APPLICATION OF UNSTRUCTURED GRID AND ADAPTIVE GRID TECHNIQUES TO 2D AND 3D ELLIPTIC PROBLEMS

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Two techniques to solve 2D and 3D differential boundary problems on an arbitrary unstructured grid and to adapt this grid to satisfy some desirable properties are presented.

To solve the differential boundary problem a family of special nonconformal finite elements was constructed to produce high order accuracy of the numerical solution. Three of the finite element schemes respectively provide approximations of 2D (3D) second order differential operator of order O(h), O(h^2), O(h^3) on an arbitrary grid. A finite support (stencil) for each finite element is a priori unknown and is determined during the solution of the problem. A collocation method, in which the residual function is set equal to zero at each grid point, is used to reduce the original differential problemto a matrix problem.

An adaptation (movement) of the grid points is managed by a general form monitor function which corresponds to a minimum of a special functional. This functional includes three parts which are summed with manager weights. The first functional plays the role of a regulator and gives the ‘‘string analogy’’ for convex domains. The second part controls the movement the grid points in the region where the function gradients are large. The third part is constructed on the basis of the second derivatives tensor and can reduce approximation errors. The monitor function is managed by the weight constants and can provide grid adaptation and a numerical solution on the grid with desirable properties.

Because there is no connection between points except for the initial numbering, each point can be moved (adapted) separately without any connection restrictions. Keeping only the grid point numbering is also convenient from point of view of the grid refinement (or coarsening) procedure.

Both these techniques can be used separately or can be combined to solve a particular problem. For example, an initial problem can be solved on a given unstructured grid without any further grid adaptation or on a given numerical grid with a known function can be adapted to the structure of the function.

Both techniques can be naturally extended to 3D problems. 2D and 3D versions of the code had been developed. Test results of these approaches to solve the 2D plasma equilibrium problem with analytic Solov'jev's equilibria and equilibria with a separatrix are presented. Applications to 2D and 3D Poisson fixed boundary elliptic problems, 2D eigenvalue problem for Laplace operator are also presented. Steady state solutions were also found for the 2D non-linear parabolic heat conduction problem with carbon-like radiation.

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