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Modification of Lower Hybrid Current Drive Due to Non-Thermal Electrons Produced by Large Angle Collisions

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Abstract

It is demonstrated by numerical solution of the Fokker-Planck equation including large angle Coulomb scattering that significant enhancement of the lower hybrid current drive efficiency over quasilinear prediction is possible. This presents an alternate explanation of the long standing spectral gap problem. The mechanism is effective at high density and low temperature, where most non-inductive current drive schemes are least efficient. It potentially can have applications in transformer-assisted start-up in tokamaks and current profile control at the edge of magnetic confinement devices.

Introduction

Lower hybrid (LH) current drive remains one of the most efficient radiofrequency current drive schemes for steady-state tokamak operation and for current profile modification in magnetic confinement devices. Although the theory of lower hybrid current drive is quite well-developed and agreement with experiments is in general good [1], an issue having to do with the discrepancy between the need of low $\omega / k_{\|}$ velocity to justify the prediction of quasilinear theory and the low $n_{\|} = k_{\|}c / \omega$ values of the spectra launched in the experiment has not been resolved with complete satisfaction [2]. Most of the proposed explanations have to do with filling the so-called spectral gap, the range of $n_{\|}$ between the launched $n_{\|}$ and the larger $n_{\|}$ needed to drive the observed rf current according to quasilinear theory. These include scattering of LH waves by low frequency density fluctuations [3], nonlinear mode coupling leading to upward cascading in k-space [4], $n_{\|}$ upshift due to multiple passes and reflections of ray trajectories [5], and anomalous Doppler instabilities of low $\omega / k_{\|}$ electrostatic waves [6]. Because these mechanisms involved some uncertainties in assumptions, or introduction of nonlinear fluctuations difficult to measure, it is still not possible to completely ascertain the validity of these mechanisms. In this paper, another possibility is investigated which does not involve filling the spectral gap with waves. Instead, we take into account large-angle Coulomb scatterings between the energetic lower hybrid electron tail (up to 1 MeV) and the thermal population which results in a ‘‘knock-on’’ source. With plasma parameters typical for normal tokamak operation, the combined interaction of the source, a small Ohmic electric field, and the lower hybrid waves is capable of significantly enhancing the noninductive current, beyond the quasilinear prediction, and beyond even the LH enhanced Dreicer runaway current prediction [7].

The ‘‘knock-on’’ source is a key ingredient in recent studies of secondary electron production by runaway avalanche during tokamak disruptions [8] and disruption mitigation using giant pellet injection [9]. The avalanche depends on the existence of some trace amount of very energetic electrons (MeV and above) in the plasma. These electrons can knock out bulk electrons to super-critical energies (*i.e.*, above the energy where the electric field acceleration balances the collisional drag) through large angle collisions. They become runaways and can positively feedback on the knock out process resulting in an avalanche effect. This process is distinct from the process discussed by Dreicer [10] by the much smaller threshold electric field E_{c} (smaller than the Dreicer field E_{D} by v_{t}^2 / c^2) and the relatively long growth time for the knock-on runaway to build up. A comprehensive, relativistic theory of knock-on avalanche has been developed, and has been validated by Monte-Carlo simulations [11] and Fokker-Planck calculations [12].

The present study adds the lower hybrid waves to the mix of the Ohmic electric field and the ‘‘knock-on’’ source. Even when the minimum phase velocity $\omega / k_{\|}$ of the lower hybrid spectrum is significantly larger than the electron thermal velocity (hence is not expected to produce a large current according to quasilinear theory), the waves can still effectively generate an initial high energy tail with much higher density than any considered in the disruption scenario. One would then expect the growth of the runaway avalanche current to be much more expedient, and should increase to observable magnitude within the duration of present day tokamak operation. In the following, we present a model describing the characteristics of the runaway avalanche current, and the dependence of its amplitude and growth rate on wave and electric field parameters. This model is compared against Fokker-Planck simulations which clearly demonstrate the existence of an enhancement effect. Relevance of this to the spectral gap problem and possible applications of this effect in magnetic confinement devices will be discussed at the conclusion.

