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[£]Lehigh University, Bethlehem, Pennsylvania

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ABSTRACT

A new gyro-Landau fluid model for the Landau damping of the circulating particles is presented. Since trapped particles can bounce average low frequency waves they are not affected by Landau damping. The loss of bounce averaging when the mode frequency exceeds the trapped particle bounce frequency is included in the model by changing the phase space integration boundary. An excellent fit to the kinetic response for the circulating particle density and the parallel and perpendicular pressure is obtained with six moment equations.

1. INTRODUCTION

The theory based transport model GLF23, published in 1997 [1], has proven to be an accurate predictor of the core energy confinement in L–mode and H–mode tokamak plasmas. The strength of the model is that it closely approximates the linear growth rates of the dominant drift-wave instabilities by using a gyro-Landau fluid (GLF) model to compute the growthrates. Since the temperature profiles are often close to the marginal linearly stable profiles for these modes, the quasi-linear GLF23 model can succeed with only a crude model for the saturated amplitude of the turbulence. The original GLF23 model was valid for only a limited range of magnetic shear and Shafranov shift and used a shifted circle magnetic geometry. Hence, it could not be used for modeling the plasma edge. It has also been observed [2] that experimental plasma condition in the outer part of the plasma can have very strongly unstable electron temperature gradient modes (ETG). These modes normally are not unstable in the same range of poloidal wavenumber as the ion temperature gradient mode (ITG). However, in the outer 20% of the plasma, kinetic linear stability analysis has found the unstable wavenumber range of ETG and ITG modes can overlap. This situation violates the assumption of a stable gap between ETG and ITG wavenumber ranges used in GLF23. It also opens up the possibility that ETG modes can produce significant particle, ion heat and momentum transport in addition to the electron thermal transport in the overlapping region.

In order to address these limitations of GLF23, and with the aim of making a transport model which is valid closer to the separatrix , we have begun development of a new model. The new model aims to treat electrons and ions together on the same footing, so both ETG and ITG modes can co-exist. This paper focuses on the development of a new GLF model for the passing particles which includes the loss of Landau damping due to bounce averaging of trapped particles. Such a model is required in order to be capable of treating the overlapping ion and electron instability domain. Only the parallel response to zero beta electrostatic potential perturbations is considered in this paper.

The original GLF model of Hammett and Perkins [3] used a cleaver closure of the fluid moments of the gyro-kinetic equations which gave an excellent approximation to the kinetic effect of Landau damping. This model was for circulating particles only. The 4-moment

Hammett-Perkins model was extended to by adding two perpendicular moments in Ref. 4. Gyro-Landau fluid models with toroidal drifts have been developed [5,6] using the methodology of fitting closure coefficients to approximate the exact kinetic response to an electrostatic potential perturbation. Gyro-fluid equations for trapped electrons have also been developed [7,8]. In Ref. 8 fluid equations are derived from the bounce averaged gyrokinetic equation. The bounce average assumes the frequency of the instability is less than the bounce frequency. This assumption breaks down for the high-k ETG modes. Hence, it is not possible to treat ETG modes by coupling the bounce average equations for trapped electrons to equations for circulating electrons. An alternative way of including the mirror force in a GLF system has been given in Ref. 6 However, this approach does not capture the bounce averaging of the trapped particles. In Ref. 7 the fluid equations are derived by integrating over the trapped and passing regions of velocity space separately. This is the approach taken here but without the restriction to small trapped fraction assumed in Refs. 5 and 1.

In this paper the 4+2 moment model for Landau damping of Ref. 4 is extended to include the exclusion of bounce averaging trapped electrons from the moment equations. The trapped particles modify the parallel wavenumber terms in the GLF equations. A model for the reduction in Landau damping due to trapped particles which is fit to the kinetic response will be determined. This model will in the future be included in a full GLF model including the toroidal drift terms and the trapped particles.

