# Toroidal Rotation and 3D Nonlinear Dynamics in the Peeling-Ballooning Theory of ELMs


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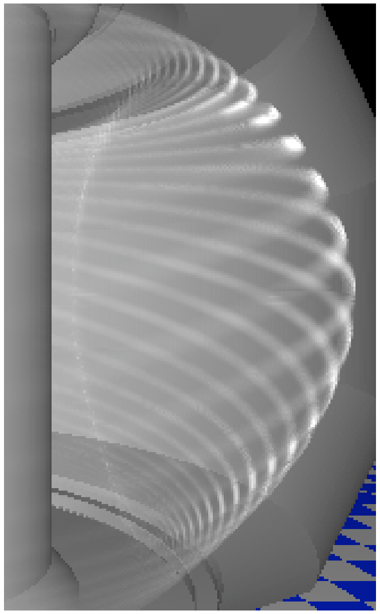
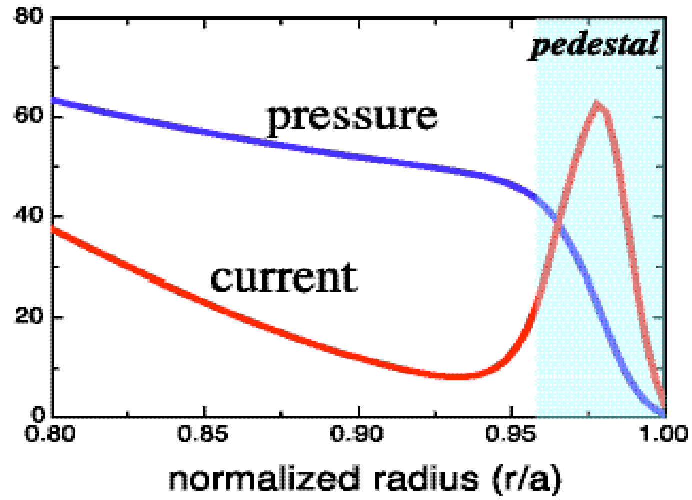


*APS Division of Plasma Physics Meeting*

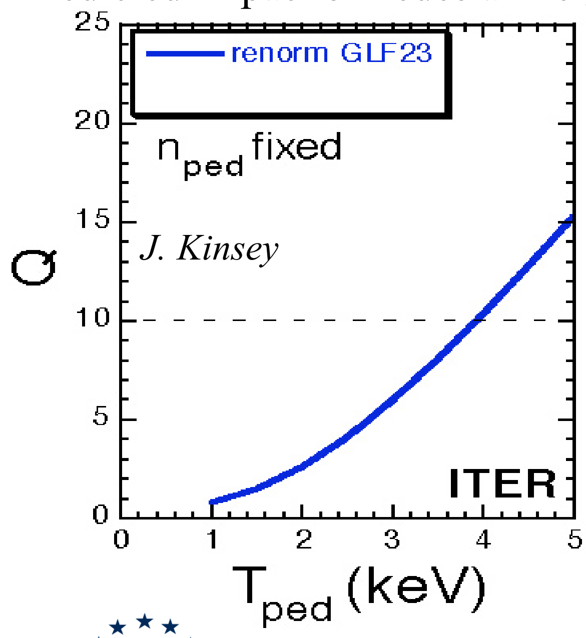
*Savannah, GA, 17 November 2004*

# Motivation and Background

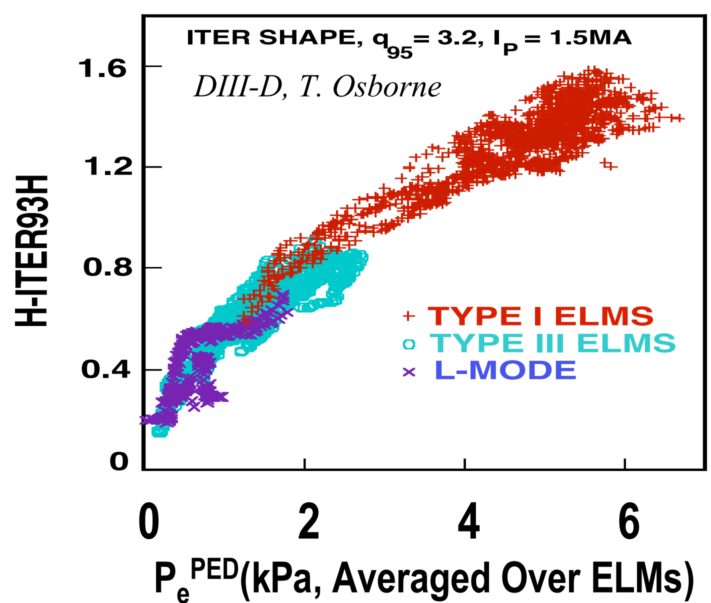
- ELMs and the edge pedestal are key fusion plasma issues
  - "Pedestal Height" controls core confinement and therefore fusion performance (Q)
  - ELM heat pulses impact plasma facing materials




Predicted Impact of Pedestal Height




Observed Impact of Pedestal Height

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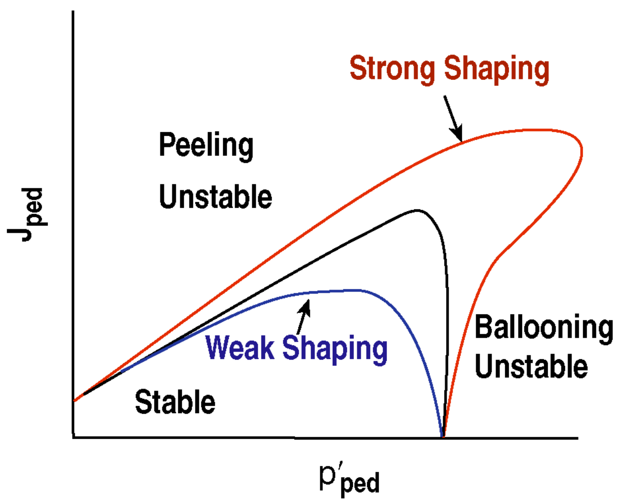
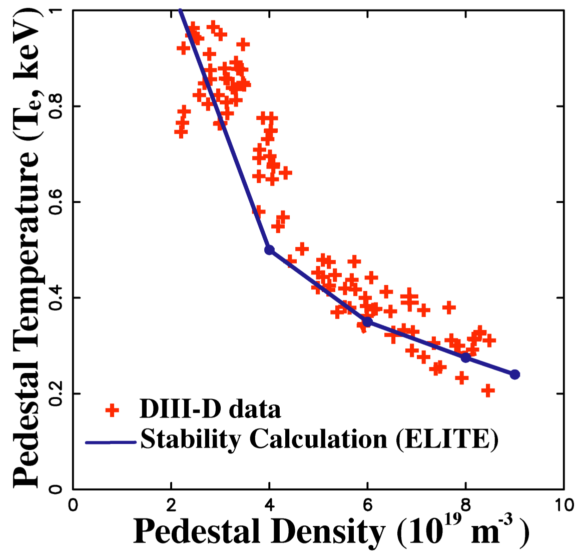
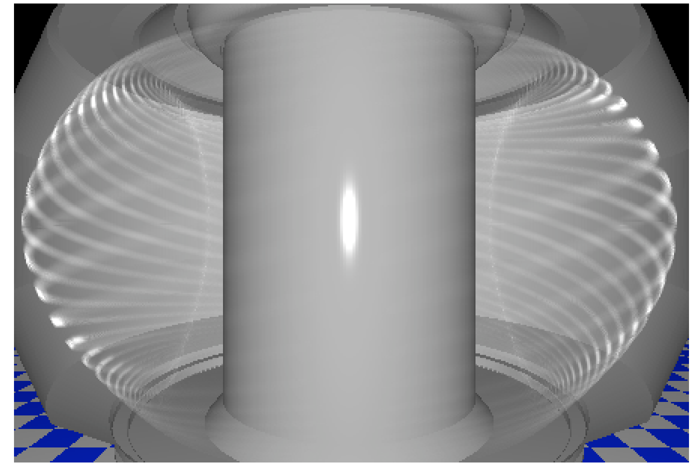
# Background: Extending the Peeling-Ballooning Model



ELITE, n=18 mode structure

- Peeling-Ballooning Model of ELMs - significant successes
  - ELMs caused by intermediate wavelength (n~3-30) MHD instabilities
    - Both current and pressure gradient driven
    - Complex dependencies on $\nu_*$, shape etc due to bootstrap current and “2nd stability”
  - Successful comparisons to experiment both directly and in database studies
- Need to understand sources and transport to get profile shapes (“pedestal width”)
- Rotation and non-ideal effects to precisely characterize P-B limits, nonlinear dynamics for ELM size and heat and particle loading on material surfaces

