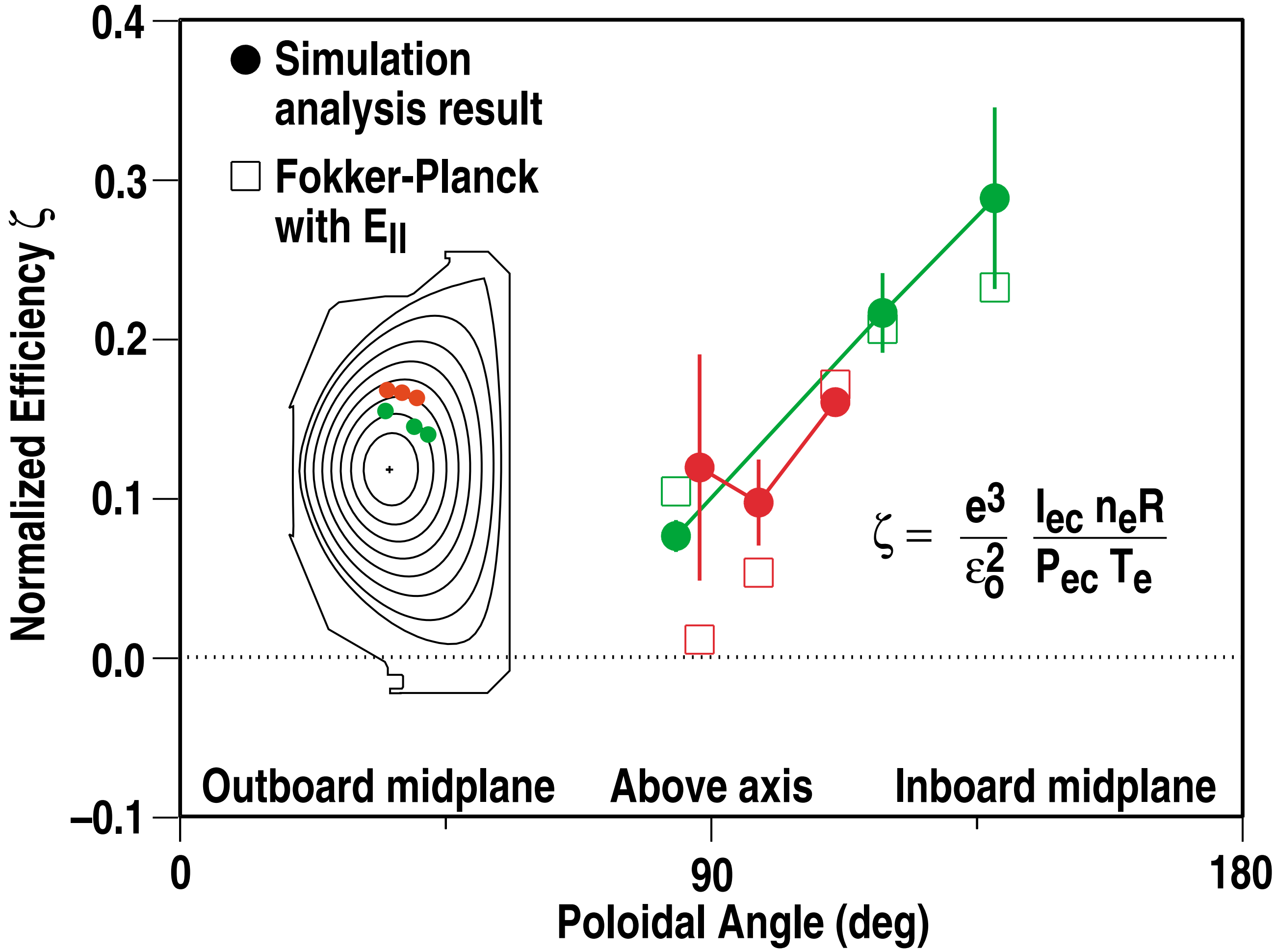
ECCD EFFICIENCY DECREASES WITH RADIUS (FOR POLOIDAL ANGLE ≈ 90 deg) AS EXPECTED FROM THEORY DUE TO TRAPPING EFFECTS



- Anomalously high ECCD is observed at largest radius
  – Need to verify this at higher ECH power with smaller error bars