2. GYRO-LANDAU FLUID MODEL

Only the parallel wavenumber terms in the GLF equations need to be modified by trapped particles so it is sufficient to consider a reduced kinetic equation with only motion along the magnetic field. The reduced kinetic equation is the one dimensional Vlasov equation

$$\frac{\partial f}{\partial t} + v_{\parallel}\nabla_{\parallel} f + \frac{e}{m} E_{\parallel} \frac{\partial f}{\partial v_{\parallel}} = 0 \quad . \tag{1}$$

Linearizing this equation for a periodic perturbation of the form

$$E_{\parallel} = -ik_{\parallel} \frac{T_0}{e} \tilde{\Phi} e^{i\left(k_{\parallel} z - \omega t\right)} \quad , \tag{2}$$

$$f = f_0 + \tilde{f} \;\; e^{i\left(k_{\parallel} z - \omega t\right)} \quad , \tag{3}$$

yields

$$-\sqrt{2}\zeta v_t \tilde{f} + v_{\parallel}\tilde{f} + v_{\parallel}\tilde{\Phi} f_0 = 0 \quad , \tag{4}$$

where $v_t = \sqrt{T_0 / m}$, $\zeta = \omega / \sqrt{2}\, v_t k_{\parallel}$. The static distribution function is a Maxwellian

$$f_0 = \frac{n_0}{\left(2\pi v_t^2\right)^{3/2}} e^{-\frac{v^2}{2 v_t^2}} \quad . \tag{5}$$

The solution to Eq. (4) is

$$\tilde{f} = \frac{v_{\parallel} f_0}{\sqrt{2} v_t \zeta - v_{\parallel}} \tilde{\Phi} \backslash \quad . \tag{6}$$

In order to construct the GLF system of moment equations the velocity moments of the kinetic equation [Eq. (4)] will be taken. The solution of the fluid moment equations will then be found taking the highest velocity moment as a linear combination of the lower moments to close the system of equations. The closure coefficients for the fluid moment equations are chosen to give a good fit to the corresponding moments of the kinetic solution Eq. (6).

The normalized moments of the perturbed distribution functions and the corresponding response functions are defined by:

density

$$\tilde{n} = \frac{1}{n_0}\int d^3v\tilde{f} = -R_n\tilde{\Phi} \quad , \tag{7}$$

parallel velocity

$$\tilde{u}_{\parallel} = \frac{1}{n_0 v_t}\int d^3v v_{\parallel}\tilde{f} = -R_u\tilde{\Phi} \quad , \tag{8}$$

parallel pressure

$$\tilde{p}_{\parallel} = \frac{1}{n_0 v_t^2}\int d^3v v_{\parallel}^2\tilde{f} = -R_{p_\parallel}\tilde{\Phi} \quad , \tag{9}$$

parallel energy flux

$$\tilde{Q}_{\parallel} = \frac{1}{n_0 v_t^3}\int d^3v v_{\parallel}^3\tilde{f} = -R_{Q_\parallel}\tilde{\Phi} \quad , \tag{10}$$

parallel thermal stress

$$\tilde{r}_{\parallel,\parallel} = \frac{1}{n_0 v_t^4}\int d^3v v_{\parallel}^4\tilde{f} = -R_{r_{\parallel,\parallel}}\tilde{\Phi} \quad . \tag{11}$$

The first four fluid moments of Eq. (4) are

$$-\sqrt{2}\zeta\tilde{n} + \tilde{u}_{\parallel} = 0 \quad , \tag{12}$$

$$-\sqrt{2}\zeta\tilde{u}_{\parallel} + \tilde{p}_{\parallel} + g_{p_\parallel}\tilde{\Phi} = 0 \quad , \tag{13}$$

$$-\sqrt{2}\zeta\tilde{p}_{\parallel} + \tilde{Q}_{\parallel} = 0 \quad , \tag{14}$$

$$-\sqrt{2}\zeta\tilde{Q}_{\parallel} + \tilde{r}_{\parallel,\parallel} + g_{r_{\parallel,\parallel}}\tilde{\Phi} = 0 \quad , \tag{15}$$

where

$$g_{p_\parallel} = \frac{1}{n_0 v_t^2}\int d^3v v_\parallel^2 f_0 \quad , \tag{16}$$

and

$$g_{r_{\parallel,\parallel}} = \frac{1}{n_0 v_t^4}\int d^3v v_\parallel^4 f_0 \quad . \tag{17}$$

