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Calculation of Direct Coupling to the Electron Bernstein Wave With A Waveguide Antenna

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Abstract. Conventional electron cyclotron heating using the O- and X-modes to carry energy from the plasma edge to the cyclotron resonance layer is not possible for high density, low magnetic field devices (RFPs and STs, for example), since these modes are evanescent in most of the plasma. As an alternative, we consider coupling to the electron Bernstein wave (EBW) with a single waveguide, the mouth of which is inserted to the vicinity of the upper hybrid resonance.

INTRODUCTION

'Conventional' electron cyclotron heating and current drive scenarios use the two cold plasma modes propagating at frequencies above the electron cyclotron resonance frequency, conventionally referred to as "O" and "X" modes, to carry the wave energy from the vacuum region to the fundamental or 2nd harmonic of the cyclotron resonance. However, at the high densities and low magnetic fields characteristic of present generation spherical torus or reversed-field-pinch experiments, and the correspondingly large values of the plasma dielectric constant ($\omega_{pe}^2/\Omega_e^2 \gg 1$) in the core of the plasma, neither cold plasma mode can propagate to the core. For this reason, a great deal of interest has arisen in coupling to the electron Bernstein wave (EBW). This nearly electrostatic mode does not correspond to any wave that propagates in a cold plasma, has a wavelength across the magnetic field on the order of the electron gyroradius, and experiences no high density cutoff. Propagation of the EBW of interest begins at densities higher than the upper hybrid resonance (UHR) density, where $\omega_{pe}^2|_{\mathrm{UHR}} = \omega^2 - \Omega_e^2$.

Previous calculations of coupling to the EBW in this context[1] have computed the efficiency with which a launched X- or O-mode propagating from the low density region can be converted to the EBW near the UHR. Ranges of toroidal wavenumbers in which efficient mode conversion to the EBW is obtained have been identified. For the parameters characteristic of the low-field machines being considered here, the UHR layer can be at very low density - on the order of $10^{10} - 10^{11}\mathrm{cm}^{-3}$. In this case, the distance between the right-hand cutoff for the X-mode and the UHR, which is the critical parameter identified in the mode conversion theory, can be very much shorter than the vacuum wavelength. The wave launcher's near fields can overlap the mode conversion region in this case, so that effects of the mode conversion process on the observable coupling properties of the coupler might reasonably be expected. In the present work, the linear coupling to the EBW from an open-ended waveguide coupler is calculated in a slab geometry. The calculation is closely related to the coupling problem in the lower hybrid range of frequencies[2, 3] and the algorithm used in that work carries over directly to the present problem. In this paper, we consider a coupler consisting of only a single waveguide, oriented to excite the X-mode, and of infinite width so that the index of refraction along the static magnetic field, $n_{\|}$, is identically zero. Generalization to a phased array of identical waveguides of finite dimensions is straightforward, and will be left to a future paper.

SOLUTION OF THE BRAMBILLA PROBLEM

The slab geometry of the problem is illustrated in Fig. 1. The waveguide mouth is taken to form an aperture in an infinite, perfectly conducting ground plane, and in this paper is assumed to have an infinitely long dimension along the static magnetic field, which defines the z-direction. The density increases in the dimension normal to the ground plane with some profile $n_e(x)$. The waveguide opening dimension in the direction orthogonal to x and z (the poloidal direction in tokamak geometry) is a. We neglect any variation in the direction or magnitude of the static magnetic field, which has magnitude B_o. The electric field of the propagating waveguide mode points across the short dimension of the waveguide, normal to B_o, thus exciting only the X-mode or the EBW. Note that with $n_{\|} = 0$, the O- and X-modes are completely decoupled. Furthermore, we neglect all evanescent waveguide modes. Carrying out the matching between the forward and reflected modes in the waveguide and the transverse electric and magnetic fields in the plasma at x = 0 according to the procedure of Brambilla[2], we obtain the integral that determines the reflection coefficient in the waveguide as