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# Outline

- Toroidal Flow Shear
  - How toroidal rotation complicates ballooning theory (1D⇒2D)
  - Eigenvalue formulation
  - Impact on peeling-ballooning modes in the tokamak edge region
- Nonlinear ELM Simulations
  - General challenges
  - 2 fluid reduced Braginskii (BOUT) simulation results
    - Expected peeling-ballooning characteristics in linear phase
    - Explosive, radially propagating filaments in nonlinear phase
  - Comparison to Observations
  - Proposals for dynamics of the full ELM crash
- Summary and Future Work

# Ballooning mode theory with rotation

**Static:**

For large $n$ and solutions $\sim e^{\gamma t}$, derive the ballooning equation:

$$L\left(\frac{\partial}{\partial\theta}, q'(\theta-\theta_0), \gamma(\theta_0)\right)\hat{\xi} = 0$$

A 2nd order ODE, 1D eigenvalue problem

Higher order theory⇒**choose $\theta_0$ to maximize $\gamma(\theta_0)$**

**With sheared toroidal flow:** $\mathbf{v} = R^2\Omega(\psi)\nabla\phi$ $\quad R\Omega/C_s \sim n^{-1} << 1 \quad \frac{1}{q'}\frac{\partial(R\Omega/C_s)}{\partial\psi} \sim 1$

Using a time dependent eikonal approach

$$L\left(\frac{\partial}{\partial\theta}, q'(\theta-\theta_0+\Omega' t/q'), \frac{\partial}{\partial t}\right)\hat{\xi} = 0$$

A 2D initial value problem

*Cooper, PPCF **30**, 1805 (1988)*

Low flow shear, separable solution ⇒ **average of $\gamma(\theta_0)$ over $\theta_0$**

$$\gamma = \frac{1}{2\pi}\oint \gamma(\theta_0)\, d\theta_0$$

*Waelbroeck and Chen Phys Fluids B**3** 601 (1991)*

– There is a **discontinuity in the theory**, which we would like to understand

– Suggests that **flow shear could in principle have a big effect on ballooning modes**

# Flow shear and the Eigenmode Formalism

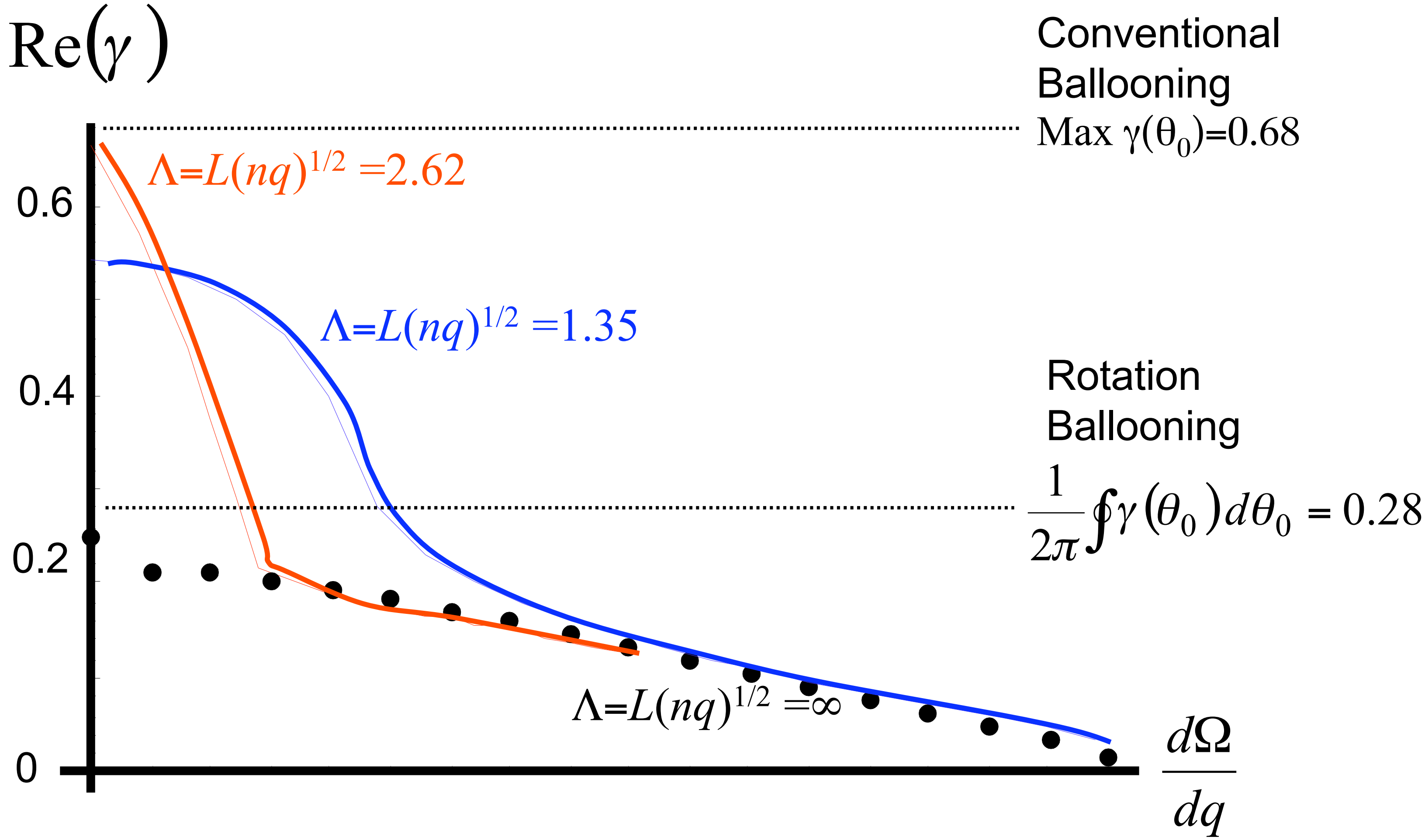
- Would like to develop an eigenmode formalism for the effect of flow shear on ballooning modes:
  - Smoothly connect to the conventional ballooning modes as $\Omega' \to 0$ and understand this 'discontinuity'
  - Calculate the radial eigenmode structure
  - Provides an eigenmode frequency
  - Enables consideration of finite $n$ corrections
  - Permits flow shear to be incorporated into ELITE (an eigenmode code)
    - Evaluate impact on P-B modes in experimental equilibria