These equations [Eqs. (12)–(15)] are identical to the four moment system of Hammett and Perkins [3] if $g_{p_\parallel} = 1$ and $g_{r_{\parallel,\parallel}} = 3$ which is the case of no trapped particles. They included a viscous stress term in Eq. (13) but showed that such a term is unphysical.

In toroidal geometry, the kinetic equation [Eq. (1)] is still valid for perturbations which have a finite parallel wavenumber but vanishing perpendicular wavenumber (i.e. no diamagnetic or toroidal drift terms). In a torus, the parallel velocity is a function of the total velocity v^2 and magnetic moment $\mu = v_\perp^2/2B$ which are constants of the guiding center motion. The magnitude of the magnetic field $B = |\vec{B}|$ varies on a flux surface with the poloidal angle $B = B(\theta)$. Thus, the parallel velocity varies on a flux surface. It is convenient to introduce the ratios

$$\varepsilon = \frac{v_\parallel^2}{v^2},\ \lambda = \frac{2\mu}{v^2} \quad . \tag{18}$$

These are related by $\varepsilon = 1 - \lambda B$. Trapped particles have $\lambda > \lambda_t = 1/B(\pi)$. At the trapped-passing boundary $\lambda = \lambda_t$ and the ratio ε is restricted to $\varepsilon < \varepsilon_t = 1 - \lambda_t B(0)$. The equilibrium circulating particle fraction is found by integrating only over the circulating particle region of velocity space (i.e. $\varepsilon \geq \varepsilon_t$)

$$g_n = \frac{1}{n_0}\int_{\varepsilon \geq \varepsilon_t} d^3v f_0 = 1 - \sqrt{\varepsilon_t} \quad . \tag{19}$$

Thus, $\sqrt{\varepsilon_t}$ is the fraction of trapped particles. The higher moment fractions [Eqs. (16),(17)] evaluate to $g_{p_\parallel} = 1 - \varepsilon_t^{3/2}, g_{r_{\parallel,\parallel}} = 3\left(1 - \varepsilon_t^{5/2}\right)$. For now, it will be assumed that all trapped particles bounce average the Landau resonance, so they do not contribute to the response functions. Later a more precise restriction to only those trapped particles which can bounce average a given wave will be made. Restricting the integration in Eq. (7) to the passing region of velocity space and using the exact kinetic solution [Eq. (6)] for the perturbed distribution function gives the modified kinetic density response [9].

$$R_n = g_n + \zeta\left(Z(\zeta) - Z\left(\zeta/\sqrt{\varepsilon_t}\right)\right) \quad , \tag{20}$$

where $Z(\zeta)$ is the plasma dispersion function [9].

The higher moment kinetic response functions [Eqs. (8)–(11)] are

$$R_u = \sqrt{2}\zeta R_n \quad , \tag{21}$$

$$\quad , \tag{22}$$

$$R_{Q_{\parallel}} = \sqrt{2}\zeta R_{p_{\parallel}} \quad , \tag{23}$$

$$R_{r_{\parallel,\parallel}} = g_{r_{\parallel,\parallel}} + 2\zeta^2 R_{p_{\parallel}} \quad , \tag{24}$$

It is easy to verify that these exact kinetic response functions satisfy the moment equations [Eqs. (12)–(15)].