POLOIDAL SCANS SHOW SYSTEMATIC INCREASE IN ECCD EFFICIENCY TO HIGH FIELD SIDE

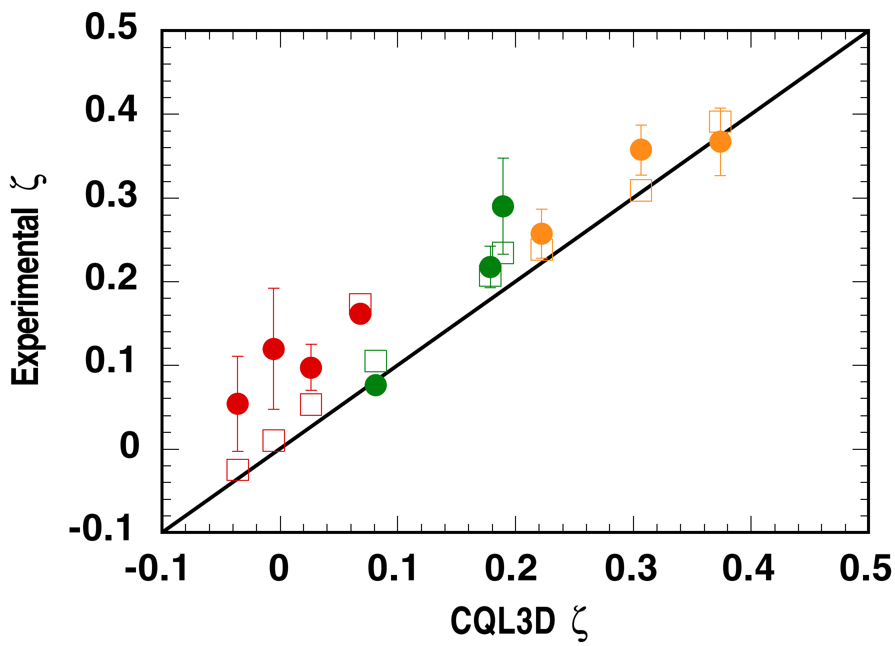


- Theoretically the increase in ECCD efficiency with poloidal angle is due to (a) reduced trapping effects and (b) wave absorption on higher energy elections from $N_{||}$ upshift

BOUNCE AVERAGED FOKKER-PLANCK CALCULATIONS UNDERESTIMATE ELECTRON CYCLOTRON CURRENT DRIVE EFFICIENCY

- **The bounce averaged calculations are based on the zero-collisionality theory, i.e., $\tau_b << \tau_e$ for all energies; the assumption is clearly not valid for low energy electrons**

- **Collisionality effectively reduces trapped electron fraction and increases current drive efficiency**



Open squares correspond to the calculated ECCD efficiency in the presence of the measured Ohmic field $E_{||}$

THEORETICAL MODEL FOR ECCD

- **Quasilinear Fokker-Planck equation:**

$$v_{\parallel}\hat{b}\nabla f - C_e f = S_{rf}(f) + eE_{\parallel}\frac{\partial f}{\partial p_{\parallel}}$$

where f is the electron distribution function, $v_{\parallel}$ and $p_{\parallel}$ are respectively the parallel velocity and momometum.

C_e: Coulomb collision operator

S_{rf}: quasilinear rf diffusion operator

$E_{\parallel}$: Ohmic electric field

- **In tokamak geometry, the driven current has a simple poloidal angle dependence:**

$$j_{\parallel} \equiv -e\int d\Gamma\, f_1\, v_{\parallel}$$

$$= \frac{\langle j_{\parallel}B\rangle}{\langle B^2\rangle}B$$

i.e., $j_{\parallel}/B$ is a flux-surface quantity

STANDARD THEORETICAL TREATMENTS

- **Collisionless model** $\left(\tau_b \approx \frac{qR}{v} << \tau_e\right)$:

$$\hat{b} \cdot \nabla f = 0 \;; \qquad f = f(\varepsilon, \mu)$$

$$-\left\langle \frac{B}{v_\parallel} C_e \right\rangle f = \left\langle \frac{B}{v_\parallel} S_{rf} \right\rangle (f) + \left\langle eE_\parallel B \right\rangle \frac{\partial f}{\partial \varepsilon}$$

where ε is the particle energy and μ is the magnetic moment. For a given flux surface, to solve the bounce-averaged equation is a 2-D problem

- **Linear Regime:**

$$f = f_M + f_1^{ohm} + f_1^{rf} + \ldots$$

$$j_\parallel = j_\parallel^{ohm} + j_\parallel^{rf} + \ldots$$

Here f_M is the Maxwellian distribution, f_1^{ohm} is the equivalent Spitzer function in toroidal geometry, and

$$v_\parallel \hat{b} \nabla f_1^{rf} - C_e^{\ell} f_1^{rf} = S_{rf}(f_M)$$

We expect the linear approximation to be justified for

$$p_{rf}\,\tau_e << n_e T_e$$

where p_{rf} is absorbed rf power density

ADJOINT TECHNIQUES CAN BE USED TO EXAMINE COLLISIONALITY EFFECTS ON ECCD IN LINEAR REGIME

- **Introducing the adjoint equation:**

$$-\nu_{\parallel}\hat{\mathbf{b}}\nabla\chi - \mathbf{C}_{\mathbf{e}}^{\ell+}\chi = \frac{\nu_{\parallel}\mathbf{B}}{\langle \mathbf{B}^2 \rangle}$$

Here, $\mathbf{C}_{\mathbf{e}}^{\ell+}$ is the adjoint collision operator defined by

$$\left\langle \int \mathbf{d}\Gamma \mathbf{f}\, \mathbf{C}_{\mathbf{e}}^{\ell}\, \mathbf{g} \right\rangle = \left\langle \int \mathbf{d}\Gamma \mathbf{g}\, \mathbf{C}_{\mathbf{e}}^{\ell+}\, \mathbf{f} \right\rangle$$

