The experimental determination of a thermal diffusivity due to turbulent transport*

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General Atomics

In plasmas where the turbulent processes are dominated by turbulence, it is not always straight forward to identify the magnitude of the experimental transport diffusion coefficients. This is primarily due to the fact that with turbulent transport it is not possible to unambiguously separate the convective from the conductive or diffusive parts of the transport. For the energy transport it is not just a matter of deciding whether the convection term is 5/2 or 3/2 times the product of the particle flux and the temperature. The expression for the convection term depends upon the type of turbulence which is causing the transport. In cases where the turbulence induced stresses cause significant radial flows, the definition of the conductive term must be modified from the usual definition.

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In order to have a correct definition of χ_i, which is consistent with DIII-D experimental results, it is necessary to include the effects of the turbulence induced stresses in both the particle and energy fluxes.

Start From First Four Moments of General Kinetic Equation (ignore explicit collisional terms and consider only single ion fluid)

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{\boldsymbol{u}}) = S_p; \qquad \vec{\boldsymbol{u}} \equiv \frac{1}{n}\int d^3v\, f\,\vec{\boldsymbol{v}}$$

$$mn\left(\frac{\partial \vec{\boldsymbol{u}}}{\partial t} + (\vec{\boldsymbol{u}}\cdot\nabla)\vec{\boldsymbol{u}}\right) + \nabla p - en\left(\vec{\boldsymbol{E}} + \vec{\boldsymbol{u}}\times\vec{\boldsymbol{B}}/c\right) = \vec{\boldsymbol{S}}_m; \quad p \equiv \frac{m}{3}\int d^3v f\left|\vec{\boldsymbol{v}} - \vec{\boldsymbol{u}}\right|^2$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \frac{1}{2}\frac{\partial m u^2}{\partial t} + \nabla\cdot\vec{\boldsymbol{Q}} - en\vec{\boldsymbol{u}}\cdot\vec{\boldsymbol{E}} = S_\varepsilon \;; \qquad \vec{\boldsymbol{Q}} \equiv \frac{m}{2}\int d^3v\, f\, v^2\vec{\boldsymbol{v}}$$

$$\frac{\partial\vec{\boldsymbol{Q}}}{\partial t} + \nabla\cdot\overleftrightarrow{\boldsymbol{R}} - \frac{5}{2}\frac{e}{m}p\vec{\boldsymbol{E}} - \frac{1}{2}enu^2\vec{\boldsymbol{E}} - \frac{e}{mc}\vec{\boldsymbol{Q}}\times\vec{\boldsymbol{B}} = 0\;; \quad \overleftrightarrow{\boldsymbol{R}} \equiv \frac{m}{2}\int d^3v\, f\, v^2\vec{\boldsymbol{v}}\vec{\boldsymbol{v}}$$

"Conventional" Approach

Ensemble average, take Steady State and neglect some terms

$$\nabla\langle p\rangle - e\left\langle n\left(\vec{\boldsymbol{E}} + \vec{\boldsymbol{u}} \times \vec{\boldsymbol{B}}/c\right)\right\rangle = \left\langle \vec{\boldsymbol{S}}_m \right\rangle; \qquad \times\hat{\boldsymbol{b}} \equiv \times\vec{\boldsymbol{B}}/B \text{ , } then \ \cdot\hat{\boldsymbol{r}}$$

$$\nabla\cdot\left\langle\vec{\boldsymbol{Q}}\right\rangle - \left\langle\vec{\boldsymbol{u}}\cdot\nabla p\right\rangle = \left\langle S_{\boldsymbol{\varepsilon}} - \vec{\boldsymbol{u}}\cdot\vec{\boldsymbol{S}}_m\right\rangle \equiv S_{\varepsilon m}$$

$$\nabla\langle R\rangle + \frac{5}{2}\frac{e}{m}\left\langle p\vec{\boldsymbol{E}}\right\rangle - \frac{e}{mc}\left\langle\vec{\boldsymbol{Q}}\times\vec{\boldsymbol{B}}\right\rangle = 0; \qquad \times\hat{\boldsymbol{b}} \equiv \times\vec{\boldsymbol{B}}/B \text{ , } then \ \cdot\hat{\boldsymbol{r}}$$

