Growth Rate of Knock-on Runaway Electron Generation in Tokamaks[1] S.C. CHIU, V.S. CHAN, General Atomics, M.N. ROSENBLUTH, ITER JCT, San Diego, R.W. HARVEY, CompX — The ability to tolerate disruptions is an important issue for high current tokamaks. To minimize the electro-mechanical stresses that can be produced during current quench in a disruption, it has been proposed to inject killer pellets to quench the plasma thermally and thus allow a rapid current decay. A serious concern is that a high electric field will develop which could produce long-lived runaway electrons. Specifically, knock-on collisions can cause an avalanche of runaways. The key quantities insuch an event are the growth rates of the avalanche and the energy spectrum of the runaways. Previous works[2,3] have used the approximation that the runaway distribution generating the knock-ons have zero pitch angle when calculating the knock-on source. Here, we abandon this approximation and use the actual electron distribution to calculate the knock-on source. Using the bounce-averaged Fokker-Planck code CQL3D, the growth rates and energy spectrum of runaways are calculated and compared with previous results.

[1]Work supported by U.S. DOE Grant DE-FG03-95ER54309.
[2]M.N. Rosenbluth, A. Putvinski, submitted to Nuclear Fusion (1997).
[3]S.C. Chiu *et al.*, oral talk, Sherwood Theory Conference, Madison, Wisc. (1997).

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MOTIVATION

- Injection of ‘killer pellet’ to induce rapid current decay is thought to be effective in mitigating a vertical displacement event resulting from a disruption.

- Rapid cooling and increase of Z_{eff} cause rapid electric field buildup and runaway generation.

- Existing traces of energetic electrons can knock out bulk electrons to cause an avalanche.

- A crucial quantity in modeling of knock-on avalanche of runaways is the growth rate. Previous calculations of avalanche growth rates used a zero pitch-angle approximation of the runaway distribution when calculating the knock-on source. However, the distribution for the lower energy runaways is less peaked around zero pitch angle, and contribution in this lower energy range to knock-on source is large.

- Here <u>we abandon the zero pitch-angle approximation</u> and <u>use the actual electron distribution to check the accuracy of previous calculations</u>.

OUTLINE

(1) Introduction.

(2) Description of simulation model and an analytic expression of growth rate.

(3) Comparison of growth rates using general source function and those using approximate source function which assumes primary electrons have zero pitch-angle.

(4) Summary and conclusions.

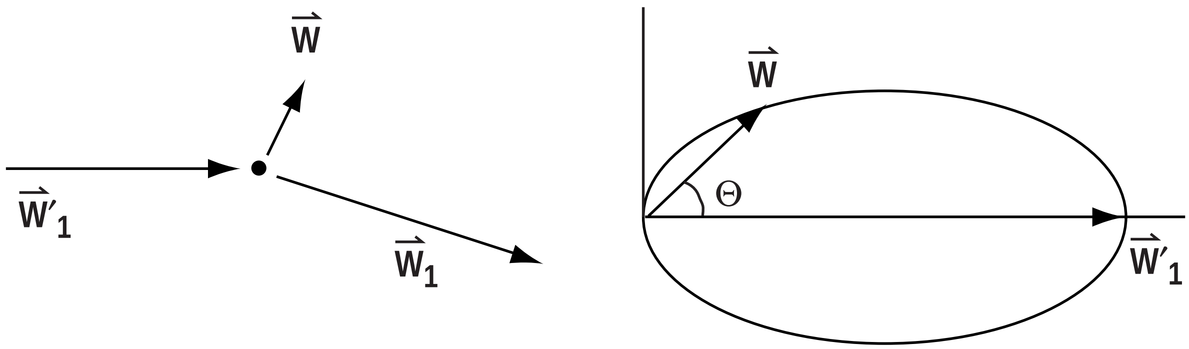
SUMMARY OF RESULTS

- Growth rates are insensitive to energy of primary electrons.

- Growth rates with general source function and those with source function using zero pitch-angle primary distribution agree very well.

- An analytic expression similar to the Rosenbluth-Putvinski expression gives very accurate growth rates.

