GA–A27406

# EULERIAN SIMULATIONS OF NEOCLASSICAL FLOWS AND TRANSPORT IN THE TOKAMAK PLASMA EDGE AND OUTER CORE

by
E.A. BELLI, J.A. BOEDO, J. CANDY, R.H. COHEN,
P. COLELLA, M.A. DORF, M.R. DORR, J.A. HITTINGER,
P.W. McCORQUODALE, T.D. ROGNLIEN and P.B. SNYDER

SEPTEMBER 2012

GENERAL ATOMICS

GA–A27406

# EULERIAN SIMULATIONS OF NEOCLASSICAL FLOWS AND TRANSPORT IN THE TOKAMAK PLASMA EDGE AND OUTER CORE

by

E.A. BELLI, J.A. BOEDO,* J. CANDY, R.H. COHEN,†
P. COLELLA,‡ M.A. DORF,† M.R. DORR,† J.A. HITTINGER,†
P.W. McCORQUODALE,‡ T.D. ROGNLIEN† and P.B. SNYDER

This is a preprint of a paper to be presented at the 24th IAEA Fusion Energy Conference, October 8–13, 2012 in San Diego, California and to be published in Proceedings.

*University of California San Diego, La Jolla, California USA
†Lawrence Livermore National Laboratory, Livermore, California USA
‡Lawrence Berkeley National Laboratory, Berkeley, California USA

Work supported by the
U.S. Department of Energy under
DE-FG02-95ER54309, DE-FG02-07ER54917,
DE-AC52-07NA27344, DE-AC03-76SF00098
and DE-FC02-06ER54873

GENERAL ATOMICS PROJECT 03726
SEPTEMBER 2012

GENERAL ATOMICS

## ABSTRACT

The drift-kinetic code NEO is used to study the neoclassical transport for parameters relevant in the plasma edge and outer core. NEO includes multiple ion species, general geometry, rapid toroidal rotation, and full linearized Fokker-Planck collisions. Comparisons are made between the NEO neoclassical simulations and experimental measurements of the deuterium parallel velocity profiles and carbon impurity flow profiles in the edge for DIII-D L-mode discharges. The accuracy of commonly-used model collision operators in the edge is assessed. Extensions of the NEO studies further into the tokamak boundary region are explored via simulations with COGENT, a full $f$ Eulerian code, and UEDGE, a 2D fluid code with neoclassical and anomalous transport models, which both include closed and open field line regions.

## I. INTRODUCTION

Several aspects of neoclassical dynamics are believed to be important in explaining enhanced edge flows, current, and confinement in tokamaks. For example, substantial ion flow in the tokamak boundary region - the pedestal and scrape-off layer (SOL) − can be important for stabilization of instabilities, interpreting temporal fluctuation spectra, and for radial transport of plasma, especially its toroidal momentum. In these studies, the $\delta f$ drift-kinetic code NEO[1,2] is used to explore the neoclassical transport for parameters relevant in the plasma edge and outer core.

The NEO code was developed as a practical predictive tool for high-accuracy neoclassical calculations. NEO improves the local neoclassical physics of NCLASS,[3] mainly by using a direct kinetic approach, rather than a fluid-moment approximation, and including toroidal rotation and general geometry. Comparisons between NEO and NCLASS find that NEO provides a 30% correction to NCLASS for the neoclassical flows for typical DIII-D plasmas.[2]

One of the unique features of NEO is the implementation of the full linearized Fokker-Planck collision operator. Accurate modeling of collisional effects, particularly in the highly collisional plasma edge, is essential for studies of neoclassical transport. However, inclusion of the full linearized operator in analytic and numerical calculations has been limited due to the complexity of the operator. Thus, most neoclassical analysis relies on reduced or model operators, particularly for multi-species plasmas. A key aspect of the implementation in NEO is the development of a sophisticated numerical algorithm based on a spectral expansion in velocity space which can accurately treat the disparate velocity scales that arise in the case of multi-species plasmas. Using NEO, extensive comparisons of neoclassical transport levels predicted by the exact collision operator and those from various commonly used model operators have been made to assess the physical accuracy and limitations of the latter over a range of collisionality regimes. In general, we found that the error in the model collision operators is 20-30% for the ion energy flux, 10-15% for both the electron energy flux and ambipolar particle flux, and 5-10% for the bootstrap current. Here we further extend these studies to more realistic edge regimes via comparisons of various operators in the outer core and edge of DIII-D plasmas.