- Eigenmode formalism derived and implemented

# Including finite-*n* via eigenmode formulation resolves small rotation discontinuity

*A. Webster et al PRL* ***92*** *(2004) 165004*

*s*- α *geometry: s*=1.0  α=1.7

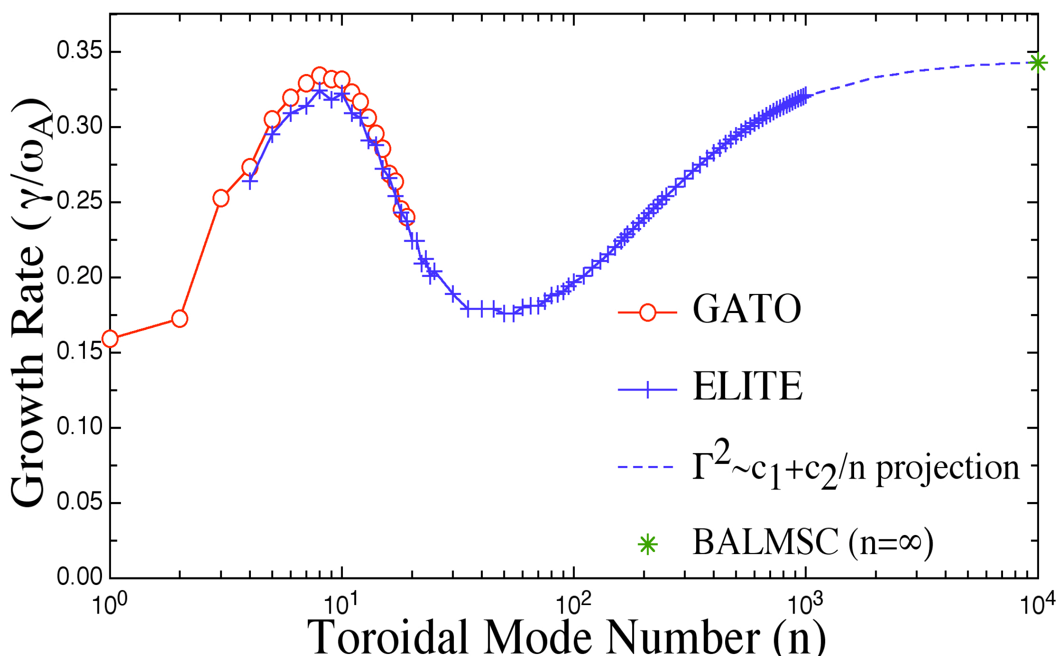


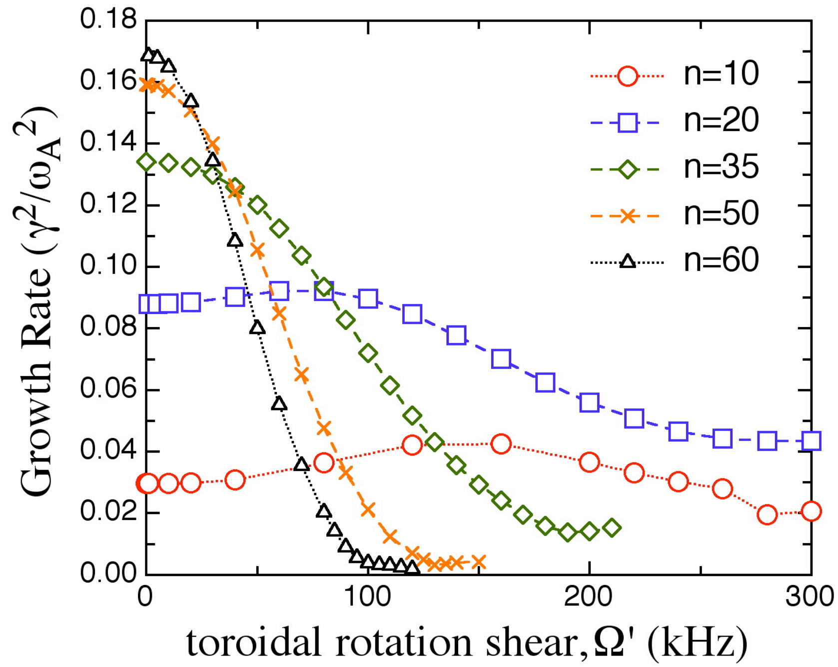

Discontinuity resolved, transition from static ballooning slows with decreasing *n*

# ELITE is a Highly Efficient MHD Stability Code for n>~5

ELITE is a 2D eigenvalue code, based on ideal MHD (amenable to extensions):

- -Generalization of ballooning theory:
  1) incorporate surface terms which drive peeling modes
  2) retain first two orders in 1/*n* (treats intermediate *n*>~5)
- -Makes use of poloidal harmonic localization for efficiency
- -Successfully benchmarked against GATO, MISHKA, MARS, BAL-MSC
- -Code extended to include leading order (nΩ~1, Ω'~1) sheared toroidal flow and compression - results qualitatively similar to s-α

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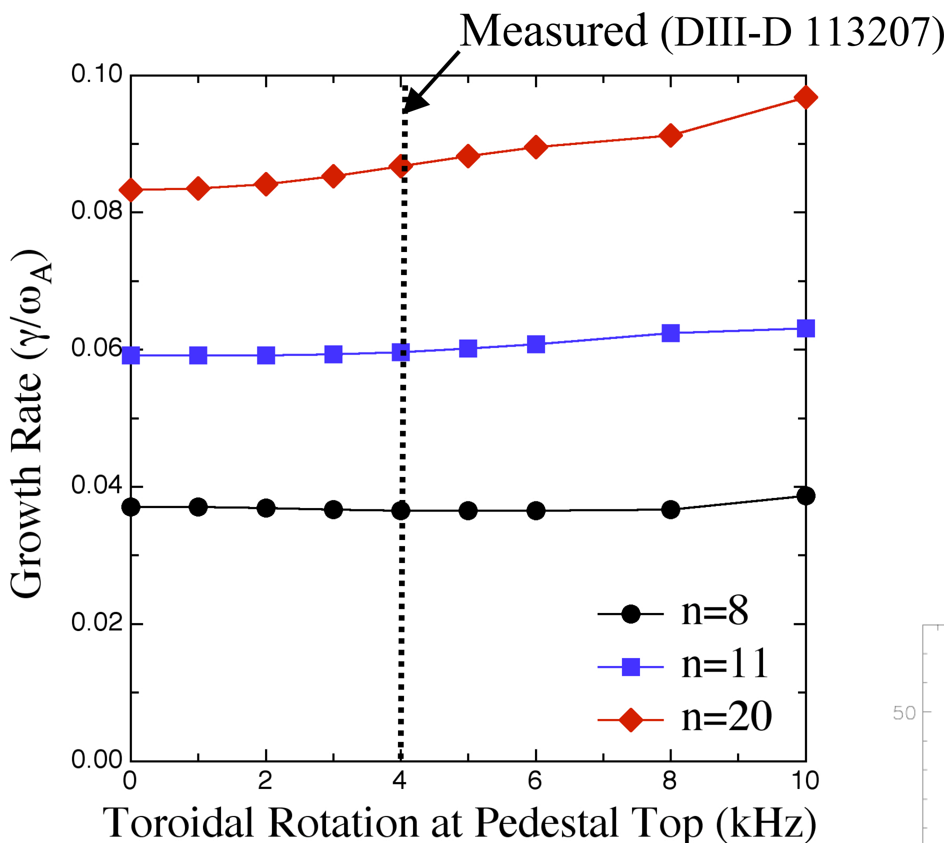
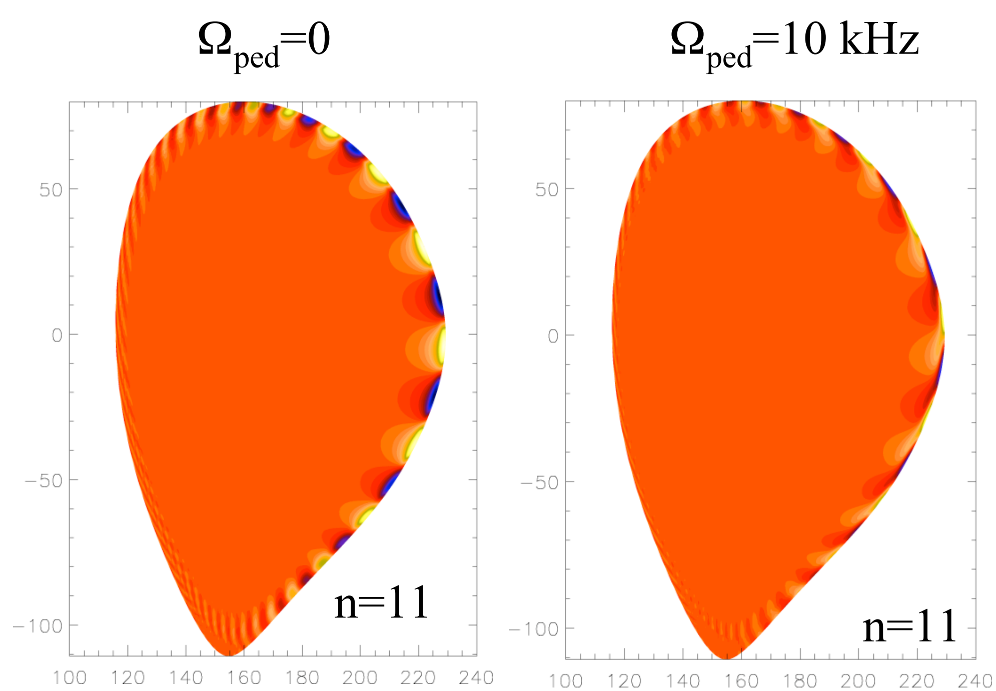
# Flow Shear Effect on Growth Rates is Modest in Standard ELMing Discharges, Mode Structure Does Change