INTRODUCTION



$$\left(\vec{w} = \frac{\vec{u}}{c} \ , \quad \gamma^2 = 1 + w^2 \right)$$

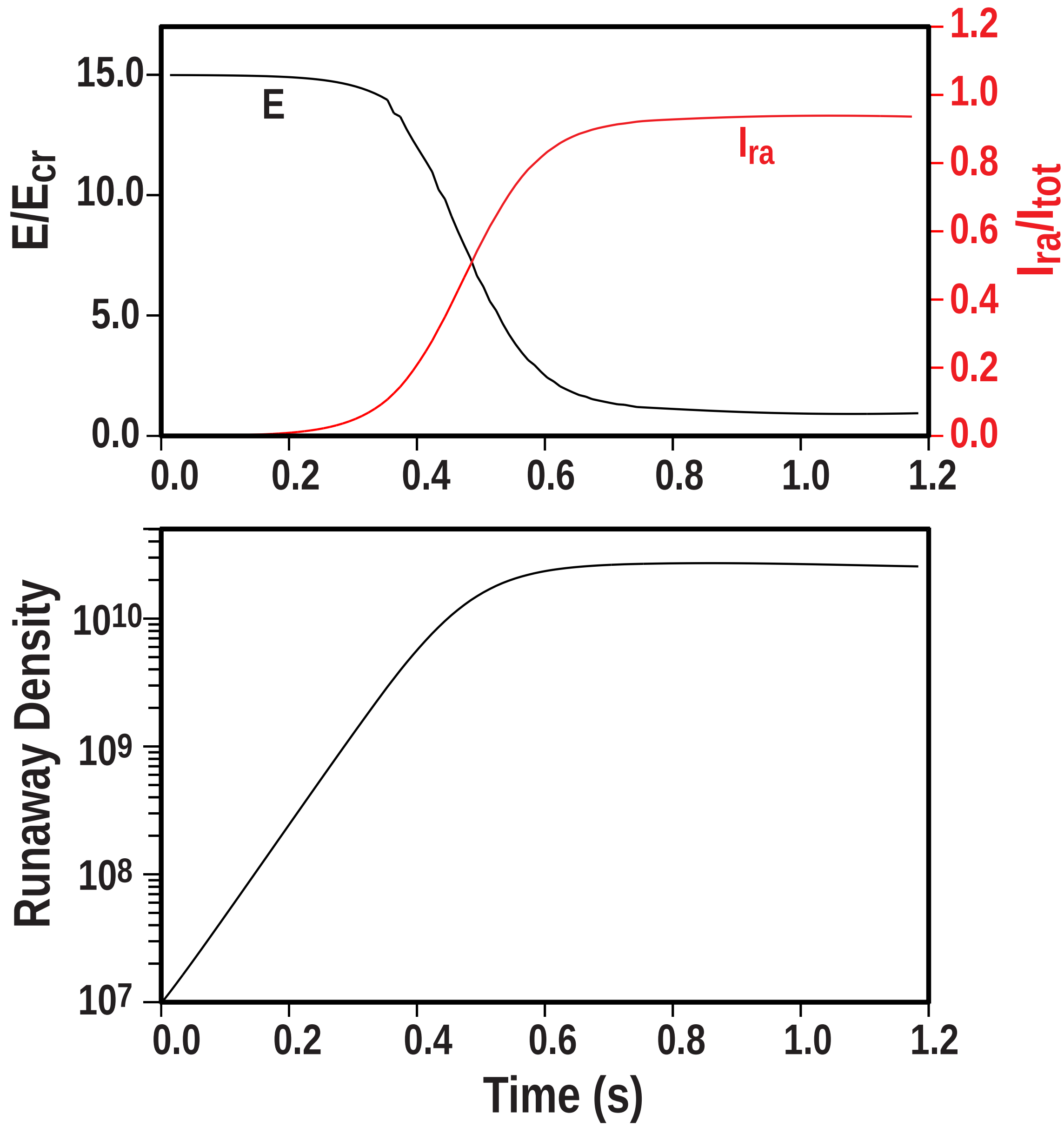
(a) **A high energy electon of momentum $\vec{w}'_1$ (primary electron) is scattered to momentum $\vec{w}_1$ knocking out a bulk electron to momentum $\vec{w}$ (secondary electron).**

(b) **Locus of momentum vector of the secondary electron falls on an ellipse:**

$$\frac{w_\perp^2}{2(\gamma'_1 - 1)} + \left[\frac{w_\parallel}{\gamma'_1 - (1/2\gamma'_1)} - \frac{1}{2} \right]^2 = \frac{1}{4}$$

(c) **Pre-existing high energy electrons can knock out bulk electrons to runaway energies which are accelerated by electric field and knock out more electrons and cause a runaway avalanche.**

Runaways Saturate at Around E/E_{cr} = 1



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SIMULATION MODEL AND ANALYTIC EXPRESSION

(A) Description of Simulation Model:

