Toroidal Rotation and 3D Nonlinear Dynamics in the Peeling-Ballooning Theory of ELMs

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Motivation and Background

- ELMs and the edge pedestal are key fusion plasma issues
  - “Pedestal Height” controls core confinement and therefore fusion performance (Q)
  - ELM heat pulses impact plasma facing materials

Predicted Impact of Pedestal Height

Observed Impact of Pedestal Height

- J. Kinsey
- DIII-D, T. Osborne

General Atomics
**Background: Extending the Peeling-Ballooning Model**

- **Peeling-Ballooning Model of ELMs** - significant successes
  - ELMs caused by intermediate wavelength (n~3-30) MHD instabilities
    - Both current and pressure gradient driven
    - Complex dependencies on $\nu^*$, shape etc due to bootstrap current and “2nd stability”
  - Successful comparisons to experiment both directly and in database studies
- **Need to understand sources and transport to get profile shapes** (“pedestal width”)
- **Rotation and non-ideal effects to precisely characterize P-B limits, nonlinear dynamics for ELM size and heat and particle loading on material surfaces**
Outline

• Toroidal Flow Shear
  – How toroidal rotation complicates ballooning theory (1D⇒2D)
  – Eigenvalue formulation
  – Impact on peeling-balloonning modes in the tokamak edge region
• Nonlinear ELM Simulations
  – General challenges
  – 2 fluid reduced Braginskii (BOUT) simulation results
    • Expected peeling-balloonning characteristics in linear phase
    • Explosive, radially propagating filaments in nonlinear phase
  – Comparison to Observations
  – Proposals for dynamics of the full ELM crash
• Summary and Future Work
Ballooning mode theory with rotation

**Static:**

For large $n$ and solutions $\sim e^{nt}$, derive the ballooning equation:

$$L \left( \frac{\partial}{\partial \theta}, q'(\theta - \theta_0), \gamma(\theta_0) \right) \xi = 0$$  
A 2nd order ODE, 1D eigenvalue problem

Higher order theory $\Rightarrow$ choose $\theta_0$ to maximize $\gamma(\theta_0)$

**With sheared toroidal flow:**  
$v = R^2 \Omega(\psi) \nabla \phi$  
$\frac{R\Omega}{C_s} \sim n^{-1} \ll 1$  
$\frac{1}{q'} \frac{\partial (R \Omega / C_s)}{\partial \psi} \sim 1$

Using a time dependent eikonal approach

$$L \left( \frac{\partial}{\partial \theta}, q'(\theta - \theta_0 + \Omega't / q'), \frac{\partial}{\partial t} \right) \xi = 0$$  
A 2D initial value problem

Cooper, PPCF 30, 1805 (1988)

Low flow shear, separable solution $\Rightarrow$ average of $\gamma(\theta_0)$ over $\theta_0$

$$\gamma = \frac{1}{2\pi} \int \gamma(\theta_0) d\theta_0$$  
Waelbroeck and Chen Phys Fluids B3 601 (1991)

—There is a discontinuity in the theory, which we would like to understand
—Suggests that flow shear could in principle have a big effect on ballooning modes

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Working with Europe

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Flow shear and the Eigenmode Formalism

• Would like to develop an eigenmode formalism for the effect of flow shear on ballooning modes:
  — Smoothly connect to the conventional ballooning modes as $\Omega' \rightarrow 0$ and understand this ‘discontinuity’
  — Calculate the radial eigenmode structure
  — Provides an eigenmode frequency
  — Enables consideration of finite $n$ corrections
  — Permits flow shear to be incorporated into ELITE (an eigenmode code)
    • Evaluate impact on P-B modes in experimental equilibria

• Eigenmode formalism derived and implemented
Including finite-$n$ via eigenmode formulation resolves small rotation discontinuity

$s$- $\alpha$ geometry: $s=1.0$ $\alpha=1.7$

\[ \text{Re}(\gamma) \]

\[ \Lambda = L(nq)^{1/2} \]

Conventional Ballooning
Max $\gamma(\theta_0)=0.68$

Rotation Ballooning
\[
\frac{1}{2\pi} \int \gamma(\theta_0) d\theta_0 = 0.28
\]

Discontinuity resolved, transition from static ballooning slows with decreasing $n$
ELITE is a Highly Efficient MHD Stability Code for $n > \sim 5$