Note: "conventional" approach assumes that R is a scalar and ignores Reynolds stress

This yields;

$$\left\langle\vec{\Gamma}\right\rangle\cdot\hat{r} = c\left\langle\tilde{n}\left(\tilde{\boldsymbol{E}}\times\vec{\boldsymbol{b}}\right)/B\right\rangle = \left\langle\tilde{n}\tilde{\mathbf{v}}_E\right\rangle\cdot\hat{r} \ ; \ with \ \ \vec{\Gamma} \equiv n\vec{\boldsymbol{u}}$$

$$\left\langle\vec{\boldsymbol{Q}}\right\rangle\cdot\hat{r} = \frac{5}{2}c\left\langle\tilde{p}\left(\tilde{\boldsymbol{E}}\times\vec{\boldsymbol{b}}\right)/B\right\rangle = \frac{5}{2}\left\langle\tilde{p}\tilde{\mathbf{v}}_E\right\rangle\cdot\hat{r} = \frac{5}{2}\bar{n}\left\langle\tilde{T}\tilde{\mathbf{v}}_E\right\rangle_r + \frac{5}{2}\bar{T}\left\langle\tilde{n}\tilde{\mathbf{v}}_E\right\rangle_r$$

The “conventional” approach

$$\left\langle \vec{\Gamma} \right\rangle \cdot \hat{r} = \left\langle \tilde{n} \tilde{\boldsymbol{v}}_E \right\rangle \cdot \hat{r}$$

$$\left\langle \vec{\boldsymbol{Q}} \right\rangle \cdot \hat{r} = \frac{5}{2} \left\langle \tilde{p} \tilde{\boldsymbol{v}}_E \right\rangle \cdot \hat{r}$$

neglects some important terms

For "correct" calculation of PARTICLE FLUX you must use,

$$\langle mn(\vec{\boldsymbol{u}}\cdot\nabla)\vec{\boldsymbol{u}}\rangle + \langle\nabla\cdot\overleftrightarrow{\boldsymbol{p}}\rangle - e\left\langle n\left(\vec{\boldsymbol{E}}+\vec{\boldsymbol{u}}\times\vec{\boldsymbol{B}}/c\right)\right\rangle = \left\langle\vec{\boldsymbol{S}}_m\right\rangle\ ; \qquad \times\hat{\boldsymbol{b}},\ \ then\ \cdot\hat{\boldsymbol{r}}$$

include Reynolds stress
and gyro-viscous terms

$$\left\langle\vec{\Gamma}\right\rangle\cdot\hat{r} = \left\langle\tilde{n}\tilde{\boldsymbol{v}}_E\right\rangle\cdot\hat{r} - \frac{1}{\Omega}\left\langle n\left\{\left[(\vec{\boldsymbol{u}}\cdot\nabla)\vec{\boldsymbol{u}}\right]\times\hat{\boldsymbol{b}}\right\} + \frac{(\nabla\cdot\overleftrightarrow{\boldsymbol{p}})\times\hat{\boldsymbol{b}}}{m}\right\rangle\cdot\hat{r}$$

$$\left\langle\vec{\Gamma}\right\rangle\cdot\hat{r} = \left\langle\vec{\Gamma}_E\right\rangle\cdot\hat{r} + \left\langle\vec{\Gamma}_S\right\rangle\cdot\hat{r}\ \ ;$$

$$where\ \ \vec{\Gamma}_E = \tilde{n}\tilde{\boldsymbol{v}}_E\ \ \ and\ \ \vec{\Gamma}_S = \frac{1}{\Omega}n\left\{\hat{\boldsymbol{b}}\times\left[(\vec{\boldsymbol{u}}\cdot\nabla)\vec{\boldsymbol{u}}\right]\right\} + \frac{\hat{\boldsymbol{b}}\times(\nabla\cdot\overleftrightarrow{\boldsymbol{p}})}{m\Omega}$$

Note: $\vec{\Gamma}_S$ is the flux due to the Reynolds and gyro-viscous stresses

Compare approximate sizes of $\langle \Gamma_S \rangle_r$ and $\langle \Gamma_E \rangle_r$

