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NONLINEAR METHODS FOR CURRENT LIMIT CONSTRAINT SATISFACTION IN TOKAMAK PLASMA SHAPE CONTROL

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ABSTRACT

The DIII–D tokamak is capable of supporting a wide variety of plasma equilibria because of its relatively large number of coils and their proximity to the plasma. To support its advanced tokamak mission, the DIII–D experimental program continues to push the envelope of this capability, frequently encountering limits imposed by allowable currents in poloidal shaping coils. Violation of current constraints is presently dealt with by operator adjustment of control targets and gains between plasma discharges. At the same time, demands for more precise and stable control have motivated efforts to develop and install advanced multivariable algorithms for control of plasma shape in DIII–D and other devices. There is currently no way to ensure respect of nonlinear current constraints in a multivariable linear controller design and no practical way to manually tune these fully coupled controllers between discharges after installation. Various linear minimization schemes can be implemented to encourage currents to remain within limits, but adherence to these limits cannot be guaranteed by linear methods alone. In this paper, we describe ongoing efforts to provide methods that guarantee currents will not exceed preset limits, and that simultaneously achieve the best obtainable quality of control subject to current limit constraints.

1. INTRODUCTION

Experimental operations of tokamaks seek to extract maximum performance from the available power supplies and PF coil set. This often leads to operation near or at power supply current and voltage limits. The DIII–D tokamak operates in proximity to one or more current limits in some part of virtually every discharge. With standard DIII–D control algorithms based on approximately one coil controlling each boundary control point, violation of these current constraints is currently dealt with by operator intervention between discharges, modifying the control or even adding an additional power supply so as to increase the current limit. Accuracy in control is sometimes sacrificed for adherence to current limits in these highly tuned controllers, since violation of a current limit usually causes a premature end to the plasma discharge.

Problems of this type must be expected in an experimental device, where new and untested equilibria are continually created according to the needs of the experimental program. However, a worthwhile goal is to provide a “soft landing” for these cases, include a graceful degradation of reference tracking in the face of impending current limits, define how this degradation should take place, and provide a warning rather than prematurely end the plasma discharge in the event of incompatible equilibrium difficulties.

Since this problem is primarily caused by requesting a reference shape that is not physically realizable by the device, what seems to be an obvious “fix” to the problem is to specify only reference shapes that are compatible with all constraints of the device. A necessary prerequisite for this approach is the ability to compute in advance reference equilibria that are completely compatible with the device constraints. However, these equilibrium calculations are dependent on current profiles, which can change in an uncontrolled manner during a plasma discharge. These calculations are also sensitive to the accuracy of the plasma response and constraint models used. The models of voltage constraints imposed by the DIII–D poloidal power systems, for example, will never be completely accurate because of the extreme nonlinearity of the chopper power supplies.

Thus, some real time methods are still desirable to handle cases where coil currents or voltages are near their limits. An ideal real time solution to this problem would have the following characteristics:

- Guarantee that the control would never cause the system to violate device constraints
- Impose no constraints on system performance in the absence of proximity to those constraints

- Provide a systematic design procedure, with guarantees for performance and stability
- Allow for operator specification of the relative importance of control of individual error signals.

The reason for the last objective is that some quality of control must be sacrificed as constraints are approached, in order to prevent actually violating the constraint. Experimentalists need to have some mechanism for specifying which control parameters should be sacrificed first in these situations.

Some of the methods developed are necessarily nonlinear and include both constrained and unconstrained minimization algorithms, calculated in real time. These methods have been generalized to the multiple control circuit configurations and plasma equilibria supported by DIII–D. Results of experimental implementations are described.

2. CONTROL DEGREES OF FREEDOM

The issue of controllable number of plasma axisymmetric equilibrium degrees of freedom (d.o.f) in a tokamak has been examined before, e.g., for C–Mod [1], ITER [2], DIII–D [3], and generically in [4] where the number of d.o.f. is reflected in a condition number of the mapping from coils to quantities equivalent to control parameters. There has always been a desire to be able to characterize this property as a single number. It is understood, however, that this number actually reflects a prerestricted class of plasmas consistent with a given device. Thus, the number of controllable d.o.f. is not an intrinsic property of the device, but a nonlinear function depending both on the device and the selected envelope of equilibria. This function clearly has a maximum value less than or equal to the number of PF coils.

