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Abstract. A velocity-space connection formula is proposed to estimate the collisionality effect on electron cyclotron current drive efficiency. The collisionality correction gives modest improvement in agreement between theoretical and recent DIII–D experimental results.

Electron Cyclotron Current Drive (ECCD) is considered a leading candidate for current profile control in Advanced Tokamak (AT) operation. Localized ECCD has been clearly demonstrated in recent proof-of-principle experiments on DIII–D [1]. The measured ECCD efficiency near the magnetic axis agrees well with standard theoretical predictions [2,3]. However, for off-axis current drive the normalized experimental efficiency does not decrease with minor radius as expected from the standard theory; the observed reduction of ECCD efficiency due to trapped electron effects in the off-axis cases is smaller than theoretical predictions. The standard approach of modeling ECCD in tokamaks has been based on the bounce-average calculations, which assume the bounce frequency is much larger than the effective collision frequency for trapped electrons at all energies. The assumption is clearly invalid at low energies. Finite collisionality will effectively reduce the trapped electron fraction, hence, increase current drive efficiency. In this work, we propose an analytical interpolation of ECCD efficiency between collisional and collisionless limits to examine the collisionality effect.

We will use the adjoint techniques to calculate the ECCD efficiency. The theoretical formulation as presented in Ref. [4] is valid in general tokamak geometry and applicable in all collisionality regimes. The current drive efficiency $\eta$ is written as

$$\eta = \frac{n_e \langle j_\parallel \rangle}{2\pi Q} = \frac{4e_0^2}{e^3/nA} \left( B_{\max} \right) \left\langle \int d\Gamma \chi S_{\text{rf}} (f_M) \right\rangle \left( \int d\Gamma (w/mv_e^2) S_{\text{rf}} (f_M) \right),$$

(1)

where $\langle \ldots \rangle$ denotes the flux surface average, $j_\parallel$ the parallel driven current density, and $Q$ the absorbed rf power density. $S_{\text{rf}}$ is the rf quasilinear diffusion operator and $f_M$ is the Maxwellian distribution function. We have assumed that the rf power density is not too high such that interactions between EC waves and electrons can be represented by
$S_{\text{rf}}(f_M)$. The function $\tilde{\chi}$ is related to the adjoint function $\chi$ by $\chi = \nu e_0^{-1}(v_e/B_{\text{max}})\tilde{\chi}$ with $v_e = (2T_e/m)^{1/2}$ and $\nu e_0 = (e^4n_e\ell n\Lambda)/(4\pi\varepsilon_0^2 m^2 v_e^3)$, the characteristic frequency of Coulomb collisions: $\chi$ satisfies

$$-v_\| \cdot \nabla \chi - C_e^{\ell+} \chi = \frac{v_B}{\langle B^2 \rangle},$$

where $C_e^{\ell+}$ is the adjoint collision operator. Note that $\chi$ is a function of energy ($w = \gamma mc^2 = mc^2[1 + (u/c)^2]^{1/2}$), magnetic moment ($\mu = mu_\perp^2 / 2B$), and poloidal angle $\theta_p$. Here $\tilde{u}$ denotes momentum per unit mass, $\tilde{u} = \gamma u = \gamma \cdot v$. To solve Eq. (2) with the full Coulomb collision operator is a three-dimensional numerical problem.

Using Fisch's relativistic high-velocity collision model [5], analytic solutions of the adjoint equation, Eq. (2), are possible in the collisionless and collisional limits. The model collision operator is given as

$$C_e f = [v_{ei}(u) + v_D(u)]L f + \frac{1}{u^2} \frac{\partial}{\partial u} u^2 \lambda_s(u) f,$$

where $L$ is the pitch-angle scattering operator, $v_{ei}(u) = Z_{\text{eff}} v_e \gamma (u_e / u)^3$, $v_D(u) = v_e \gamma (u_e / u)^3$, and $\lambda_s(u) = v_e \gamma (u_e / u)^2$ with $u_e = v_e$. By keeping only the first Legendre harmonics of the slowing-down operator, the collisionless (bounce-averaged) adjoint function can be written as

