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SIMILARITY IN H–MODE ENERGY CONFINEMENT: ν* RATHER THAN $n/n_{\lim it}$ SHOULD BE KEPT FIXED

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ABSTRACT

While most of the dimensionless parameters that govern energy transport are readily apparent, there is some controversy as to whether the collisionality should be represented by the collision frequency normalized to the bounce frequency (ν_*) or the density normalized to the Greenwald density limit (n/n_{limit}). To help resolve this question, experiments on the DIII–D and JET tokamaks have compared the normalized energy transport in ELMing H–mode plasmas with matched dimensionless parameters. When ν_* was kept fixed, the normalized energy transport on JET and DIII–D was in good agreement, but when n/n_{limit} was kept fixed, the normalized energy confinement time on DIII–D was 20% lower than on JET (for all cases, $n/n_{limit} < 1$). Therefore, ν_* appears to be the correct form of the normalized density, and scaling transport properties from present day tokamaks to ITER at fixed n/n_{limit} can result in incorrect predictions.

1. INTRODUCTION

The principle of similarity is a powerful tool for understanding complex physical phenomenon because it dictates that the physics is independent of the size scale [1]. Similarity occurs when for a fixed set of dimensionless parameters, a family of systems can exist with different sets of dimensional parameters. In this situation, tokamaks of different physical size will have the same energy transport (normalized to the Bohm diffusivity) when the dimensionless parameters like relative gyroradius ($\rho_* \sim m_i^{0.5} T^{0.5} / aB_T$), ratio of kinetic pressure to magnetic field pressure ($\beta \sim nT / B_T^2$), safety factor ($q \sim aB_T / RB_p$), etc., are matched. Here m_i is the hydrogen isotope mass, T is the plasma temperature, n is the plasma density, a is the minor radius, R is the major radius, and B_T and B_p are the toroidal and poloidal magnetic field strengths. While many of the important dimensionless parameters are apparent, there seems to be some debate among the fusion energy science community as to whether the collisionality (i.e., the dimensionless form of the density) should be represented by the collision frequency normalized to the bounce frequency ($\nu_* \sim Z_{eff}\, nq / T^2$, where Z_{eff} is the effective ion charge), or by $n/n_{\lim it}$, the density normalized to the Greenwald density limit [2,3]. This is not merely an academic question since it strongly affects how demonstration discharges on present day machines are scaled to experiments such as ITER. To resolve this question, experiments on the DIII–D and JET tokamaks in deuterium have compared the normalized energy transport in dimensionally identical ELMing H–mode plasmas at (1) fixed ν_*, and (2) fixed $n/n_{\lim it}$. Only plasmas with $n/n_{\lim it} < 1$ are examined in this paper. Of course, other dimensionless forms of the collisionality besides ν_* and $n/n_{\lim it}$ exist, but these two are representative of the other forms and, according to the Buckingham Π theorem [1], can be related to the other forms by a transformation of dimensionless variables.

First, it is instructive to apply dimensional analysis to the Greenwald density limit, which is given by the empirically derived relation [2]

$$n_{\text{limit}}\left(10^{20} m^{-3}\right) = \frac{I_p(MA)}{\pi a(m)^2} \quad , \tag{1}$$

where I_p is the plasma current. It has been noted previously [3,4] that the ratio $n/n_{\lim it}$ is not "dimensionally correct" since it cannot be constructed from the dimensionless variables believed to be important for plasma physics. This is the reason why ν_* and $n/n_{\lim it}$ cannot both be matched between otherwise self-similar plasmas of different physical size. However, if one assumes that $n_{\lim it}$ is dependent upon edge atomic physics, then the density ratio can be made dimensionally correct by introducing Planck's constant ($\hbar$) and the corresponding atomic unit of energy $\xi_o = m_e e^4 / \hbar^2 \approx 27\ eV$ [5],

$$\frac{n}{n_{\text{limit}}} \propto \frac{\beta q}{\varepsilon \rho_*} \sqrt{\frac{\xi_o}{T}} \quad , \tag{2}$$

where m_e is the electron mass, e is the electron charge, and ε is the inverse aspect ratio. Planck's constant can also be used to define a Greenwald density limit that is dimensionally correct,

$$n_{\text{limit}} = 35 \frac{\hbar}{e^3} \frac{I_p}{\pi a^2} \quad . \tag{3}$$

Thus, if similarity is obeyed between plasmas with identical values of ν_*, ρ_*, β, q, etc., but different n/n_{limit}, then this would indicate that edge atomic physics does not significantly influence the plasma properties.