Enhanced LH Current by Electric Field and “Knock-On” Avalanche

The Landau-Fokker-Planck collision operator $C(f)$ treats only the dominant small-angle Coulomb scattering contribution. To study ‘‘knock-on’’ effects one has to modify the Fokker-Plank equation by adding a source term S to account for the electrons knocked out from the bulk to large energies through large-angle scattering. Furthermore, at least a weakly relativistic treatment (justified for $T / mc^2 \ll 1$) is required. The following assumptions are used in the derivation of the source. In a two particle collision process, the slower particle from the bulk is taken to have zero initial momentum, while the high energy (runaway) particle has finite momentum only parallel to the electric field, *i.e.*, $\vec{w}_1' = w_1'\hat{z}$ initially. The high energy particle is scattered to momentum $\vec{w}_1$, knocking out the idle electron to momentum $\vec{w}$ as shown in Fig. 1. The relationship of the initial and final momenta can be derived using conservation of momentum and energy, via an intermediate transformation to the center-of-momentum (CM) frame. This relationship is geometrically expressed by the equation of an ellipse

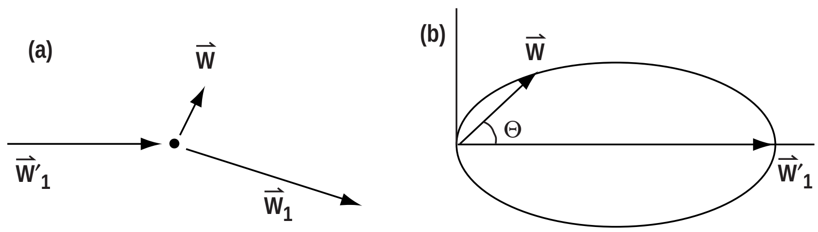

Fig. 1. Kinematics of Coulomb scattering; (a) primary of momentum $\vec{w}_1'$ is scattered by a cold electron to momentum $\vec{w}_1$, knocking out the cold electron to above critical momentum $\vec{w}$; (b) locus of the knock-on secondaries lie on an ellipse in momentum space.

$$\frac{w_\perp^2}{2(\gamma_1'-1)} + \left(\frac{w_\parallel}{\sqrt{\gamma_1'^2-1}} - \frac{1}{2}\right)^2 = \frac{1}{4} \ , \tag{1}$$

where $\gamma^2 = 1 + w^2$, $w = u/c$. In the limit $\gamma_1' \to \infty$, Eq. (1) becomes parabolic: $w_\parallel = w_\perp^2/2$. Using the conservation of particles in the laboratory and CM frame, the source S is related to the Møller cross-section [13] which accounts for large-angle scattering. The complete form is given in Ref. 12. In the limit $\gamma_1' \to \infty$

$$S = 4 n_\mathrm{r}\, c\, n_\mathrm{e}\, r_0^4 \, \frac{\delta(w_\parallel - w_\perp^2/2)}{w_\perp^4} \ , \tag{2}$$

where r_0 is the classical electron radius $= e^2/mc^2$, n_r is the instantaneous runaway density, and n_e is the bulk density. It is useful for analytic evaluation of the growth rates to express

$$S = \frac{4 n_\mathrm{r}}{\tau} \ln(\Lambda) \, \frac{\delta(w_\parallel - w_\perp^2/2)}{w_\perp^4} \ , \tag{3}$$

with $\tau = n_\mathrm{e} 4\pi e^4 m \ln(\Lambda)/(mc)^3$.

The Fokker-Planck equation with the source S has been solved analytically [11] for the runaway avalanche growth rate γ_ra in three limits: weak pitch-angle scattering, strong pitch-angle scattering, and near the critical field $E = E_\mathrm{c}$. A matching formula is obtained to fit all three limits which has been validated against Monte-Carlo and Fokker-Planck calculations. A slightly modified expression obtained from fitting against a Fokker-Planck code is also available [12].