$$\chi_b = \text{sgn}(u_\|) H(\lambda) G(u, Z_{\text{eff}}, f_t),$$

$$H(\lambda) = \frac{\theta(\lambda_c - \lambda)}{2} \int_{(1 - \lambda B^2)^{1/2}} \frac{d\lambda'}{(1 - \lambda' B^2)^{1/2}},$$

$$G(u, Z_{\text{eff}}, f_t) = \left(\frac{c^2}{v_e u_e^3}\right) \frac{1}{(1 - f_t)} \left(\frac{\gamma + 1}{\gamma - 1}\right)^{\hat{\rho}/2} \int_0^{\gamma} d\gamma' \left(\frac{u'}{\gamma'}\right)^{2} \left(\frac{\gamma' - 1}{\gamma' + 1}\right)^{\hat{\rho}/2},$$

with $\lambda = B^{-1}(u_\perp / u)^2 = B^{-1}(1 - \xi^2)$, $\lambda_c = B_{\text{max}}^{-1}$, $\hat{\rho} = (Z_{\text{eff}} + 1) / (1 - f_t)$, and $f_t$ is the well-known effective trapped particle fraction

$$f_t = 1 - \frac{3}{4} \langle B^2 \rangle \int_{0}^{\hat{\lambda}} \frac{\lambda' d\lambda'}{(1 - \lambda B^2)^{1/2}}.$$

We have found that $\chi_b$ given in Eq. (4) gives similar predictions as those of Ref. 2 for ECCD efficiency. In the collisional limit, where the transit term in the left-hand-side of Eq. (2) can be ignored, the adjoint function is given by
\[ \chi_c = \left( B / \langle B^2 \rangle \right) \xi G(u, Z_{\text{eff}}, 0). \]  

(8)

In the finite collisionality regime, we approximate the adjoint function by a velocity-space interpolation formula:

\[ \chi \approx \chi_c + \left[ 1 + \alpha \sqrt{\nu_c} \left( \frac{u_e}{u} \right)^2 \right]^{-1} (\chi_b - \chi_c), \]

(9)

\[ \nu_c = \sqrt{2} \left( qR \right) / \left( 8^{3/2} \tau_e v_e \right), \]

(10)

where \( \tau_e \) is the Braginskii collision time, \( \delta \) is the inverse aspect ratio, and \( \alpha \) is an adjustable parameter. From the boundary-layer theory of Hinton and Rosenbluth [6], we expect the collisionality effect to have a square root dependence on \( \nu_c \). \( \alpha \) is a constant of an order of one. We take \( \alpha = 2 \). In the Lorentz-gas limit (\( Z_{\text{eff}} \gg 1 \)), this gives \( L_{31} = f_t / (1 + \sqrt{\nu_c}) \) and \( \sigma_{\text{neo}} / \sigma_{\text{sp}} = 1 - f_t / (1 + 0.59 \sqrt{\nu_c}) \) where \( L_{31} \) is the density-gradient bootstrap coefficient, and \( \sigma_{\text{neo}} \) and \( \sigma_{\text{sp}} \) are respectively the neoclassical and Spitzer conductivities. Both agree well with a recent numerical calculation of these coefficients [7].

The connection formula was implemented in a current drive package which is coupled to the raytracing code TORAY. Using the kinetic profiles and the ECH system parameters of recent DIII–D experiments [1], we have calculated the collisionality correction to the ECCD efficiency as well as the corresponding values in both the collisional (\( \nu_c \gg 1 \)) and collisionless (\( \nu_c = 0 \)) limits. Comparison of the theoretical and experimental results is shown in Fig. 1(a) and (b). Fig. 1(a) shows the cases near the magnetic axis and good agreement is observed. In Fig. 1(b), we show the cases of off-axis. The collisionality correction gives modest improvement in the current drive efficiency in comparison with the collisionless value. Experimental data for off-axis current drive appear to be more consistent with the theoretical results of \( \nu_c \gg 1 \).

In summary, we have found a velocity-space connection formula to estimate the collisionality effect on ECCD efficiency. The collisionality correction gives a modest improvement for agreement between experimental and previous theoretical results.

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FIGURE 1. Comparison of theoretical and experimental values of normalized ECCD efficiency $\eta / T_e$ as functions of poloidal angle: (a) near the magnetic axis, (b) off-axis. The inset plasma cross-sections show the ECH deposition locations.