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Collisionality Effects on Electron Cyclotron Current Drive Efficiency

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Abstract: Lorentz gas model is used to examine collisionality modification of the trapped electron effects on electron cyclotron current drive efficiency. Appreciable collisionality enhancement of the current drive efficiency appears to be possible in the strong trapping cases in present-day current drive experiments.

Reduction of rf current drive efficiency due to trapped electrons in tokamaks is a widely studied subject in the literature [1]. Nevertheless, most theoretical investigation of the effect invoked the bounce average approximation [2]. This approximation is based on the assumption that the effective collision frequency is much smaller than the bounce frequency such that the trapped electrons are allowed to complete the banana orbits at all energies. The low-collisionality assumption neglects the de-trapping process of electrons due to collision and will underestimate current drive efficiency. Collisionality enhancement of current drive efficiency might be small in high temperature reactor-grade tokamak plasmas, but the situation is less clear for present-day experiments, especially in the case of off-axis electron cyclotron current drive (ECCD) experiments [3,4]. In this work, we use a Green's function formulation [5] to study collisionality effects on ECCD efficiency. To gain semi-quantitative understanding of the effects and to simplify numerical work involved, we perform a non-bounce averaged calculation of ECCD efficiency for Lorentz gas model, in which the Coulomb collision operator is simplified by considering pitch-angle scattering only. Results of the numerical calculation clearly indicate the $(\nu_{e*})^{1/2}$ dependence of collisionality enhancement of the current drive efficiency. Using recent DIII–D experimental parameters, we have found that appreciable enhancement of the current drive efficiency appears to be possible in the case of strong trapping.

By using a Green's function formulation to discuss current drive efficiency, we are assuming that the electron distribution function is close enough to the Maxwellian, *i.e.*, $f \sim f_M$, for Coulomb collision operator to be linearized, and that the rf power density is not too high such that interactions between EC waves and electrons can be described by $S_{rf}(f_M)$, where S_{rf} denotes the rf quasilinear diffusion operator. The perturbed distribution function f_1 satisfies the linearized Fokker-Planck equation:

$$v_\parallel \hat{b} \cdot \nabla f_1 - C_e^\ell f_1 = S_{rf}(f_M) \ , \tag{1}$$

The parallel driven current density is given as $j_{\|} = -e \int d\Gamma\, f_1 v_{\|}$ and $j_{\|} / B$ can be shown as a flux surface quantity. Instead of solving Eq. (1) directly, we consider the "adjoint" problem [5]:

$$-v_{\|}\hat{b} \cdot \nabla \chi - C_e^{\ell +} \chi = \frac{v_{\|} B}{\langle B^2 \rangle} \ , \tag{2}$$

where $C_e^{\ell +}$ is the adjoint collision operator and $\langle \ldots \rangle$ denotes the flux surface average. For a given flux surface, χ is a function of energy [$w = (\gamma - 1) mc^2$] magnetic moment ($\mu = mu_\perp^2 / 2B$), $\sigma = \mathrm{sgn}(u_{\|})$, and poloidal angle θ, where $\vec{u} \equiv \vec{p} / m = \gamma \vec{v}$ is momentum per unit mass. The rf driven current density is then given as $j_{\|} = -eB \langle \int d\Gamma \chi S_{\mathrm{rf}}(f_{\mathrm{M}}) \rangle$. We measure the current drive efficiency using the dimensionless quantity [3, 4],

$$\varsigma \equiv \frac{e^3}{\varepsilon_0^3} \left[\frac{n_{\mathrm{e}} \langle j_{\|} \rangle}{2\pi Q} \right] = -\frac{4}{\ell n \Lambda} \left\langle \frac{B}{B_{\max}} \right\rangle \frac{\langle \int d\Gamma \tilde{\chi} S_{\mathrm{rf}}(f_{\mathrm{M}}) \rangle}{\langle \int d\Gamma (w / m v_{\mathrm{e}}^2) S_{\mathrm{rf}}(f_{\mathrm{M}}) \rangle} \ , \tag{3}$$

where Q is the absorbed rf power density, and $\tilde{\chi} = \nu_{\mathrm{e}0} (B_{\max} / v_{\mathrm{e}}) \chi$ with and $\nu_{\mathrm{e}0} = (e^4 n_{\mathrm{e}} \ell n \Lambda) / (4 \pi \varepsilon_0^2 m^2 v_{\mathrm{e}}^3)$. Note that the formulation given above is applicable in any regime of collisionality and in general tokamak geometry. To evaluate ς, we use a simplified quasilinear operator for EC waves [1]:

$$S_{\mathrm{rf}}(f) = \delta(\vec{x} - \vec{x}_{\mathrm{R}}) \Lambda D_0\, \delta\!\left(\omega - k_{\|} v_{\|} - \ell \frac{\omega_{\mathrm{c}}}{\gamma} \right) \Lambda f \ . \tag{4}$$