The conventional approach to handling this nonlinearity is to split the overall equilibrium control problem into a nonlinear scenario (i.e., sequence of equilibria) definition and a (nominally) linear control problem. In scenario definition, equilibria consistent with the device are determined. Currents required for equilibria from previous discharges are well understood. However, equilibria defined by a priori calculations have never been completely accurate, presumably because of differences in plasma internal profiles between experiment and calculation and, in the case of DIII–D, because of the nonlinearity and complexity of the external shaping circuit, including nonlinear power supplies. Therefore, some portion of the nonlinear scenario definition work, including ensuring conformance to current limits, usually must be done via operator tuning of the shape control between discharges. This is also often true of shapes that are simply variations of previously produced equilibria. Occasionally, equilibria that are predicted to be achievable by the off-line calculations cannot in fact be reliably produced at all.

Most often, target shapes defined by the scenario method are only approximately produced, but with an accuracy that is more than adequate for experimental purposes. For example, Fig. 1 illustrates a shape that would be considered well-controlled for experimental purposes.

Some small errors are maintained at each of the isoflux control target points throughout the discharge. A linear perturbation calculation (Fig. 2) can be used to show that making all shape errors identically zero requires currents that are outside of the device limits. The experimental currents that achieve a close approximation to the equilibrium (Fig. 1) lie within the accessible region in Fig. 2, but in several coils are far from the ideally required currents. When controlling 13 control points, more typically used in DIII–D shape control, the required currents can be as much as 10 times the allowable currents [5]. A basic problem is that when perturbing from a nominal equilibrium, it is easy to produce target shapes that are not compatible with current limits, even when the original nominal equilibrium is compatible. On the other hand, it is not practical (or desirable) to recalculate offline for relatively minor variations of a shape.

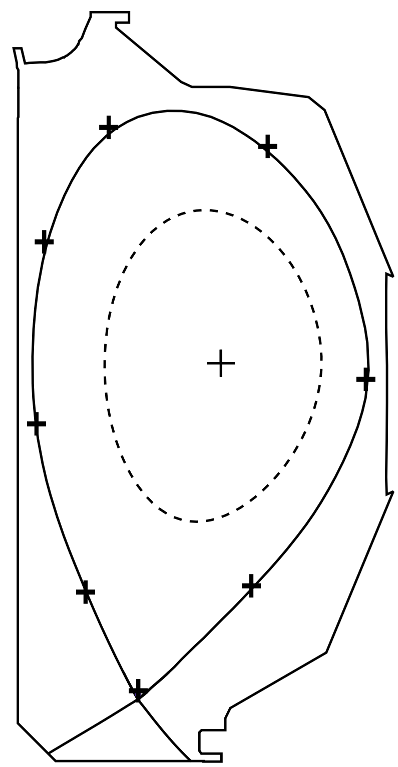

Fig. 1. Typical achieved LSN equilibrium in DIII–D and target control points (+).

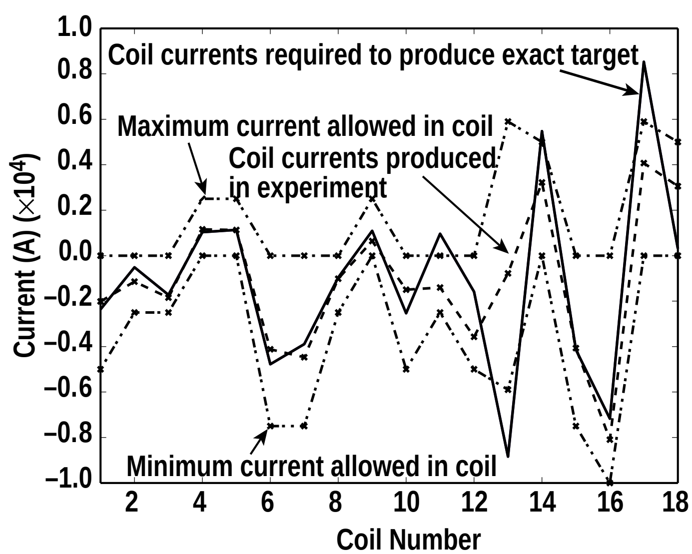

Fig. 2. Coil currents required to exactly achieve the equilibrium when using 7 control points and 9 total d.o.f. Required values are outside of limits for 4 of 18 coils.