It is difficult to estimate the size and direction of $\langle \Gamma_E \rangle_r$,
Since they depend critically upon the phase between $\tilde{n}$ and $\tilde{\boldsymbol{v}}_E$.
But it is a reasonable assumption that $\langle \Gamma_E \rangle_r$ is outward and is not larger than the total particle flux,

Thus, $\langle \vec{\Gamma}_E \rangle_r = \langle \tilde{n} \tilde{\boldsymbol{v}}_E \rangle_r \leq \langle \vec{\Gamma} \rangle_r \equiv \langle n \vec{\boldsymbol{u}} \rangle_r \equiv \overline{n}\,\overline{u}_r^{\,*}$

We define $\overline{n}\,\overline{u}_r^{\,*}$ as the total outward particle flux.

We estimate the size of $\left\langle \vec{\Gamma}_S \right\rangle_r$ from one of its largest terms,

(note: $\tilde{u}_r$ *and* $\partial \tilde{u}_\theta / \partial r$ are approximately in phase)

$$\left\langle \vec{\Gamma}_S \right\rangle_r \sim \frac{\overline{n}}{\Omega} \left\{ \left| \tilde{u}_r \right| \frac{\partial \left| \tilde{u}_\theta \right|}{\partial r} \right\} \sim \overline{n}\, \overline{u}_r^{\,*} k_\perp \rho_i \frac{\left| \tilde{u}_r \right| \left| \tilde{u}_\theta \right|}{\overline{u}_r^{\,*} v_{thi}}$$

$$\tilde{u}_r \sim \tilde{u}_\theta \sim \left| \tilde{v}_E \right| \sim c\, k_\perp \tilde{\phi} / B \sim k_\perp \rho_i \left(e\tilde{\phi} / T \right) v_{thi} \sim \Delta \delta\, v_{thi}$$

$$\left\langle \vec{\Gamma}_S \right\rangle_r \sim \left\langle \vec{\Gamma} \right\rangle_r \Delta^3 \delta^2 \frac{v_{thi}}{\overline{u}_r^{\,*}}; \;\; \Delta \sim 0.3,\;\; 10^{-3} \le \delta \le 0.15,\;\; 10^7 \ge \frac{v_{thi}}{\overline{u}_r^{\,*}} \ge 10^5$$

$\therefore\;\; \left\langle \vec{\Gamma}_S \right\rangle_r \sim \left\langle \vec{\Gamma} \right\rangle_r \ge \left\langle \vec{\Gamma}_E \right\rangle_r$ **, over large regions of the plasma**

“Correct” ENERGY FLUX

The energy flux also has stress related terms
Instead of using R (a scalar), **we must use,**

$$\overleftrightarrow{\boldsymbol{R}} \equiv \frac{m}{2}\int d^3v\, f\, v^2 \vec{\boldsymbol{v}}\vec{\boldsymbol{v}} \ .$$

Then the radial component of the energy flux is,

$$\left\langle \vec{\boldsymbol{Q}} \right\rangle_r = \frac{1}{\Omega}\left\langle \hat{\boldsymbol{b}} \times \nabla \cdot \overleftrightarrow{\boldsymbol{R}} \right\rangle_r + \frac{5}{2}\left\langle \tilde{p}\tilde{\boldsymbol{v}}_E \right\rangle_r ,$$

or

$$\left\langle \vec{\boldsymbol{Q}} \right\rangle_r = \left\langle \vec{\boldsymbol{Q}}_S \right\rangle_r + \frac{5}{2}\left\langle \tilde{p}\tilde{\boldsymbol{v}}_E \right\rangle_r \equiv \left\langle \vec{\boldsymbol{Q}}_S \right\rangle_r + \left\langle \vec{\boldsymbol{Q}}_E \right\rangle_r$$

Analogous to the particle flux, we conclude that

Q_S is not negligible compared to Q_E

Steady State Energy Conservation Equation

$$\nabla \cdot \left\langle \vec{\boldsymbol{Q}} \right\rangle - \left\langle \vec{\boldsymbol{u}} \cdot \nabla p \right\rangle = S_{\varepsilon m}$$

$$\left\langle \vec{\boldsymbol{u}} \cdot \nabla p \right\rangle = \nabla \cdot \left\langle p \vec{\boldsymbol{u}} \right\rangle - \left\langle p \nabla \cdot \vec{\boldsymbol{u}} \right\rangle$$

$$\vec{\boldsymbol{u}} \equiv \vec{\boldsymbol{u}}_S + \tilde{\boldsymbol{v}}_E \approx \tilde{\boldsymbol{v}}_E \ \ ; \ \ and \ \ note \ \ \nabla \cdot \tilde{\boldsymbol{v}}_E = 0$$