$$\frac{1-\rho}{1+\rho} \equiv \Lambda = \frac{\gamma}{\pi\sqrt{\varepsilon_w}} \int dn_y \;\; Y(n_y) \frac{\sin^2(\gamma n_y)}{(\gamma n_y)^2} \tag{1}$$

in which $\gamma \equiv \omega a/(2c)$ and the definition of the surface admittance $Y(n_y)$ is the ratio of the tranverse magnetic and electric fields of a plane wave with wavenumber n_y at x = 0+, i.e., $Y(n_y) \equiv B_z(n_y)/E_y(n_y)|_{x=0+}$. The second factor in the integrand represents the spectral characteristics of the antenna, while the first factor has embedded all of the information about the plasma that is relevant to the coupling problem.

Two different models of the plasma's dielectric properties have been numerically integrated to obtain the surface admittance. In the first, the GLOSI code[4] is used, in which the finite difference method is used to solve a sixth-order wave equation incorporating a second order FLR expansion to model the Bernstein wave. Though this code was initially written to study mode conversion problems for the ion cyclotron range of frequencies, it can be equally well used in the electron cyclotron range. The density profile is taken to be of the form $n_e(x) = (n_{max}/2)\{1 - \tanh([x_{in} - x]/L)\}$, in which the density gradient scale length is L, the asymptotic density for large x is n_{max} and the distance from the wall to the point of inflection is x_{in}. We choose n_{max} to be larger than the left-hand cutoff density, so that the only wave that can carry energy to higher density regions is the Bernstein wave.

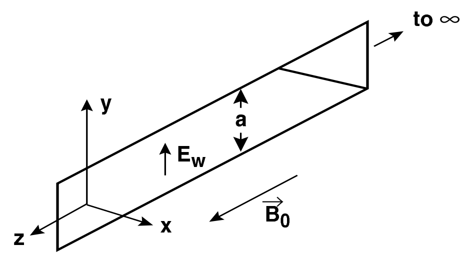

FIGURE 1. Slab geometry of the coupling calculation; density varies only in the x-direction.

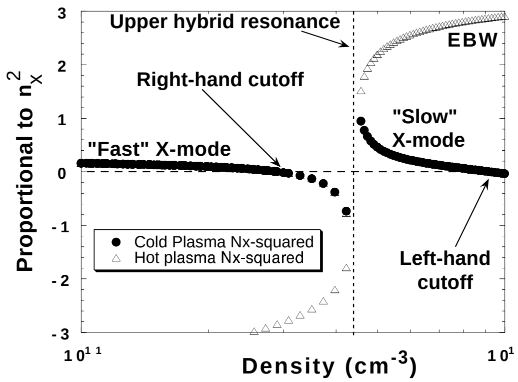

FIGURE 2. The square of the wavenumber n_x (for $n_y = n_z = 0$) as a function of density for $f = 7$ GHz, $B_o = 0.13$ T, $T_e = 10$ eV.

In the second model, we use the cold plasma theory. Just as in the mode conversion theories, e.g. Ref. [1], the addition of weak collisions to the model permits integration through the upper hybrid resonance, at which all of the power that has not reflected back at the right-hand cutoff is then dissipated by collisions. It has been shown in previous work[1, 5] that the power dissipated in the cold plasma model with collisions is very nearly equal to the power mode converted to the Bernstein wave in a kinetic model. To assess the differences between the predictions of the two models, we compare these two calculations of the surface admittance for a specific case, with parameters chosen to be relevant to CDX-U experiments[6]. The static magnetic field (in the z-direction) is 0.13 T and the frequency is 7 GHz ($\omega/\Omega_e = 1.92$) so that coupling to the lowest order EBW (the only one included in the present version of GLOSI) is possible near the upper hybrid resonance. The density profile parameters are taken to be $n_{max} = 2.0 \times 10^{12}$ cm^{-3}, the gradient scale length $L = 0.5$ cm, and the position of the inflection point $x_{in} = 1$ cm ($n_e[x = 0] = 3.6 \times 10^{10}$ cm^{-3}). For the GLOSI model, the electron temperature is relevant, and is taken to be 10 eV. The dispersion characteristics of the X-mode and EBW for these parameters over the density range from 1×10^{11} cm^{-3} to 1×10^{12} cm^{-3} is shown in Fig. 2. For these parameters, the right-hand cutoff for the X-mode occurs at a density of 2.92×10^{11} cm^{-3}, the upper hybrid resonance density occurs at 4.43×10^{11} cm^{-3} and the left-hand cutoff for the "slow" X-mode is at 9.23×10^{11} cm^{-3}.