This requirement for unobtainable currents to produce exact fits is typical. This does not pose a significant problem for the present hand-tuned shape control, but it does create a problem for linear multivariable control design, since a good linear controller will push the currents toward the illegal values in order to zero out the isoflux errors. Thus, more often that not, a plasma discharge controlled by a good multivariable controller would terminate due to a current limit violation.

The off-line approach to defining scenarios and open-loop trajectories imposes a significant demand on models to provide accurate representations of the nonlinear process of forming a shape and long-term predictions of plasma evolution. For this reason, one might consider performing a portion of the scenario definition by an automated mechanism, a portion of which may be on-line. Abstracting the present (scenario) approach, this process can be thought of as a two layer control, having a linear controller as its kernel with a nonlinear controller wrapped around it, the nonlinear control being supplied presently by an operator manually tuning the control. Thus, at some level, nonlinear control is required to guarantee that currents produced by linear controllers remain within allowable current constraints.

In the following, we describe some nonlinear approaches to this problem that have been implemented to various degrees. We also discuss some of the linear methods that have been developed to reduce the severity of this problem, and simultaneously to make the nonlinear controls more effective. The objective of the present work is to obtain the best possible control, with an assumed linear multivariable controller, subject to the current constraints of the device.

3. NONLINEAR CONTROLS

A simple method that has been tested experimentally and is fairly robust involves modification of the voltage control of the shaping power supplies. When a coil current passes a predefined threshold, the command for the power supply on that coil is modified to force the current back inside the threshold. This guarantees that no shot will be lost to current limit violations, but does not address the consequent controller windup [5].

Another nonlinear approach that has been implemented in the DIII–D real time PCS and tested [6] using hardware-in-the-loop simulations [7], is the use of a form of model predictive control (MPC) [8] to construct an error governor. The error governor is an old concept [9], with several more recent variations (e.g. [10], [11], [12]) proposing use of on-line optimization to compute the modified reference (or equivalently, modified error) signal such that predicted future values of states and inputs satisfy certain constraints.

Nearly all of the standard methods of this type involve a significant amount of on-line computation — often too much to be practical for a problem of this size. However, the combination of improved optimization algorithms and installation of a new, faster real time computer system on DIII–D [3] have made it feasible to consider some version of on-line optimization methods. A simple on-line optimization approach that was implemented for evaluation with DIII–D shape control [6] seeks an on-line constrained replacement e_c of the original computed error signal vector e_0 via the constrained minimization problem

$$\min_{e_c} \left\| W(e_c - e_0) \right\|^2 \quad , \quad \text{subject to}$$
$$I_{min} \le I_{eq} + G_{e,I}(0)\, e_c \le I_{max} \quad ,$$

where I_{eq} is the present time equilibrium current vector, $G_{e,I}(0)$ is the steady state open loop gain from errors to PF-coil currents, and W is a diagonal weighting matrix that can be used to increase relative importance of matching certain errors over others. This work demonstrated the feasibility of using on-line optimization, since it was implemented and tested on the DIII–D real time Intel Xeon processors, requiring about 30 μs to compute a sufficiently converged solution. However, use of the steady state gain in (1) tends to be conservative. The result can be a sluggish initial response to changes in a reference signal.

Although such an approach serves the important purpose of preventing future constraint violations, it is important as well to provide methods that will allow the linear controller to be more effective. For example, we would like to minimize the need for such nonlinear adjustments by providing enough actuator “headroom” to allow linear commands to take effect. This is the topic of the following section.

4. IMPROVING NOMINAL TRAJECTORIES

Open loop programming of PF coil currents has often been used to provide a nominal trajectory of currents that approximately produce a desired trajectory of equilibria. To accomplish this, the trajectory must be generated by off-line scenario calculations as described above.

