Equations of non-ideal magnetohydrodynamics

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Abstract. In existing derivations[1-4] of fluid equations for strongly magnetized collisional plasmas from kinetic equations, the component of mass velocity perpendicular to the magnetic field is treated as an independent fluid variable with its own equation for temporal variation. This is not in strict accord with the Chapman-Enskog method that forms the basis of these derivations, as the plasma does not relax to a state with arbitrary perpendicular mass velocity in the rapid time scale that includes gyro-motions together with Coulomb collisions. It is shown that this difficulty can be circumvented if the equations for the plasma variables are coupled to the equations for the electromagnetic field, in which displacement current is neglected and quasi-neutrality is assumed. In the hydrodynamic time scale, ideal magnetohydrodynamic equations are obtained. In the next order, the equations include transport fluxes. Although in the latter case, the resulting equations do not differ substantially from those in existing works, they are expressed in forms that more clearly exhibit dynamical consistency. Thus, the generalized Ohm's law is in a form that emphasizes its role for eliminating the electric field from Faraday's law. The electric current density in the $\vec{j} \times \vec{B}$ term of the perpendicular momentum equation is related to the magnetic field through Ampere's law, obviating its determination from the second order kinetic equation. The heat flux depends on electric current density similar to the Peltier effect for metals and semiconductors. In addition to simple plasmas, where comparison with existing works is made, the transport fluxes are also obtained for plasmas with two ion species. The limit of small mass velocity leads to classical transport, wherein the elimination of the electric field from the $\vec{E} \times \vec{B}$ motion causes the local particle flux to depend on boundary conditions.

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