Scattering rate given by Moller cross-section

$$\frac{d\sigma}{d\gamma} = 2\pi r_0^2 \, \Sigma(\gamma, \gamma_1')$$

$$\Sigma(\gamma, \gamma_1') = \frac{\gamma_1'^2}{(\gamma_1'^2 - 1)(\gamma - 1)^2 (\gamma_1' - \gamma)^2}$$
$$\times \left\{ (\gamma_1' - 1)^2 - \frac{(\gamma - 1)(\gamma_1' - \gamma)}{\gamma_1'^2} \left[2\gamma_1'^2 + 2\gamma_1' - 1 - (\gamma - 1)(\gamma_1' - \gamma) \right] \right\}$$

Note: $\lim\limits_{w_1' >> w} \Sigma(\gamma, \gamma_1') = \Sigma(\gamma, \infty) = \dfrac{1}{(\gamma - 1)^2}$;

this limit is reached quickly.

Let (Θ, Φ) be the scattering angle and orientation of the scattering plane. Then the secondary and primary pitch angles are related by

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$$\cos\theta = \cos\Theta\cos\theta_1' - \sin\Theta\sin\theta_1'\cos\Phi$$

The general source function is given by $S(w,\theta)$,

$$S(w,\theta)\delta\xi\delta\phi = \frac{ncr_0^2 w_1'}{w\gamma\gamma_1'}\Sigma(\gamma,\gamma_1')f(\vec{w}_1,t)w_1'^2 dw_1 d\xi_1' d\phi_1' d\Phi$$

When $f(\vec{w}_1,t)$ is sharply peaked around $\theta_1' \approx 0$, one has the reduced source function:

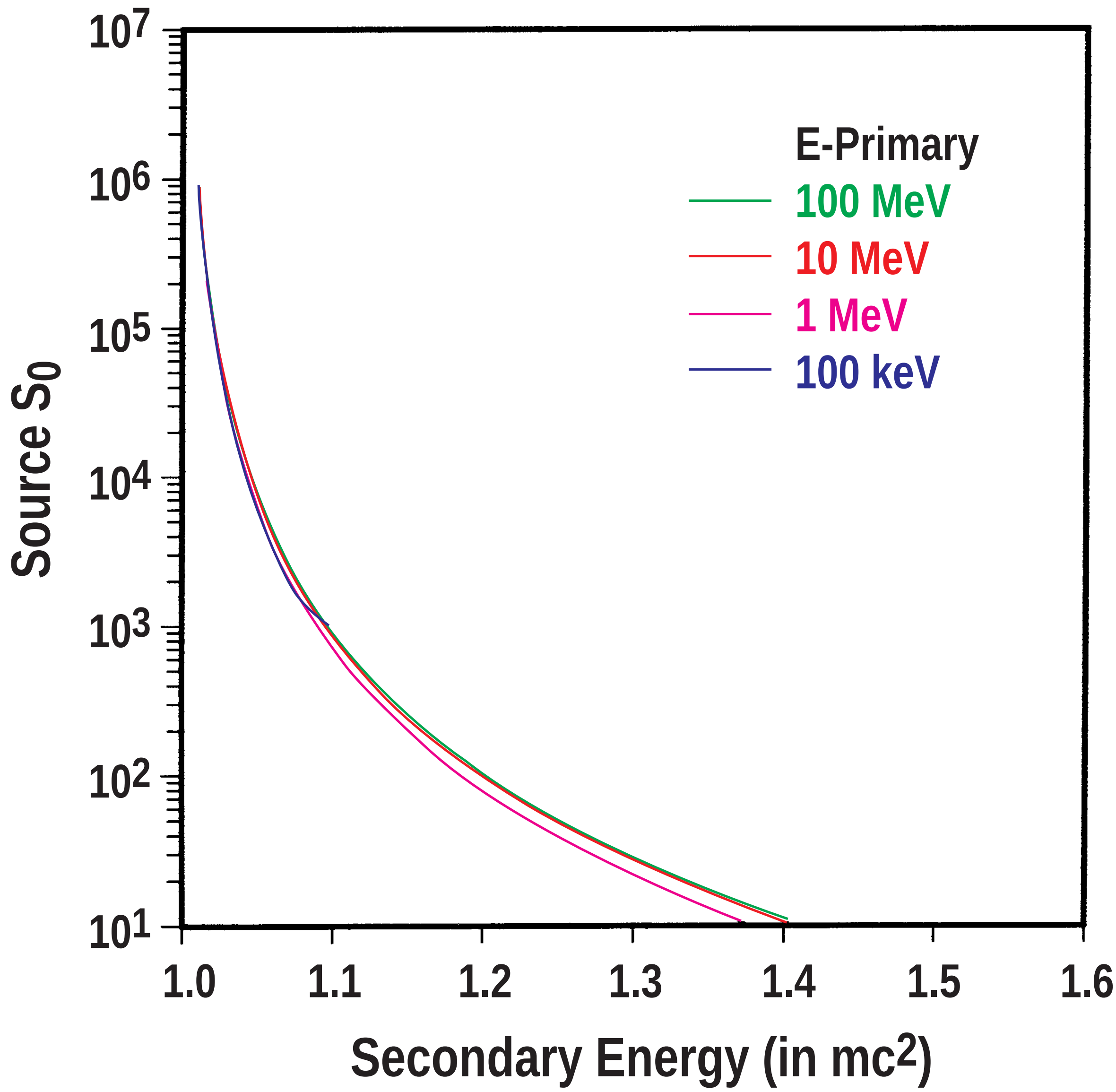
$$S_r(w,\theta) = ncr_0^2\bar{f}(w_{\|1}',t)w_1'^2 S_0(\gamma,\gamma_1')$$

