Sturmian Behavior of the Unstable Ideal MHD Spectrum in Axisymmetric Toroidal Equilibria*

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Numerical calculations in widely different examples of axisymmetric toroidal equilibria with q below 1 have revealed sequences of unstable ideal modes that are global eigenmode manifestations of Mercier instability. The numerical calculations were performed with a new version of the full ideal MHD code GATO [1] in which the continuum has been restabilized [2] to yield marginal points to high accuracy; previously, the continuum was numerically destabilized by the Finite Hybrid Element Method [3]. Calculations were performed for the ideal n = 1 internal kink mode in equilibria with q below 1 and slightly inverted shear as well as for an SSPX Spheromak equilibrium for intermediate n = 8. These modes exhibit the Sturmian behavior expected from spectral theory for the ideal MHD operator that the eigenvalues are ordered strictly with the number of zero crossings [4]. Numerically, the eigenvalues extend into the stable continuum and full convergence studies are needed to confirm if they actually have an accumulation point at marginal stability. For the SSPX example, the exponential accumulation near the marginal point is clearly exhibited; 18 separate modes on the unstable side of the continuum were resolved, becoming increasingly localized around the innermost q = 5/8 surface, with the first mode on the stable side of the continuum being the resonant m/n = 5/8 continuum mode. In the tokamak examples, the first few are of the same order of magnitude growth rate as the ideal internal kink. With nonideal effects, the more localized modes near marginal stability presumably become irrelevant in practice. Nevertheless, the existence of multiple unstable modes may influence the interpretation of the tokamak sawtooth instability. There are also likely confinement implications even when the modes are stabilized by nonideal effects.

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