Rotation Damping and ITG Modes

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Recent advances in gyro-fluid simulation of Ion Temperature Gradient modes in tokamaks have shown that the predominant saturation mechanism for the instability is the production of m=n=0 primarily poloidal flows which vary with radius and serve to shear-stabilize the instability. Thus the damping of such poloidal flows is critically important in determining the turbulence level to be expected, and the adequacy of gyro-fluid models for calculating the damping is an issue. We solve kinetically a relevant model problem, and suggest it as a benchmark for gyro-fluid simulations. We calculate the linear collisionless damping of poloidal rotation with particular interest in the level of buildup of such rotation as fed by ITG modes. We find that, after a transient of a few ion transit times, the kernel relating the rotation to the nonlinear source asymptotes to a plateau value which would then slowly damp according to neoclassical collisional damping. This plateau value is compared with gyro-fluid predictions. A higher value would imply a stronger shear-stabilizing effect, and hence a lower level of ITG turbulence.

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1. Introduction

Recent advances in gyro-fluid simulation of Ion Temperature Gradient modes in tokamaks have shown that the predominant saturation mechanism for the instability is the production of radial modes, which are m=n=0 primarily poloidal flows[1,2,3] which vary with radius and serve to shear-stabilize the instability. For example, Fig.5.7 of Ref.1 indicates more than an order of magnitude decrease in turbulent diffusion when the gyrofluid approximation to collisionless damping of the poloidal rotation is turned off. Thus the damping of such poloidal flows is critically important in determining the turbulence level to be expected. The adequacy of gyro-fluid models for calculating the damping is an important issue, especially in view of claims that such ITG turbulence would severely limit confinement in reactor sized tokamaks[4]. In this note, we solve kinetically a relevant model problem, the linear collisionless damping of poloidal rotation with particular interest in the level of buildup of such rotation as fed by ITG modes. Our result is that the poloidal rotation, even if driven by a rapidly fluctuating source, is not damped by collisionless magnetic pumping. This implies a larger level of poloidal flows, a stronger shear-stabilizing effect, and hence a lower level of ITG turbulence than predicted by the gyro-fluid simulations.

2. Gyrokinetic Equation with Source

We will consider the linearized gyrokinetic equation with a "source" representing the nonlinear driving by ITG turbulence. In order to find out what the source looks like, we first derive the gyrokinetic equation and identify the source with the nonlinear terms.

We start with the Vlasov eqn. for electrons and ions:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \frac{e}{m}(-\nabla \phi + \vec{v} \times \vec{B}/c) \cdot \frac{\partial f}{\partial \vec{v}} = 0 \tag{1}$$

with the potential ϕ determined by quasineutrality:

$$\sum_j e_j \int \, d^3 v \; f_j = 0 \tag{2}$$

We write the distribution function as the sum of equilibrium and perturbation:

$$f = F + \delta f \tag{3}$$

where F is a solution of

$$\vec{v} \cdot \nabla F + \Omega \vec{v} \times \hat{b} \cdot \frac{\partial F}{\partial \vec{v}} = 0 \tag{4}$$

with $\Omega = eB/(mc)$, the gyrofrequency.

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The equilibrium F varies slowly perpendicular to $\vec{B}$: $\rho/L_n \ll 1$, where ρ is the gyroradius and L_n is the equilibrium scale length. To lowest order in ρ/L the equilibrium is

$$F = F_0(\vec{x}_\perp - \vec{\rho}, \mathcal{E}) \tag{5}$$

where $\vec{\rho} = \hat{b} \times \vec{v}/\Omega$ with $\hat{b} = \vec{B}/B$ and $\mathcal{E} = (m/2)v^2$. It will be chosen to be a Maxwellian at temperature T.