where

$$S_0(\gamma,\gamma_1') = \frac{\Sigma(\gamma_1'-1)(\gamma+1)}{w^2 w_1'\gamma}$$

Note: The function S_0 is only weakly dependent on the primary energy

Source Depends Weakly on Primary Energy



Bounce averaged Fokker-Planck equation:

$$\left\langle \frac{\partial f}{\partial t} - \frac{eE}{mc}\frac{\partial f}{\partial w_\parallel} \right\rangle = \langle C(f) \rangle + \langle S(w,\theta) \rangle$$

solved by CQL3D. For high energies, only drag and pitch angle scattering are important. Then the collision operator reduces to

$$\langle C(f) \rangle \approx \frac{eE_c}{mc}\left[\frac{\zeta(\lambda)}{w^2}\frac{\partial}{\partial w}\gamma^2 f + \frac{2(Z-1)\gamma}{w^3}\frac{\partial}{\partial \lambda}\lambda\eta\frac{\partial}{\partial \lambda} f \right]$$

where

$$b(\ell) = \frac{B(\ell)}{B_{\max}} \;, \qquad \lambda = \frac{w_\perp^2}{w^2 b(\ell)} \;,$$

$$\zeta(\lambda) = \oint \frac{d\ell}{L}\frac{1}{\sqrt{1-\lambda b(\ell)}} \;, \qquad \eta(\lambda) = \left\langle \frac{1}{b(\ell)} - \lambda \right\rangle \;.$$

The critical electric field E_c is defined as

$$E_c = \frac{4\pi n e^3 \ln\lambda}{mc^2} \approx 0.102\, n_{14}\ (\mathrm{V/m})$$

For $E \le E_c$, there can be no runaways.

(B) Analytical expression of growth rate:

$$\gamma_{ra} = \frac{eE}{mc \ln\Lambda} \left\{ \hat{E} \left[\hat{E} - 1 - \frac{0.2(Z+1)}{(Z+5)} \right] \left[\frac{f_p}{(Z+5)} \right] \times \left[\frac{1}{\hat{E}^2 + 3(Z+1)^2/(Z+5)} \right] \right\}^{1/2}$$

