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Poloidal Pressure Gradients, Divertor Detachment and Marfes

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Abstract

Because the radiation power density from a marfe scales approximately as the square of its plasma pressure, and since increased radiation would aid divertor detachment for high power tokamaks, this paper identifies regions that might permit locally increased plasma pressure in steady state. The magnetic and dynamic (flow) constraints of magneto-hydrodynamics (MHD) are examined for self-consistent locally increased pressure equilibria, in both the magnetically open tokamak scrape-off layer (SOL) and the closed surfaces just inside the last closed flux surface. In most tokamak geometries it is difficult to recycle particles at a sufficient rate to sustain high pressure marfes, but they might be possible near a divertor X–point.

1. Introduction

Plasma in the magnetically open tokamak scrape-off layer (SOL) and divertor obeys the equations of MHD equilibrium with flow. “Cross-field-static” equilibrium, in which only the flow parallel to **B** is rapid, requires the presence of Pfirsch-Schlüter currents in order to satisfy $\nabla \cdot \mathbf{J} = 0$. Experimentally, SOL Pfirsch-Schlüter currents are observed to close through the electrically conducting targets of present tokamaks when the divertor plasmas are attached [1]. The SOL poloidal pressure gradients, which are localized in front of the targets in attached operation, are equilibrated by currents crossing the toroidal magnetic magnetic field normal to the magnetic surfaces. The Pfirsch-Schlüter currents flow parallel to **B** (force-free), both around the common SOL and to the target, to redistribute current from the high–B side to the low–B side, where extra current is needed to maintain equilibrium in the weaker magnetic field. When divertor plasmas detach in JET [1] and DIII–D, the Pfirsch-Schlüter target currents become very small, possibly due to increased target plasma impedance. Despite the blocked divertor equilibrium currents, there are no experimental signs of a loss of the cross-field-static equilibrium, such as an enhanced poloidal flow from the inner to the outer divertors in typical single-null top- or bottom-diverted tokamaks. Previous work showed that without target currents, cross-field-static equilibria in this geometry *must* have at least one poloidal pressure gradient region on the SOL upstream of the divertor gradients [2]. The present paper shows that poloidal pressure gradients are allowed, but not required, on closed magnetic surfaces as well. However, because $\nabla \cdot \mathbf{J} = 0$, current and pressure cannot be arbitrarily distributed. Likewise, the poloidal pressure gradients, which drive parallel equilibrium flows—the dynamic part of the MHD equilibrium—cannot be ignored.

Marfes commonly occur in or near the X–point, both inside and outside of the magnetic separatrix during detachment [3]. In this paper "marfe" means any localized region of strongly radiating, low-temperature, high-density plasma, regardless of the mechanism(s) involved. Marfes are usually assumed to exist with little or no pressure gradient on each affected magnetic surface. However, some preliminary DIII–D divertor Thomson scattering data show locally increased p_e near the X–point, both in and outside the separatrix. (Ion pressure cannot yet be measured there.)

It is worth considering whether marfes might be associated with any of the higher pressure equilibrium regions, because, since a marfe radiates as $n^2 \sim p^2$ at a fixed temperature set by atomic processes, the increased radiation would aid detachment at high tokamak power. This paper discusses edge-plasma MHD constraints in detached plasmas and their relations to the poloidal pressure distribution and to marfes. Only single-null divertors are treated.

2. Magnetic Equilibria

Simplified multispecies MHD equilibrium equations with flow are derived in [1] in a form useful for open and closed axisymmetric nested toroidal magnetic surfaces. In cross-field-static equilibrium only the parallel and poloidal momentum equations are interesting. In the tokamak limit the poloidal inertial terms are negligible [1], and the poloidal equation simplifies to

$$\frac{B_\phi}{r} \nabla_p \Im = J_n B_\phi = -\nabla_p \, p . \tag{1}$$

Coordinate directions are identified in Fig. 1, and $\nabla_p = \mathbf{e}_p \cdot \nabla$. The plasma pressure is p, but subscript p denotes a vector component in the poloidal direction. The poloidal current stream function, $\Im(r,z) = \Im\left(x_n, x_p\right) = rB_\varphi / \mu_0$, yields the divergence-free current in the poloidal plane: $J_n = r^{-1} \nabla_p \Im$, $J_p = -r^{-1} \nabla_n \Im$. Here $\Im$ is the poloidal current per radian in the torus with $\Im = 0$ at the major axis and $\nabla_n = \mathrm{e}_{\hat{n}} \nabla$. Equation 1 states that a poloidal pressure gradient is equilibrated by current crossing the toroidal magnetic field in the normal direction. Further simplification follows if the poloidal pressure changes are approximated as steps and the J_n as current sheets [2]. Then,

$$\Delta p = -\frac{B_\phi I}{r} , \quad \text{where} \quad I = \Delta \Im = r \int J_n \, dx_p . \tag{2}$$