ELITE is a 2D eigenvalue code, based on ideal MHD (amenable to extensions):
- Generalization of ballooning theory:
  1) incorporate surface terms which drive peeling modes
  2) retain first two orders in $1/n$ (treats intermediate $n > \sim 5$)
- Makes use of poloidal harmonic localization for efficiency
- Successfully benchmarked against GATO, MISHKA, MARS, BAL-MSC
- Code extended to include leading order ($n\Omega \sim 1$, $\Omega' \sim 1$) sheared toroidal flow and compression - results qualitatively similar to s-α
Flow Shear Effect on Growth Rates is Modest in Standard ELMing Discharges, Mode Structure Does Change

Rotation Shear on P-B Modes:
- Stabilizing near marginality
- Finite $n$ and large $\gamma$ dramatically reduce effect
- Does not measurably change expected ELM onset time in typical ELMing discharges

Measured (DIII-D 113207)

- Mode structure strongly altered
  - Narrowing and phase changes
  - May impact dynamics, ELM size
Calculated Mode Rotation Agrees with Observation during ELM

- Measured rotation profile strongly sheared just before the ELM, becomes ~flat at ~45km/s across pedestal region at ELM onset
- Study with ELITE finds peeling-ballooning unstable just before ELM - most unstable mode (max \( \gamma/\omega_* \)) is n=9
- Calculated frequency for this n=9 mode is \( \omega/\omega_A=0.0082 \), \( V_{\text{rot}}=45\text{km/s} \)
- Suggests “locking” of pedestal region to the mode during initial phase of ELM crash \( \Rightarrow \) edge barrier collapse
Summary of Toroidal Flow Shear Impact on P-B Modes

- Toroidal flow shear generally stabilizing at high $n$, effect reduced with decreasing $n$
- For experimental profiles:
  - Stabilization near marginal point, weak effect on growth rate away from marginal point (except for high $n$)
  - Slightly delay ELM onset time, and reduce most unstable $n$ value
  - Effect stronger at low $s$ (high $\Omega'/q'$), e.g. where local shear is reduced by high bootstrap current (low $\nu^*$, high pedestal). May play a significant role in QH and “grassy ELM” regimes
- Substantial radial narrowing of eigenmode
- Mode eigenfrequency matches plasma $\Omega$ near top of pedestal
  - Observations suggest “locking” of bulk rotation during early ELM crash
- Both of the above effects can have important impacts on the dynamics of the ELM crash
Nonlinear Edge/Pedestal Simulations

• Many challenges for nonlinear simulations of the edge region
  – Broad range of overlapping scales and physics (L-H transition, sources and transport, ELMs, density limit..)
  – Many techniques used to simplify core simulations not applicable in edge
  – Long term goal is to unite full set of physics into massive scale simulations

• Here we focus on the fast timescales of the ELM crash event itself
  – Goal is to understand physics determining ELM size and heat deposition
  – Initialize with P-B unstable equilibria, evolve dynamics on fast timescales

• Reduced Braginskii 2 fluid simulations with the 3D BOUT code [X Q Xu et al Nucl Fus 42 21 2002]
BOUT Simulation Geometry

- BOUT incorporates 2 fluid/diamagnetic physics and uses field line following coordinates
  - Bundle of lines (left) wraps around $2\pi$ poloidally
  - A group of such bundles (right) spans the flux surface
  - For ELM simulations, generally go 1/5 (or 1/2) of the way around the torus, ie treat $\Delta n=5$ (or $\Delta n=2$), $n=0,5,10,\ldots,160$, $0.9 < \Psi < 1.1$ both closed and open flux surfaces
  - Equilibrium current (kink term) added for ELM studies
Fast ELM-like Burst Seen in BOUT Simulations

- High density (small ELM), DIII-D LSN case, $0.9 < \psi < 1.1$
- Initial linear growth phase, then fast radial burst begins at $t \sim 2000$, can see positive density (light) moving into SOL and negative density perturbations near pedestal top
- Radial burst has filamentary structure, extended along B field
Expected Peeling-Ballooning Character in Early Phase

- Plots show projections of bundles of field lines onto the RZ plane - field lines extend into and out of page (radial vs parallel)

- Linear phase: Mode has ~expected characteristics of linear mode, radial and poloidal extent, $n \sim 20$, $\gamma/\omega_A \sim 0.15$
  - Reducing gradients slightly stabilizes the mode- abrupt onset near P-B boundary

- Fast Burst: Filaments extended along the field, but irregular
Fast ELM Burst Shows Toroidal Localization, “finger”

- R,φ plots on outer midplane
- Linear phase, n=20. Burst occurs asymmetrically at a particular toroidal location.
- Burst location is point of maximum resonance between dominant linear mode (n=20) and dominant nonlinearly driven beat wave
- “Finger” is an extended filament along the field, which propagates rapidly into the open field line region
Similarities to Nonlinear Ballooning Theory

  - Nonlinear terms weaken field-line bending, accelerating mode growth
  - In nonlinear regime, perturbation grows like $\sim 1/(t_0-t)^{r}$

- Perturbed density in nonlinear simulations grows like $\sim 1/(t_0-t)^{0.5}$ (theory $r\sim 1.1$)
- Growth rate increases with time, increases rapidly during burst
  - Significant complexity, characteristic lull in growth rate prior to radial burst
  - Possible association with symmetry-breaking event when $\delta n \sim n_{0 loc}$
One Filament or Many?