2. FIXED ν_* COMPARISONS

Several previous publications have reported experimental tests of similarity in ELMing H–mode plasmas on DIII–D and JET while keeping ν_* fixed [6,7], which are briefly reviewed here. The principle of similarity states that two plasmas that have identical values for the important dimensionless parameters must also have the same normalized confinement $(B_T\tau_{th})$ and normalized transport χ/χ_{Bohm} despite having different physical parameters [8]. Here τ_{th} is the energy confinement time of the thermal plasma component, χ is the thermal diffusivity, and $\chi_{Bohm} = T/eB_T$ is a Bohm-like diffusion coefficient. To match the local dimensionless parameters ρ_*, β, q, and ν_* between two plasmas of different physical size but identical plasma shape, the quantities $B_T a^{5/4}$, na^2, $Ta^{1/2}$, and $I_p a^{1/4}$ need to be kept fixed (note that n/n_{limit} is not kept fixed in this case). As shown in Table 1 of Ref. 6 and Table 1 of Ref. 7, the normalized confinement times on JET and DIII–D agreed to within $\approx 5\%$ under these conditions, which is within the measurement uncertainties.

Table 1: Comparison of the global engineering and dimensionless matching criteria for similarity tests at fixed Greenwald density ratio.

	JET	DIII–D
Engineering		
a (m)	0.91	0.55
R (m)	2.91	1.72
B_T (T)	1.18	1.98
I_p (MA)	1.28	1.27
$\bar{n}(10^{19}\ m^{-3})$	3.6	10.2
P_{abs} (MW)	5.5	6.7
τ_{th} (s)	0.32	0.15
Dimensionless		
R/a	3.20	3.10
κ	1.82	1.77
$B_T\, a$	1.07	1.10
$\bar{n}\, a^2$	3.0	3.1
β_{th} (%)	2.8	2.7
q_{95}	3.4	3.4
$B_T\, \tau_{th}$	0.38	0.30

The good agreement in $B_T\tau_{th}$ between these two tokamaks for self-similar conditions can also be seen in Fig. 1 (open circles) of this paper. In addition, the normalized one-fluid diffusivities for DIII–D and JET were shown to be in good agreement for dimensionally identical discharges in Fig. 4 of Ref. 7, where the scaling of the Bohm diffusion coefficient $\chi_{Bohm} \propto a^{3/4}$ for self-similar plasmas has been utilized. Therefore, the principle of similarity has been validated for tokamak energy transport in ELMing H–mode plasmas using fixed ρ_*, β, q, and ν_*. This indicates that effects not considered, such as Debye length scale physics and atomic physics, do not play a dominant role in the energy transport process.

3. FIXED n/n_{limit} COMPARISONS

New tests of similarity in ELMing H–mode plasmas have been done between DIII–D and JET while keeping constant the density normalized to the Greenwald density limit rather than ν_*. To match the local dimensionless parameters ρ_*, β, q, and n/n_{limit} between two plasmas of different physical size but identical plasma shape, the quantities $B_T a$, na^2, Ta^0, and $I_p a^{\upsilon}$ need to be kept fixed. This will result in the collisionality scaling like $\nu_* \propto a^{-1}$. Global parameters from a matched pair of ELMing H–mode plasmas on JET and DIII–D with $n/n_{\text{limit}} \approx 0.75$ are shown in Table 1. Both plasmas were heated mainly by co-neutral beam injection such that the toroidal rotation effects were similar. The normalized global parameters that were experimentally controlled, including n/n_{limit}, were matched to within 5%. However, Table 1 shows that similarity was not well satisfied since the normalized thermal confinement time on DIII–D was 20% smaller than on JET. Figure 1 plots $B_T\tau_{th}$ for a couple of JET/DIII–D similarity comparisons with fixed n/n_{limit} (closed squares), demonstrating that the normalized confinement for this type of comparison was not as well matched as for the fixed ν_* comparisons described in Section 2 (open circles).