Also, let $B_\phi / r = R_0 B_0 / r^2$.

Figure 1 shows a detached SOL equilibrium having inner and an outer regions with possibly different constant pressures p_{in} and p_{out}, respectively. Pressure below the

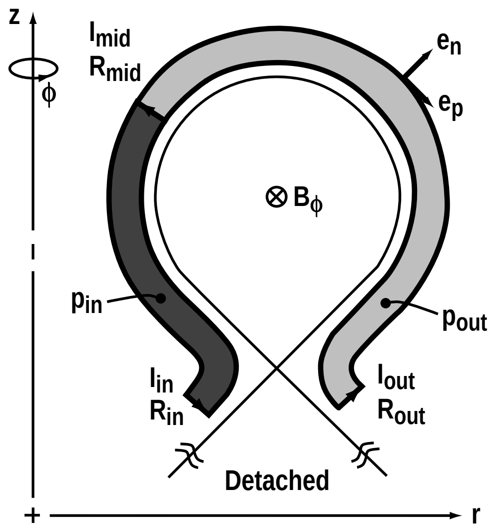

Fig.1. Detached single-null-diverted tokamak with elementary SOL current layer.

detachment region is small and is taken as zero. Then, application of Eq. (2) to each pressure discontinuity yields

$$\frac{R_0 B_0 I_{in}}{R_{in}^2} = p_{in}, \qquad \frac{R_0 B_0 I_{out}}{R_{out}^2} = p_{out}, \qquad \frac{R_0 B_0 I_{mid}}{R_{mid}^2} = p_{in} - p_{out} . \tag{3}$$

The normally directed currents satisfy $\nabla \cdot \mathbf{J} = 0$ which globally is

$$I_{out} + I_{mid} - I_{in} = 0 \quad . \tag{4}$$

Equations (3) and (4) can be solved to show

$$\frac{p_{out} - p_{in}}{p_{out} + p_{in}} = \frac{\frac{1}{2}\left(R_{out}^2 - R_{in}^2\right)}{R_{mid}^2 - \frac{1}{2}\left(R_{out}^2 + R_{in}^2\right)} \ . \tag{5}$$

It is apparent from Eq. (5) that $p_{out} \neq p_{in}$ except when $R_{out} = R_{in}$, i.e. when there is no divertor. The pressure difference is large if I_{mid} is free to flow at a radius R_{mid} that makes the denominator small. If the left side of Eq. (5) is to be in its allowable range (-1,1), then R_{mid} is either inward of R_{in} or outward of R_{out}. The reason is that, when $R_{mid} < R_{in}$, I_{mid} crosses a larger B_ϕ and more effectively equilibrates pressure than I_{in}, thereby supplementing the less effective I_{out} [2]. The resulting equilibrium has $p_{in} > p_{out}$. In the other case, when $R_{mid} > R_{out}$, the sign of I_{mid} reverses and $p_{out} > p_{in}$. Now I_{mid} is less effective than even I_{out}, but it combines with the more effective I_{in} to equilibrate p_{out}. These equilibria impose unequal inner and outer pressures during detachment, especially when $R_{out} - R_{in}$ is large, but they do not exihibit localized pressure nonuniformities unless the geometry is like Fig. 2.