Here $\vec{x}_{\mathrm{R}}$ is the spatial location of wave deposition; $\Lambda = (\partial / \partial w) + [(k_{\|} / \omega)(\partial / \partial p_{\|})]$ is a differential operator in velocity space; ω and $k_{\|}$ are, respectively, the frequency and parallel wave number of the EC wave; ω_{c} is the local cyclotron frequency and ℓ denotes the cyclotron harmonics; D_0 is the quasilinear diffusion coefficient. In the small gyro-radius limit ($k_\perp \rho \ll 1$), we approximate D_0 by

$$D_0 \approx \left(\frac{\pi e^2 \tilde{E}_-^2}{2} \right) \left(\frac{k_\perp u_\perp}{2 \omega_{\mathrm{c}}} \right)^{2\ell - 2} \frac{u_\perp^2}{\gamma^2} \ , \tag{5}$$

where $\tilde{E}\_$ is right-hand circularly polarized wave electric field. Substituting Eqs. (4) and (5) into Eq. (3), we obtain

$$\varsigma = -\frac{4}{\ell n \Lambda} \left\langle \frac{B}{B_{\max}} \right\rangle \frac{m v_{\mathrm{e}}^3 \langle \int d\gamma (u_\perp)^{2\ell} f_{\mathrm{M}} \Lambda \tilde{\chi} \rangle}{\langle \int d\gamma (u_\perp)^{2\ell} f_{\mathrm{M}} \Lambda \tilde{\chi} \rangle} \ . \tag{6}$$

Here both $u_{\|}$ and $u_\perp$ are regarded as functions of γ through the cyclotron resonance condition, $\gamma \omega - k_{\|} u_{\|} - \ell \omega_{\mathrm{c}} = 0$

To obtain χ by solving Eq. (2) is a three dimensional numerical problem. To simplify the numerical work involved and to gain semi-quantitative understanding of collisionality effects, we consider Lorentz gas model in which electron-electron collisions are neglected at the outset and only the electron-ion pitch-angle scattering is retained, *i.e.*, $C_e f \approx \nu_{ei}(u) L f$, where L is the pitch-angle scattering operator,

$$L = \frac{1}{2} \frac{\partial}{\partial \xi} \left(1 - \xi^2\right) \frac{\partial}{\partial \xi} \ , \tag{7}$$

$\xi \equiv u_\parallel / u$ and $\nu_{ei}(u) = Z_{eff} \nu_{e0} \gamma (v_e / u)^3$. We parameterize the magnetic field using $B = B_0 / h(\theta)$, where B_0 is a constant and $h(\theta)$ is a function of poloidal angle θ. Write $\chi = \chi_c + \chi_t = [v B_0 / \nu_{ei} \langle B^2 \rangle](G_c + G_t)$, where χ_c satisfies the equation, $-C_e^{\ell +} \chi_c = v_\parallel B / \langle B^2 \rangle$, and χ_t can be identified as the trapped electron contribution. In Lorentz gas model, $G_c = \xi / h(\theta)$ and G_t satisfies

$$-\sigma \frac{\partial}{\partial \theta} G_t - \nu \frac{\partial}{\partial \lambda} \left(\lambda |\xi|\right) \frac{\partial}{\partial \lambda} G_t = \alpha \ , \tag{8}$$

where $\sigma = \mathrm{sgn}(u_\parallel)$, $\lambda \equiv (1 - \xi^2) h$, $\nu \equiv (2 r B_0 / B_\theta v) \nu_{ei} \approx (2qR / v) \nu_{ei}$, q the safety factor, and $\alpha \equiv (\partial / \partial / \theta)(|\xi| / h)$. We note that the energy variable w (or u) appears only as a parameter in Eq. (8). In the banana regime, *i.e.,* $\nu \to 0$, analytic solution is possible; we have

$$G \equiv G_c + G_t = \frac{1}{2} \mathrm{sgn}\left(u_\parallel\right) \int_{\lambda}^{\lambda_c} \frac{d\lambda'}{\left\langle \sqrt{(1 - \lambda' / h)} \right\rangle} \ , \tag{9}$$