Then

$$\nabla \cdot \left\langle p \vec{\boldsymbol{u}} \right\rangle = \nabla \cdot \left(\left\langle \tilde{p} \tilde{\boldsymbol{v}}_E \right\rangle + \left\langle p \vec{\boldsymbol{u}}_S \right\rangle \right) \approx \nabla \cdot \left(\left\langle \tilde{p} \tilde{\boldsymbol{v}}_E \right\rangle + \vec{\Gamma}_S \overline{T} \right),$$

and

$$\left\langle \vec{\boldsymbol{u}} \cdot \nabla p \right\rangle \approx \nabla \cdot \left(\left\langle \tilde{p} \tilde{\boldsymbol{v}}_E \right\rangle + \vec{\Gamma}_S \overline{T} \right)$$

And since, $\left\langle \vec{\boldsymbol{Q}} \right\rangle = \left\langle \vec{\boldsymbol{Q}}_S \right\rangle + \frac{5}{2} \left\langle \tilde{p} \tilde{\boldsymbol{v}}_E \right\rangle$ **, we obtain,**

$$\nabla \cdot \left(\left\langle \vec{\boldsymbol{Q}}_S \right\rangle + \frac{3}{2} \left\langle \tilde{p} \tilde{\boldsymbol{v}}_E \right\rangle - \vec{\Gamma}_S \overline{T} \right) = S_{\varepsilon m}$$

This result assumes that $\nabla \cdot \left(\overline{n} \left\langle \tilde{T} \vec{\boldsymbol{u}}_S \right\rangle \right) - \left\langle p \nabla \cdot \vec{\boldsymbol{u}}_S \right\rangle \approx 0$

Since,

$$\nabla \cdot \left(\left\langle \vec{\boldsymbol{Q}}_S \right\rangle + \frac{3}{2} \left\langle \tilde{p} \tilde{\boldsymbol{v}}_E \right\rangle - \left\langle \vec{\Gamma}_S \right\rangle \overline{T} \right) = S_{\varepsilon m} \ ,$$

we can write, (using the usual tokamak symmetries)

$$\left\langle \vec{\boldsymbol{Q}}_S \right\rangle_r + \frac{3}{2} \left\langle \tilde{p} \tilde{\boldsymbol{v}}_E \right\rangle_r - \left\langle \vec{\Gamma}_S \right\rangle_r \overline{T} = \frac{1}{V} \int dV \, S_{\varepsilon m}$$

V is local plasma volume

or

$$\overline{Q}_S + \frac{3}{2} \overline{n} \left\langle \tilde{T} \tilde{\boldsymbol{v}}_E \right\rangle_r + \frac{3}{2} \overline{\Gamma}_E \overline{T} - \overline{\Gamma}_S \overline{T} = \frac{1}{V} \int dV \, S_{\varepsilon m} \ ,$$

where we define

$$\overline{Q}_S \equiv \left\langle \vec{\boldsymbol{Q}}_s \right\rangle_r , \ \ \overline{\Gamma}_E \equiv \left\langle \vec{\Gamma}_E \right\rangle_r , \ \ and \ \ \overline{\Gamma}_S \equiv \left\langle \vec{\Gamma}_S \right\rangle_r$$

TRANSP defines

$$q_{cond} \equiv \frac{1}{V'}\int dV\, S_{\varepsilon m} - \frac{3}{2}\overline{\Gamma}\,\overline{T}$$

where, $\overline{\Gamma} \equiv \overline{\Gamma}_E + \overline{\Gamma}_S$

(In TRANSP the 3/2 can be replaced by any number, but it is usually taken to be 3/2)

.