Therefore,

$$\begin{aligned}
\frac{\mathbf{j}_{\parallel}}{\mathbf{B}} &= -\mathbf{e}\left\langle \int \mathbf{d}\Gamma\, \mathbf{f}_1\, \frac{\nu_{\parallel}\mathbf{B}}{\langle \mathbf{B}^2 \rangle} \right\rangle \\
&= -\mathbf{e}\left\langle \int \mathbf{d}\Gamma\, \mathbf{f}_1 \left(-\nu_{\parallel}\hat{\mathbf{b}}\nabla\chi - \mathbf{C}_{\mathbf{e}}^{\ell+}\chi \right) \right\rangle \\
&= -\mathbf{e}\left\langle \int \mathbf{d}\Gamma\, \chi \left(\nu_{\parallel}\hat{\mathbf{b}}\nabla \mathbf{f}_1 - \mathbf{C}_{\mathbf{e}}^{\ell}\mathbf{f}_1 \right) \right\rangle \\
&= -\mathbf{e}\left\langle \int \mathbf{d}\Gamma\, \chi \mathbf{S}_{\mathbf{rf}}(\mathbf{f}_{\mathbf{M}}) \right\rangle
\end{aligned}$$

- **Absorbed power density:**

$$\mathbf{Q} = \left\langle \int \mathbf{d}\Gamma \varepsilon \mathbf{S}_{\mathbf{rf}}(\mathbf{f}_{\mathbf{M}}) \right\rangle$$

ADJOINT FORMULATION: AN EFFICIENT METHOD FOR MODELING ECCD IN LINEAR REGIME

- **Current drive efficiency:**

write $$\chi = \nu_{e0}^{-1}\left(\frac{v_e}{B_{max}}\right)\tilde{\chi} \quad ,$$

with $$v_e \equiv \left(\frac{2T_e}{m}\right)^{1/2} \text{ and } \nu_{e0} = \frac{e^4 n_e \ell n \Lambda}{4\pi\varepsilon_0^2 m^2 v_e^3}$$

$$\varsigma \equiv \frac{e^3}{\varepsilon_0^2}\left[\frac{n_e \langle j_{\|} \rangle}{2\pi Q}\right]$$

$$= -\frac{4}{\ell n \Lambda}\left\langle \frac{B}{B_{max}} \right\rangle \frac{\left\langle \int d\Gamma \tilde{\chi} S_{rf}(f_M) \right\rangle}{\left\langle \int d\Gamma \left(\varepsilon / m v_e^2\right) S_{rf}(f_M) \right\rangle}$$

- **Note that there is no dependence on S_{rf} in χ; once evaluated, it can be used to calculate ς for any given S_{rf}**

- **Moreover, χ is the Spitzer function in toroidal geometry. It can be used to evaluate σ_{neo} and the bootstrap coefficients**

SIMPLIFIED COLLISION MODEL

- **To solve the adjoint equation with the full coulomb collision operator for arbitrary collisiisionality is a 3-D numerical problem**

- **Small inverse aspect ratio limit ($\delta \equiv \frac{a}{R} \rightarrow 0$)**

  - **pitch-angle scattering dominant**

  - **analytic solutions possible in the banana regime**

  - **using Hinton-Rosenbluth boundary layer analysis the leading order collisionality corrections to ECCD +**

$$\Delta j \cong \sqrt{\delta \nu_{*e}}\, j_c$$

where j_c is the ECCD in the no-trapping limit

- **The collisionality correction to ECCD efficiency estimated by the velocity-space connection formula appears to be consistent with the above scaling law**

+ V.S. Chan (unpublished, 1981 APS)

LORENTZ GAS MODEL

- **Lorentz gas model (e-i pitch-angle scattering only):**

$$C_e f \approx \nu_{ei} L f$$

$$L = \frac{1}{2}\frac{\partial}{\partial\xi}\left(1-\xi^2\right)\frac{\partial}{\partial\xi} \quad ; \qquad \xi \equiv \frac{u_\parallel}{u}$$

$$\nu_{ei}(v) = Z_{eff}\,\nu_{e0}\,\gamma\left(\frac{u_e}{u}\right)^3; \qquad \vec{p} \equiv m\vec{u} = m\gamma\vec{v}$$

- **Adjoint equation (2-D numerical problem):**

$$B = \frac{B_0}{h(\theta)}; \qquad \hat{b}\cdot\nabla = \frac{B_\theta}{rB}\frac{\partial}{\partial\theta}$$

$$\chi = \chi_c + \chi_t = \frac{vB_0}{\nu_{ei}\left\langle B^2\right\rangle}\left(\frac{\xi}{h} + G_t\right)$$

$$-\sigma\frac{\partial}{\partial\theta}G_t - \nu\frac{\partial}{\partial\lambda}\left(\lambda|\xi|\right)\frac{\partial}{\partial\lambda}G_t = \alpha$$

with $\lambda \equiv \left(1-\xi^2\right)h; \qquad \sigma = \mathrm{sgn}\left(u_\parallel\right)$

$$\nu \equiv \frac{2rB_0}{B_\theta v}\nu_{ei} \approx \frac{2qR}{v}\nu_{ei}; \qquad \alpha \equiv \frac{\partial}{\partial\theta}\frac{|\xi|}{h}$$