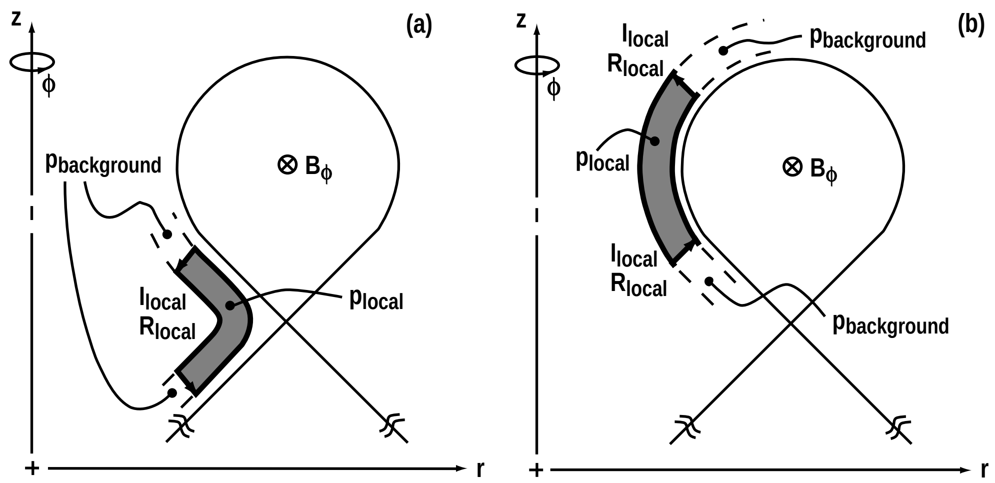

Fig.2. Two examples with equal and opposite normal currents at the same radius.

Figure 2 illustrates two examples with the normally directed currents equal and opposite on a magnetic surface at different poloidal locations at the same radius. In this case the currents maintain a local pressure $p_{\rm local}$ different from the pressure $p_{\rm background}$ immediately adjacent on the same surface. The closed current loop structure automatically satisfies $\nabla \cdot \mathbf{J} = 0$, and $p_{\rm local} - p_{\rm background} = R_0 B_0 I_{\rm local} / R_{\rm local}^2$ can take any value, positive or negative. Unlike the other poloidal nonuniformities discussed in this paper, this one does not couple to any other current. It couples to the equilibrium only by the inertia of parallel flow. The figure 2 examples can couple, respectively, to marfes beside the X–point and at the minimum major radius.

Figure 3 is analogous to Fig. 1, except it is for closed magnetic surfaces. Application of Eq. (2) to each current sheet gives

$$\frac{R_0 B_0 I_1}{R_1^2} = p_{31} - p_{12}, \qquad \frac{R_0 B_0 I_2}{R_2^2} = p_{12} - p_{23}, \qquad \frac{R_0 B_0 I_3}{R_3^2} = p_{23} - p_{31}. \tag{6}$$

Equation (6) plus the $\nabla \cdot \mathbf{J} = 0$ condition $I_1 + I_2 + I_3 = 0$ yield

$$\left(R_1^2 - R_2^2\right) p_{12} + \left(R_2^2 - R_3^2\right) p_{23} + \left(R_3^2 - R_1^2\right) p_{31} = 0. \tag{7}$$

Equation (7) constrains less than Eq. (5), because there is no reference to zero pressure. Equation (7) requires that the pressure bounded by the greatest and least of the three current radii is intermediate between the other two pressures. For a possible marfe at the p_{31} location of Fig. (3), if $R_2 < R_1 < R_3$, then either $p_{31} > p_{23} > p_{12}$ or $p_{31} < p_{23} < p_{12}$, corresponding to high and low pressure marfes, respectively. If $R_1 < R_3 < R_2$, then either $p_{31} > p_{12} > p_{23}$ or $p_{31} < p_{12} < p_{23}$ again corresponding to high and low pressure marfes. But, if $R_1 < R_2 < R_3$, then p_{31} is intermediate between the other two pressures.

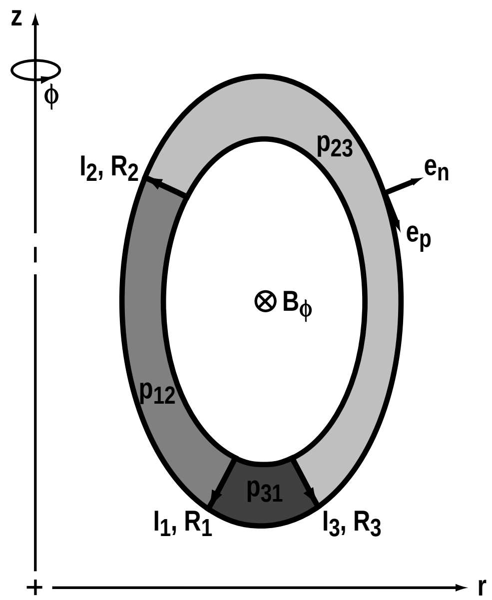

Fig.3. Elementary current layer with poloidal pressure gradients on a closed magnetic surface.