Similarly, the problem of Dreicer runaway enhancement by LH waves has been studied by a number of authors. In Ref. 7, a minimum $\omega / k_{\|} = v_{\min}$ was imposed and an explicit dependence of the modified Dreicer runaway flux Γ_{D} on the wave power and spectrum was derived

$$\Gamma_{\mathrm{D}} \sim \exp\left\{ -\frac{1}{2\alpha^2} - \frac{2}{\alpha} - \frac{v_{\min}^2}{2} \right\} , \tag{4}$$

with $\alpha^2 = E(1 + D_0) / E_{\mathrm{D}}$, $E_{\mathrm{D}} = 2\pi e^3 n_{\mathrm{e}} \ln(\Lambda) / T$, and D_0 and $v_{\min}$ LH quasilinear diffusion coefficient normalized by the collisional diffusion coefficient. The dependence on D_0 is in good agreement with numerical Fokker-Planck calculations.

We are primarily interested in a situation when E is closer to the critical electric field $E_{\mathrm{c}} = (mc / e\tau)$ than E_{D} typical for normal tokamak operation. Since the $n_{\|}$ values of the LH waves are greater than 1 due to the accessibility condition [14], the maximum parallel phase velocity $v_{\max}$ of the LH spectrum is $< c$. Hence the electric field is largely responsible for accelerating the electrons to multi-MeV energies. A first condition is that avalanche will occur only when $(E_{\mathrm{c}} / E)^{1/2} < v_{\max} / c$. To describe the temporal evolution of the runaway density (and equivalently the runaway current J_{ra}), the two processes are combined to give

$$\frac{dn_{\mathrm{r}}}{dt} = \gamma_{\mathrm{ra}} n_{\mathrm{r}} + \Gamma_{\mathrm{D}} \ . \tag{5}$$

Assuming that γ_{ra} and Γ_{D} are time independent, the solution is

$$n_{\mathrm{r}} = n_{\mathrm{r}0} \exp\{\gamma_{\mathrm{ra}} t\} + \frac{\Gamma_{\mathrm{D}}}{\gamma_{\mathrm{ra}}} \exp\{\gamma_{\mathrm{ra}} t\} \left[1 - \exp\{-\gamma_{\mathrm{ra}} t\}\right] , \tag{6}$$

and at large time,

$$n_{\mathrm{ra}} = \frac{\Gamma_{\mathrm{D}}}{\gamma_{\mathrm{ra}}} \exp\{\gamma_{\mathrm{ra}} t\} ,$$

and

$$\frac{d \ln(J_{\mathrm{ra}})}{dt} = \gamma_{\mathrm{ra}} \ .$$

The separation of time scales between the saturation of the Dreicer runaway tail, which takes several collision times, and the avalanche growth time which is much longer, justifies the assumption that Γ_{D} and γ_{ra} is time independent. Also, when E is close to E_{c} only particles with small pitch angles are accelerated by the electric field beyond the LH interaction region. In this case, we expect the growth rate $\gamma_{\rm ra}$ to be only weakly sensitive to the LH waves, and can use previous result for the growth rate [12]

$$\gamma_{\rm ra} = \frac{eE}{mc\ln\Lambda}\sqrt{\hat{E}\left[\hat{E}-1-\frac{0.2(Z+1)}{(Z+5)}\right]\left[\frac{f_{\rm p}}{(Z+5)}\right]\frac{1}{\left[\left(\hat{E}\right)^2+3(Z+1)^2/Z+5\right]}} \ . \tag{7}$$

To evaluate the accuracy of this model and to explore any variation for $E \gg E_{\rm c}$, we proceed to solve the Fokker-Planck equation with the ‘‘knock-on’’ source.