In addition to the NEO analysis, in this work extensions of the NEO studies further into the tokamak boundary region are explored via simulations with COGENT, a 4D full $f$ continuum electrostatic gyrokinetic code which, unlike NEO, describes both closed and open field-line regions. With COGENT, the generation of intrinsic plasma flows due to neoclassical particle losses in the tokamak edge (e.g. from thermal ion orbit losses and X-point losses) can be investigated. COGENT is distinguished by a fourth-order finite-volume (conservative) discretization independent of grid choice, hence providing no loss of accuracy order in going to a non-uniform grid.[4,5] This exploits arbitrary mapped multiblock grid technology[5] (nearly field-aligned on blocks) to handle the complexity of divertor geometry without loss of accuracy. Another distinguishing feature of the code is the use of the Colella-Sekora flux-limiter to suppress unphysical oscillations about discontinuities while maintaining high-order accuracy elsewhere.[6] Finally, the code is written in $v_\parallel$-$\mu$ (parallel velocity - magnetic moment) variables, which avoids "cut-cell" issues appearing, for instance, when $E$-$\mu$ (energy - magnetic moment) variables are used such that the $v_\parallel = 0$ phase-space boundary does not align with the mesh. A succession of increasingly comprehensive collision operators is implemented in the code and have been verified in neoclassical simulations carried out in closed flux-surface geometry. Development of the divertor version of the code is underway.

Simulations are also performed with UEDGE,[7] a 2D fluid code with a neoclassical transport model for both closed and open field lines allowing general collisionality, to assess the adequacy of the fluid transport model. UEDGE has previously been used to compute the electrostatic potential in the pedestal and SOL regions, showing a deep negative radial electric field well associated with the H-mode in tokamaks.[8] However, these simulations could not adequately differentiate the parallel flow for the deuterium and carbon species. A recent upgrade to the parallel velocity and current equations includes magnetic field gradients along B and the full neoclassical viscous terms arising from gradients in velocity and heat-flux moments with coefficients that account for the collisionality regime.[9] UEDGE includes neutrals arising from plasma recycling at the divertor plate and the subsequent ion charge-exchange friction with neutrals. In the future, comparisons with the results of NEO and COGENT will give a clear picture of the adequacy of this generalized fluid transport model with the more detailed and computationally intensive kinetic models in the long mean-free path regime, as well as provide a target for the kinetic codes in the collisional regime.

## II. NEO NEOCLASSICAL RESULTS

In this section, we present results from analysis of neoclassical transport for edge-relevant parameters using NEO. Recent experimental measurements of the deuterium parallel velocity profiles by Mach probes and CER spectroscopic measurements of the carbon impurity toroidal velocity in the edge for DIII-D L-mode discharges[10] have allowed for comparisons with NEO neoclassical flow simulations. In these studies, a new method has been adopted to determine the shift in the parallel flow due to the radial electric field. The equilibrium-scale radial electric field, which is an input in NEO, is usually determined via the pressure balance equation, using carbon impurity toroidal and poloidal flow measurements along with the measured radial pressure gradients. However, the carbon poloidal flow measurements often have large uncertainties and are usually neglected from this equation under the assumption that the poloidal flow term is small compared to the toroidal flow term. This is generally true in the core but not in the edge. Thus, for these studies, in which we are interested in comparing the deuterium ion parallel flows, we have used the carbon toroidal flow measurements as a calibration to determine the radial electric field by choosing the value such that the NEO-computed carbon toroidal flow matches the measured value at at $\theta = 0$. The value of $\theta$ chosen for the calibration is arbitrary since, according to the standard neoclassical relation, the poloidal variation of the parallel flow is determined by the poloidal variation of $B$. This recalibration method is valid only in the weak rotation limit (applicable to most DIII-D plasmas), since then the neoclassical flow coefficient does not depend on the radial electric field.

The results are shown in figure 1. The measurements and analysis were done for two plasma densities, $n_e = 3 \times 10^{13}$ cm$^{-3}$ (#134074) and $n_e = 1.8 \times 10^{13}$ cm$^{-3}$ (#134046) during co- (2505 ms) and counter- (3505 ms) NBI injection phases. For all cases, the results show that the NEO calculations tend to agree with the measurements upon approach to the last closed flux surface. Specifically, the deuterium velocity, which follows the carbon velocity in the core measurements, rapidly rises toward the edge. This indicates that deviations between the deuterium and carbon flows in the edge can be explained by neoclassical physics.