The GLF equations are constructed to give a good approximation to the kinetic response functions. This is accomplished by closing the moment equations by expressing the highest moment as a linear combination of the lower moments. First introducing the heat flux $\tilde{q}_{\parallel} = \tilde{Q}_{\parallel} - \Gamma_{\parallel}\tilde{u}_{\parallel}$, where $\Gamma_{\parallel} = g_{r_{\parallel,\parallel}}/g_{p_{\parallel}}$ then eliminating the electrostatic potential term from Eq. (15) using Eq. (13) gives

$$-\sqrt{2}\zeta\tilde{q}_{\parallel} + \tilde{r}_{\parallel,\parallel} - \Gamma_{\parallel}\tilde{p}_{\parallel} = 0 \quad . \tag{25}$$

The adiabatic limit of the thermal stress moment is $\tilde{r}_{\parallel,\parallel} - \Gamma_{\parallel}\tilde{p}_{\parallel} \rightarrow 0$. Thus, the closure model is taken to have the form

$$\tilde{r}_{\parallel,\parallel} = \Gamma_{\parallel}\tilde{p}_{\parallel} + \left(\Gamma_{\parallel} + \beta_{\parallel}\right)\left(\tilde{p}_{\parallel} - g_{T_{\parallel}}\tilde{n}\right) - i\sqrt{2}D_{\parallel}\frac{|k_{\parallel}|}{k_{\parallel}}\tilde{q}_{\parallel} \quad , \tag{26}$$

where $g_{T_{\parallel}} = g_{p_{\parallel}}/g_n$ ensures the correct adiabatic limit. This closure reduces to the one of Hammett and Perkins for no-trapped particles (note the correspondence to Ref. 3 $\beta_{\parallel} = 2\beta_1, D_{\parallel} = D_1$) . Only two real coefficients $\left(\beta_{\parallel}, D_{\parallel}\right)$ are free to be fit to the kinetic response functions. The density response function obtained from the 4-moment equations with the closure [Eq. (26)] is (for $k_{\parallel} > 0$)

$$R_{n,4} = \frac{g_{p_{\parallel}}\left[\Gamma_{\parallel} + \beta_{\parallel} - 2\zeta\left(\zeta + iD_{\parallel}\right)\right]}{\left(\Gamma_{\parallel} + \beta_{\parallel}\right)\left(g_{T_{\parallel}} - 2\zeta^2\right) + 2\zeta\left(\zeta + iD_{\parallel}\right)\left(2\zeta^2 - \Gamma_{\parallel}\right)} \quad . \tag{27}$$

Both the adiabatic limits ($\zeta = 0$) and the high phase velocity limits ($|\zeta| >> 1$) of the kinetic density response function are correctly fit by this solution to the GLF equations. The higher moment response functions can be found from the density response $R_{n,4}$ using the relations analogous to Eqs. (21)–(23) except for the thermal stress Eq. (24). The adiabatic and high phase velocity limits of the higher moments up to $R_{Q_{\parallel},4}$ are also found to match the limits of the corresponding kinetic response functions. The two closure coefficients $\left(\beta_{\parallel}, D_{\parallel}\right)$ were determined by Hammett and Perkins by fitting to the small phase velocity limit of the density response. They found $\beta_{\parallel} = \beta_{HP} = (32 - 9\pi)/(3\pi - 8)$ and $D_{\parallel} = D_{HP} = 2\sqrt{\pi}/(3\pi - 8)$. The subtraction of the scaled argument plasma dispersion function in Eq. (20) greatly changes the small phase velocity behavior of the density response. It is not possible to choose fitting coefficients $\left(\beta_{\parallel}, D_{\parallel}\right)$ based on a small $|\zeta|$ expansion for non-zero trapped fraction due to the singular limit of Eq. (20) as $\varepsilon \to 0$. However, a reasonable fit is obtained by choosing $\left(\beta_{\parallel}, D_{\parallel}\right)$ to give an exact fit at the point $\zeta = 1.2$. The equation $R_{n,4}(1.2) = R_n(1.2)$ is solved for $\left(\beta_{\parallel}, D_{\parallel}\right)$. The resulting functions of ε_t can be approximated by

$$D_{\parallel} = D_{HP}\left[0.59 + 0.41\left(1 - \sqrt{\varepsilon_t}\right)^5\right] \quad , \tag{28}$$

and

$$\beta_{\parallel} = \left(\beta_{HP} + 3\right)\left[1.0 - 0.54\sqrt{\varepsilon_t} + 0.59\varepsilon_t\right] - \Gamma_{\parallel} \quad , \tag{29}$$

where $\left(\beta_{HP}, D_{HP}\right)$ are the Hammett-Perkins values.