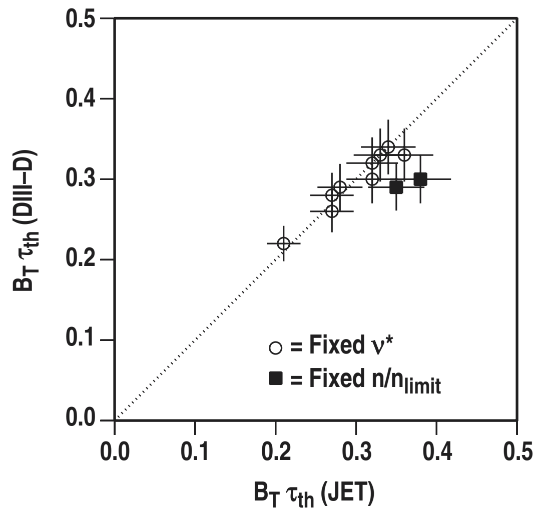

Fig. 1. Comparison of normalized thermal confinement times of JET and DIII–D H–mode discharges with fixed ν_* (open circles) or fixed n/n_{limit} (filled squares). The dotted line shows the expected locus for pairs with perfect similarity. Fixed uncertainties of 10% are shown.

The normalized plasma profiles were also not as well matched for the fixed n/n_{limit} comparisons between DIII–D and JET, compared to fixed ν_*, indicating that the plasmas were not as self-similar. Figure 2 shows the profiles of normalized electron density, electron and ion temperatures, and effective ion charge as a function of the normalized toroidal flux coordinate (ρ). Although the volume average values were reasonably well matched (except for Z_{eff}, where JET reported a higher value than DIII–D for both the fixed ν_* and fixed $n/n_{\lim it}$ comparisons), the profile shapes were more peaked on DIII–D compared to JET. In addition, the heights of the H-mode pedestals at $\rho \approx 0.8$ were not as well matched for the fixed $n/n_{\lim it}$ case in Fig. 2 as for the fixed ν_* case seen in Fig. 3 of Ref. [7].

Note that while T_e/T_i was not matched exactly between the two tokamaks, the difference was in the wrong direction to explain the mismatch in the normalized thermal confinement times [9]. A more stringent test of similarity using energy transport is shown in Fig. 3, where the normalized thermal diffusivity is given by χa^{-1} because the Bohm diffusion coefficient scales like $\chi_{Bohm} \propto a$ for this fixed $n/n_{\lim it}$ comparison. The experimental value of χ is obtained by solving the energy continuity equation, which can be written schematically as

$$\frac{3}{2} n \frac{\partial T}{\partial t} + \nabla \cdot Q = S \quad , \tag{4}$$

where Q is the energy flux and S represents the net value of the various sources and sinks of energy. The thermal diffusivity is determined experimentally by dividing the conductive portion of Q by the quantity $-n\nabla T$. The density was sufficiently high on DIII–D that separation of the plasma into distinct ion and electron fluids with individual diffusivities was not possible. The transport coefficients in Fig. 3 are not plotted near the plasma center and edge because the effects of sawteeth and ELMs could not be properly taken into account. Figure 3 shows that the normalized diffusivities on JET and DIII–D for the inner regions of the plasma were the same to within the displayed uncertainties; however, for $\rho > 2/3$ the normalized diffusivity on DIII–D was significantly greater than on JET. The displayed uncertainties were evaluated by taking a population standard deviation of the diffusivities using the time history. The agreement between the JET/DIII–D normalized diffusivities appear to be worse for the fixed $n/n_{\lim it}$ comparison than for the fixed ν_* comparison discussed in Section 2, although owing to the relatively small size difference between these two tokamaks a definitive conclusion on this point cannot be reached.