As an alternative to this off-line approach, algorithms have been developed to adaptively compute a nominal coil current trajectory vector to minimize the proximity to limits while maintaining good shape control. A key point is that in the case of more PF coils than parameters to be controlled, there exists a subspace of coil current perturbations away from the equilibrium current vector that will not affect the controlled plasma shape parameters. Coil current vectors in this “shape nullspace” can be added to the equilibrium current vector to move it away from current limits.

We define I_{center} to be the vector of currents that are midway between the minimum and maximum current values for each PF coil. Given a measured current I_{meas}, we wish to find a minimizing nominal current vector $I_{nom} = \arg\min \| W(I - I_{center}) \|$ such that it produces the same error signal as I_{meas}. The weight W is used to account for the fact that different coils have different allowable coil current ranges. This problem reduces to solving the optimization problem

$$\min_{q} \left\| W\left[P_{N^\perp}(I_{meas} - I_{center}) + X_N\,(q - q_{center})\right] \right\|^2 , \tag{2}$$

where X_N is the matrix of orthonormal basis vectors for the shape nullspace N, $N^\perp$ refers to the current vector space which does affect the shape, $P_{N^\perp}$ is the projection onto $N^\perp$, q is the vector of coefficients of basis vectors for the shape nullspace, and $q_{center} = X_N^T I_{center}$. The problem (2) has the solution $q^* = Q(I_{meas} - I_{center}) + q_{center}$ where $Q = -(W X_N)^\dagger W P_{N^\perp}$, the dagger representing the pseudo inverse. Then $I_{nom} = P_{N^\perp} I_{meas} + X_N q^*$ is the desired nominal current.

Simulink [13] simulations were run to evaluate the effectiveness of this nominal trajectory calculation. A recent implementation of the real time EFIT [14] code in Simulink (using the same code as executes on the real time computers) has enabled a more accurate representation of DIII–D operation. For the first time, all of the principal nonlinearities of the DIII–D shape control plant (which includes the real time EFIT calculation) are now accessible in the Simulink environment. Figure 3 shows the plasma shape far from the targets at the start of the simulation (t = 1502 ms) and well controlled after convergence to an approximately steady state condition (t = 3185 ms). Figure 4 shows the evolution of the shape control errors during plasma current flat top as well as parts of rampup and rampdown.

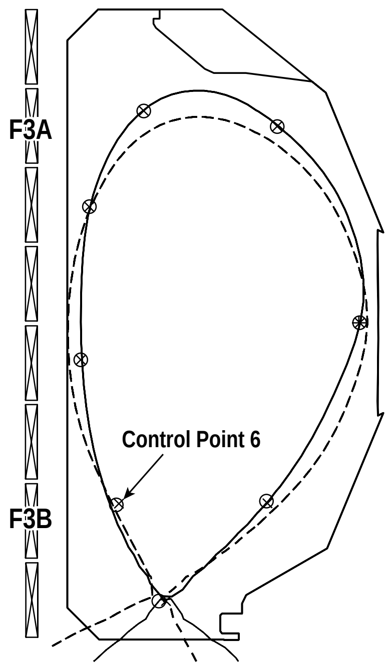

Fig. 3. Plasma equilibrium from simulation: at initial time, t = 1502 ms (dash), and just prior to plasma current rampdown, t = 3185 ms (solid). Target shape at various times is indicated by symbols: t = 1502 ms (O), t = 3185 ms (×).

