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PROBLEMS IN NONLINEAR RESISTIVE MHD*

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Linear ideal MHD stability has been remarkably successful in describing many of the general features of DIII–D operation, for example disruption and β limits, resistive wall mode onset, and the general features of the observed modes (whether internal, global, or edge localized for example). Nevertheless, there are several instances where linear ideal magnetohydrodynamics (MHD) does not provide an adequate description of observed phenomena. This is becoming increasingly more common as improved diagnostics provide an unprecedented detailed description of the plasma. The success of ideal MHD in reproducing the general MHD observations gives confidence that the lowest order operation is well understood and that the addition of various nonlinear non-ideal effects to theoretical predictions will greatly expand our predictive capabilities.

A number of research areas and specific problems that are likely to further this goal have been identified. The categories are somewhat arbitrary, but are useful in organizing the individual problems. The list is not intended to be all-inclusive but it does reflect the highest priority topics for DIII–D. These are as follows (not necessarily in priority order).

1. β limit disruptions and β crashes
    a. Nonlinear development of ideal and ideal-like MHD modes near the β limit.
    b. Nonlinear coupling and interaction of multiple MHD modes near the β limit.
    c. Development of overlapping islands into full-scale turbulence and relation to disruptions.
2. Sawtooth physics
    a. Simulation of Kadomtsev and Wesson-type reconnection with observed stability thresholds and fast crash times.
    b. Nonlinear coupling of the $m/n = 1/1$ sawtooth mode with higher n (gongs) and development of seed islands outside $q = 1$.
3. Resistive wall physics
    a. Nonlinear evolution of resistive wall mode and its relation to disruptions and β crashes.
    b. Interaction of resistive wall modes with tearing modes and plasma rotation.
    c. Effect of error fields and mode locking.
4. Neoclassical tearing modes
    a. Nonlinear evolution of neoclassical modes, saturation amplitudes, and role of polarization term.

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5. Fast ion modes
    a. Fast particle stabilization of MHD branch and destabilization of non-MHD fast particle driven modes (fishbones, TAE, BAE, KBM, …).
    b. Nonlinear development of fast particle driven modes and fast particle expulsion.

Each of these research topics requires specific nonlinear nonideal effects to be included for an adequate description. Table I shows each of the topics with the physics believed to be crucial given by an asterisk (∗) and that believed to be useful denoted by a check (✓). The first four rows are clearly necessary in all cases for any realistic success at providing a quantitative comparison with observations. Hence the focus is really on the bottom seven rows. This table can be used in two ways. First, if one assumes a code is given with certain physics elements already modeled, one can use the table to search for appropriate research problems to tackle. We have done this and identified two particular problems that will require minimal additional physics beyond the top four rows for a useful comparison to be made. These are shown highlighted and will be discussed in more detail below.

Alternatively, one can use this table from the point of view of a code developer and look for the physics elements that, if added to a given code, would provide the highest leverage in increasing the applicability of the code to the maximum number of problems. The result of this exercise is clear. Addition of plasma rotation (equilibrium and perturbed) would provide the highest leverage followed closely by the addition of vacuum boundary conditions. This, at least, is the highest priority for DIII–D.

From Table I it is clear that some additional physics beyond the first four rows is necessary in order to treat real problems of experimental interest. Two problems, however, can be identified that could be attacked with only the addition of plasma rotation. These are discussed below.

1. *Nonlinear coupling and interaction of multiple MHD modes near the β limit*

The essence of this problem is the nonlinear coupling between MHD modes that are simultaneously linearly unstable near the β limit. A specific example is of interest here, namely the interaction of localized resistive interchange modes and ideal-like global modes [1] in DIII–D L–mode negative central shear (NCS) discharges near $\beta_{\mathrm{N}} \sim 2$. The latter are believed to be responsible for the observed disruptions. However, the localized resistive interchange modes are also usually present and are likely to play a role also. The localized mode appears often as an early MHD burst at around 50 kHz corresponding to the measured rotation in the core of these discharges where $q' < 0$. The final terminating global mode, however, typically has a lower real frequency around 15 kHz near the measured rotation speed of the minimum in q.