The potential ϕ is considered small in the sense $e\phi/T \ll 1$ and so the distribution function response, δf is also small: $\delta f/F \ll 1$. The perturbation δf and ϕ vary rapidly perpendicular to $\vec{B}$: $k_\perp \rho \sim 1$. We write the perturbation as

$$\delta f = -\frac{e\phi}{T} F_0 + g \tag{6}$$

which defines the nonadiabatic part g, and then change variables to facilitate removing the fast gyration from the problem: $(\vec{x}, \vec{v}) \to (\vec{R}, v_\perp, v_\|, \alpha)$, where the guiding center position is defined by

$$\vec{R} = \vec{x} - \vec{\rho} = \vec{x} - \frac{\hat{b} \times \vec{v}}{\Omega} \tag{7}$$

with

$$\vec{v} = \hat{b} v_\| + v_\perp(\hat{e}_1 \cos\alpha + \hat{e}_2 \sin\alpha) \tag{8}$$

The function g is expanded in $\epsilon = \rho/L_n \sim \omega/\Omega \sim k_\parallel \rho$: $g = g_0 + g_1 + \cdots$. The leading order equation is $\Omega \partial g_0/\partial\alpha = 0$, while the next order equation yields the constraint equation (the gyrokinetic equation) for g_0 (now called g):

$$\frac{\partial g}{\partial t} + (v_\parallel \hat{b} + \vec{v}_d) \cdot \nabla_{\vec{R}}\, g = \frac{e}{T} F_0 \frac{\partial \langle\phi\rangle_\alpha}{\partial t} - \vec{v}_E \cdot \nabla F_0 + \langle S\rangle_\alpha \tag{9}$$

where $\nabla_{\vec{R}}$ is the gradient taken with respect to $\vec{R}$ and where $\vec{v}_d$ is the guiding center drift velocity:

$$\vec{v}_d = -v_\parallel \hat{b} \times \nabla \left(v_\parallel/\Omega\right) \tag{10}$$

with $v_\parallel = [2(\mathcal{E}/m - \mu B)]^{1/2}$. The independent velocity variables used now are $\mathcal{E} = m(v_\perp^2 + v_\parallel^2)/2$, the energy, $\mu = v_\perp^2/(2B)$, the magnetic moment, and α, the gyroangle. The average over gyroangles holding guiding center position fixed is defined by

$$\langle\phi\rangle_\alpha = \oint \frac{d\alpha}{2\pi}\, \phi(\vec{R} + \vec{\rho}, t) \tag{11}$$

and the averaged ExB drift velocity is

$$\vec{v}_E = \frac{c}{B} \hat{b} \times \nabla_{\vec{R}} \, \langle\phi\rangle_\alpha \tag{12}$$

The nonlinear terms have been included in

$$\langle S\rangle_\alpha = \oint \frac{d\alpha}{2\pi} \frac{e}{m} \nabla\phi(\vec{R} + \vec{\rho}, t) \cdot \left(\frac{\partial}{\partial \vec{v}'} + \frac{\hat{b}}{\Omega} \times \nabla_{\vec{R}} \right) \delta f(\vec{R} + \vec{\rho}, \vec{v}', t) \tag{13}$$

where $\partial/\partial\vec{v}'$ represents the velocity derivatives holding $\vec{R}$ fixed. These velocity derivatives will be omitted because they do not affect the density moment, holding $\vec{R}$ fixed. This nonlinear term will be treated as a known source.

We now use a Fourier representation in the coordinates perpendicular to the magnetic field:

$$\phi(\vec{x}, t) = \sum_{\vec{k}} \phi_k \exp(i\vec{k}_\perp \cdot \vec{x}) \tag{14}$$

where ϕ_k depends on distance parallel to $\vec{B}$ and time. Averaging over gyroangle keeping $\vec{R}$ fixed gives

$$\langle\phi\rangle_\alpha = \sum_{\vec{k}} J_0(k_\perp \rho)\phi_k \exp(i\vec{k}_\perp \cdot \vec{R}) \tag{15}$$

where J_0 is a Bessel function. Then g has a corresponding Fourier representation in the guiding center position:

$$g(\vec{R}, t) = \sum_{\vec{k}} g_k \exp(i\vec{k}_\perp \cdot \vec{R}) \tag{16}$$

and so does the averaged source term:

$$\langle S\rangle_\alpha = F_0 \sum_{\vec{k}} S_k \exp(i\vec{k}_\perp \cdot \vec{R}) \tag{17}$$