The real and imaginary parts of the 4-moment density response [Eq. (27)] and kinetic density response [Eq. (20)] are compared in Fig. 1. The four moment model gives a good fit for all trapped particle fractions. The fit for the parallel pressure response function is of similar quality. The choice of fitting point $\zeta = 1.2$ is not unique. A more accurate fit could be obtained by adjusting the coefficients to minimize the error between the model and the kinetic density response as was done in Ref. 6. As long as the coefficient $D_{\parallel}$ is positive the poles in the GLF density response model will be in the lower half frequency plane (i.e. damped). In this case fitting the response function along the real axis is sufficient to insure that the fit will be good in the upper half plane which is the region of interest with positive growth rates.

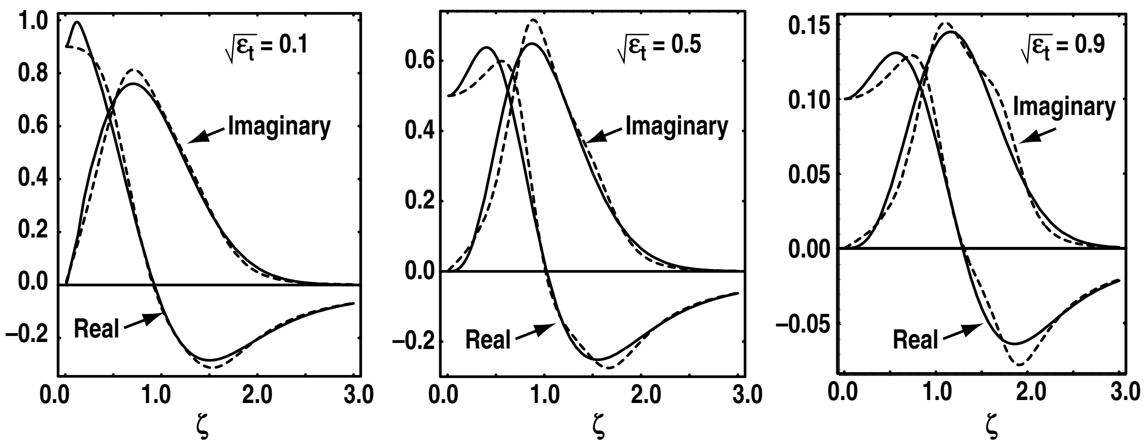

Fig. 1. Comparison of the kinetic (solid) and GLF (dashed) density response for three values of $\sqrt{\varepsilon_t}$.

If there are no trapped particles, the Landau resonance does not produce any mixing between the parallel and perpendicular velocities. Hence, the perpendicular pressure moment of the perturbed distribution function [Eq. (6)] is the same as the density moment. In other words, there is no perpendicular temperature perturbation induced by the parallel electrostatic wave Eq. (2). In the presence of trapped particles bounce averaging the Landau resonance, there is mixing between perpendicular and parallel velocities due to the trapped passing boundary in velocity space. This effect yields a finite perpendicular temperature perturbation from the parallel electrostatic wave perturbation. In order to model this effect the perpendicular pressure and energy flux moments are needed. They are defined by

$$\tilde{p}_\perp = \frac{1}{2n_0 v_t^2}\int d^3 v v_\perp^2 \tilde{f} = -R_{p_\perp}\tilde{\Phi} \quad , \tag{30}$$

$$\tilde{Q}_\perp = \frac{1}{2n_0 v_t^3}\int d^3 v v_\parallel v_\perp^2 \tilde{f} = -R_{Q_T}\tilde{\Phi} \quad . \tag{31}$$