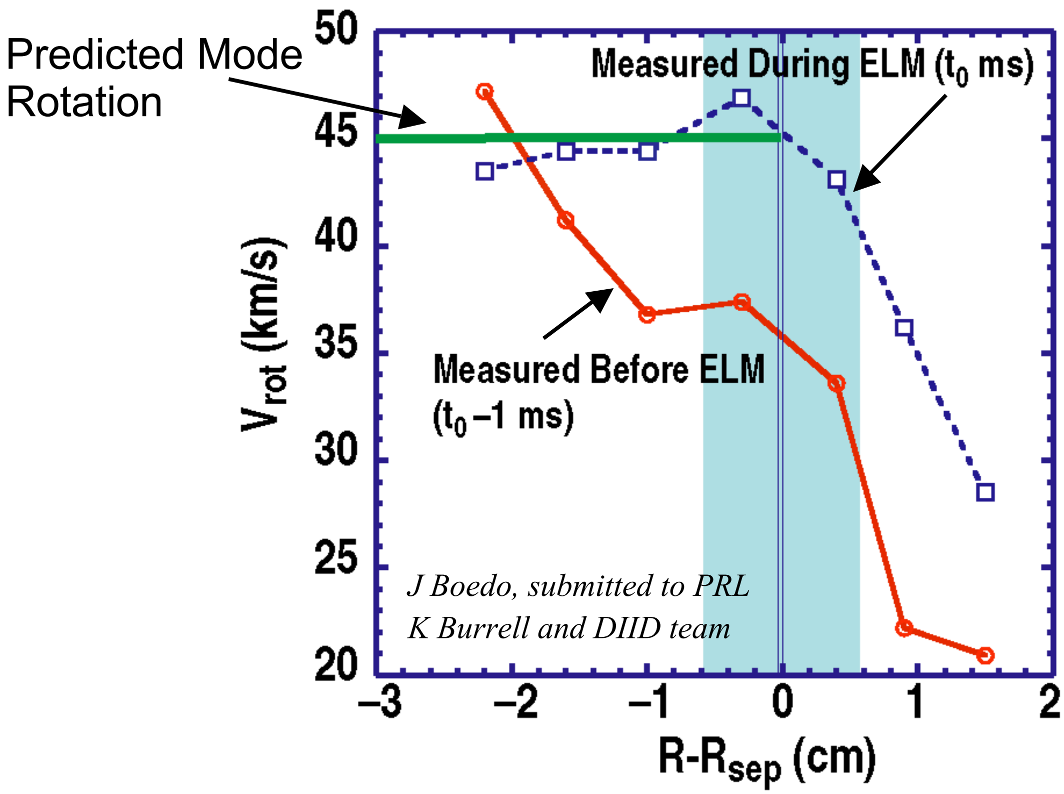

Rotation Shear on P-B Modes:

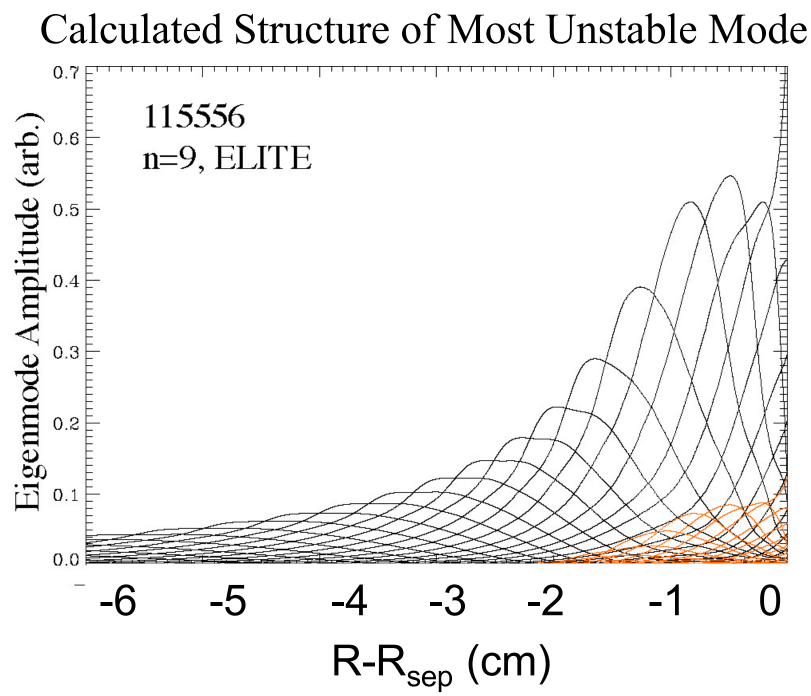
- Stabilizing near marginality
- Finite *n* and large $\gamma$ dramatically reduce effect
- Does not measurably change expected ELM onset time in typical ELMing discharges




- Mode structure strongly altered
  - Narrowing and phase changes
  - May impact dynamics, ELM size

# Calculated Mode Rotation Agrees with Observation during ELM




- Measured rotation profile strongly sheared just before the ELM, becomes ~flat at **~45km/s** across pedestal region at ELM onset
- Study with ELITE finds peeling-ballooning unstable just before ELM - most unstable mode (max $\gamma/\omega_*$) is n=9
- Calculated frequency for this n=9 mode is $\omega/\omega_A$=0.0082, **$V_{rot}$=45km/s**
- Suggests "locking" of pedestal region to the mode during initial phase of ELM crash $\Rightarrow$ edge barrier collapse

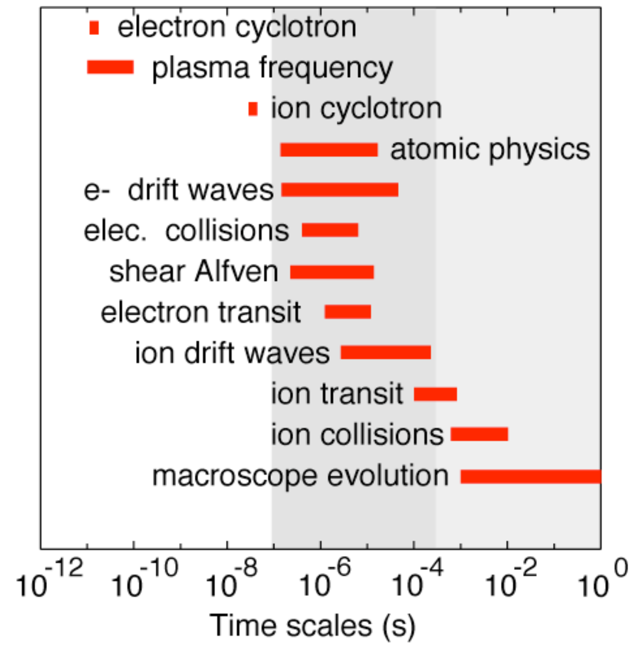
# Summary of Toroidal Flow Shear Impact on P-B Modes

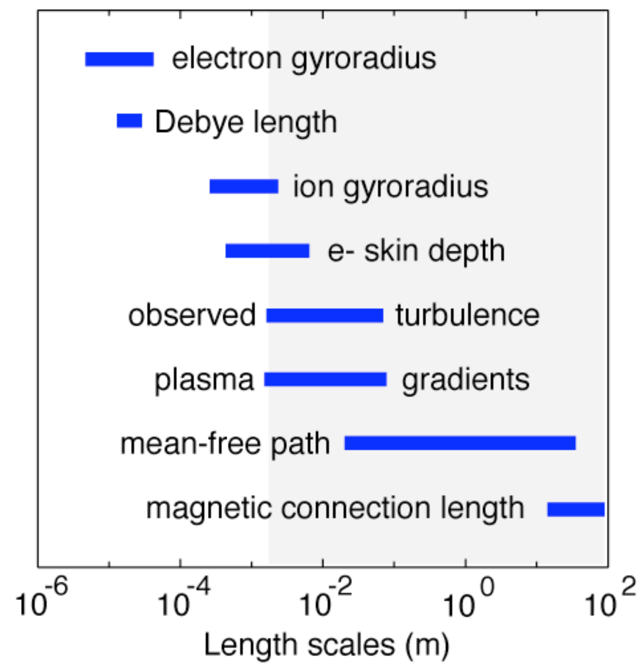
- Toroidal flow shear generally stabilizing at high *n*, effect reduced with decreasing *n*
- For experimental profiles:
  - Stabilization near marginal point, weak effect on growth rate away from marginal point (except for high *n*)
  - Slightly delay ELM onset time, and reduce most unstable n value
  - Effect stronger at low s (high $\Omega$'/q'), *e.g.* where local shear is reduced by high bootstrap current (low $\nu^*$, high pedestal). May play a significant role in QH and "grassy ELM" regimes
- Substantial radial narrowing of eigenmode
- Mode eigenfrequency matches plasma $\Omega$ near top of pedestal
  - Observations suggest "locking" of bulk rotation during early ELM crash

- **Both of the above effects can have important impacts on the dynamics of the ELM crash**