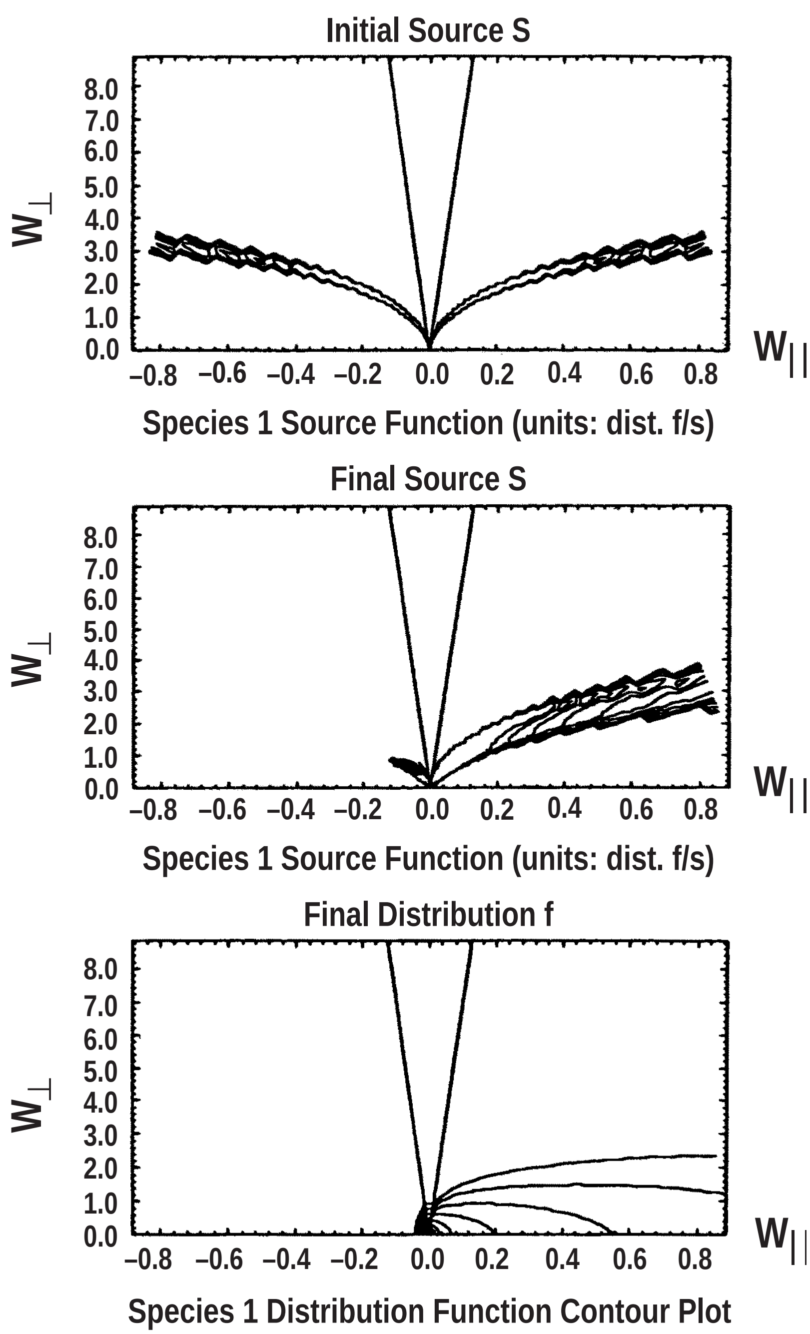
where $\hat{E} = E/E_c$, and the trapping effect is in

$$f_p = 1 - 1.46\sqrt{\varepsilon}\ .$$

Growth rate decreases with increasing Z.
For $Z >> \hat{E} >> 1$,

$$\gamma_{ra} \to \frac{eE\hat{E}}{mc \ln\Lambda\,(Z+1)} \sqrt{\frac{f_p}{3}}$$

REDUCED



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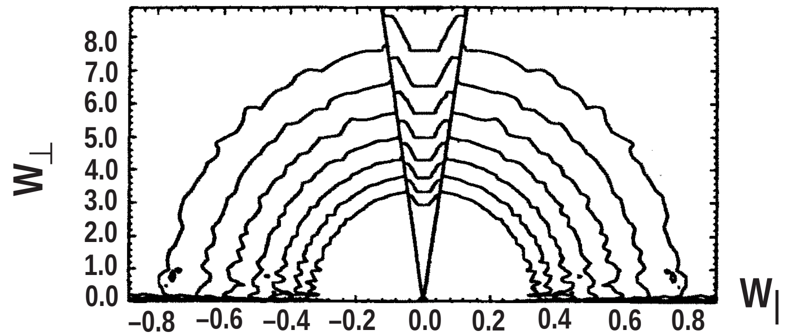
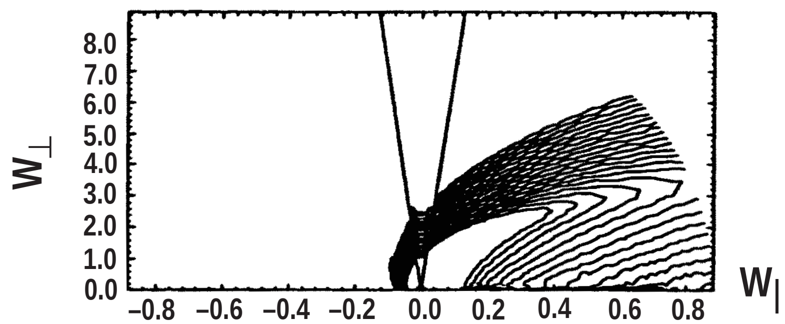
GENERAL