LORENTZ GAS MODEL (NUMERICAL SCHEME)

- Following Hinton and Rosenbluth,[+]

Define $G_t^{\pm} = \frac{1}{2}\left\{G_t(\sigma = -1) \pm G_t(\sigma = +1)\right\}$,

$$\frac{\partial}{\partial\theta} G_t^- - \nu \frac{\partial}{\partial\lambda}(\lambda|\xi|)\frac{\partial}{\partial\lambda} G_t^+ = \alpha$$

$$\frac{\partial}{\partial\theta} G_t^+ - \nu \frac{\partial}{\partial\lambda}(\lambda|\xi|)\frac{\partial}{\partial\lambda} G_t^- = 0$$

Introduce $\Phi = \int_0^{\lambda} d\lambda' \, G_t^+(\lambda', \theta)$, which satisfies

$$\frac{\partial}{\partial\theta}\left(\frac{\partial\Phi}{\partial\theta} \Big/ \nu\lambda|\xi|\right) - \nu \frac{\partial^2}{\partial\lambda^2}(\lambda|\xi|)\frac{\partial^2\Phi}{\partial\lambda^2} = \frac{\partial\alpha}{\partial\lambda}$$

The equation is 2nd order in θ and 4-th order in λ

- The problem of solving the above equation can be formulated in terms of a variational principle, and

$$G_t^+ = \frac{\partial\Phi}{\partial\lambda}; \qquad \frac{\partial G_t^-}{\partial\lambda} = \frac{1}{\nu\lambda|\xi|}\frac{\partial\Phi}{\partial\lambda}$$

[+]Hinton and Rosenbluth (1973)

NUMERICAL SCHEME (CONTINUED)

- Numerical solution:
  - — finite difference in θ and $|\xi|$.
  - — ADI iteration scheme based on Hinton-Rosenbluth variational principle
  - — use a primitive multi-grid scheme
  - — simplified magnetic geometry ($B = \dfrac{B_0}{1+\delta\cos\theta}$)

- Analytic solution in the banana regime:

write $\chi = \dfrac{vB_0}{\nu_{ei}\langle B^2\rangle}\left(\dfrac{\xi}{h}+G_t\right) = \dfrac{vB_0}{\nu_{ei}\langle B^2\rangle}G$

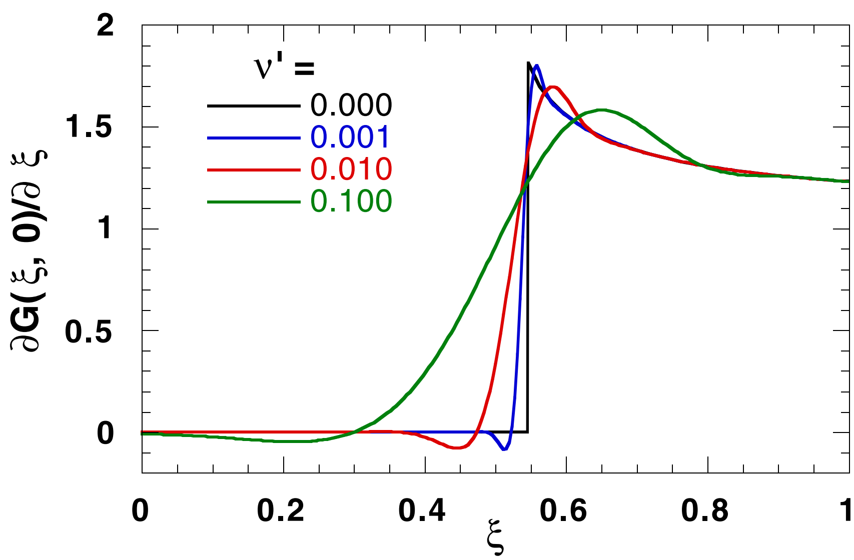
$$G = \frac{1}{2}\,\mathrm{sgn}(u_\parallel)\int_\lambda^{\lambda_c}\frac{d\lambda'}{\left\langle\sqrt{(1-\lambda'/h)}\right\rangle} \qquad (\lambda < \lambda_c)$$