for $\lambda < \lambda_c \equiv 1 - \delta$, and $G = 0$ for $\lambda > \lambda_c$, where λ_c gives the trapped-passing boundary and δ is the inverse aspect ratio. The leading order corrections to the banana regime results can be obtained using a boundary-layer analysis in the case of $\delta \ll 1$ [6]. The collisionality enhancement to the ECCD efficiency was estimated to be of the order of $(\nu_{e*} \delta)^{1/2}$, where $\nu_{e*} = \sqrt{2} (qR) / (\delta^{3/2} \tau_e v_e)$ with τ_e the Braginskii collision time [7].
To obtain quantitative results for finite aspect ratio, we have solved Eq.(8) numerically using an Alternating Direction Implicit (ADI) scheme based on Hinton-Rosenbluth variational principle [6] for a simple equilibrium model, $B = B_0 / (1 + \delta \cos \theta)$. An example of the numerical results is shown in Fig. 1. We take $\delta = 0.175$, which corresponds to the inverse aspect ratio at half radius of the DIII–D tokamak. The quantity $\partial G(\xi, 0) / \partial \xi$ is shown as a function of ξ at the outboard midplane ($\theta = 0$) for the collisionality parameter $\nu' = 0.001$, 0.01, 0.1, where $\nu' \equiv (qR / \delta^{3/2} v) \nu_{ei}$. Also shown in the figure is the analytic solution of the banana regime. For $\nu' = 0$, the electron response function vanishes in the trapped region of the phase space, *i.e.*, $G = 0$ and $\partial G / \partial \xi = 0$ for $\xi < \xi_c = [2\delta / (1 + \delta)]^{1/2}$; $\partial G / \partial \xi$ is discontinuous at $\xi = \xi_c$. From Fig. 1, it can be seen clearly that collisionality modification of the electron response is localized near the trapped-passing boundary for $\nu' \ll 1$. Making use of the numerical results obtained above and Eq. (6), we perform the energy integration to evaluate the current drive efficiency ζ. We take plasma parameters similar to those of recent off-axis ECCD experiments on DIII–D ($\omega = 2\omega_c$, $n_\parallel = k_\parallel c / \omega = 0.5$, and $T_e = 1.0$ keV) and calculate ζ as a function of ν_{e*} for the wave deposition at outboard midplane. In Fig. 2, we show the normalized current drive efficiency ζ / ζ_c as a function of ν_{e*}, where ζ_c is the current drive efficiency in the limit $\nu_{e*} \gg 1$ evaluated with χ_c using Eq. (6). The discrete

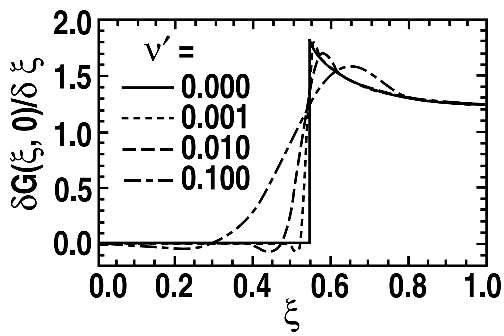

FIGURE 1. $\partial G / \partial \xi$ as a function of ξ at poloidal angle $q = 0$ (outboard midplane) for various ν' at inverse aspect ratio $\delta = 0.175$.

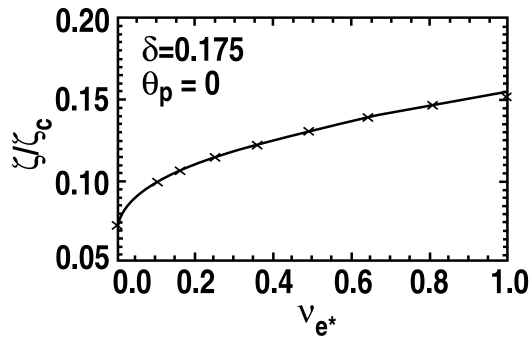

FIGURE 2. Normalized current drive efficiency ζ / ζ_c as a function of ν_{e*} at poloidal angle $\theta = 0$ (outboard midplane) for plasma parameter of $\omega = 2\omega_c$, $n_{\|} = k_{\|} c / \omega = 0.5$, and $T_e = 1.0$ keV; crosses are the numerical results from Lorentz gas model, and the solid line curve fitting with a dependence of $(\nu_{e*})^{1/2}$.

points shown in the figure are results of the numerical calculation and the solid line corresponds to a curve fitting using a dependence of $(\nu_{e*})^{1/2}$. It is interesting to note that the $(\nu_{e*})^{1/2}$ dependence describes the numerical results quite well even for $\nu_{e*} \sim 1.0$. In the parameter range of interest to current experiments, $\nu_{e*} \sim 1.0–0.2$, collisionality enhancement of the current drive efficiency is about 30%–50%.

In summary, we have performed non-bounce averaged calculations of ECCD efficiency using Lorentz gas model to gain semi-quantitative understanding of collisionality effects. Appreciable collisionality enhancement of the current drive efficiency seems to be possible in present off-axis ECCD experimental conditions. Finally, we note that the numerical results obtained here can be used as a benchmark case for more complete three-dimensional calculations using the full Coulomb collision operator.

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