Initial Source S



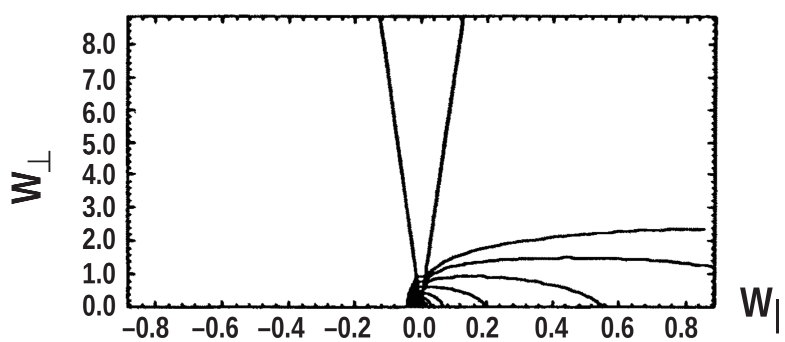
Species 1 Source Function (units: dist. f/s)

Final Source S



Species 1 Source Function (units: dist. f/s)

Final Distribution f



Species 1 Distribution Function Contour Plot

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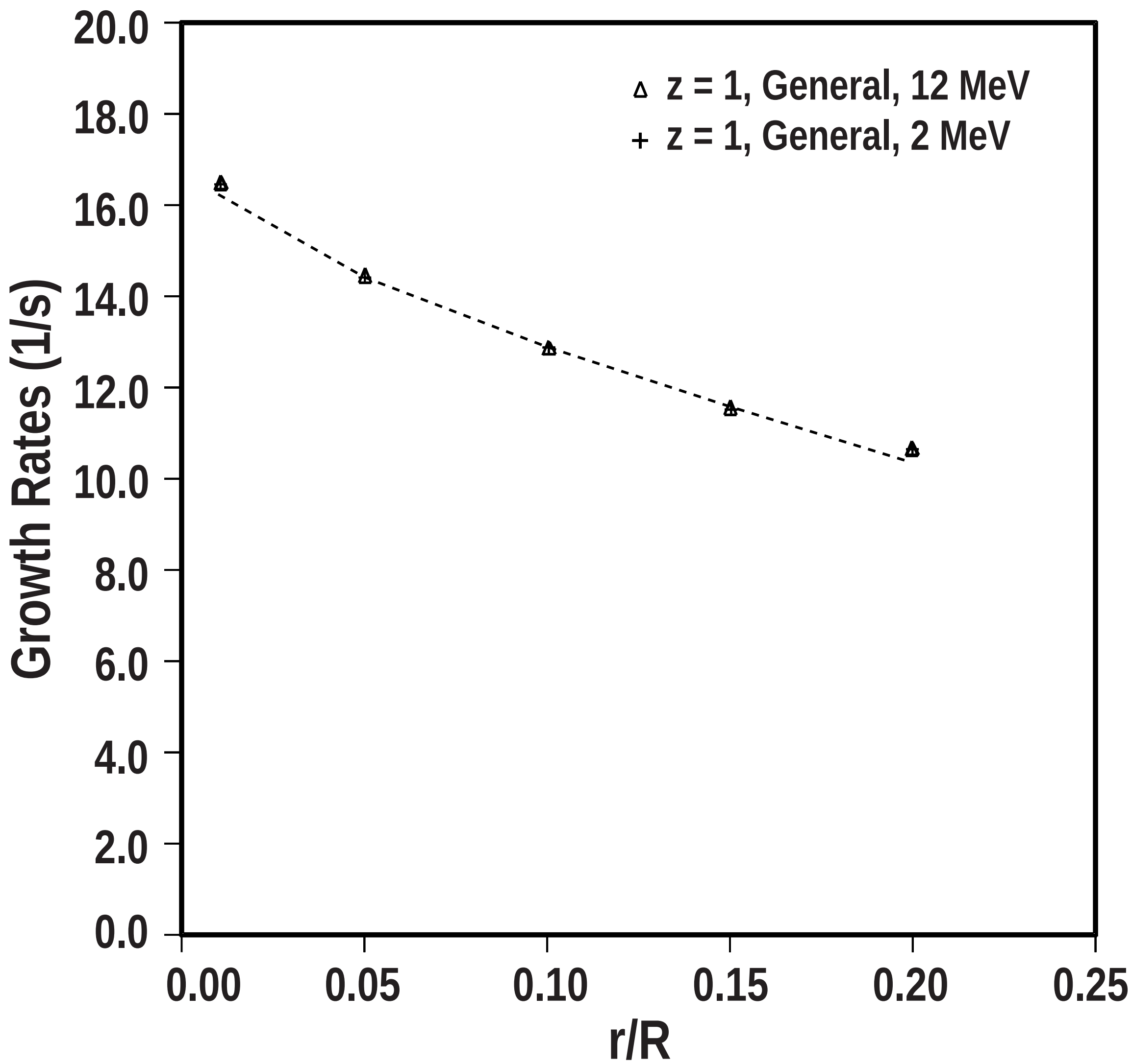
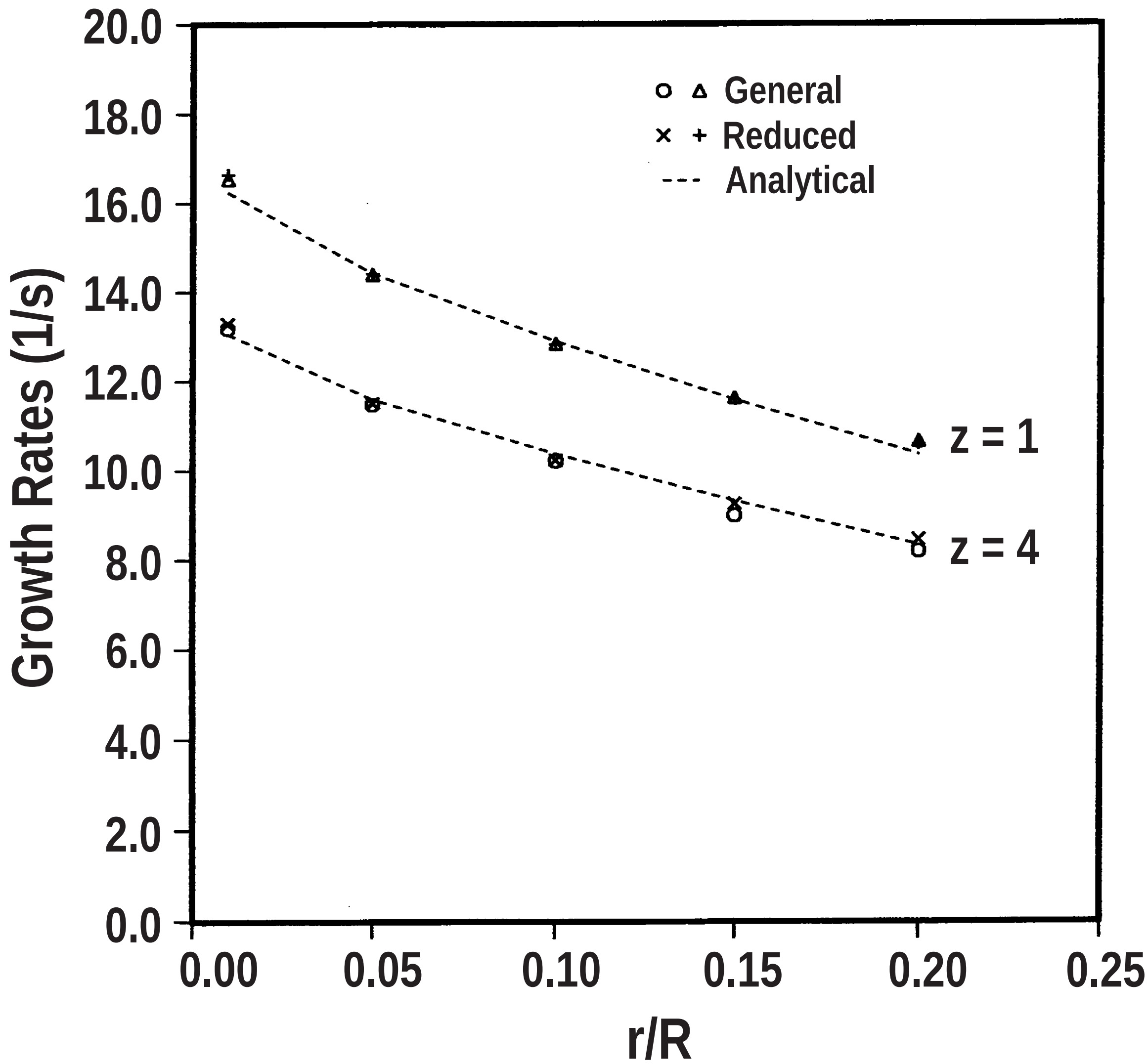
Growth Rate Depends Weakly on Primary Energy