The linearized gyrokinetic equation becomes

$$\frac{\partial g_k}{\partial t} + v_\parallel \hat{b} \cdot \nabla g_k + i\omega_D g_k = \frac{e}{T} F_0 J_0 \frac{\partial \phi_k}{\partial t} - \vec{v}_{E\vec{k}} \cdot \nabla F_0 + S_k F_0 \tag{18}$$

where the drift frequency is defined by

$$\omega_D = \vec{k}_\perp \cdot \vec{v}_d \tag{19}$$

and

$$\vec{v}_{E\vec{k}} = \frac{ic}{B} \hat{b} \times \vec{k} J_0 \phi_k \tag{20}$$

3. Properties of the Source

Using the definition of the nonadiabatic part g,

$$\delta f(\vec{x},\vec{v},t) = -\frac{e}{T}F_0\phi(\vec{x},t) + g(\vec{x}-\vec{\rho},\vec{v},t) \tag{21}$$

and the gyroaverage (keeping R fixed),

$$\langle\delta f\rangle_\alpha = -\frac{e}{T}F_0\langle\phi\rangle_\alpha + g(\vec{R},\vec{v},t) \tag{22}$$

the source can be written as

$$\langle S\rangle_\alpha \simeq -\frac{c}{B}\hat{b}\times\nabla_{\vec{R}}\,\langle\phi\rangle_\alpha\cdot\nabla_{\vec{R}}\,\langle\delta f\rangle_\alpha \tag{23}$$

Since $\langle\delta f\rangle_\alpha$ is the guiding center distribution function (the gyrophase-independent part) the source represents the convection of the guiding center density in phase space by the gyroaveraged ExB motion. The electron and ion sources cause a buildup of the electrostatic potential on a magnetic surface, because for finite ion gyroradius, the electron and ion terms in the quasineutrality condition do not cancel.

Using the Fourier representation for ϕ and a similar one for δf, the Fourier coefficient of the source is given by

$$S_kF_0 = \frac{c}{B}\hat{b}\cdot\sum_{\vec{k}'}\vec{k}'_\perp\times\vec{k}_\perp J_0(k'_\perp\rho)J_0(|\vec{k}_\perp-\vec{k}'_\perp|\rho)\phi_{\vec{k}}\delta f_{\vec{k}-\vec{k}'} \tag{24}$$

In the limit of zero gyroradius, quasineutrality can be used to show that the density moments of the electron and ion sources are equal:

$$\sum_j e_j \int d^3v \, \langle S_j \rangle_\alpha \to 0 \tag{25}$$

4. Response to Axisymmetric Part of Source

We now consider the response to that part of the source which has no dependence on toroidal angle, i.e. the $n = 0$ part. The Fourier sum may be interpreted as a sum of terms of the form

$$\phi(\vec{x}, t) = \phi_k \exp(i\mathcal{S}) \qquad (26)$$

where the eikonal function $\mathcal{S}$ contains the fast spatial variation perpendicular to $\vec{B}$. Then the wave vector is $\vec{k} = \nabla\mathcal{S}$, so $\omega_D = \vec{v}_d \cdot \nabla\mathcal{S}$. To correspond to $n = 0$, the eikonal must be a function of ψ only: $\mathcal{S} = \mathcal{S}(\psi)$, so $\vec{k} = \nabla\psi\mathcal{S}'(\psi)$. The ExB drift term is zero: $\vec{v}_{E\vec{k}} \cdot \nabla F_0 = 0$, and the linearized gyrokinetic equation is

$$\frac{\partial g_k}{\partial t} + v_\parallel \hat{b} \cdot \nabla g_k + i\omega_D g_k = \frac{e}{T} F_0 J_0 \frac{\partial \phi_k}{\partial t} + S_k F_0 \qquad (27)$$

The drift frequency is

$$\omega_D = (\vec{v}_d \cdot \nabla\psi)\mathcal{S}'(\psi) = v_\parallel \hat{b} \cdot \nabla Q \qquad (28)$$

where

$$Q = I \frac{v_\parallel}{\Omega} \mathcal{S}'(\psi) \qquad (29)$$

with $I \equiv RB_\phi$.