The surface admittances calculated using the two different models for this example are compared in the leftmost and center panels of Fig. 3. The two models agree quite well despite the substantial differences in the behavior of the wave fields at densities around and higher than the upper hybrid resonance density, thus showing that the differences in the reactive fields in the two models do not strongly affect the fields at the plasma surface a few millimeters away. A pronounced asymmetry between positive and negative n_y results from strong gradients in the off-diagonal elements of the dielectric tensor. This asymmetry implies that improved coupling can be obtained with a more sophisticated launching structure, such as a phased array with the phasing in the y-direction with which waves could be launched in the 'preferred' direction.

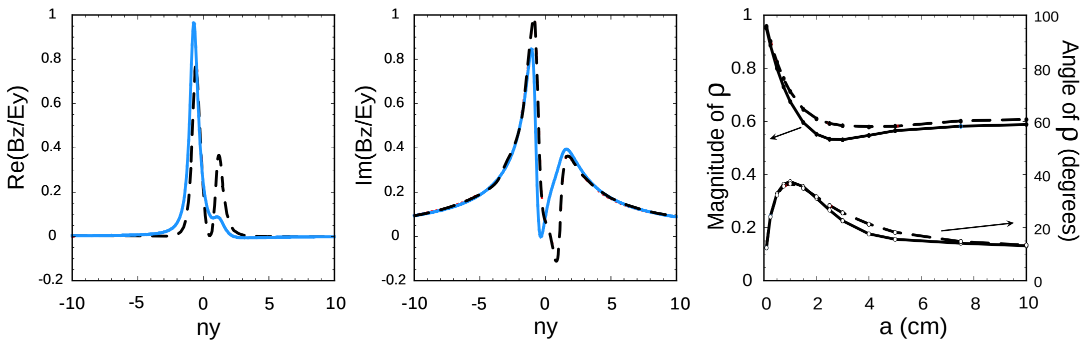

FIGURE 3. Comparison of the real (left panel) and imaginary (center panel) parts of the surface admittance for the CDX-U-relevant case in the GLOSI model (dashed curves) and the cold plasma model (solid curves). The rightmost panel shows the reflection coefficient (amplitude and phase) for a single waveguide of variable opening height *a* calculated from these surface admittances with the two models.

The spectrum of the surface admittance is not a directly observable quantity – only the convolution of the admittance with the spectrum launched by a coupling structure can be measured. In the case of a waveguide launcher, the amplitude and phase of the reflection coefficient, ρ, that can be readily observed. For a single, infinitely wide waveguide, this quantity is calculated from Eq. 1. The comparison between the reflection coefficients obtained using the two different surface admittances is also shown in the rightmost panel of Fig. 3, in which the extent in the y-direction of the waveguide opening is varied. The integration washes out the already small differences between the two models. The fact that the minimum reflection coefficient is greater than 0.5 (power reflection coefficient $>$ 25%, marginally practical without an impedance-matching network) suggests that the following options might be considered in the search for a launcher which would not require a matching network for a high power experiment: (1) an n_y-selective launcher to capitalize on the significant structure in the surface admittance spectrum, (2) a more complicated choice of the waveguide mode[2], such as TM_{11}, might yield an improved coupling for steep density gradients, (3) O-mode launch might be considered. While the results of the simple theory described here are promising, the evaluation of these possible improvements requires a more complex model. These extensions of the theory are under study in our ongoing work.

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