EFFECT OF ROTATION ON IDEAL AND RESISTIVE MHD MODE*

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Experimental observations suggest that toroidal plasma rotation can play a crucial role in the determination of the stability of a tokamak. However, conventional MHD stability analysis ignores rotation. In this paper, we report on the results of both numerical and analytic analysis of a number of tokamak stabilities including the effect of toroidal plasma rotation. Depending on the plasma and rotation profiles, the effect of flow can be both stabilizing and destabilizing; it could also have substantial effect on the observed mode structure. Studied issues include low n global ideal and resistive MHD modes, the double kink mode, the localized interchange and the effect of sheared rotation on magnetic island structure.

For the numerical study, the MHD equilibria are computed with the inclusion of sheared toroidal rotation effects. The low mode number, MHD stability of these rotating equilibria is determined by an extended version of the MARS code [1], which solves for the complex growth rates of these modes. The new effects included in this extended version of MARS code include inertial effects of the equilibrium and toroidal rotation shear within the plasma. Of these two effects, the stability of the plasma is affected most by the Doppler shift of the rotation frequency between different flux surfaces due to rotational shear. For normal plasma safety factor with positive shear, the rotational shear has been found to be slightly stabilizing for both the ideal and resistive modes. For the resistive modes, the unstable eigenfunction can be substantially modified by the rotational shear.

The effect of rotational shear on MHD modes in a tokamak with reversed magnetic shear is analyzed. We found that: (1) rotational shear has a stabilizing effect on the finite beta double kink [2] localized between two mode resonant surfaces; and (2) the rotational shear has a destabilizing effect on MHD modes localized within the central reversed shear region. In tokamaks with low magnetic shear, the Alfvén wave can be substantially modified by the variation of the toroidal rotation velocity across the plasma surface. First, the location of Alfvén resonance of the MHD mode within the plasma is modified by the rotation shear. The location is determined by both the overall mode structure and the local rotational shear, and the region which provides instability drive to the MHD mode can be enlarged or reduced. Second, the Alfvén wave restoring force is substantially weakened over a low shear region, when the shearing rate of the rotation frequency matches with the shearing rate of the local Alfvén frequency.

The analysis of Ref. [2] for the double kink in a large aspect ratio circle is extended to include the effect of toroidal rotation. Frieman and Rotenberg's non-self-adjoint formulation [3] is found to be related to a self-adjoint formulation with the inclusion of a constraint condition. Modification to the Alfvén wave mentioned above stabilizes the double kink mode, whereas the centrifugal force destabilizes the mode. Numerical results

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from the MARS code also corroborate the above analytic predictions. Because the plasma rotation frequency is usually much smaller than the Alfvén frequency, substantial modifications of the Alfvén stability only occur in weak magnetic shear tokamak plasmas driven by strong tangential neutral beam injection.

Stability to localized MHD interchange modes is studied in a tokamak with a small toroidal rotation but with a non-negligible shearing rate. The variational principle of Frieman and Rotenberg is applied to the localized plasma motion around a rational surface. Modification to the localized interchange stability criterion is obtained by maximizing the growth rate. The rotational shear couples to both the Alfvén and sound waves and reduces the stabilizing effect of these waves. This coupling allows the plasma motion to tap the energy associated with the rotational shear via the Kelvin-Helmholtz process. A new interchange criterion [4] is obtained. In this new criterion, the effect of the rotational shear appears through the flow mach number and the effect of sound wave appears through the plasma beta.

Of special interest is the nature of the coupling of the sound wave and the Alfvén wave in a rotating plasma. Although this coupling is formally of order a/R smaller than the usual Mercier index [5], it has a significant effect on the interchange criterion [4] since the coupling has a resonant feature. To clarify this resonant coupling, a kinetic theory approach is employed. Specifically, the fluid resonance is replaced by the weaker wave-particle resonance. The effect of ion Landau damping is also taken into account.

In static tokamaks, one of the criteria for identifying the presence of a magnetic island is the characteristic 180 degree phase shift of the temperature fluctuations across the magnetic island surfaces. However, in a rotating tokamak, due to viscous drag, plasma temperature fluctuations are not 180 degrees out of phase. A two-dimensional plasma equilibrium equation is derived which includes the effect of plasma flow and a bulk plasma viscosity. Solution of this equation reveals that plasma viscosity together with the curvature in the plasma flow, give rise to a phase shift “anomaly.” This equation is solved analytically for a model equilibrium with the inclusion of a magnetic island. A scaling relationship of this phase shift anomaly with respect to the viscosity and flow curvature is obtained. The equation is also solved numerically with extensive plasma current, pressure and rotation profiles and compared with experiment.

In summary, the effect of rotation and rotational shear on the ideal and resistive MHD stabilities has been studied. The effect of rotational shear is most important when it can modify the shear Alfvén wave spectrum. This can be utilized to stabilize the double kink mode. The rotational shear introduces Kelvin-Helmholtz drive to localized plasma instabilities. It can reduce the pressure gradient threshold of ideal localized interchanges . The centrifugal force has found to be always destabilizing. Finally, the effect of rotational shear couples with the plasma viscosity can significantly affect the shape of the magnetic island.

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