Fokker-Planck Simulations

The CQL3D Fokker-Planck code [15] is a general purpose code for studying the interactions of charged particles via Coulomb collisions and under auxiliary heating conditions to which a ‘‘knock-on’’ source has been added. It solves the bounce-averaged Fokker-Planck equation in velocity space, and can accommodate multiple flux surfaces, general geometry and trapped and passing particles. For the present study, we will neglect particle trapping by considering the magnetic axis of a tokamak. The LH waves are modeled by a quasilinear diffusion operator in momentum space

$$\left.\frac{df}{dt}\right|_{\rm ql} = \frac{d}{d\vec{w}}\cdot\left[\vec{D}_{\rm ql}\cdot\frac{df}{d\vec{w}}\right] \ . \tag{8}$$

For simplicity, we choose a constant $D_{\rm ql}$ The relativistic Landau resonance condition is given by

$$\frac{(n_\parallel^2-1)w_\parallel^2}{c^2}-\frac{w_\perp^2}{c^2}=1 \ . \tag{9}$$

The range of resonance strip ($n_{\parallel\max} > n_{\parallel\min} > 1$) is as shown schematically in Fig. 2. As mentioned, at small pitch angle, particles with $w_\parallel > c\,/\,(n_{\parallel\min}^2-1)^{1/2}$ can only be accelerated by the E field.

For this study, we wish to compare the results with those of previous study [12], hence the plasma parameters are chosen to be the same: $n_{\rm e} = 1\times10^{14}$ cm^{-3}, $T_{\rm e} = 100$ eV. Also, Z is set to be 1, and it is useful to remember $E_{\rm c} = 0.102\,n_{14}$ V/m. A number of fine points are worthy of mentioning. The approximate expression for the source given by Eq. (2) does not depend on the energy of the primary particle which greatly simplifies the analytic solution of the growth rate. This is validated by computing the complete expression which has been incorporated in CQL3D. Also, in deriving the source, it is assumed that the primary electron has zero pitch angle. This is quite well satisfied in the case with Ohmic field only,

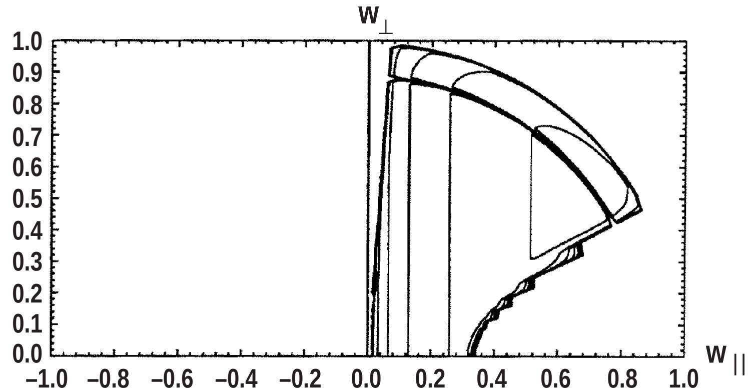

Fig. 2. Contours of the rf diffusion coefficient.

and as shown in Fig 3, still approximately satisfied when lower hybrid is added. The contour plot of the source S is shown in Fig. 4 clearly depicting the parabolic shape described by Eq. (1). The S is somewhat arbitrarily cut-off at low velocity (a few times v_{th}) to avoid double counting small and large angle collisions, but the results are found to be insensitive when this cut-off is varied.

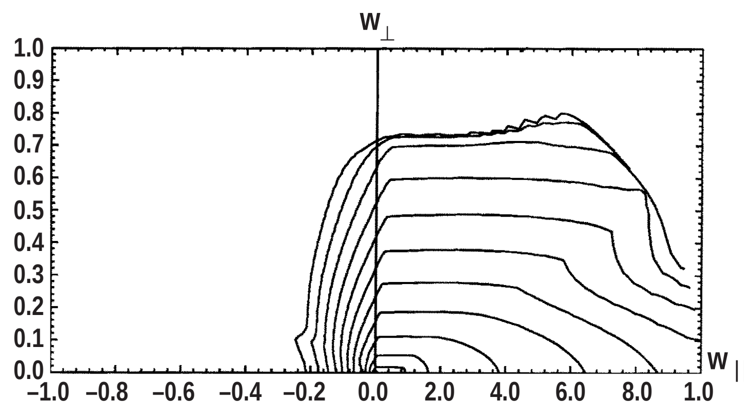

Fig. 3. Distribution function contour plot with LH waves and electric field.