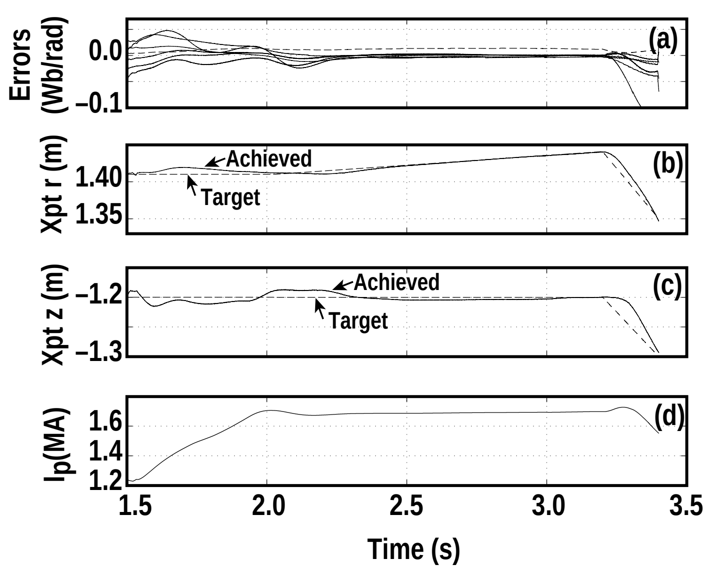

Fig. 4. Convergence of (a) isoflux shape errors, (b) X-point R position, and (c) X-point z position to targets, along with (d) plasma current during control period.

Figure 5 shows the calculated nominal current values I_{nom} as computed by (2), the smoother nominal trajectory $I_0(t)$ that was actually commanded, and the simulated coil current evolution for a single coil (F9A). The slow approach of $I_0(t)$ towards I_{nom} is the result of intentional "filtering" to avoid large disturbances to the control. The initial value of $I_0(t)$ is computed to provide bumpless transfer to the new controller. For this shape, this coil often operates near the lower current limit using the standard isoflux control. In the simulation, the nominal value I_{nom} was recomputed whenever the shape program changed significantly. The computed nominal currents are difficult to use as a nominal trajectory, because they are not continuous across successive calculations. The smoother piecewise linear commanded trajectory $I_0(t)$ is produced on-line by interpolating between the commanded value at the recompute time and the newly computed nominal value I_{nom},

$$I_0(t_k+t)=\alpha(t-t_k)\,I_{nom}(t_k)+[1-\alpha(t-t_k)]\;I_0(t_k) \quad ,$$

for $t_k \le t < t_{k+1}$, where t_k is the time at which recalculation of I_{nom} takes place, $\alpha(t)=t$, $0 \le t < 1$, and $\alpha(t)=1,\ t \ge 1$.

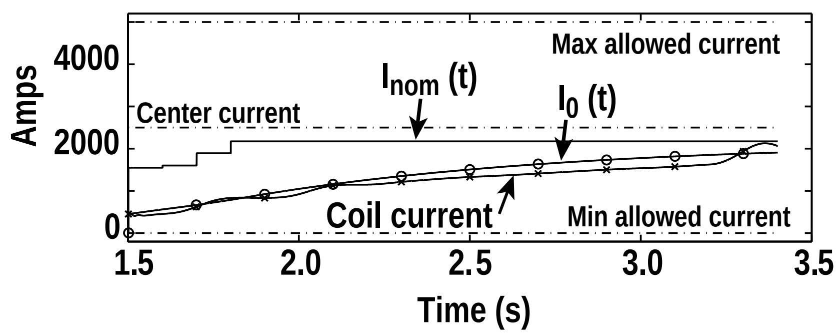

Fig. 5. Simulation illustrating calculation of desired nominal trajectory for PF coil F9A in DIII–D which tends to drift toward zero current under standard isoflux control. In simulation, the commanded trajectory $I_0(t)$ moves steadily toward the centered value of 2500 A, with the actual coil current following.

The times $t_k, k=1,2,...,\mathrm{n}$ at which to recompute $I_{nom}(t_k)$ are determined in off-line calculations after the desired evolution of the shape for the discharge is programmed. They are chosen such that the reference shape is nearly the same for times t near to each calculation time. In this simulation, we have also added additional calculation times for purpose of illustration. These additional calculation times can also serve the purpose of adjusting for changes in the equilibrium profiles, especially if linear models can be generated online from real time reconstructed equilibria.

5. TRADING LINEAR DEGREES OF FREEDOM FOR ADHERENCE TO CONSTRAINTS

The minimization problem in the previous section was posed so as to preserve the measured shape errors using I_{nom}, because this led to a linear (and, therefore, more easily realizable in realttime) solution. However, the linear controller will attempt to zero out the remaining errors, and currents will therefore diverge from the open loop trajectory. These currents can be influenced to remain nearer to the commanded trajectory at the same time by further exploiting the shape null space.