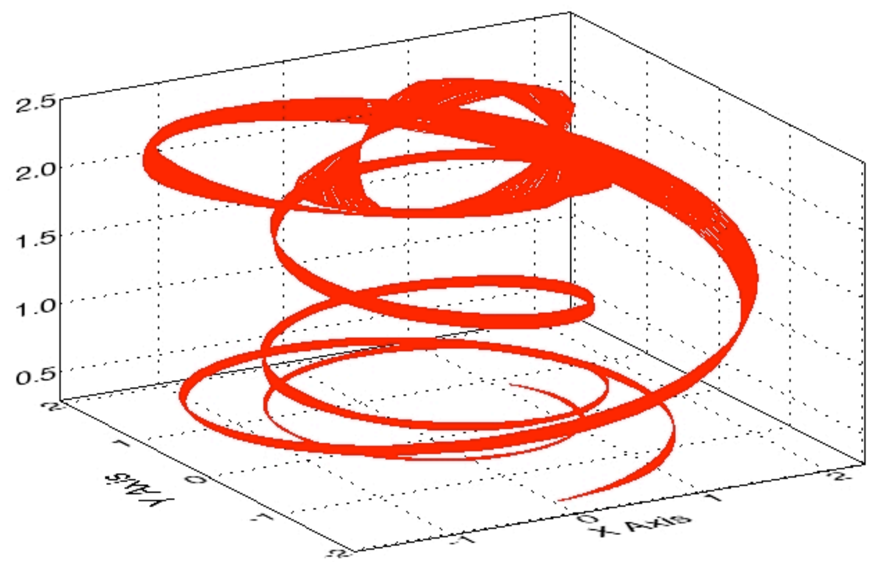
# Nonlinear Edge/Pedestal Simulations

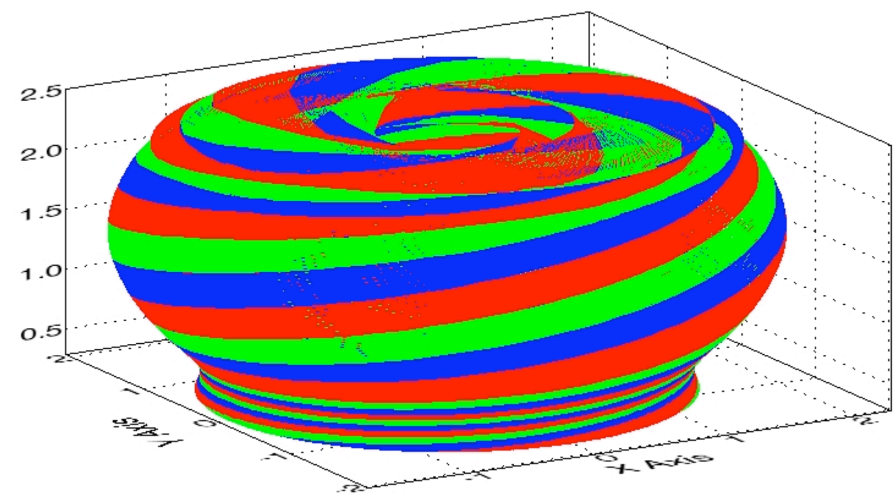
- Many challenges for nonlinear simulations of the edge region
  - Broad range of overlapping scales and physics (L-H transition, sources and transport, ELMs, density limit..)
  - Many techniques used to simplify core simulations not applicable in edge
  - Long term goal is to unite full set of physics into massive scale simulations
- Here we focus on the fast timescales of the ELM crash event itself
  - Goal is to **understand physics determining ELM size and heat deposition**
  - Initialize with P-B unstable equilibria, evolve dynamics on fast timescales
- Reduced Braginskii 2 fluid simulations with the 3D BOUT code *[X Q Xu et al Nucl Fus **42** 21 2002]*



# BOUT Simulation Geometry




- BOUT incorporates 2 fluid/diamagnetic physics and uses field line following coordinates
  - –Bundle of lines (left) wraps around $2\pi$ poloidally
  - –A group of such bundles (right) spans the flux surface
  - –For ELM simulations, generally go 1/5 (or 1/2) of the way around the torus, ie treat $\Delta n=5$ (or $\Delta n=2$), n=0,5,10…~160, 0.9 <~ $\Psi$ ~< 1.1 both closed and open flux surfaces
  - –Equilibrium current (kink term) added for ELM studies

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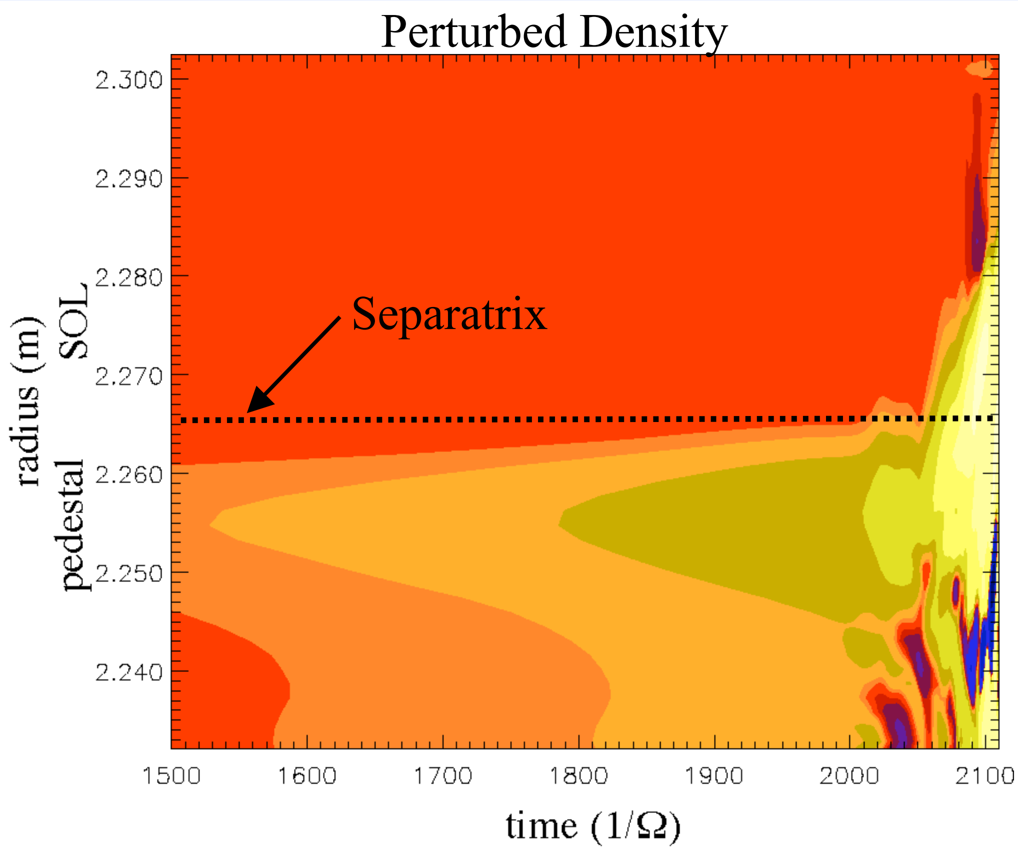
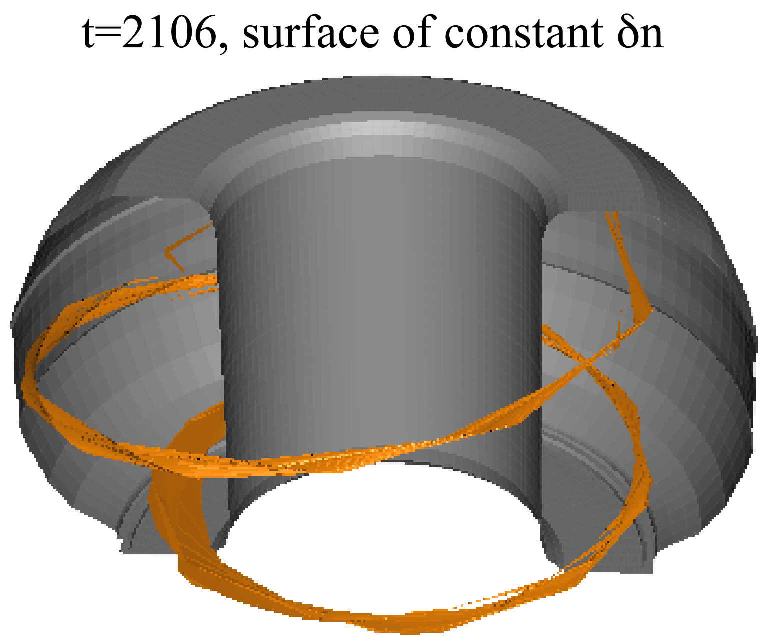
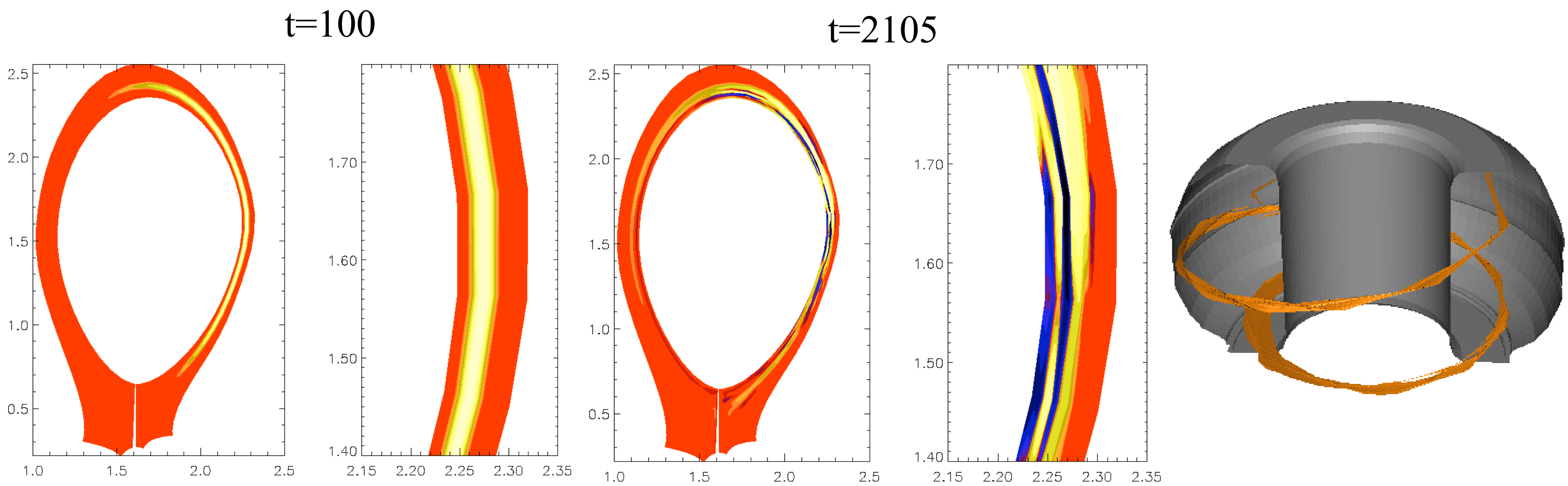
# Fast ELM-like Burst Seen in BOUT Simulations