5. Integral Equation for Long Time Response

For times long compared to the ion bounce period, $\omega/\omega_b \ll 1$ with ω the inverse of the time scale and $\omega_b = v_i/L_\parallel$ is the bounce frequency, we let $g_k = g_0 + g_1 + \cdots$, We shall take $\omega_D \sim \omega_b$, since Q may be of order unity. The zeroth order equation is

$$v_\parallel \hat{b} \cdot \nabla g_0 + i\omega_D g_0 = 0 \tag{30}$$

whose solution has the form

$$g_0 = he^{-iQ} \tag{31}$$

where $Q = I(v_\parallel/\Omega)\mathcal{S}'(\psi)$ and $\hat{b} \cdot \nabla h = 0$. The first order equation is

$$v_\parallel \hat{b} \cdot \nabla g_1 + i\omega_D g_1 = -\frac{\partial g_0}{\partial t} + \frac{e}{T} F_0 J_0 \frac{\partial \phi_k}{\partial t} + S_k F_0 \tag{32}$$

which yields the solubility condition which determines h:

$$\frac{\partial h}{\partial t} = \frac{e}{T} F_0 \overline{\left(e^{iQ} J_0 \frac{\partial \phi_k}{\partial t} \right)} + \overline{(e^{iQ} S_k)} F_0 \tag{33}$$

where the bounce average is defined by

$$\overline{A} = \frac{\oint \frac{dl}{v_\parallel} A}{\oint \frac{dl}{v_\parallel}} \tag{34}$$

with $dl = B dl_p / B_p$. For trapped particles, the integral goes over a closed orbit, while for untrapped particles, it goes once around the poloidal circumference.

Thus, g_k is given to lowest order for long times by integrating in time:

$$g_k = e^{-iQ} \left[\frac{e}{T} F_0 \overline{(e^{iQ} J_0 \phi_k)} + \overline{\left(e^{iQ} \int dt \; S_k \right) F_0} \right] \tag{35}$$

The electron expression is given by setting $J_0 = 1$ and $Q = 0$.

The quasineutrality condition is

$$-\frac{e}{T_i} n_0 \phi_k + \int \; d^3v \; J_0 g_{ik} = \frac{e}{T_e} n_0 \phi_k + \int \; d^3v \; g_{ek} \tag{36}$$

which then yields the integral equation for ϕ_k:

$$n_0 e \left(\frac{1}{T_i} + \frac{1}{T_e} \right) \phi_k - \frac{e}{T_i} \int d^3v \; F_{0i} e^{-iQ} J_0 \overline{(e^{iQ} J_0 \phi_k)} - \frac{e}{T_e} \int d^3v \; F_{0e} \overline{\phi_k} = s_k \tag{37}$$

where the source terms are combined in the expression

$$s_k = \int dt \; \left\{ \int d^3v \; F_{0i} e^{-iQ} J_0 \overline{(e^{iQ} S_{ik})} - \int d^3v \; F_{0e} \overline{S_{ek}} \right\} \tag{38}$$

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6. Variational Principle

The integral equation can be solved with the use of a variational principle, which is constructed by multiplying the terms in the integral equation by ϕ_k^* and carrying out the integral over a magnetic surface, using

$$\oint \frac{dl_p}{B_p} \int d^3v \ \cdots = \int 2\pi d\mathcal{E} d\mu \oint \frac{dl}{v_\|} \cdots \tag{39}$$

where $dl = dl_p B/B_p$. It is $\delta V = 0$, where $V = (V_i + V_e)/|V_s|^2$ where

$$V_i = \frac{e}{T_i} \int 2\pi d\mathcal{E} d\mu F_{0i} \left[\oint \frac{dl}{|v_\||} |\phi_k|^2 - \frac{\left| \oint \frac{dl}{|v_\||} e^{iQ} J_0 \phi_k \right|^2}{\oint \frac{dl}{|v_\||}} \right] \tag{40}$$

$$V_e = \frac{e}{T_e} \int 2\pi d\mathcal{E} d\mu F_{0e} \left[\oint \frac{dl}{|v_\||} |\phi_k|^2 - \frac{\left| \oint \frac{dl}{|v_\||} \phi_k \right|^2}{\oint \frac{dl}{|v_\||}} \right] \tag{41}$$

$$V_s = \oint \frac{dl_p}{B_p} \phi_k^* s_k \tag{42}$$

Both V_i and V_e can be shown to be positive definite, using the Schwartz inequality and $1 - J_0^2 \geq 0$. It is straightforward to show that V is minimized for the exact solution of the integral equation, and that the minimum value is $V = 1/V_s$. Because a positive minimum exists, it follows that a nontrivial solution of the integral equation exists. Therefore the potential response to the axisymmetric part of the source is not damped by collisionless processes.