Then

$$q_{cond} \approx \frac{3}{2}\overline{n}\left\langle \tilde{T}\tilde{\mathbf{v}}_E \right\rangle_r + \overline{Q}_S - \frac{5}{2}\overline{\Gamma}_S\,\overline{T}\;;$$

$If \;\; \overline{Q}_S \;\; and \;\; \overline{\Gamma}_S = 0.0, \;\; then \;\; q_{cond} \approx \frac{3}{2}\overline{n}\left\langle \tilde{T}\tilde{\mathbf{v}}_E \right\rangle_r,$ (this is the “conventional” result)

$But \;\; if \;\; \overline{Q}_S \;\; and \;\; \overline{\Gamma}_S \;\; dominate, \;\; then$ (by usual TRANSP definition)

$$q_{cond} \approx \overline{Q}_S - \frac{5}{2}\overline{\Gamma}_S\,\overline{T}$$

$and \;\; this \;\; q_{cond} \;\; can \;\; be \;\; negative.$ (See discussion of Experimental Results)

It can be shown that $\overline{Q}_S \approx -\frac{3}{2} D\overline{n}\frac{\partial \overline{T}}{\partial \rho} + \frac{3}{2}\overline{\Gamma}_S \overline{T}$, (from Drift Kinetic Equation).

This results in

$$-\frac{3}{2} D\overline{n}\frac{\partial \overline{T}}{\partial \rho} + \frac{1}{2}\overline{\Gamma}_S \overline{T} + \frac{3}{2}\overline{n}\left\langle \tilde{T}\tilde{\mathbf{v}}_E \right\rangle_r + \frac{3}{2}\overline{\Gamma}_E \overline{T} = \frac{1}{V}\int dV\, S_{\varepsilon m}$$

$$Then \ \ if \ \ \overline{Q}_S \ \ and \ \ \overline{\Gamma}_S \ \ dominate,$$

$$-\frac{3}{2} D\overline{n}\frac{\partial \overline{T}}{\partial \rho} + \frac{1}{2}\overline{\Gamma}_S \overline{T} = \frac{1}{V}\int dV\, S_{\varepsilon m},$$

and it is better to define q_{cond} **by,**

$$q_{cond} \equiv \frac{1}{V'}\int dV\, S_{\varepsilon m} - \frac{1}{2}\overline{\Gamma}\overline{T}.$$