Figure 5 presents the runaway current from simulation under different conditions with $E / E_c = 1.5$. The lowest two curves are with Ohmic field alone, without and with the ‘‘knock-on’’ source. Computationally, one can still distinguish the Dreicer runaway and the avalanche effect, although both are very small compare with the Ohmic current and not observable within this time duration. The middle two curves are with lower hybrid turned on at $D_{ql} / D_{coll} = 5$, $E / E_c = 0$, and with S on and off. Even though the source exists below $v_{min} = v_1$, there is no mechanism to accelerate these particles to the resonant region, and no avalanche takes place. The resulting current is as predicted by the quasilinear theory of lower hybrid current drive. The second from the top curve is with $E / E_c = 1.5$, LH applied but $S = 0$. The current approaches saturation, is significantly larger than with LH only and represents the LH enhanced Dreicer runaway current [7]. The top curve is with E, LH,

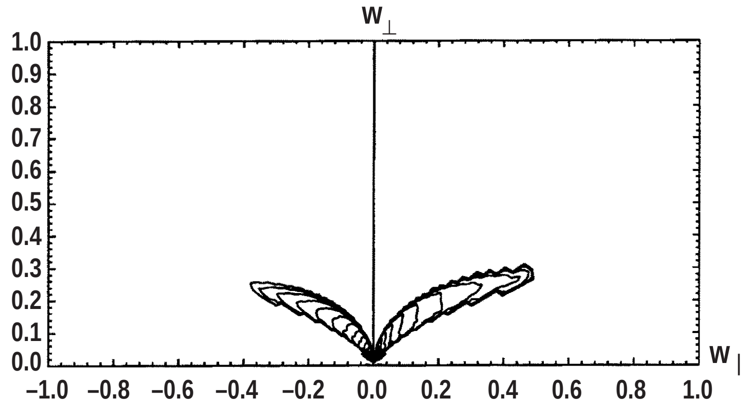

Fig. 4. ''Knock-on'' source contours.

and S all turned on. It shows an exponential growth to orders of magnitude larger than the quasilinear prediction within an interval of two seconds.

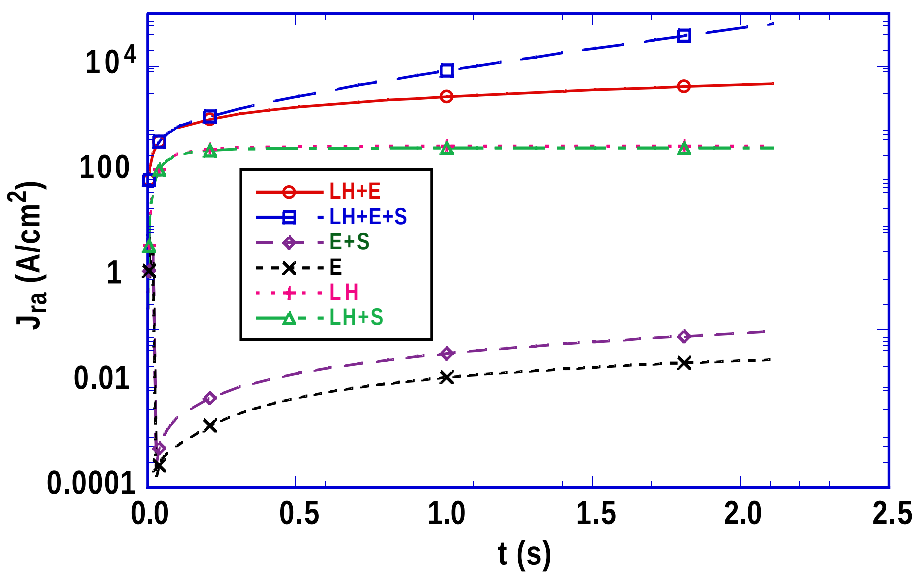

Fig. 5. Runaway currents under different conditions.