The interaction can be direct or indirect. Indirectly, the localized interchange modes have been found to reduce rotation shear [2] which is known to be destabilizing for the global mode. They also affect the other profiles. In some cases, they appear to be directly coupled as they grow and saturate and slow down until their frequency matches that of the global mode, at which point the global mode rapidly grows and results in disruption [3]. Figure 1 shows the observed frequency spectrum for both types of interaction. Clearly, rotation is crucial for describing the

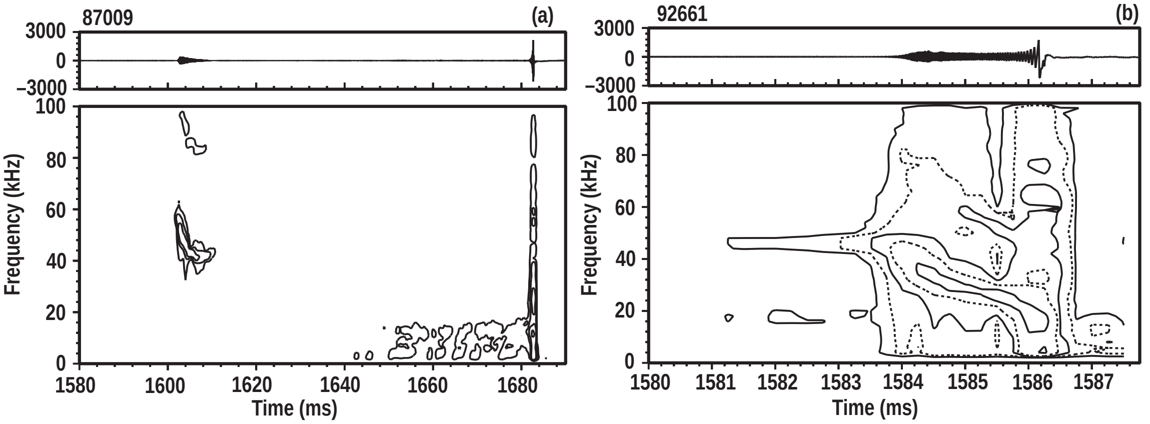

Fig. 1. Coupling between localized resistive interchange mode and global ideal-like mode in L–mode NCS discharges. (a) Indirect coupling: prior localized burst uncoupled but modifies rotation and q profile. (b) Direct coupling: localized mode grows, saturates, and slows to global mode frequency $\Rightarrow$ fast disruption.

interaction. Vacuum boundary conditions would be useful for describing the final disruption but are probably not essential for understanding the essence of the interaction between the modes in the early phase.

2. Nonlinear coupling of the m/n = 1/1 sawtooth mode with higher n gongs and development of seed islands outside q = 1

In DIII–D, sawteeth have been observed to trigger $m/n = 3/2$ seed islands which subsequently grow and saturate at large amplitude [4]. This subsequent nonlinear evolution is well described by the modified Rutherford equation with neoclassical effects. However, a key unresolved problem is the initial $3/2$ island excitation mechanism and its relation to the sawtooth crash. An example is shown in Fig. 2 which shows the $3/2$ mode appearing on the Mirnov diagnostic immediately after the large sawtooth crash [soft x–ray (SXR) signal] at 2275 ms. Nonlinear mode coupling between the predominantly $m/n = 1/1$ kink associated with the sawtooth crash and the $n = 2$ mode is clearly essential to model this correctly. The $1/1$ mode is thought to nonlinearly drive an $n/m = 2/2$ mode (and probably higher n ‘‘gongs’’) at $q = 1$. This should have an $m/n = 3/2$ component as well, simply from linear coupling of poloidal harmonics through toroidicity, noncircular cross-section, and finite β. At the crash, the plasma reconnects at $q = 1$ but leaves the $3/2$ island. Rotation shear is almost certainly important in modeling this process correctly since it is the most likely cause of the decoupling of the $3/2$ island at $q = 1.5$ from the $n/m = 2/2$ gong at $q = 1$. There may also be a lot of additional effects required to reproduce all the details of the sawtooth crash. However, it does not appear to be essential to correctly model all the details of the crash — the dominant process of interest here is the coupling between different toroidal modes, which should be well described by nonlinear MHD codes.