3. Parallel Equilibrium and Plasma Flow

We are interested in conditions under which a marfe might exist at a significantly higher or lower pressure than plasma elsewhere on the same magnetic surface. Even when the presssure difference is equilibrated poloidally by $J_n B_\phi$, plasma still flows along **B**. The parallel component of the multispecies momentum equation is derived and discussed in [1]. Here we simplify it to uniform B, equivalent to a straight magnetic flux tube, and two species: a single-fluid plasma and a fluid of charge-exchange (cx) neutrals that on average exit the plasma with the plasma parallel velocity. Incoming low-velocity neutrals do not affect the momentum balance. Then, if the cx neutral pressure is much smaller than the plasma pressure p,

$$(d/ds)(\Gamma v + p) \approx -\Gamma_{\mathrm{cx}} v / w_{\mathrm{cx}} , \tag{8}$$

where s = distance along B, $\Gamma = \rho v$ = plasma parallel mass flux, ρ and v are its mass density and velocity, respectively, and $\Gamma_{\mathrm{cx}} = \rho_{\mathrm{cx}} v_{\mathrm{cx}n}$ = normal component of cx neutral flux exiting the plasma from a layer of thickness w_{cx}. Other viscous terms, such as those due to poloidal flow, are neglected. In the absence of nozzle expansion, the full set of equations limits $M = v/c$ to < 1, as is well known.

Now consider parallel flow out of a marfe. Let the marfe be fueled by recycling neutrals and powered by a net deposition of energy from hotter plasma regions. The flow starts with $\Gamma = 0$ in the marfe. With no charge exchange, Eq. (8) gives the well known result

$$(p_1 - p_2)/p_2 = \gamma M^2 \tag{9}$$

where p_1 and p_2 are pressures inside and outside the marfe, respectively, γ is the specific heat ratio, $M = v/c$ = Mach number and $c^2 = \gamma p/\rho$. For $M < 1$ the marfe pressure is limited to about two times the pressure outside, regardless of fueling and heating, which affect the final Γ and v, respectivley. With charge exchange the pressure ratio can be larger [4].

4. Discussion and Conclusions

The local high or low pressure zones of Fig. 2 match to typical marfe locations at the lowest radius region of a magnetic surface (closed or open) and just inward or outward of the X–point in the SOL [3]. Because $\nabla \cdot \mathbf{J} = 0$ can be satisfied locally at these locations, there are no unexpected MHD equilibrium constraints on marfe pressure. Whether the marfe pressure can exceed the pressure on the rest of the surface then depends just on the particle flow. Application of Eq. (8) to DIII–D (R_0 = 1.7 m, a = 0.6 m) yields a (not unreasonable) particle source requirement of $\dot{N} \sim 10^{21}$ s^{-1} to sustain $p_1/p_2 \sim 2$. Similar particle source considerations apply to the Fig. 3 closed-surface location that would be just above the X–point.

The main problem is not achieving $\dot{N}$, but how to return so many particles to the marfe. In most tokamak geometries the particles must return via lower temperature and density SOL flows or even more tenuous neutral flows, all of which are flux limited. In fact, in a recent numerical simulation of divertor detachment by the UEDGE code that had a growing marfe near the X–point just inside the separatrix [5], the marfe pressure was *lower*. The parallel flow was into the marfe, and particles exited across **B**. However, marfes near the X–point (Fig. 2a) might be resupplied by the adjacent high–recycle particle flux. To conclude, in most tokamak geometries it is difficult to recycle particles at a rate sufficient to sustain high pressure marfes, but they might exist near a divertor X–point.

The detached configuration of Fig. 1 does not offer any unique localized pressure zones that might be matched to a marfe. However, it specifies an in-out pressure difference that can be large ($R_{\rm out}^2 / R_{\rm in}^2 \sim 2$ in low aspect tokamaks) and undoubtedly affects SOL and divertor equilibria quantitatively. This fundamental effect is not in any of the commonly used analytic and numerical models.

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