Alternately, the experiments described in this section can be interpreted as a scan of the normalized collisionality at constant ρ_*, β, q, etc., since $\nu_* \propto a^{-1}$ for these fixed $n/n_{\lim it}$ comparisons. In this case, the discrepancy between $B_T\tau_{th}$ on DIII–D and JET shown by the filled squares in Fig. 1 can be explained by the factor-of-1.6 change in ν_* if the thermal confinement time has a collisionality scaling like $B_T\tau_{th} \propto \nu_*^{-0.4}$. This scaling of the thermal confinement time with normalized collisionality is in good agreement with previous measurements from DIII–D that varied ν_* by a factor of 8 in ELMing H–mode plasmas and found $B_T\tau_{th} \propto \nu_*^{-0.42\pm0.03}$ [10]. Experiments on JET found a similar ν_* scaling of the energy confinement time [11,12]. This shows that the ν_* dependence of τ_{th} is the same regardless of whether $n/n_{\lim it}$ is allowed to vary, as it did in single machine experiments, or $n/n_{\lim it}$ is held constant, as was the case for these multiple machine experiments. In addition, Ref. 10 shows that the ν_* scaling of transport increases in the outer, more collisional, regions of the plasma, which could explain why the disagreement between the normalized thermal diffusivities on JET and DIII–D are largest for $\rho > 2/3$ in Fig. 3. This discussion supports the notion that ν_* is a better dimensionless parameter than n/n_{limit} in regard to predicting the transport properties of plasmas with different physical sizes.

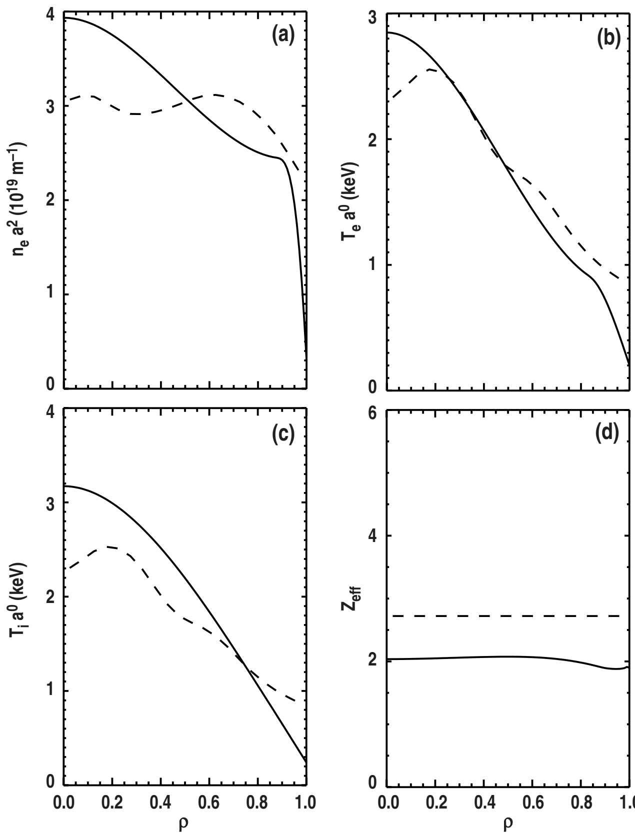

Fig. 2. Normalized profiles from DIII–D (solid lines) and JET (dashed lines) of (a) electron density, (b) electron temperature, (c) ion temperature, and (d) effective ion charge, for the test of similarity at fixed n/n_{limit}.

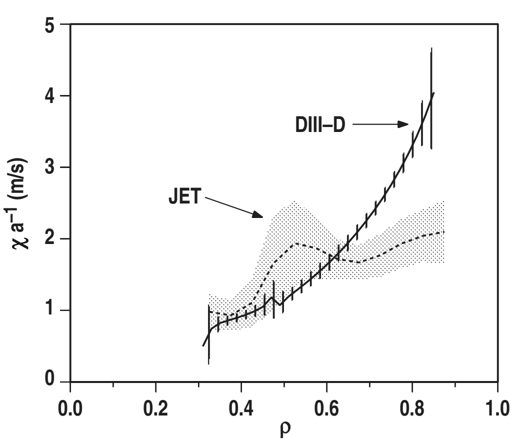

Fig. 3. One-fluid thermal diffusivity normalized to Bohm scaling as a function of the normalized toroidal flux coordinate for DIII–D (solid line) and JET (dash line).