The corresponding moment equations are

$$-\sqrt{2}\zeta\tilde{p}_\perp + \tilde{Q}_\perp = 0 \quad , \tag{32}$$

$$-\sqrt{2}\zeta\tilde{Q}_\perp + \tilde{r}_{\perp,\parallel} + g_{r_{\perp,\parallel}}\tilde{\Phi} = 0 \quad , \tag{33}$$

where

$$\tilde{r}_{\perp,\parallel} = \frac{1}{2n_0 v_t^4}\int d^3v v_\parallel^2 v_\perp^2 \tilde{f} = -R_{r_{\perp,\parallel}}\tilde{\Phi} \quad , \tag{34}$$

and the fraction in front of the potential term in Eq. (33) is

$$g_{r_{\perp,\parallel}} = \frac{1}{2n_0 v_t^4}\int_{\varepsilon\geq\varepsilon_t} d^3v v_\parallel^2 v_\perp^2 f_0 = 1 - \frac{5}{2}\varepsilon_t^{3/2} + \frac{3}{2}\varepsilon_t^{5/2} \quad . \tag{35}$$

The electrostatic potential term can be eliminated from Eq. (33) by changing variables to $\tilde{q}_\perp = \tilde{Q}_\perp - \Gamma_\perp \tilde{u}_\parallel$ where $\Gamma_\perp = g_{r_{\perp,\parallel}} / g_{p_\parallel}$ and using Eq. (13) to eliminate $\tilde{u}_\parallel$ giving

$$-\sqrt{2}\zeta\tilde{q}_\perp + \tilde{r}_{\perp,\parallel} - \Gamma_\perp \tilde{p}_\parallel = 0 \quad . \tag{36}$$

The thermal stress is closed using the same form as in Ref. 4 with factors to give the correct adiabatic limit

$$\tilde{r}_{\perp,\parallel} = \Gamma_\perp \tilde{p}_\parallel + \beta_\perp\left(\tilde{p}_\perp - g_{p_\perp}\tilde{n}\right) - i\sqrt{2}D_\perp \frac{|k_\parallel|}{k_\parallel}\tilde{q}_\perp \quad , \tag{37}$$

where $\beta_\perp, D_\perp$ are fit coefficients and $g_{T_\perp} = g_{p_\perp} / g_n$. The fraction $g_{p_\perp}$ is defined by

$$g_{p_\perp} = \frac{1}{2n_0 v_t^4}\int_{\varepsilon\geq\varepsilon_t} d^3v v_\perp^2 f_0 = 1 - \frac{3}{2}\varepsilon_t^{1/2} + \frac{1}{2}\varepsilon_t^{3/2} \quad . \tag{38}$$

This closure gives the perpendicular pressure response function (for $k_\parallel > 0$)

$$R_{P_\perp,6} = \frac{\left(2\zeta^2 - \beta_\perp g_{T_\perp} / \Gamma_\perp + 2iD_\perp\zeta\right)}{\left(2\zeta^2 - \beta_\perp + 2iD_\perp\zeta\right)}\Gamma_\perp R_{n,4} \quad . \tag{39}$$

Note that for zero trapped fraction $\Gamma_\perp = \beta_\perp = g_{T\perp} = 1$ and the ratio of polynomials in Eq. (39) becomes unity. Hence for no trapped particles the coefficients $\beta_\perp, D_\perp$ are undetermined. The kinetic response for the perpendicular pressure is found by performing the integrals in Eq. (30) over the region of passing particles using the kinetic solution [Eq. (6)] for the perturbed distribution function.