To compare with the proposed model, Fig. 6 presents three cases in which the minimum phase velocity of the LH spectrum is varied while keeping E, D_{ql}, and S on as in the top curve of Fig. 5. The curves are from bottom to top $v_1 / v_{th} = 6.5$, 6.0, and 5.0. The relative magnitude of the current is easy to understand as the lower the v_{min} the easier it is for the electric field to accelerate the knocked-out electrons into the resonance region. When the bottom two curves are scaled by the factor $\exp\{-v_1^2 / 2\}$ according to Eq. (4), they lie right on top of the top curve. This is convincing evidence that the amplitude of the runaway current is determined by the LH enhanced Dreicer flux. A similar scaling of the Dreicer flux (with S turned off) is also found.

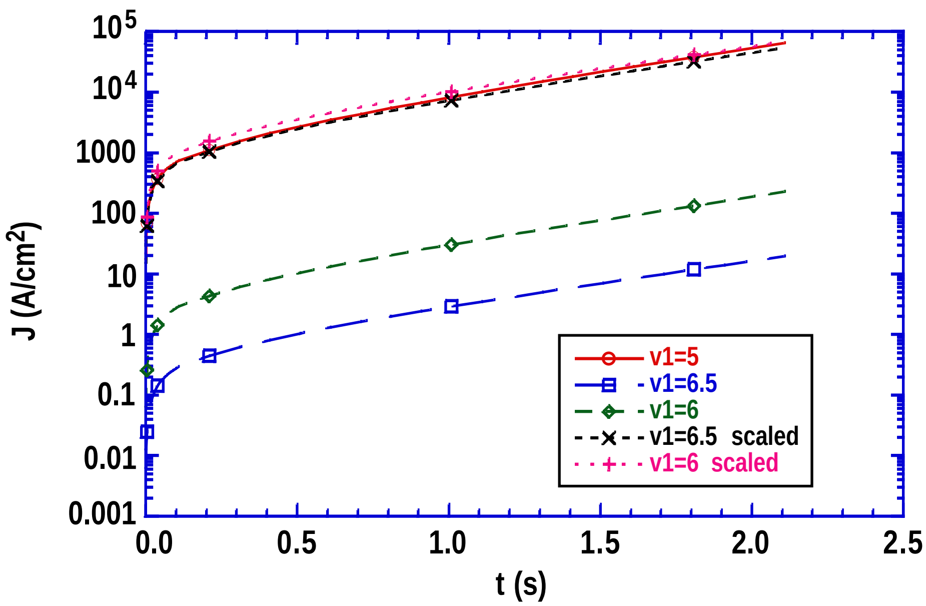

Fig. 6. LH enhanced runaway current (knock-on and Dreicer).

Figure 7 shows the exponential growth rate calculated from $d\ln(J_{\mathrm{ra}})/dt$ for the same three cases in Fig. 6. Two points can be made. The first is that the growth rate does approach a constant value after a short time which justifies the assumption in the proposed model. Secondly, the three growth rates are indistinguishable from each other which suggests that γ_{ra} does not depend on the wave spectrum. To determine whether γ_{ra} depends on the applied LH power, $D_{\mathrm{ql}}/D_{\mathrm{coll}}$ is varied. Three values, 1, 5, and 10 are plotted in Fig. 8 with the top curve corresponding to the highest D_{ql} value. The growth rate shows some increase with LH power even though E is only slightly above threshold, and one would expect the LH influence to be weak. One possible explanation from looking at Fig. 3 is that the perpendicular temperature on the high energy side of the resonance region in momentum space is influenced by the temperature in the resonance region which in turn depends on the wave power. In all the calculations, the Coulomb logarithm, $\ln(\Lambda)$ has been set to a constant in order to compare with previous results. The change in perpendicular temperature could affect this value. Finally, the roll-off at high power is due to the runaway density approaching the bulk density, thus the particle reservoir for producing the secondary electrons is depleted.