$$G = 0 \qquad (\lambda > \lambda_c)$$

where $\lambda_c \equiv 1-\delta$

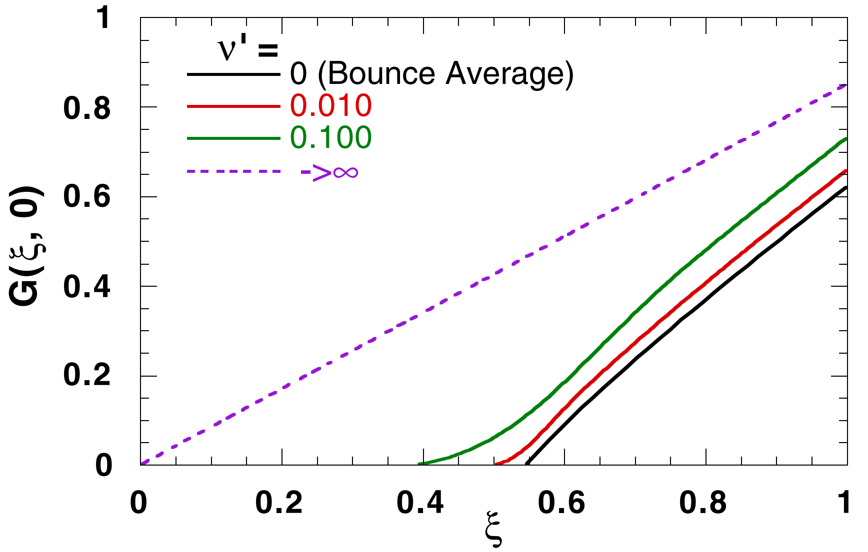
COLLISIONALITY CORRECTIONS SHOW BOUNDARY LAYER CHARACTERISTICS

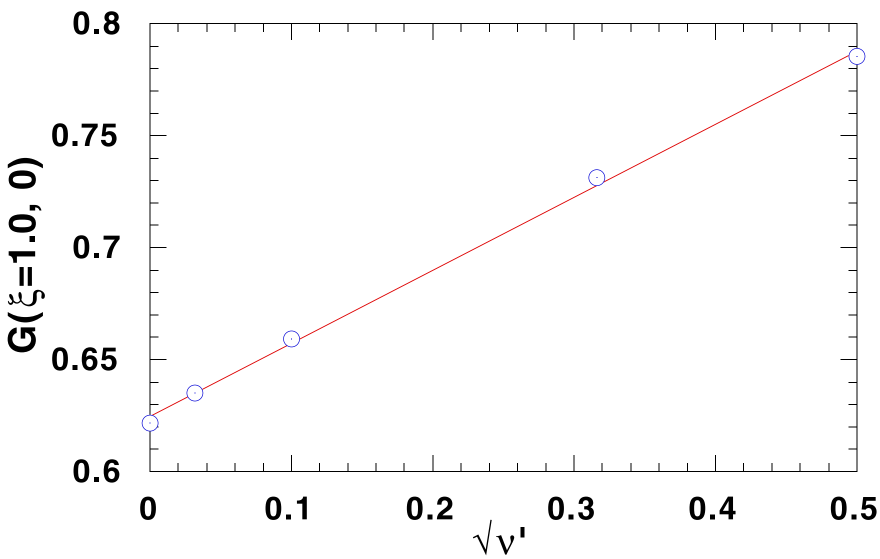
- **Numerical results for Lorentz gas model:**
  - $B = \dfrac{B_0}{1+\delta\cos\theta}$; $\delta = 0.175$
  - $\nu' \equiv \dfrac{\nu}{2\delta^{3/2}} = \dfrac{qR}{\delta^{3/2}v}\nu_{ei}$
  - $\nu' = 0$ **corresponds to the analytic banana-regime solution**



- $\partial G / \partial \xi$ **as a function of** ξ **at ploidal angle** $\theta = 0$ **(outboard midplane) for various** ν'

FINITE COLLISIONALITY ALTERS TRAPPED-PASSING BOUNDARY AND REDUCES TRAPPING EFFECT





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NUMERICAL RESULTS FOR INTEGRATED QUANTITIES AGREE WITH ANALYTIC THEORY IN LOW COLLISIONALITY LIMIT

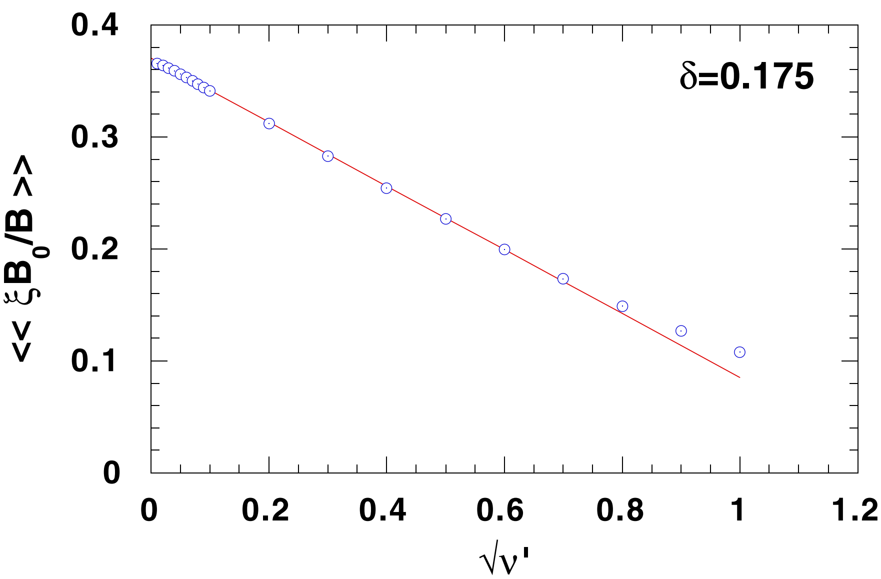
- **Define** $\left\langle\!\left\langle \xi \frac{B_0}{B} \right\rangle\!\right\rangle = h \int d\xi G_t(\xi, \theta, \nu')\xi$