The result is

$$
\begin{aligned}
R_{P_\perp} &= g_{p_\perp} + \left(1-\zeta^2\right)R_n - g_n \\
&+ \left[\zeta^2\left(1+\zeta Z(\zeta)\right) - \left(\frac{\zeta}{\sqrt{\varepsilon_t}}\right)^2\left(\sqrt{\varepsilon_t} + \zeta Z\left(\zeta/\sqrt{\varepsilon_t}\right)\right)\right] \quad .
\end{aligned}
\tag{40}
$$

Since the GLF response for the perpendicular pressure [Eq. (39)] is factorizable it is convenient to determine the fit coefficients $\beta_\perp, D_\perp$ by fitting to the ratio $R_{P_\perp} / \left(\Gamma_\perp R_n\right)$ of the kinetic response functions. A good fit is found for

$$
D_\perp = \left(\sqrt{\pi}/2\right)\varepsilon_t^{1/2}\left(1.14 + 2.66\varepsilon_t^{1/2} - 1.92\varepsilon_t\right) \quad , \tag{41}
$$

$$
\beta_\perp = \left(g_{r_{\perp,\parallel}} / g_{p_\perp}\right)\varepsilon_t\left(4.34 - 3.35\varepsilon_t^{1/2} + 0.44\varepsilon_t\right) \quad . \tag{42}
$$

The physically interesting quantity is the perpendicular temperature response $R_{T_\perp} = \left(R_{P_\perp} - g_{T_\perp} R_n\right) / g_n$. The 6-moment and kinetic perpendicular temperature response functions are compared in Fig. 2. The fit is good for all values of the trapped fraction. Particular care was taken to maintain a good fit even for small trapped fraction by adjusting the fitting point. Note that both fit coefficients go to zero as $\varepsilon_t \to 0$. This is not inconsistent with the perpendicular temperature closure coefficients $\beta_\perp = 1, D_\perp = \sqrt{\pi}/2$ determined by Dorland and Hammett [4]. They used a driven perturbation in the perpendicular temperature to determine the fit coefficients. As already noted, the parallel electrostatic wave perturbation [Eq. (2)] does not produce a perpendicular temperature perturbation unless there are trapped particles. Hence, the fit coefficients $\beta_\perp, D_\perp$ found in this paper are unrelated to the ones of Ref. 4. The trapped particle effects considered here do determine that these coefficients should vanish in the no-trapped particle limit. This calls into question the inclusion of the finite values of Ref. 4 in the GLF equations of Refs. 5 and 6. It may be that the toroidal drift terms require finite values for these coefficients due to the perpendicular temperature perturbations they induce. This question is left to consideration when the toroidal drifts are introduced into the model.

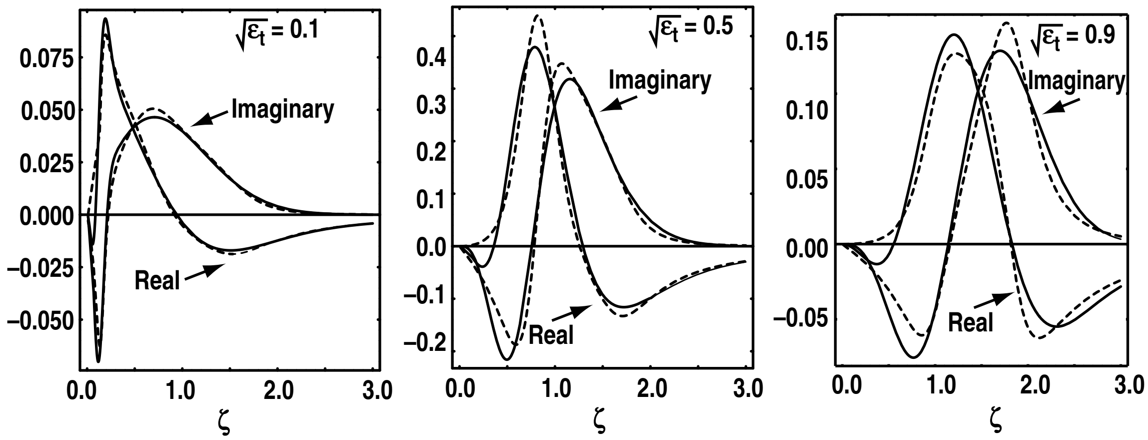

Fig. 2. Comparison of the kinetic (solid) and GLF (dashed) total pressure response for three values of $\sqrt{\varepsilon_t}$.