The final figure, Fig. 9 shows the result of varying the electric field. The growth rate for three values, $E/E_{\mathrm{c}} = 1.5$, 5, 10 at $D_{\mathrm{ql}}/D_{\mathrm{c}} = 1$ are depicted. As expected, the largest field gives the highest growth rate and takes the shortest time to reach a steady-state. The saturated values are also in qualitative agreement with the formula in Eq. (7).

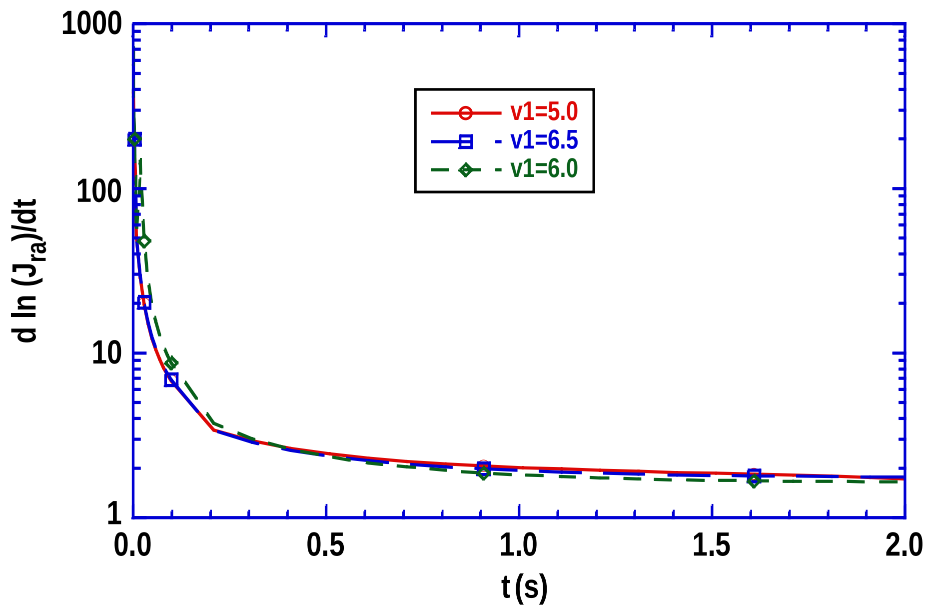

Fig. 7. LH enhanced runaway current growth rate.