  — it is independent of poloidal angle θ

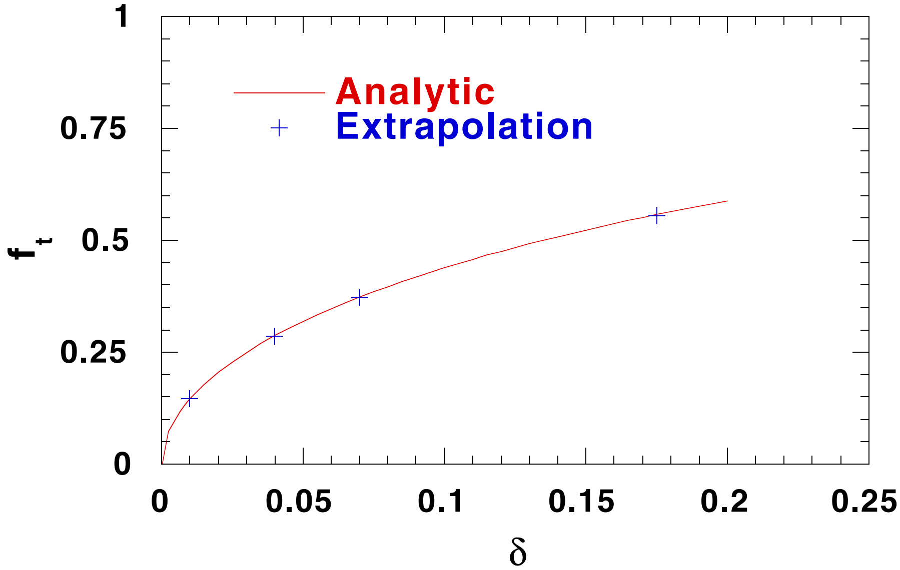
  — $\left\langle\!\left\langle \xi \frac{B_0}{B} \right\rangle\!\right\rangle \propto \left(1 - a\sqrt{\nu'} + \cdots\right)$ as $\nu' \to 0$

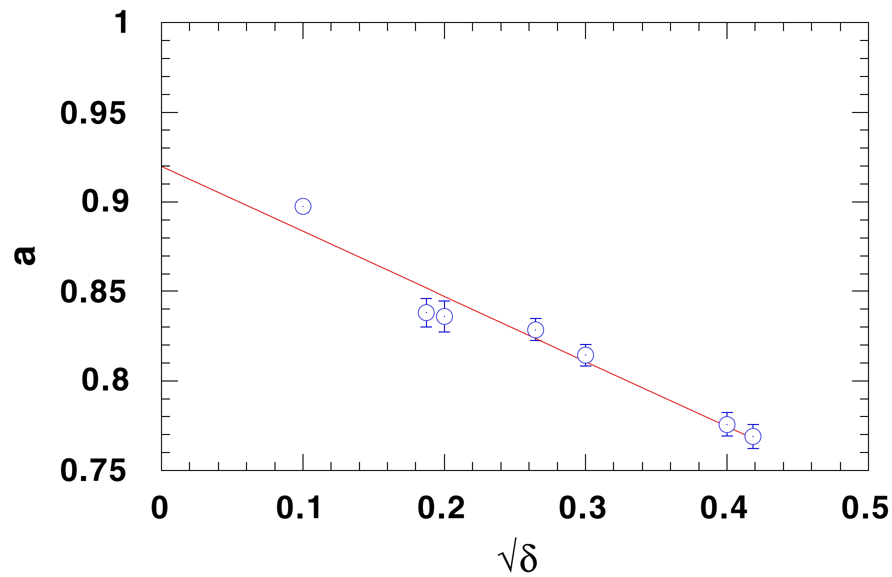
  — $f_t = \frac{3}{2} \left\langle\!\left\langle \xi \frac{B_0}{B} \right\rangle\!\right\rangle$ as $\nu' \to 0$;