7. Solution for Small Gyroradius and Banana Width

The integral equation is now solved by expanding in $\delta = (k_\psi \rho_i)^2 \sim (k_\psi \Delta_i)^2$ where k_ψ is the radial wave number, ρ_i is the ion gyroradius, and Δ_i is the ion banana width:

$$\phi_k = \phi_0 + \phi_1 + \cdots \tag{43}$$

The source term is expanded similarly; the zeroth order term is zero if either (1) S_{ik} and S_{ek} are independent of θ (m=0), or (2) S_{ik} and S_{ek} are independent of μ (isotropic). This follows from Eqs.(38) and (25). Then

$$s_k = s_1 + \cdots \tag{44}$$

The source contribution to the variational principle, V_s, is of order δ and since the exact minimum of V is given by $V = 1/V_s$, it follows that V is of order δ^{-1}. Thus, expanding in δ, $V_i + V_e = V^{(0)} + V^{(1)} + \cdots$, we have

$$V = \frac{V^{(0)} + V^{(1)} + \cdots}{|V_s|^2} \tag{45}$$

from which we conclude that

$$V^{(0)} = 0 \tag{46}$$

and

$$V^{(1)} = V_s \tag{47}$$

To zeroth order, the variational principle gives

$$V^{(0)} = e \sum_j \frac{1}{T_j} \oint \frac{dl_p}{B_p} \int d^3v \; F_{0j} \left| \phi_0 - \overline{\phi_0} \right|^2 = 0 \tag{48}$$

which shows that ϕ_0 must be uniform on a magnetic surface:

$$\hat{b} \cdot \nabla \phi_0 = 0 \tag{49}$$

The zeroth order potential is determined by the next order terms in the variational principle, i.e. the equation $V^{(1)} = V_s$. Expanding to second order in $k_\perp \rho$ and $Q \sim k_\perp v_\parallel / \Omega_\theta$ (related to banana width):

$$e^{iQ} J_0 \simeq 1 + iQ - \frac{1}{2} Q^2 - \frac{k_\perp^2 \rho^2}{4} \tag{50}$$

we have

$$V^{(1)} = \frac{e}{T_i} |\phi_0|^2 \oint \frac{dl_p}{B_p} \int d^3v \; F_{0i} \left[Q(Q - \overline{Q}) + \frac{1}{2} k_\perp^2 \rho^2 \right] \tag{51}$$

where

$$k_\perp^2 \rho^2 = |\nabla \psi|^2 \left(\mathcal{S}'(\psi) \right)^2 \rho^2 \tag{52}$$

and

$$Q(Q - \overline{Q}) = R^2 B_\phi^2 \left(\mathcal{S}'(\psi) \right)^2 \frac{v_\parallel}{\Omega} \left(\frac{v_\parallel}{\Omega} - \overline{\left(\frac{v_\parallel}{\Omega} \right)} \right) \tag{53}$$

The term $V^{(1)}$ contains the classical polarization current as well as the collisionless neoclassical polarization current.

The term $V^{(1)}$ can be written in terms of the polarization current:

$$V^{(1)} = -\phi_0{}^* \oint \frac{dl_p}{B_p} \rho_k \tag{54}$$

where

$$\rho_k = -\int dt\ i\vec{k} \cdot \vec{j}_k = -i\mathcal{S}'(\psi) \int dt\ \vec{j}_k \cdot \nabla\psi \tag{55}$$

with the current density given by $\vec{j} = \vec{j}_k^{cl} + \vec{j}_k^{nc}$ where the classical polarization current is given by

$$\vec{j}_k^{cl} = -n_i m_i c^2 \frac{\nabla\psi}{B^2} i\mathcal{S}'(\psi) \frac{\partial \phi_k}{\partial t} \tag{56}$$

and the collisionless neoclassical polarization current[5] is given by

$$\oint \frac{dl_p}{B_p} \vec{j}_k^{nc} \cdot \nabla\psi = -\frac{e^2 R^2 B_\phi^2}{T_i} \oint \frac{dl_p}{B_p} \int d^3v\ F_{0i} \frac{v_\parallel}{\Omega} \left(\frac{v_\parallel}{\Omega} - \overline{\left(\frac{v_\parallel}{\Omega} \right)} \right) i\mathcal{S}'(\psi) \frac{\partial \phi_k}{\partial t}$$
$$= -1.6 \left(\frac{r}{R_o} \right)^{3/2} n_i m_i c^2 R_o^2 i\mathcal{S}'(\psi) \frac{\partial \phi_k}{\partial t} \oint \frac{dl_p}{B_p} \tag{57}$$

where the integrals have been evaluated for large aspect ratio circular geometry[6]. The variational principle provides a derivation of the collisionless neoclassical polarization current.