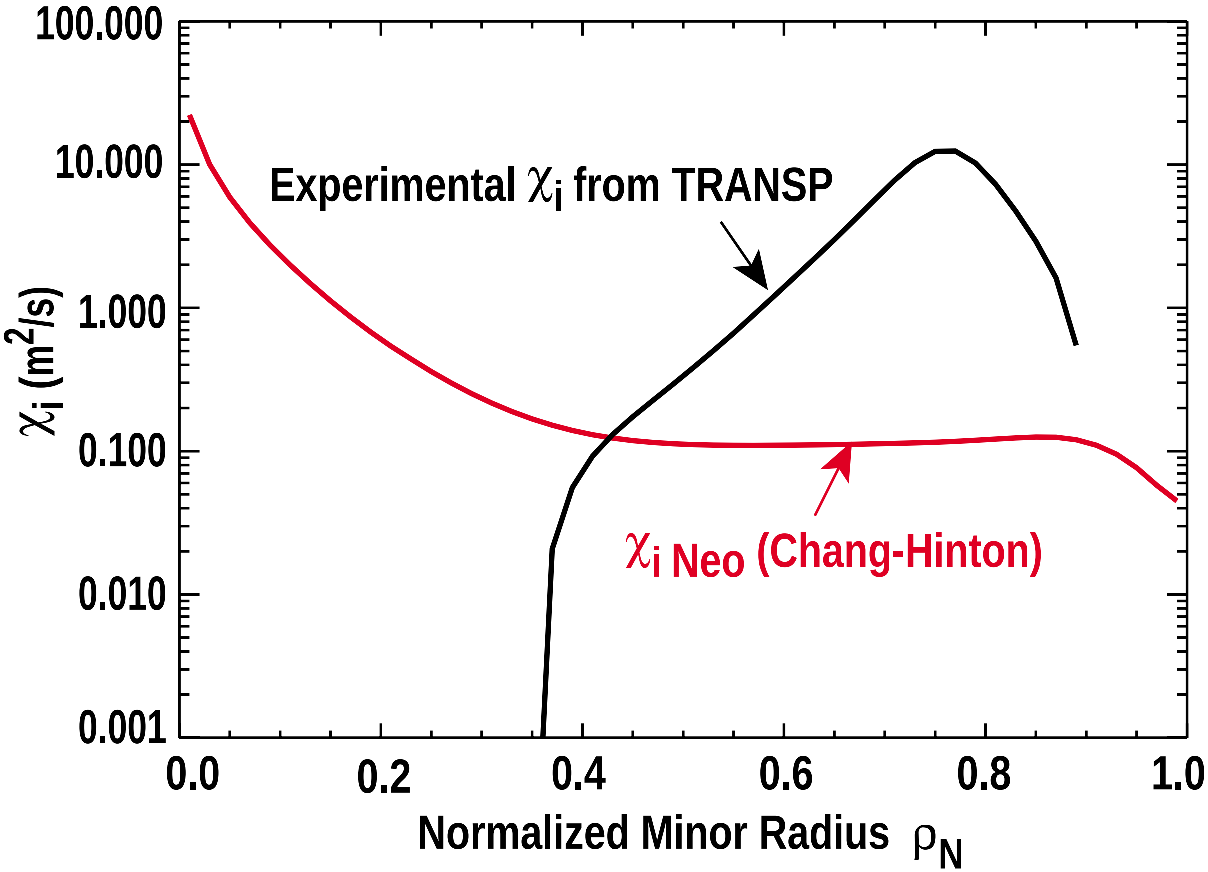
Then

$$q_{cond} \approx -\frac{3}{2} D\overline{n}\frac{\partial \overline{T}}{\partial \rho}, \ \ and \ \ \chi_i \approx \frac{3}{2} D,$$

<u>not</u> negative

EXPERIMENTAL RESULTS

For Some DIII-D Plasmas, using the usual TRANSP definition for q_{cond} $\left(q_{cond} \equiv \frac{1}{V'}\int dV\, S_{\varepsilon m} - \frac{3}{2}\overline{\Gamma}_r \overline{T}\right)$, results in negative χ_is in some regions of the plasma



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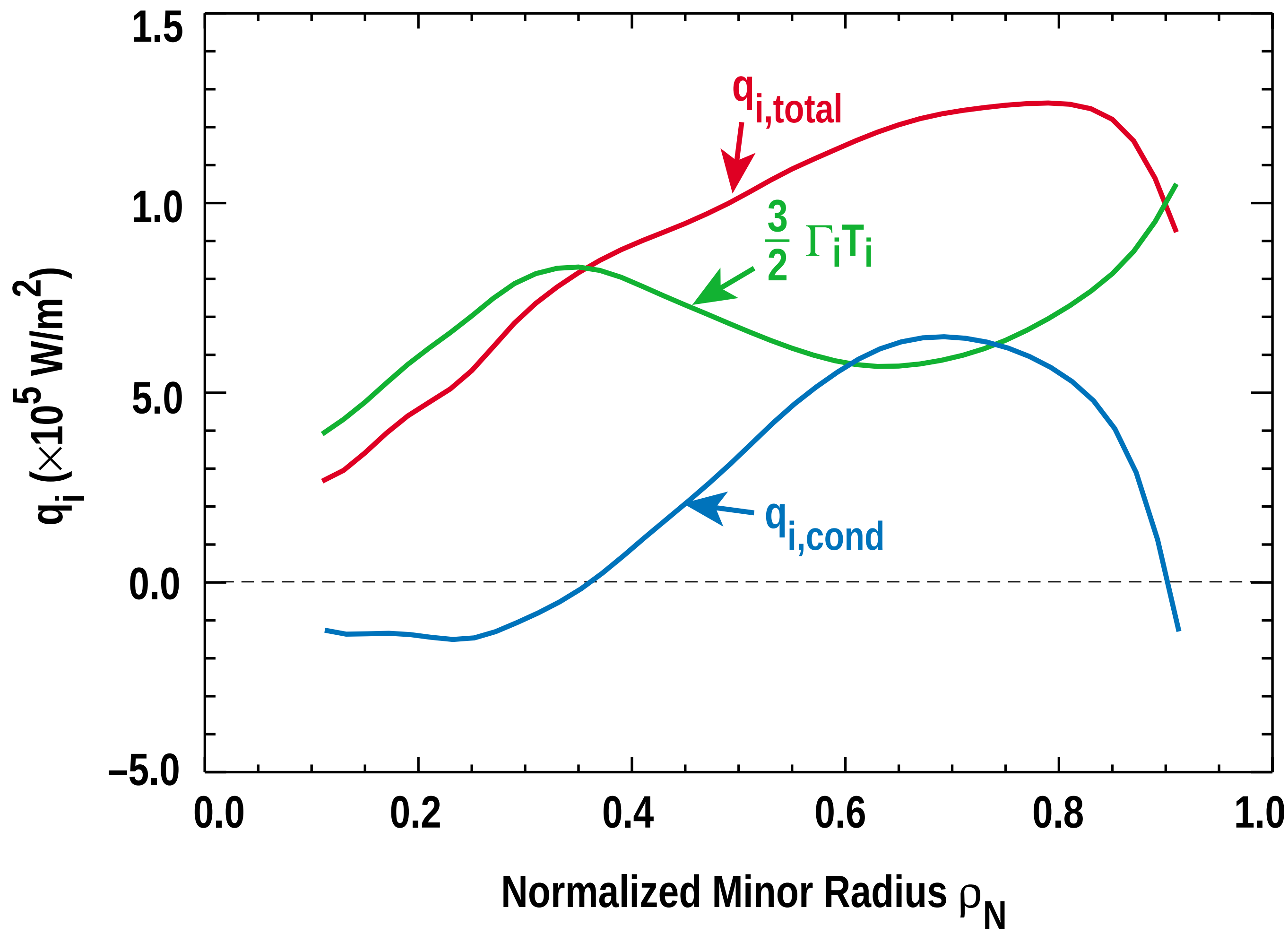
The total outward ion energy flux is positive, but the "conductive" part is negative in some regions