Trapping Decrement of Growth Rate
(E/E_{cr} = 15)



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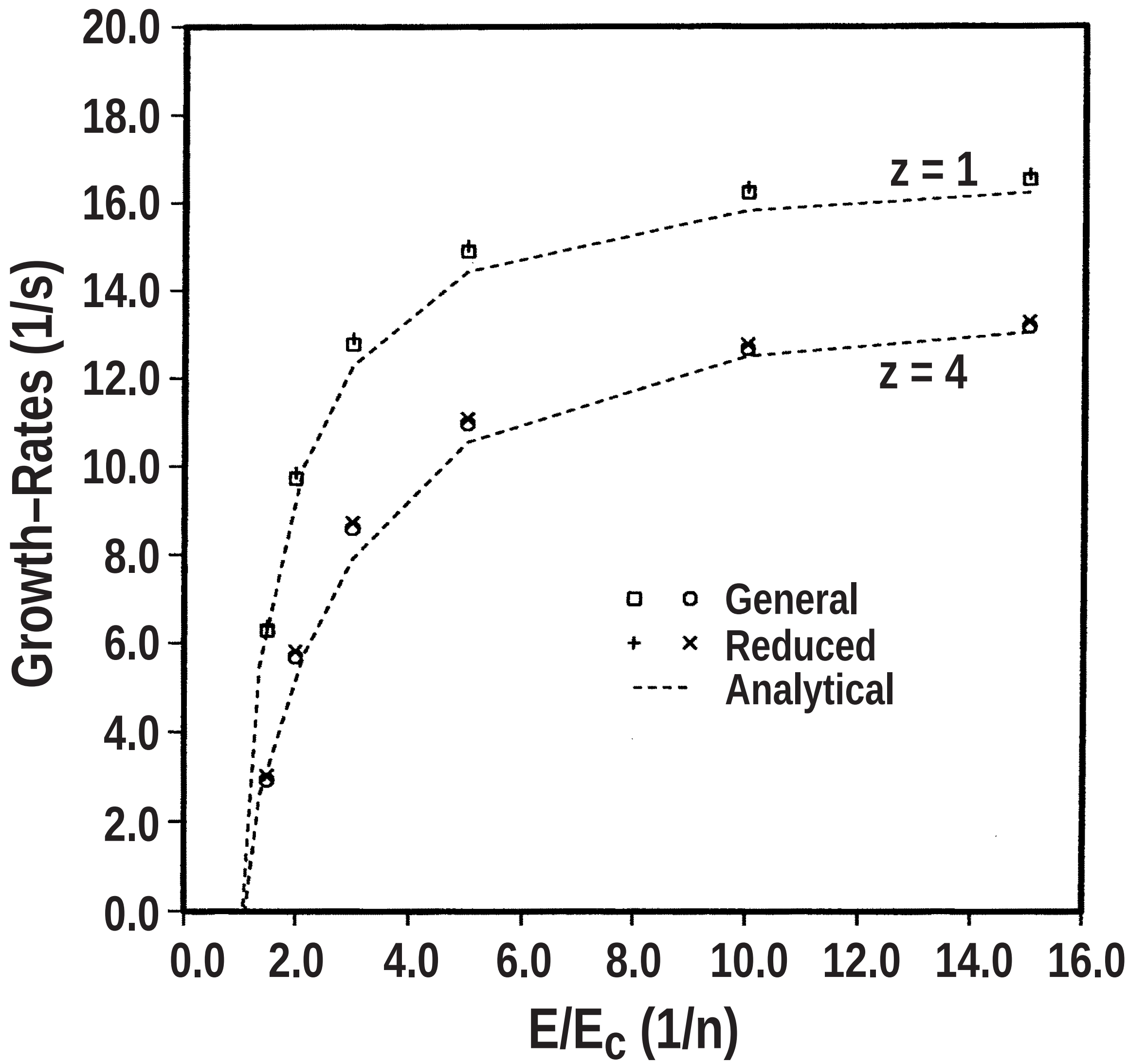
Growth Rates Decrease with Density



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SUMMARY AND CONCLUSIONS

(l) Growth rates of knock-on avalanche are found to be rather insensitive to the primary energy.

(2) Growth rates using the general source function are compared with those using the reduced source function. It was found that the reduced source function gives growth rates with remarkable accuracy. The latter is computationally about twenty times more efficient.

(3) An analytic expression which is modified from an earlier expression was found to give very accurate growth rates.