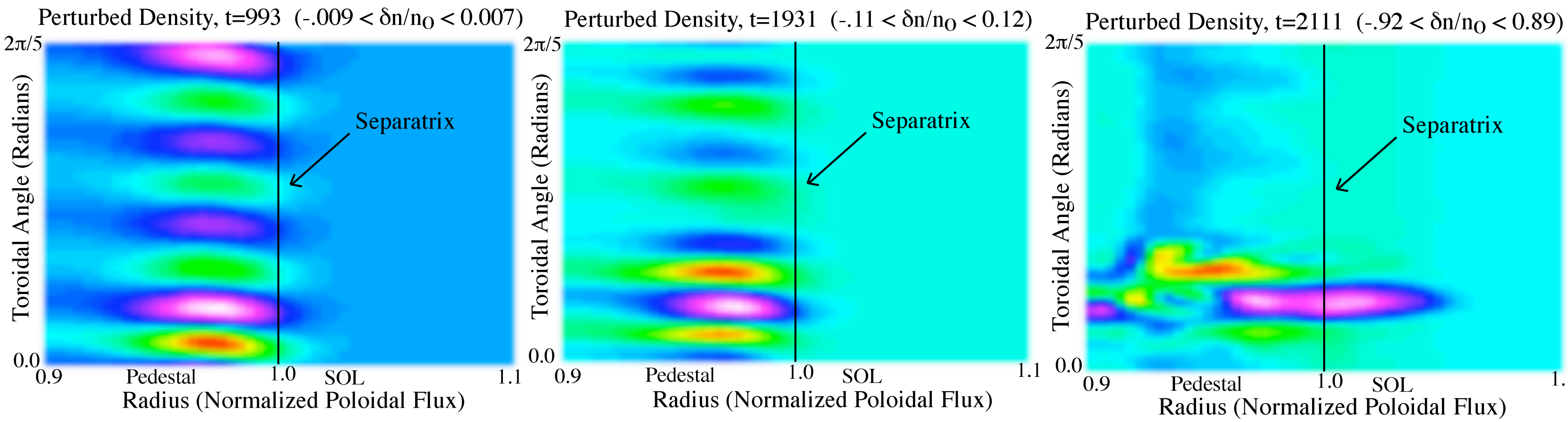

- High density (small ELM), DIII-D LSN case, $0.9 < \psi < 1.1$
- Initial linear growth phase, then fast radial burst begins at t~2000, can see positive density (light) moving into SOL and negative density perturbations near pedestal top
- Radial burst has filamentary structure, extended along B field

# Expected Peeling-Ballooning Character in Early Phase




- Plots show projections of bundles of field lines onto the RZ plane - field lines extend into and out of page (radial vs parallel)
- Linear phase: Mode has ~expected characteristics of linear mode, radial and poloidal extent, n~20, $\gamma/\omega_A$~0.15
    - Reducing gradients slightly stabilizes the mode- abrupt onset near P-B boundary
- Fast Burst: Filaments extended along the field, but irregular

# Fast ELM Burst Shows Toroidal Localization, “finger”




- R,ϕ plots on outer midplane
- Linear phase, n=20. Burst occurs asymmetrically at a particular toroidal location.
- Burst location is point of maximum resonance between dominant linear mode (n=20) and dominant nonlinearly driven beat wave
- “Finger” is an extended filament along the field, which propagates rapidly into the open field line region

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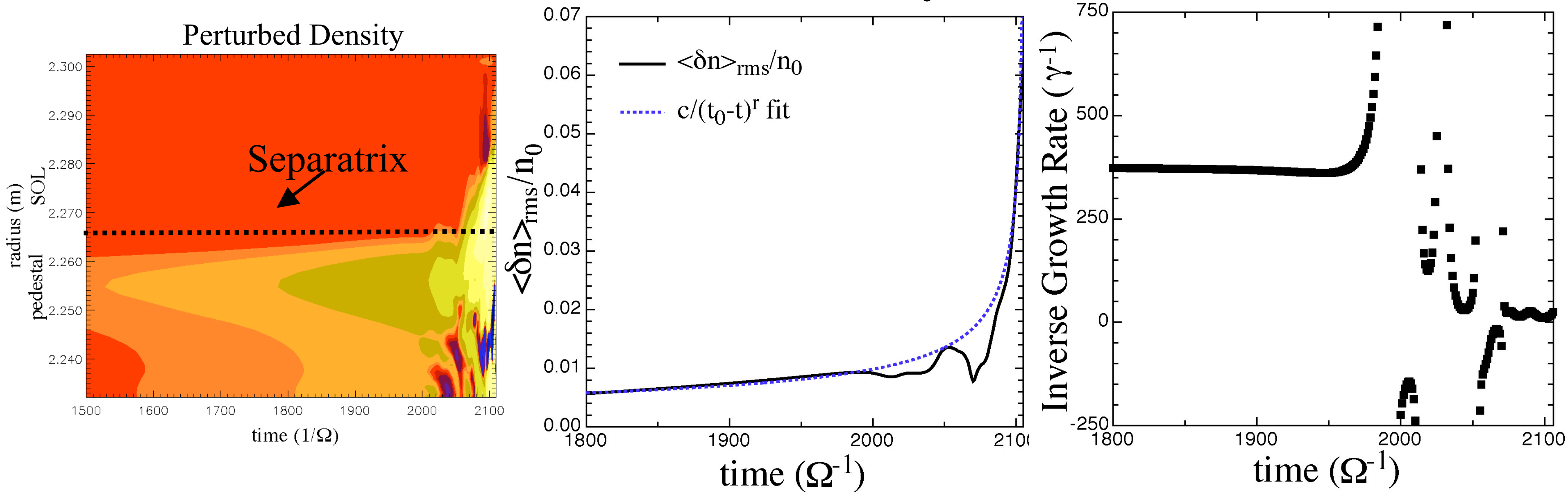
# Similarities to Nonlinear Ballooning Theory

- Nonlinear ideal ballooning theory *[Cowley and Wilson PRL **92** (2004) 175006]* predicts explosive growth of a number of filaments
  - Nonlinear terms weaken field-line bending, accelerating mode growth
  - In nonlinear regime, perturbation grows like $\sim 1/(t_0-t)^r$




- Perturbed density in nonlinear simulations grows like $\sim 1/(t_0-t)^{0.5}$ (theory r~1.1)
- Growth rate increases with time, increases rapidly during burst
  - Significant complexity, characteristic lull in growth rate prior to radial burst
  - Possible association with symmetry-breaking event when $\delta n \sim n_{0loc}$

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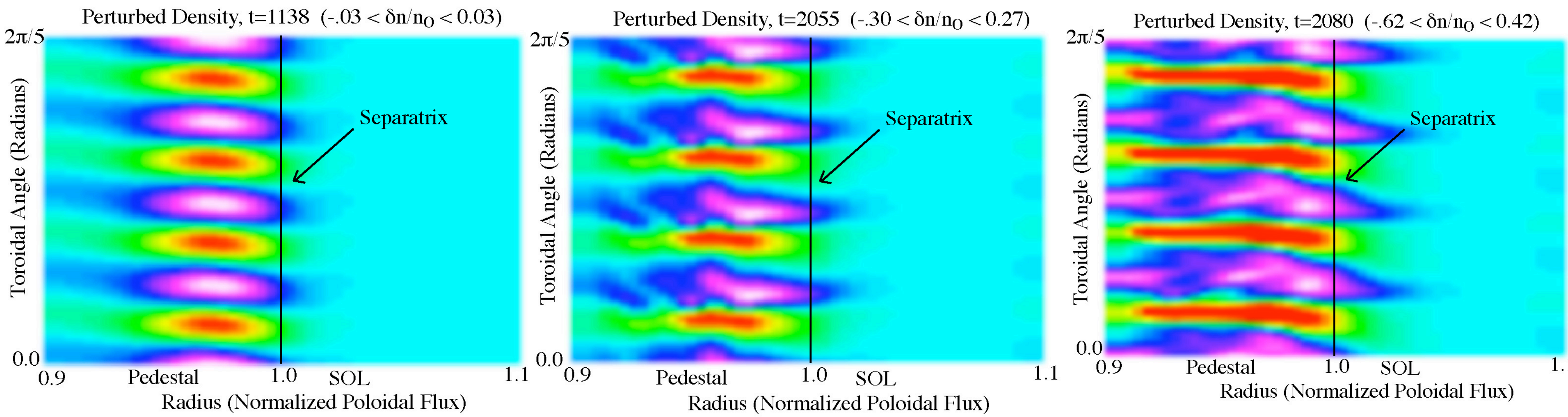
# One Filament or Many?