4. EXTRAPOLATION TO ITER

The determination as to whether ν_* or n/n_{limit} is the correct dimensionless quantity to satisfy the principle of similarity is not merely an academic question because it strongly affects how demonstration discharges on present day tokamaks are scaled to experiments such as ITER [13]. Although it is well known that the only practical path to ignition involves constructing new devices that operate at smaller ρ_* [14], some publications have projected present day confinement properties to ITER along a gyro-Bohm-like scaling path at fixed ν_* [15] while others have made confinement projections at fixed n/n_{limit} [16] (β, q, and the plasma shape are assumed to remain constant in these projections). Furthermore, some publications state that ITER demonstration discharges on present day tokamaks should have the same ν_* as projected for ITER [17] while other publications have stated that it is relevant to match n/n_{limit} instead [18]. In this section, the ramifications of these two extrapolation paths are discussed.

For the case of a constant ν_* projection to ITER by decreasing ρ_*, the Greenwald density ratio will increase by a factor of 3 from DIII–D to ITER. This is easily determined from the scalings $n \propto B_T^{4/3} a^{-1/3}$ and $I_p \propto B_T a$ needed to keep ν_*, β, and q constant [17]; therefore, $n/n_{\rm limit} \propto B_T^{1/3} a^{2/3}$. This is the reason why demonstration discharges on present day tokamaks need to operate with low values of n/n_{limit} when matching the expected collisionality of ITER. For the case of a constant n/n_{limit} projection to ITER by decreasing ρ_*, the normalized collisionality will vary along the projection path like $\nu_* \propto \rho_*^2 a^{-1} \propto B_T^{-1} a^{-2}$, which means that ν_* must be reduced by a factor of 30 or more when extrapolating DIII–D discharges to ITER at fixed n/n_{limit}. Given the ν_* scaling of H–mode energy confinement discussed in Section 3, this change in ν_* cannot be ignored because the variation in confinement is a factor of 4. Furthermore, the relative increase in confinement from present day tokamaks to ITER is larger along the fixed ν_* path than along the fixed n/n_{limit} path. Assuming that the thermal confinement time scales like $\tau_{th} \propto B_T^{-1} \rho_*^{-3} \nu_*^{-0.4}$ along a dimensionally similar path (which is different than the commonly used IPB98(y,2) scaling [15]), the confinement gain from DIII–D to ITER is a factor of 60 for fixed ν_* but only a factor of 40 for fixed n/n_{limit}. Since the former involves only an extrapolation in one dimensionless parameter (ρ_*) while the latter involves an extrapolation in two (ρ_* and ν_*), it is reasonable to believe that the fixed ν_* path will give more reliable confinement projections to ITER.

5. CONCLUSION

Experiments on the DIII–D and JET tokamaks have compared the normalized energy transport in ELMing H–mode plasmas at (1) fixed ν_*, and (2) fixed $n/n_{\lim it}$ with the other dimensionless parameters held constant and $n/n_{\lim it} < 1$. Although other forms of the dimensionless collisionality are possible, these two were chosen as representative and can be related to the other forms by a transformation of dimensionless variables [1]. When ν_* was kept fixed, the normalized energy confinement times on JET and DIII–D agreed to within $\approx 5\%$, which is within the measurement uncertainties. On the other hand, when $n/n_{\lim it}$ was kept fixed, the normalized energy confinement time on DIII–D was 20% smaller than on JET. This discrepancy in the latter comparison can be explained by a collisionality scaling of the thermal confinement time like $B_T \tau_{th} \propto \nu_*^{-0.4}$, which agrees with previous ν_* scaling experiments in ELMing H–mode discharges on DIII–D and JET. Therefore, these experiments showed that ν_* is a correct dimensionless form of the density for satisfying the principle of similarity. This does not mean that $n/n_{\lim it}$ is not an important quantity for determining the operational limits of tokamaks, but rather $n/n_{\lim it}$ is not a relevant plasma physics dimensionless parameter on par with ρ_*, β, ν_*, etc., and that scaling energy transport properties from present day tokamaks to ITER at fixed $n/n_{\lim it}$ will result in incorrect predictions unless the ν_* dependence is explicitly taken into account.

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