Until now it has been assumed that all trapped particles can bounce average the Landau resonance and hence should be excluded from the velocity integration domain. However, in order to bounce average the Landau resonance the trapped particles must change the sign of their parallel velocity within a half period of the wave. This is usually expressed [8] as the mode frequency being less than the bounce frequency (approximated by the transit frequency at the trapped-passing boundary) $\omega < v_{\|}/Rq$ where R is the major radius and q is the safety factor. Turning this around, if the mode frequency exceeds the bounce frequency then the trapped particle may be able to Landau resonate. However, it is also necessary that the parallel velocity be able to resonate with the wave $v_{\|} = \omega/k_{\|}$. Putting these two requirements together gives a condition on the parallel wavenumber $Rqk_{\|} < 1$ in order for all trapped particles to bounce average the Landau resonance. This type of argument can be used to define an effective boundary in velocity space for particles which can bounce average. Starting with the time derivative of the poloidal angle of a particle

$$\frac{d\theta}{dt} = \frac{v_{\|}}{Rq} \quad . \tag{43}$$

The angle traveled while the particle has an average velocity equal to the phase velocity of the wave for a duration τ is

$$\Delta\theta = \frac{\omega\tau}{Rqk_{\|}} \quad . \tag{44}$$

A trapped particle keeps the same sign of its velocity while traveling between bounce angles θ_B (turning points). A minimum requirement for a trapped particle to be able to bounce average is that it can travel more than half of an orbit ($2\theta_B$) in half of a wave period $\omega\tau = \pi$. This gives the condition

$$\Delta\theta = \frac{\pi}{Rqk_{\|}} > 2\theta_B \quad . \tag{45}$$

At the trapped-passing boundary the marginally trapped particles have $\theta_B = \pi$. Thus, if $k_{\|}$ is such that Eq. (45) is satisfied even for marginally trapped particles then all trapped particles can bounce average. However, if $k_{\|}$ is large enough then only trapped particles with smaller bounce angles which satisfy Eq. (45) can bounce average. The condition Eq. (45) can thus be used to define a maximum bounce angle θ_{BA} for trapped particles which can bounce average a particular wave

$$\theta_{BA} = MIN\left(\frac{\pi}{2Rqk_{\|}}, \pi\right) \quad . \tag{46}$$

This in turn can be used to define the angle in velocity space at the boundary between resonant and bounce averaging particles $\lambda_{BA} = 1/B(\theta_{BA})$ and the fraction of bounce averaging particles $\sqrt{\varepsilon_{BA}}$ where $\varepsilon_{BA} = 1 - \lambda_{BA}B(0)$. Thus, the formulas in this paper can be generalized to a bounce average rather than a trapped particle boundary simply by replacing ε_t with ε_{BA}. The moment equations in this paper then include all of the circulating particles plus those trapped particles which can Landau resonate. The moment equations for the remaining trapped particles which can bounce average will not have any odd moments or $k_{\|}$ terms.

3. SUMMARY

The 6-moment GLF model of Ref. 4 was extended to include the reduction of Landau damping due to bounce averaging of trapped particles. The model gives a good fit to the kinetic response of density and perpendicular temperature to an external electrostatic wave with only a parallel wavenumber. This model will be incorporated in an extension of the 6-moment model of Beer and Hammett [6] which has finite perpendicular wavenumber effects as well. The details of this extension including the diamagnetic and toroidal drifts will be given in a future publication. The present model, with the bounce averaging boundary rather than a simple trapped particle boundary, allows for some waves to resonate with even trapped particles. High parallel wavenumber waves have to have a high mode frequency in order to resonate with the average parallel velocity. This can result in a loss of bounce averaging when the mode frequency exceeds the bounce frequency. The present model is applicable to both low frequency trapped particle modes with bounce averaging by all trapped particles and higher frequency temperature gradient modes which have diminished bounce averaging effects.

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