Data from DIII–D [4] also shows a scaling of the initial $3/2$ island size with magnetic Reynolds number S^{-1}. This scaling is difficult to derive from the neoclassical threshold models

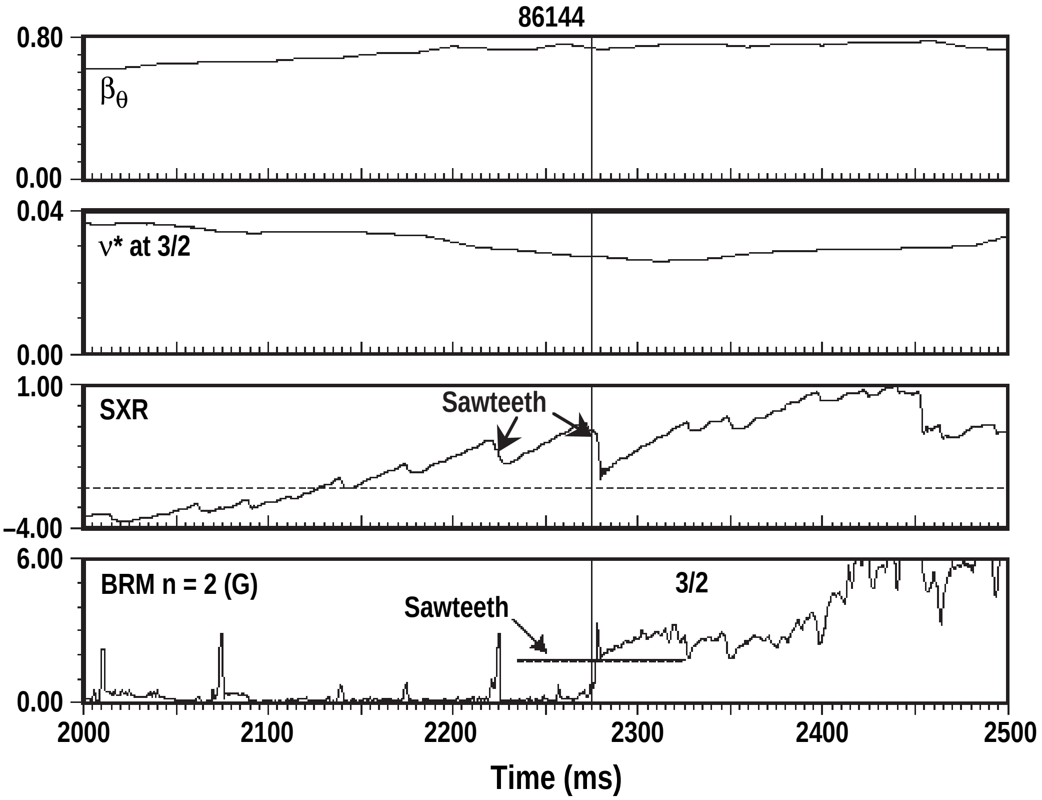

Fig. 2. $3/2$ seed island generated by a $1/1$ sawtooth crash at 2275 ms.

and, instead, seems to be a consequence of the $3/2$ seed island generation. Simulation with a nonlinear MHD code should shed some light on this scaling also.

In conclusion, two experimentally relevant problems can relatively easily be tackled by nonlinear MHD codes. Both problems require plasma rotation in addition to the nonlinear mode coupling and full geometry already incorporated into the codes, but no additional physics seems to be crucial. Addition of plasma rotation to the codes is identified as providing the most leverage for increasing the utility of these codes. Addition of vacuum boundary conditions would also provide high leverage and should be the second highest priority from the point of view of benchmarking the codes against DIII–D.

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