- Same case, initialized with a pure n=20 mode
  - Largely eliminates nearest neighbor coupling to generate beat wave
- Remains dominated by harmonics (n=0,20,40,60,80) well into nonlinear phase
- Burst occurs fairly symmetrically, multiple propagating filaments
- Evidence of secondary instability breaking up filaments

Both single and multiple filament cases are possible. Dependence on flatness of $\gamma$ spectrum and rate at which profiles are driven across the marginal point.

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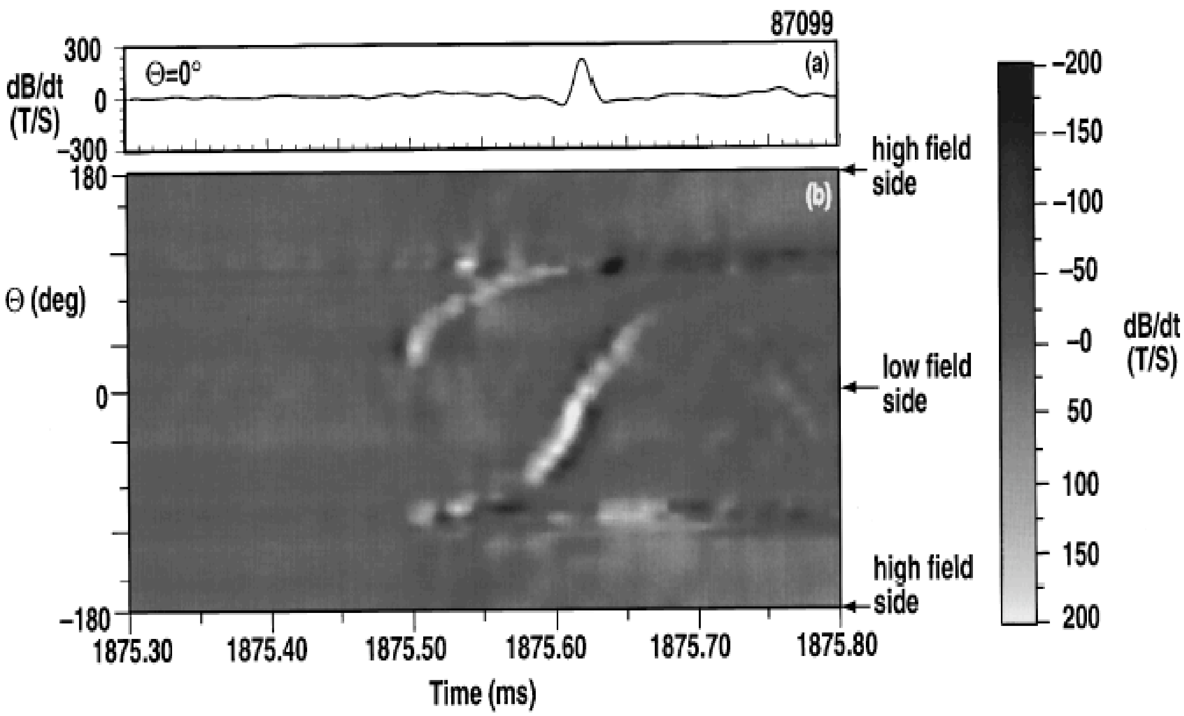
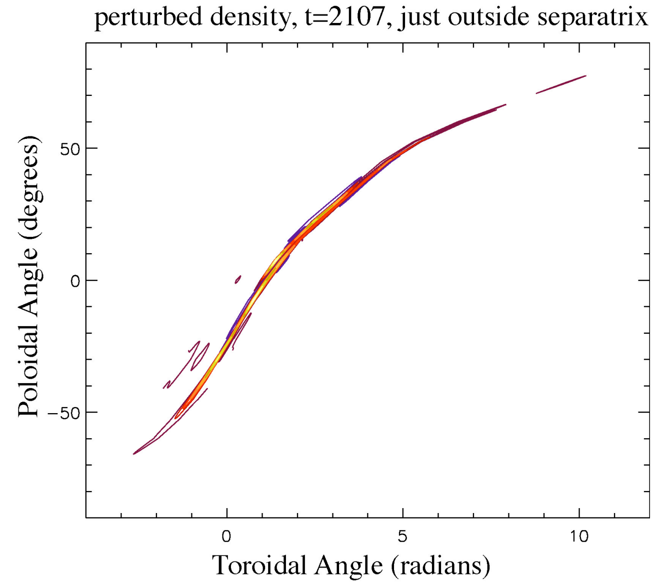
# Filaments Observed During ELMs




**DIII-D Observation** ***[E Strait, Phys Plas 1997]***




**3D Simulation**

- Filament observed in fast magnetics during ELM (left)
- Finger-like structure from simulation (right) is extended along the magnetic field
- Qualitatively similar (rotation rate consistent with toroidal extent)

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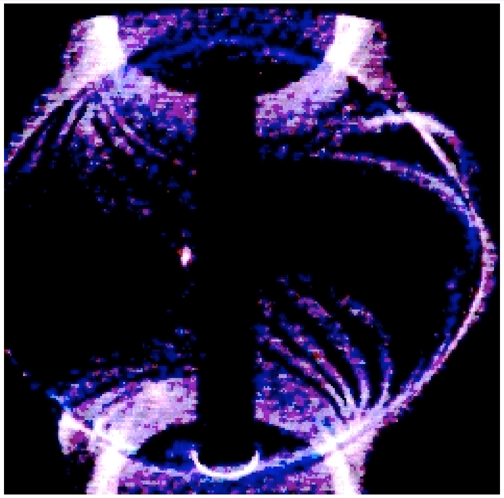
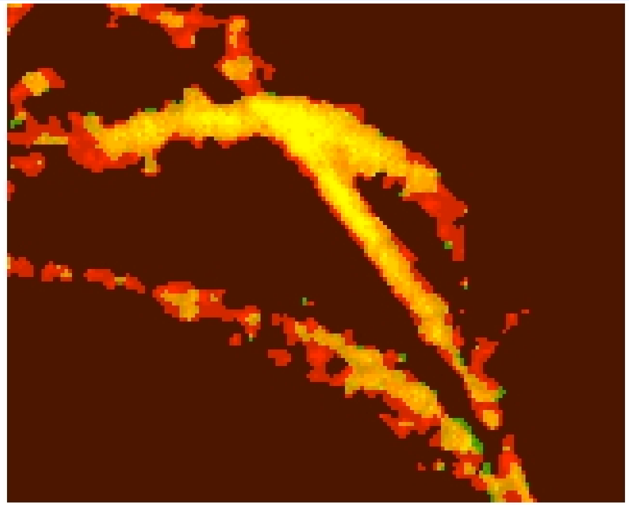
# Fast ELM Observations: Multiple Machines