  is the effective trapped particle fraction



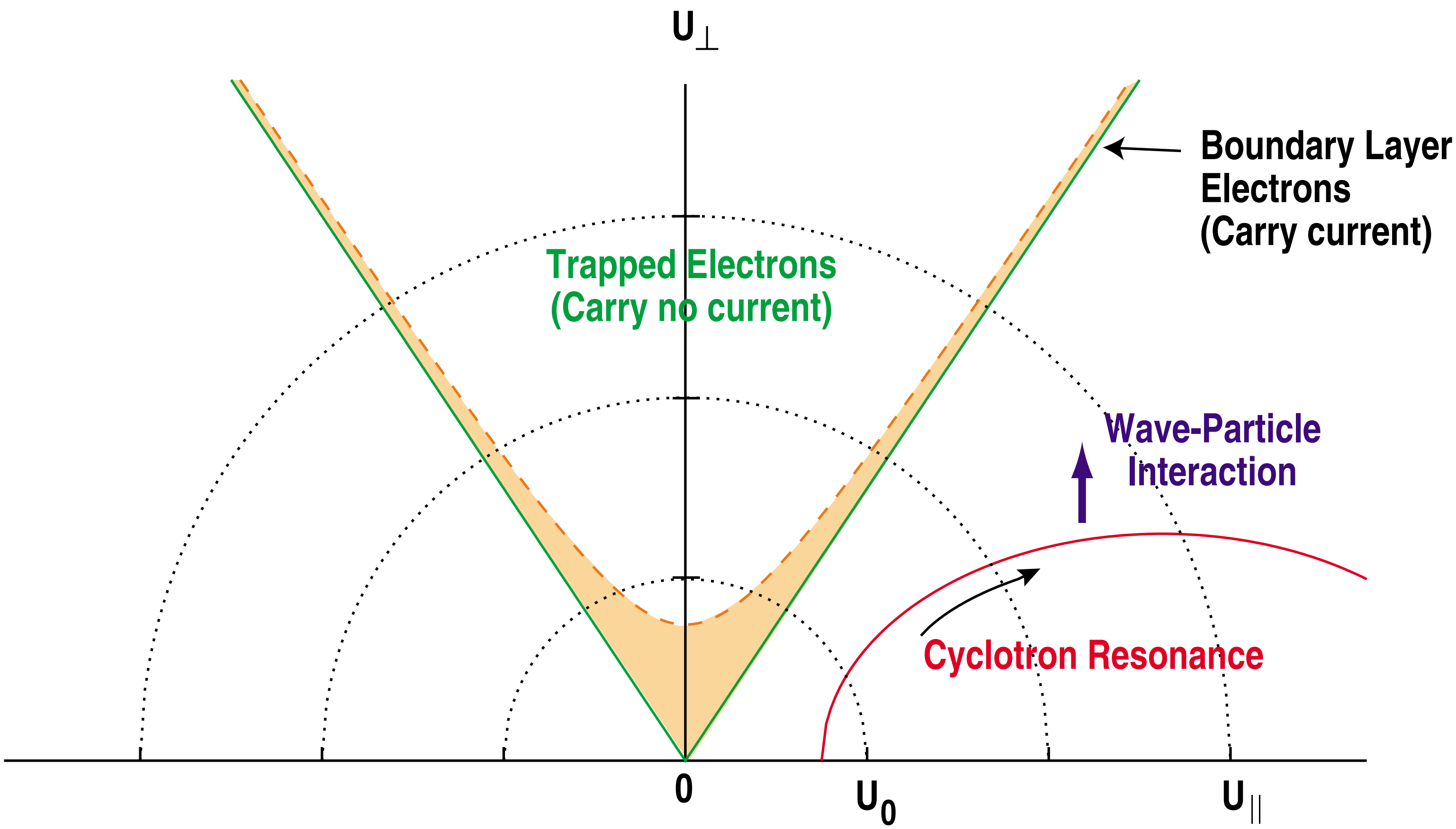
NUMERICAL RESULTS (CONTINUED)





- $a \approx 0.92$ **in the small inverse aspect ratio limit ($\delta \to 0$)**

SCHEMATIC OF FINITE COLLISIONALITY ON ECCD



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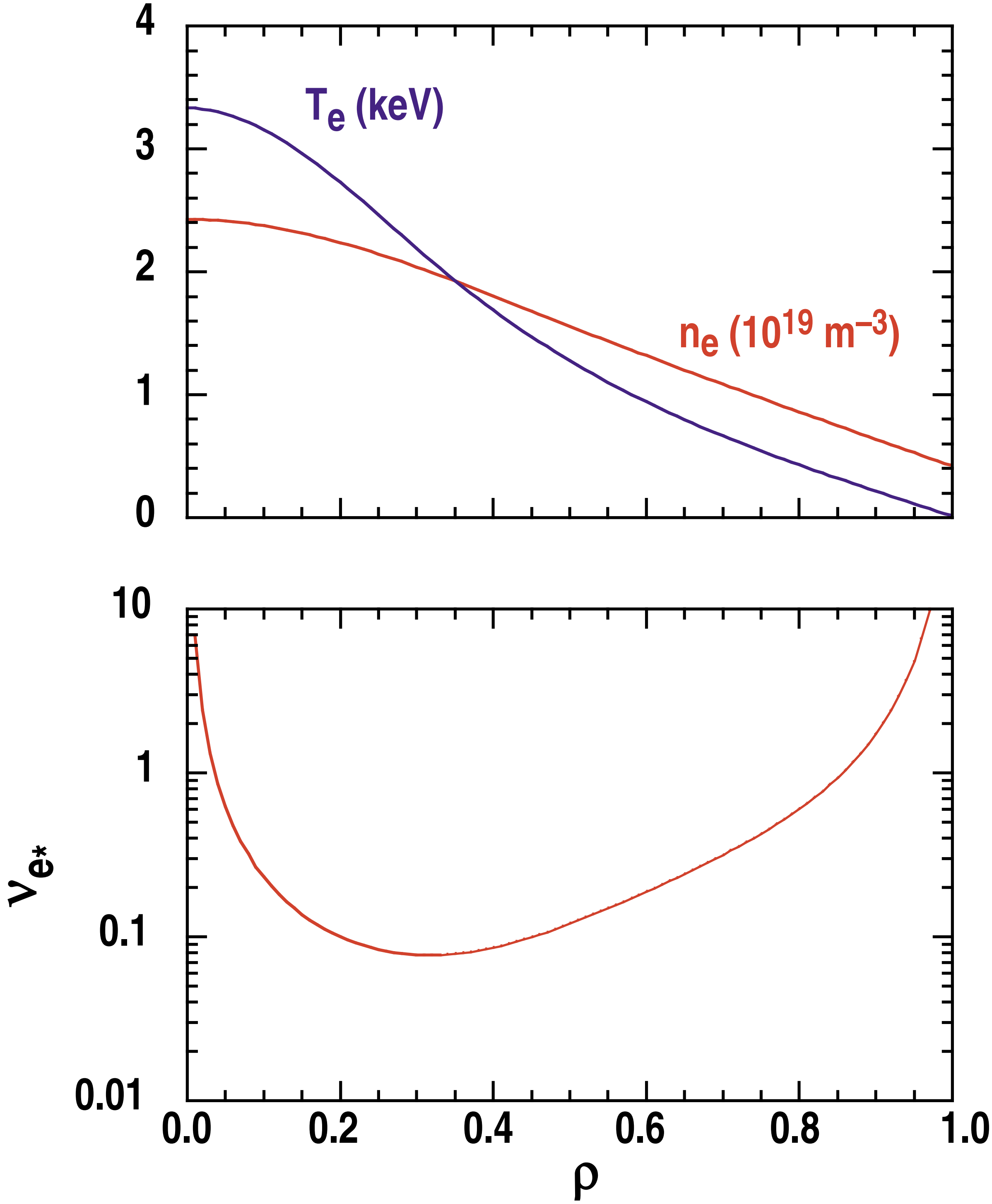
KINETIC PROFILES OF EXPERIMENTAL L–MODE DISCHARGE