Similarly, V_s can be written to next order in δ. We assume that the source is even in $v_{\|}$, so it does not add parallel momentum to the plasma. Then

$$V_s = -\phi_0{}^* \oint \frac{dl_p}{B_p} \int d^3v \; F_{0i} \left[\frac{1}{2}(\overline{Q^2} - 2Q\overline{Q} + Q^2) + \frac{1}{4} k_\perp^2 \rho^2 \right] \int dt \; S_{ik} \tag{58}$$

Thus, $V^{(1)} = V_s$ gives

$$\frac{e\phi_k}{T_i} = \frac{- \oint \frac{dl_p}{B_p} \int d^3v \; F_{0i} \left[Q(Q - \overline{Q}) - \frac{1}{2}(Q^2 - \overline{Q^2}) + \frac{1}{4} k_\perp^2 \rho^2 \right] \int dt \; S_{ik}}{\oint \frac{dl_p}{B_p} \int d^3v \; F_{0i} \left[Q(Q - \overline{Q}) + \frac{1}{2} k_\perp^2 \rho^2 \right]} \tag{59}$$

For the part of the source which is independent of poloidal angle, i.e. the $m = 0$ part, this integral can be evaluated explicitly. The potential response to the source is then

$$\mathcal{K} \equiv \frac{e\phi_k / T_i}{\int dt \; S_{ik}} \simeq -1 \tag{60}$$

neglecting the terms of order $k_\psi^2 \rho_i^2$ compared with those of order $k_\psi^2 \Delta_i^2$.

That is, there is a long-time plateau in the response function, which is not zero, showing that the radially sheared potentials driven by the ITG turbulence source are not damped by transit time damping, but only by the much weaker collisional damping[7] (not included here).

8. Kinetic Coupling to Poloidally Varying Sources

The coupling to poloidally varying sources is generally stronger than in fluid models. Assuming only that

$$\oint \frac{dl_p}{B_p} s_1 \neq 0 \tag{61}$$

that is,

$$\oint \frac{dl_p}{B_p} \int d^3v \; F_{0i} \left[\left(Q - \overline{Q}\right)^2 + \overline{\left(Q - \overline{Q}\right)^2} + \frac{1}{2}\overline{\left(k_\perp^2 \rho^2\right)} \right] S_{ik} \neq 0 \tag{62}$$

(for example, $S_{ik} \propto \cos\theta$), Eqn.(59) gives

$$\mathcal{K} \sim 1 \tag{63}$$

Thus, even for the $m \neq 0$ part of the source, the potential response is still generally strong, because of the influence of trapped ions, which strongly couple to the source.

The exceptional case is for that part of the source for which Eq.(62) is not true (for example, $S_{ik} \propto \sin\theta$ when B is even in θ). Then we find $\phi_0 = 0$, so that $\phi \sim \delta$ and hence

$$\mathcal{K} \sim \delta \tag{64}$$

In this case, the coupling is weak and so the response is weak.

9. Mean Square Potential Fluctuation

We now use our result to show that the mean square potential fluctuation, and hence the poloidal rotation, driven by the source is not damped by collisionless processes. The linear response to the source can be written in terms of the response kernel $\mathcal{K}$ as

$$\tilde{\phi}_k(t) = \int_0^t dt' \, \mathcal{K}(t-t') S_k(t') \tag{65}$$

where $\tilde{\phi}_k = e\phi_k/T_i$. An integral over the poloidal variation of $S_k(t)$ should also be included, but has been omitted for notational simplicity. The ensemble average of $|\phi_k|^2$ (related to the shear decorrelation of the ITG turbulence) is

$$\left\langle |\tilde{\phi}_k|^2 \right\rangle = \int_0^t dt' \int_0^t dt'' \, \mathcal{K}(t-t')\mathcal{K}(t-t'') \left\langle S_k(t') S_k(t'') \right\rangle \tag{66}$$