We do this by computing the null vector (coefficient) errors $q_{error} = X_N^T [I_{meas}(t) - I_0(t)]$ and feeding back to a linear controller designed to control both shape and null vector errors to zero. Note that for $t_k \le t < t_{k+1}$, $I_0(t)$ provides only an approximation to $I_{nom}(t_k)$, so the null vector reference signal $q_{ref} = X_N^T I_0(t)$ provides only an approximation to the optimized reference signal $q^*(t_k) = X_N^T I_{nom}(t_k)$.

For DIII–D, the number of d.o.f. is not taken to be the total number of coils, but a reduced number equal to the number of power supplies in the PF coil circuit, which reflects the number of independent current d.o.f. the circuit is able to maintain in steady state. The number of controlled shape errors plus the number of controlled null vector coefficients is equal to this total number of current d.o.f.

Figure 6 shows that null vector control is able to prevent current limiting for some coils, but not all. A controller that operates on null vector errors as well as shape errors produces a current trajectory for coil F3A which is well within the constraints, while a similar controller (identical weightings used in design) operating only on shape errors shows a current trajectory that violates the constraints. However, the coil F3B (not shown) violates current limits whether or not null vectors are being controlled. The effect of this can be seen in Fig. 4(a), where the isoflux control point #6 error (dashed line) does not converge to zero. Cause and effect can be clearly seen in Fig. 3, where isoflux control point #6 is located adjacent to coil F3B.

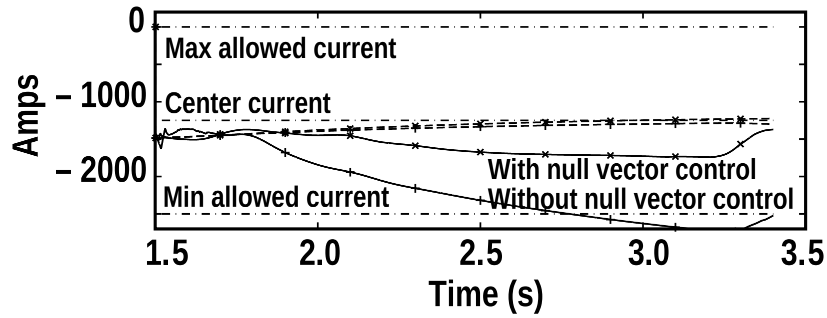

Fig. 6. Nominal trajectories produced by controllers with (×, dashed) and without (+, dashed) feedback of shape null vectors are nearly identical. The current evolution produced by a controller controlling null vectors stays far from current limits (x,solid), while the current produced by a controller without null vector control violates the current limits (+,solid). The target signals, initial currents, and linearized response model for this simulation were all derived from DIII–D discharge 107673.

6. CONCLUSIONS

In this paper, we have outlined some root causes for PF coil current limiting when using multivariable linear controllers and suggested some methods for preventing this problem. Some of these methods have been implemented in the real time control system at DIII–D and a few have been tested experimentally. The basic control approach is two layered with a nonlinear controller wrapped around the linear multivariable controller. Our basic premise is that some of the nonlinear off-line calculations presently performed under the guise of scenario or equilibrium development can be replaced by on-line calculations, resulting in a reduced operator workload and less stringent requirements on model accuracy. The tradeoff is that these calculations must be capable of being performed in real time. The overall objective of this work is to obtain the best possible control, with an assumed linear multivariable controller, while simultaneously guaranteeing adherence to the current constraints of the device. Such methods have important applications in future devices, since the ability to design and build devices with smaller control margins can mean a significant savings in cost of construction and operation.

In addition to use of nonlinear methods to guarantee adherence to coil current constraints, we have described some linear techniques for improving the ability of the linear control to simultaneously reduce shape error and to increase the “headroom” needed to operate effectively. This includes an algorithm for adaptive generation of a nominal trajectory of coil currents and a method for influencing coil currents to stay near to this trajectory. In addition to the usual benefits of a nominal trajectory, e.g., allowing control gains to remain low, these algorithms allow the current trajectory to remain as far as possible from current limits, consistent with desired shape.

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