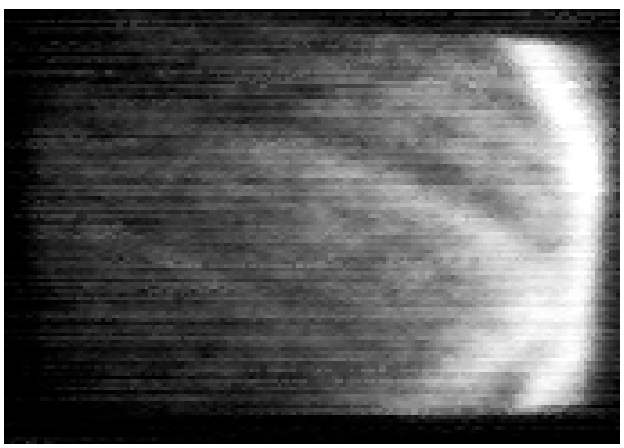


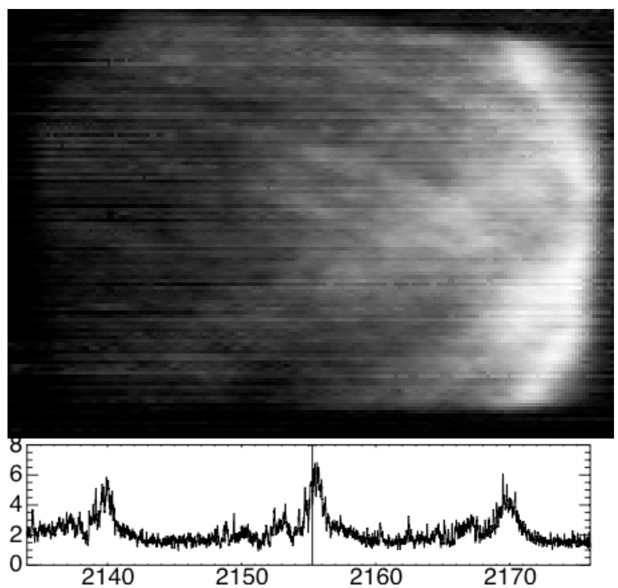


*A. Kirk, MAST, PRL 92 (2004) 245002-1*

- n=10 structure on outboard side
- Filaments moving radially outward

*M. Fenstermacher, DIII-D, IAEA 2004*






- CIII images from fast camera on DIII-D
- n~18 inferred from filament spacing

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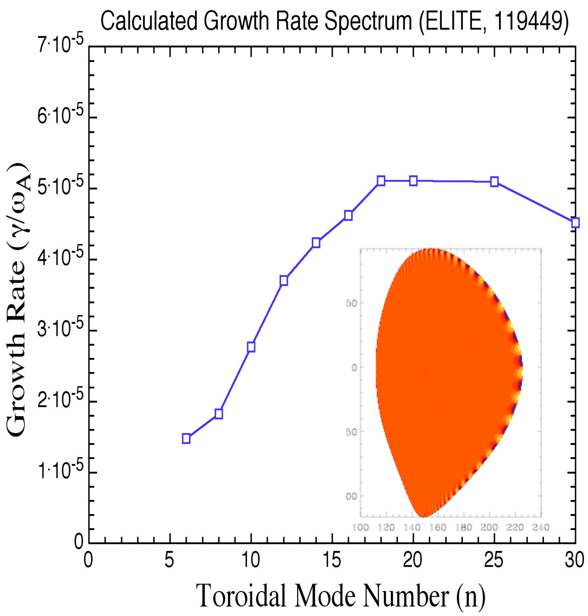
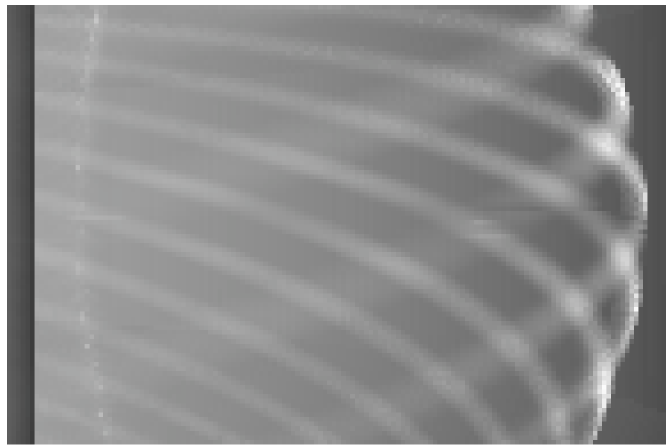
# Fast CIII Images on DIII-D show filamentary structure

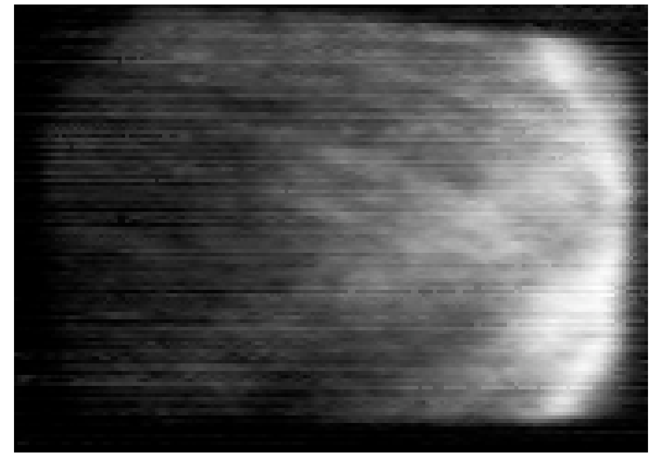



ELITE, n=18



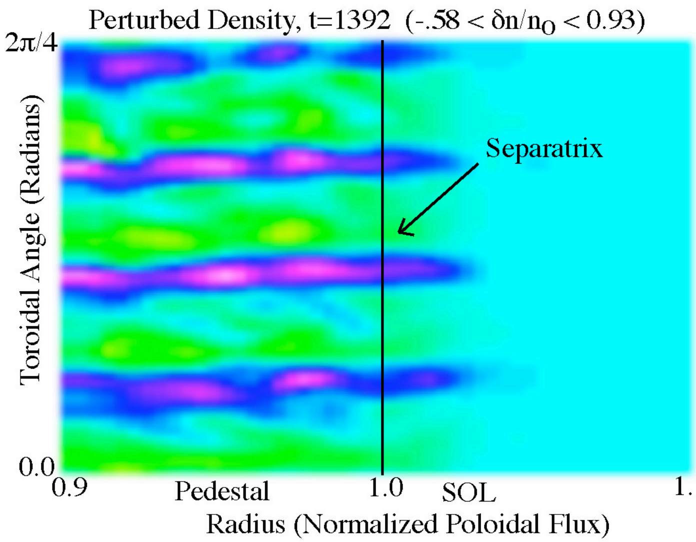
Fast CIII Image, DIII-D 119449

*M. Fenstermacher, DIII-D/LLNL*



- **ELITE linear P-B calculation on kinetic equilibrium shows peak 15 ≤ n ≤ 25; mode in this range predicted to be first to go unstable**
- **Calculated structure of n = 18 mode similar to images**
  - **Poloidal structure similar to outer midplane SOL structure in images**
  - **3D structure has similar m/n structure seen in images**
- **Nonlinear simulations show symmetric structure in early phase, extended uneven filaments later**

BOUT, nonlinear burst phase

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# Proposals for ELM Particle and Energy Losses

- Radially propagating filaments (one or many) carry only a small fraction of energy lost during an ELM
- Two possible mechanisms for the full ELM loss:

  1) **Conduits:** Heat and particles flow along filaments while ends remain connected to hot core. Fast diffusion/secondary instabilities allow flow across filament to open flux SOL plasma.

  2) **Barrier Collapse:** Radial eruption of filament (with fixed eigenfrequency) damps sheared rotation as it moves outward, collapses sheared Er and edge transport barrier. Temporary return to L-mode-like transport+. Reduced gradients re-stabilize mode, allowing shear and pedestal to be re-estabished. *[$E_r$ well collapse during ELM observed on DIII-D, see Wade CI2A.002]*

- Possible that both mechanisms are active. Collisional restriction of heat flow along filaments may explain transition to convective ELMs at high collisionality.

# Summary

- Peeling-ballooning model has achieved a degree of success in explaining pedestal constraints, ELM onset and a number of ELM characteristics
  - Extend to include rotation and nonlinear, non-ideal dynamics
- Toroidal rotation shear included in ELITE
  - Discontinuity in previous studies removed via eigenmode formulation
  - Small effect on predicted ELM onset, but significant modification of mode structure
    - Real frequency of mode matches plasma rotation near center of mode
  - Encouraging comparisons with fast CER observations, suggests mode damps flow shear
- 3D nonlinear ELM simulations carried out with BOUT
  - Early structure and growth similar to expectations from linear Peeling-Ballooning
  - Radially propagating filamentary structures, grow explosively
    - One or many filaments possible (dependence on spectral shape and heating rate; single filament due to resonance of lin & nonlin modes)
    - similar to observations (eg MAST and DIII-D), and nonlinear ballooning theory
  - Filaments acting as conduits and collapse of the edge barrier provide possible mechanisms for full ELM particle and energy losses

# Future Work

- Extend duration of existing simulations, test proposals for ELM losses, compare to expt
- Move on to larger problems:
  1) Toroidal scales – For some types of ELMs, need full torus (n=1 to $\sim\rho_i$)
  2) Radial scales – extend to wall and further into core
  3) Time scale – Include sources and drive pedestal slowly across P-B boundary
- Scale overlap and close coupling with pedestal formation (L-H) physics, inter-ELM transport and source (including atomic) physics
- Need optimal formulations (collisionless), efficient numerics and large computational resources

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