Assuming the source is statistically stationary, $\langle S_k(t') S_k(t'') \rangle$ is a function of $|t'-t''|$ only; we assume it is nonzero only for $|t'-t''| \gtrsim \tau_c$, where τ_c is the autocorrelation time of the random source. We are only interested in times $t \gg \tau_c$, so

$$\left\langle |\tilde{\phi}_k|^2 \right\rangle \simeq \int_0^t dt' \, \mathcal{K}^2(t') \int_{-\tau_c}^{\tau_c} d\tau \left\langle S_k(0) S_k(\tau) \right\rangle \simeq 2\tau_c \left\langle |S_k|^2 \right\rangle \int_0^t dt' \, \mathcal{K}^2(t') \tag{67}$$

For $t \gg \omega_b$, our result may be used to obtain

$$\left\langle |\tilde{\phi}_k|^2 \right\rangle \simeq 2\tau_c \left\langle |S_k|^2 \right\rangle t \tag{68}$$

showing that the mean square potential fluctuation grows linearly with time. It would eventually saturate, of course, but only after a collision time.

10. Random Walk of the Radial Electric Field

The mean square electric field resulting from the nonlinear ITG source can be understood as follows. For the axisymmetric part of the radial electric field,

$$\frac{\partial E_r}{\partial t} = -4\pi(j_L + j_{NL}) \tag{69}$$

where j_{NL} is the nonlinear current from ITG turbulence. The linear current j_L consists only of polarization current, since there is no dissipation:

$$j_L = \frac{(\varepsilon - 1)}{4\pi}\frac{\partial E_r}{\partial t} \tag{70}$$

and only changes the effective inertia:

$$\frac{\partial E_r}{\partial t} = -\frac{4\pi}{\varepsilon} j_{NL} \tag{71}$$

This equation is thus like the equation for the velocity of a Brownian particle, a dust particle subject to random accelerations from impacts by gas molecules. In spite of the high frequency nature of these impacts, the mean square velocity increases linearly with time, in the absence of dissipation.

The mean square electric field is thus

$$\langle E_r^2 \rangle = \left(\frac{4\pi}{\varepsilon}\right)^2 \int_0^t dt' \int_{-\tau_c}^{\tau_c} d\tau \;\; \langle j_{NL}(0) j_{NL}(\tau) \rangle \tag{72}$$

that is, linearly increasing with time, in the absence of collisions.

11. Discussion

In order to determine poloidal rotation, it is in principle also necessary to calculate the buildup of parallel flow. However, since toroidal angular momentum is conserved, the contribution of parallel flow to poloidal rotation may be neglected. A related problem is the collisionless decay of an initially poloidally rotating plasma, obtained from the Laplace transform of Eq.(27), with the source now related to initial values. Although the exact value of the rotation is sensitive to these initial details, it is clear from our solution that transit time damping of poloidal rotation does not occur.

The authors of Ref. 1,2 argue[8] that since gyrofluid theory predicts roughly correctly the short (transit time) damping of poloidal rotation, the response to the rapidly fluctuating ITG drive will damp quickly. Our results show this not to be the case. A long term buildup should occur, which might be difficult to simulate numerically. It should be limited only by collisions or, at high turbulence levels, by nonlinear effects. More recently, these authors have pointed out[9] that their simulations do show a nonzero rotation remaining after an initially damped transient, which may be consistent with our results.

It is also of interest that poloidally varying sources should have much stronger coupling to the undamped 0,0 modes kinetically than would be expected from a fluid theory since trapped ions only average over a limited poloidal distance.

We are far from understanding how important these effects are, but in the absence of collisions or nonlinear damping, they must strongly reduce the ITG turbulence level. Any "first principles" theory should be expected to treat the linear damping accurately in view of its importance in determining saturated amplitudes. We conclude that gyrofluid simulations are likely to overestimate poloidal rotation damping and hence underestimate the stabilizing effects of the m=n=0 radial modes on the amplitude of ITG turbulence, and therefore overestimate the transport from ITG turbulence.

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