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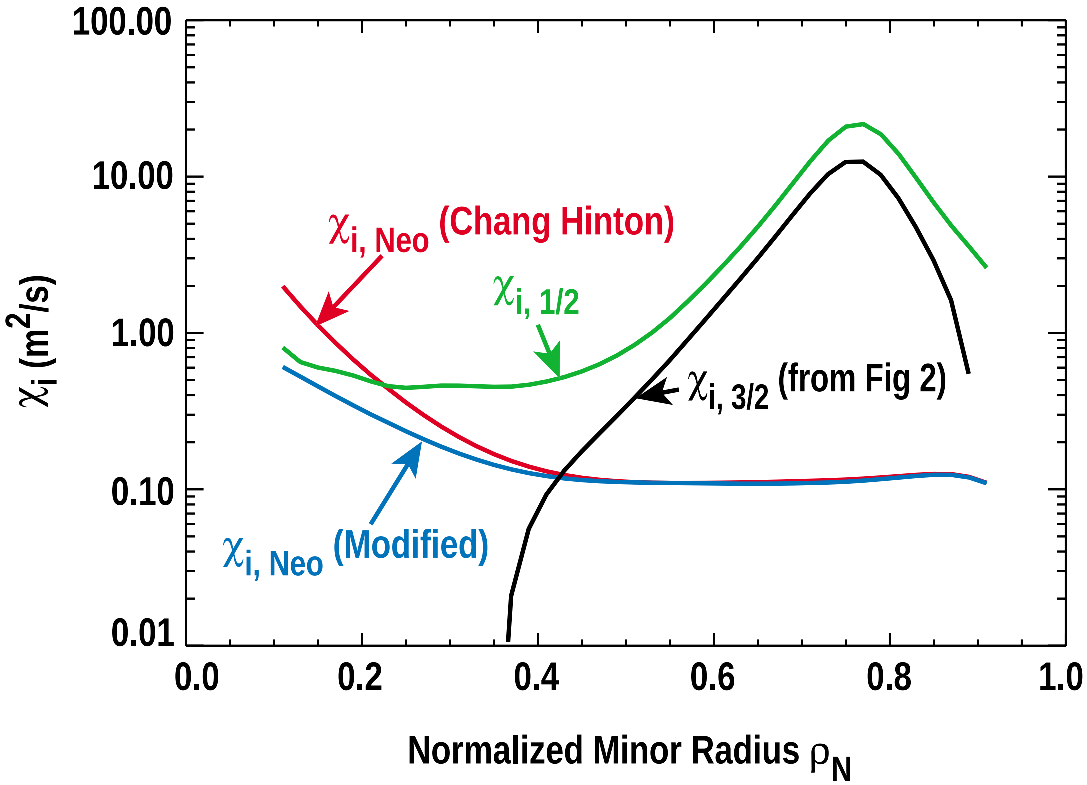
Defining q_{cond} as, $q_{cond} \equiv \frac{1}{V'} \int dV\, S_{\varepsilon m} \;-\; \frac{1}{2}\overline{\Gamma}_r \overline{T}$,