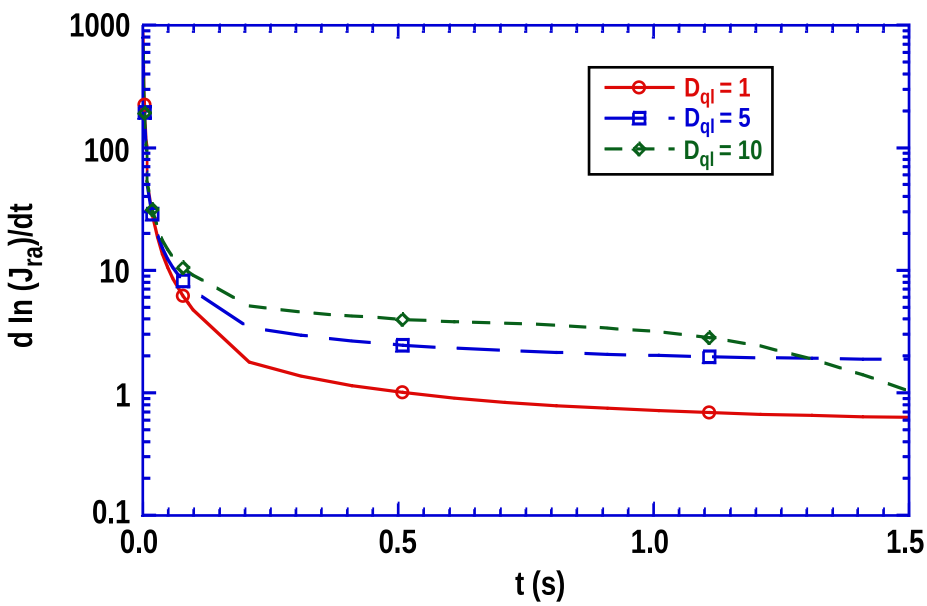

Fig. 8. Dependence of runaway current growth rate on LH power.

Discussion

This study presents the first demonstration that the non-inductive current driven by LH waves in the presence of a small Ohmic electric field can be significantly enhanced by large angle Coulomb scattering. As a comparison, the reduction in resistivity over Spitzer resistivity for LH and electric field only is $\simeq 3\times10^{-3}$ for the conditions in Fig. 5, but with

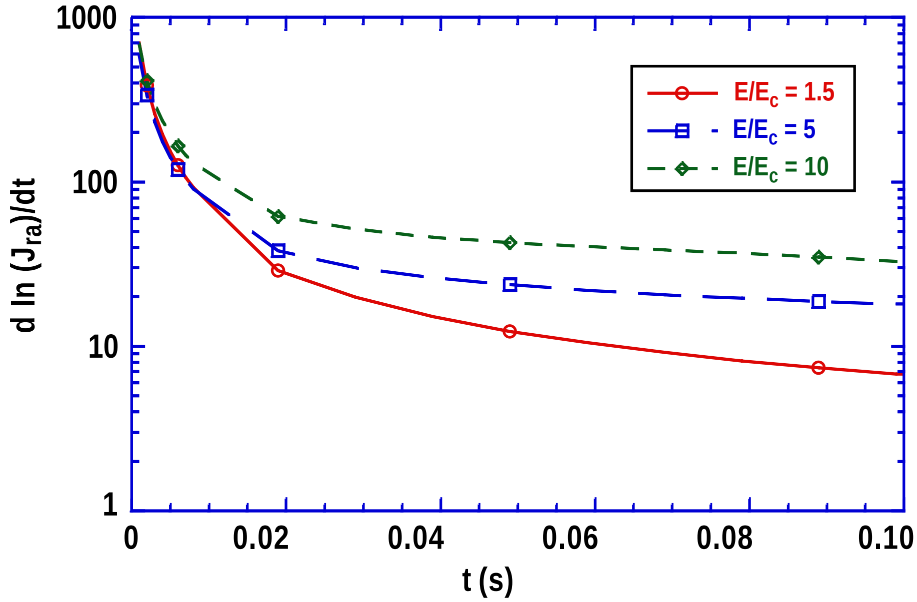

Fig. 9. Runaway current growth rate versus E / E_c.

the ''knock-on'' source added, that value is reduced further to $\simeq 2 \times 10^{-4}$ another order of magnitude difference. The total current is increased by two orders of magnitude over the quasilinear prediction, which could lead to the erroneous interpretation that v_{min} is lowered by some anomalous mechanism to 4 from the actual value of 5, thus raising the spectral gap issue. A way to ascertain the validity of this mechanism will be to examine the details of the inductive electric field, its rate of decay and whether it settles near the threshold value E_c. This mechanism is effective at high density and low temperature, a regime where non-inductive current drive schemes tend to have the lowest efficiencies. Potential applications include transformer-assisted start-up in tokamaks and poloidal current profile control in reverse-field pinches. In both cases, saving valuable inductive volt-seconds is very desirable. Quantitative simulation of these scenarios using CQL3D is in progress.

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