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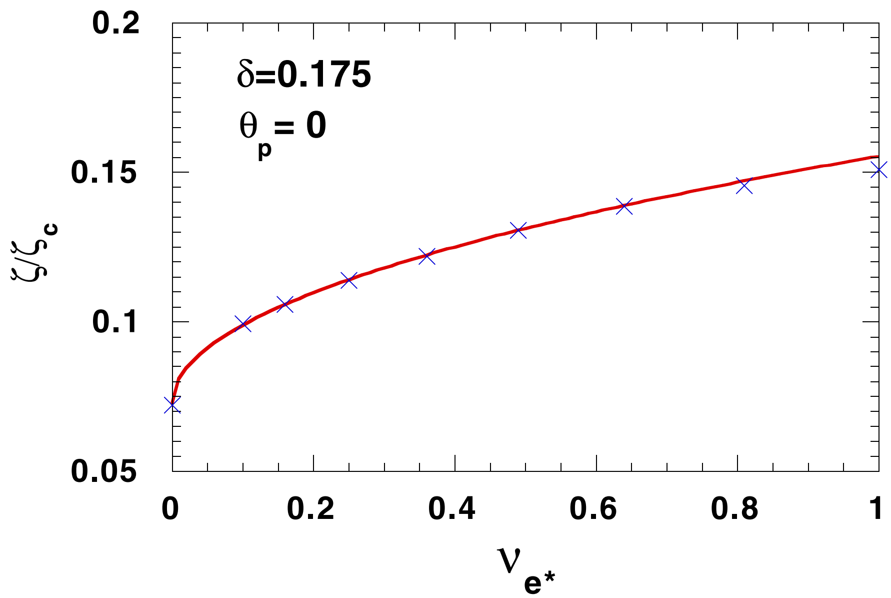
MODEST COLLISIONALITY ENHANCEMENT IN CURRENT DRIVE EFFICIENCY FOR PARAMETER REGIMES OF OFF-AXIS ECCD

- **Plasma parameters for off-axis ECCD :**
  - $B = \dfrac{B_0}{1+\delta\cos\theta}$; $\delta = 0.175$
  - $T_e \approx 1.$ kev $\nu_{*e} \approx 0.1$
  - $\omega \approx 2\omega_c$ $n_{||} = 0.5$



- **Normalized current drive efficicency as a function of ν_{*e} at ploidal angle $\theta_p = 0$ (outboard midplane) ; crosses are numerical results from the Lorentz-gas model, and the solid line curve fitting with a dependence $\sqrt{\nu_{*e}}$; ζ_c is the current drive efficiency in the limit $\nu_{*e} >> 1$**

SUMMARY

- Comparing with the measured off-axis ECCD efficiency in recent DIII–D experiments, bounce-averaged calculations give lower estimates of the efficiency

- Collisionality reduces the trapped electron effects and will increase ECCD efficiency

- The boundary layer analysis in the small inverse aspect ratio limit ($\delta \to 0$) indicates the collisionality correction is on the order of $\sqrt{\delta \nu_{*e}}$

- Non-bounce averaged calculations of ECCD effieciency were performed using the Lorentz gas model (pitch angle scattering only) to gain semi-quantitative understanding of collisionality effects

- Appreciable collisionality enhancement of current drive effciency is possible in off-axis ECCD experimental conditions