results in a χ_i that is positive everywhere and not significantly less than the Chang-Hinton neoclassical value



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The Chang-Hinton Neoclassical diffusivity may not be valid near the origin. $\chi_{i,neo}$ (modified) is the Chang-Hinton value reduced by the factor $(1-(1-r/3r_p)^2)$, as derived in:

Bergmann, A.G. Peeters and S.D. Pinches, Phys. Plasmas 8, 5192 (2001)

where, r_p is 1.6 times the potato orbit width of $\left(2q\rho_L\right)^{2/3} R_0^{1/3}$.

CONCLUSIONS

- The turbulence induced stress makes a major contribution to the particle flux

- The turbulence induced energy stress makes a major contribution to the energy flux

- q_{cond} is best defined by $q_{cond} \approx \frac{1}{V'}\int dV\left(S_{\varepsilon m}\right) - \frac{1}{2}\overline{\Gamma}_r\overline{T}$

Velocity Ordering in DIII-D

$$v_{thi} \sim 10^6 m/s$$

$$v_{tor} \sim 10^5 m/s$$

$$v_{pol} \sim 10^4 m/s \sim \left|\tilde{v}_E\right|$$

$$v_{rad} \sim (0.1-10) m/s$$

$$\text{Compare } \left\langle mn\frac{\partial\vec{\boldsymbol{u}}}{\partial t}\right\rangle \ \ with\ \ \langle mn(\vec{\boldsymbol{u}}\cdot\nabla)\vec{\boldsymbol{u}}\rangle$$

Since $\frac{\partial}{\partial t}\langle g\rangle \to 0$ by assumption, and it can be shown that $\tilde{\boldsymbol{u}} \approx \tilde{\boldsymbol{v}}_E$, we can compare

$$\left\langle \tilde{n}\frac{\partial\tilde{\boldsymbol{u}}}{\partial t}\right\rangle \quad with \quad \langle n(\vec{\boldsymbol{u}}\cdot\nabla)\vec{\boldsymbol{u}}\rangle$$

or

$$\left\langle \tilde{n}\frac{\partial\tilde{\boldsymbol{v}}_E}{\partial t}\right\rangle \quad with \quad \bar{n}\left\langle\left(\tilde{\boldsymbol{v}}_E\cdot\nabla\right)\tilde{\boldsymbol{v}}_E\right\rangle$$

or

$$\omega|\tilde{n}|\left|\tilde{\boldsymbol{v}}_{E,\theta}\right| \quad with \quad \bar{n}\left|\tilde{\boldsymbol{v}}_{E,r}\right|\left|k_r\right|\left|\tilde{\boldsymbol{v}}_{E,\theta}\right|$$

Using $\omega \approx \omega_{*e} = ck_\theta T/\left(eBL_n\right) \approx k_\theta\rho_i v_{thi}/a$, we compare

$$\left(k_\theta\rho_i v_{thi}/a\right)|\tilde{n}| \quad with \quad \bar{n}k_r\left|\tilde{\boldsymbol{v}}_{E,r}\right|$$

Since $\left|\tilde{v}_{E,r}\right| \sim k_r\rho_i\left(e\tilde{\phi}/T\right)v_{thi}$, we compare

$$\left(k_\theta\rho_i v_{thi}/a\right)|\tilde{n}|/\bar{n} \quad with \quad k_r^2\rho_i\left(e\tilde{\phi}/T\right)v_{thi}$$

or (asssuming that $k_r \sim k_\theta$)

$$1 \quad with \quad k_r a \ \ >> \ 1$$