- Same case, initialized with a pure $n=20$ mode
  - Largely eliminates nearest neighbor coupling to generate beat wave
- Remains dominated by harmonics ($n=0,20,40,60,80$) well into nonlinear phase
- Burst occurs fairly symmetrically, multiple propagating filaments
- Evidence of secondary instability breaking up filaments

Both single and multiple filament cases are possible. Dependence on flatness of $\gamma$ spectrum and rate at which profiles are driven across the marginal point.
Filaments Observed During ELMs

DIII-D Observation  [E Strait, Phys Plas 1997]

- Filament observed in fast magnetics during ELM (left)
- Finger-like structure from simulation (right) is extended along the magnetic field
- Qualitatively similar (rotation rate consistent with toroidal extent)
Fast ELM Observations: Multiple Machines

- n=10 structure on outboard side
- Filaments moving radially outward


M. Fenstermacher, DIII-D, IAEA 2004

- CIII images from fast camera on DIII-D
- n~18 inferred from filament spacing
Fast CIII Images on DIII-D show filamentary structure

- ELITE linear P-B calculation on kinetic equilibrium shows peak $15 \leq n \leq 25$; mode in this range predicted to be first to go unstable
- Calculated structure of $n = 18$ mode similar to images
  - Poloidal structure similar to outer midplane SOL structure in images
  - 3D structure has similar $m/n$ structure seen in images
- Nonlinear simulations show symmetric structure in early phase, extended uneven filaments later
Proposals for ELM Particle and Energy Losses

- Radially propagating filaments (one or many) carry only a small fraction of energy lost during an ELM

- Two possible mechanisms for the full ELM loss:
  1) **Conduits**: Heat and particles flow along filaments while ends remain connected to hot core. Fast diffusion/secondary instabilities allow flow across filament to open flux SOL plasma.

  2) **Barrier Collapse**: Radial eruption of filament (with fixed eigenfrequency) damps sheared rotation as it moves outward, collapses sheared $E_r$ and edge transport barrier. Temporary return to L-mode-like transport+. Reduced gradients re-stabilize mode, allowing shear and pedestal to be re-established. [*$E_r$ well collapse during ELM observed on DIII-D, see Wade CI2A.002*]

- Possible that both mechanisms are active. Collisional restriction of heat flow along filaments may explain transition to convective ELMs at high collisionality.
Summary

• Peeling-ballooning model has achieved a degree of success in explaining pedestal constraints, ELM onset and a number of ELM characteristics
  - Extend to include rotation and nonlinear, non-ideal dynamics

• Toroidal rotation shear included in ELITE
  - Discontinuity in previous studies removed via eigenmode formulation
  - Small effect on predicted ELM onset, but significant modification of mode structure
    • Real frequency of mode matches plasma rotation near center of mode
  - Encouraging comparisons with fast CER observations, suggests mode damps flow shear

• 3D nonlinear ELM simulations carried out with BOUT
  - Early structure and growth similar to expectations from linear Peeling-Ballooning
  - Radially propagating filamentary structures, grow explosively
    • One or many filaments possible (dependence on spectral shape and heating rate; single filament due to resonance of lin & nonlin modes)
    • similar to observations (eg MAST and DIII-D), and nonlinear ballooning theory
  - Filaments acting as conduits and collapse of the edge barrier provide possible mechanisms for full ELM particle and energy losses
Future Work

• Extend duration of existing simulations, test proposals for ELM losses, compare to expt

• Move on to larger problems:
  1) Toroidal scales – For some types of ELMs, need full torus (n=1 to ∼ρ₁)
  2) Radial scales – extend to wall and further into core
  3) Time scale – Include sources and drive pedestal slowly across P-B boundary

• Scale overlap and close coupling with pedestal formation (L-H) physics, inter-ELM transport and source (including atomic) physics

• Need optimal formulations (collisionless), efficient numerics and large computational resources
References