For this case, we also use NEO to assess the limitations of commonly used model collision operators for edge-relevant parameters by comparing the neoclassical transport levels predicted by the exact full linearized Fokker-Plank collision operator to those from the model operators. We consider four model collision operators, given as follows in order of decreasing degree of sophistication: test particle with an ad hoc field particle operator,[2] full Hirshman-Sigmar,[11] zeroth order Hirshman-Sigmar,[12] and Connor.[13] The results for the DIII-D cases in the near-edge region are shown in figure 2. For both the high (#134074) and low (#134046) density cases, the ad hoc field particle operator and the zeroth order Hirshman-Sigmar are the most accurate for the flows as well as for the bootstrap current, with less than $\sim 10\%$ error up to $\rho_N = 0.9$ for the latter and between 20-30% for the flows. The Connor model and

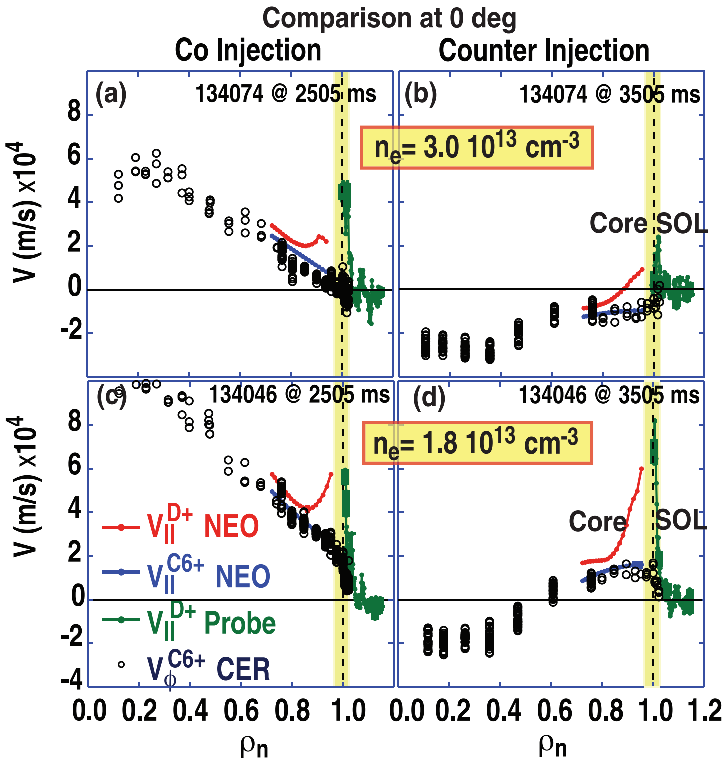


*FIG. 1. Velocity comparison of the CER toroidal carbon measurement (open circles), NEO parallel carbon calculations (blue lines), NEO parallel deuterium calculations (red lines), and Mach-probe parallel deuterium measurements (green lines) for (a,c) co- and (b,d) counter-NBI injected phases versus $\rho_n$, the normalized toroidal flux.*

the zeroth order Hirshman-Sigmar, which are most closely related in that they contain only the Lorentz operator with a simple momentum-restoring term, underestimate the bootstrap current, while the ad hoc field particle and full Hirshman-Sigmar operators, which both contain energy diffusion terms, generally overestimate the bootstrap current. All of the models underestimate the carbon flow further in the near edge. It is not surprising that the Connor model is the least accurate, as it has previously been shown that the model is generally not accurate for impure plasmas,[1] due to lack of modeling the deceleration effect due to dynamic friction. The zeroth order Hirshman-Sigmar model more accurately models the flows for the low density case, compared to the high density case, which is consistent with the model performing better at lower collision frequency, where presumably energy diffusion is less important. The large inaccuracy of the full Hirshman-Sigmar model, especially at higher collision frequency closer to the edge, is notable and surprising. While this operator does include a diffusion-type term, it is only approximate and e.g. is missing the $l = 1$ Legendre component in pitch angle, i.e. $\propto P_1(\xi)$, and has been shown to significantly underestimate the ion flow coefficient for pure plasmas at large collisionality. Overall, we note that these results show that using the full linearized Fokker-Planck operator becomes more important further into the edge.

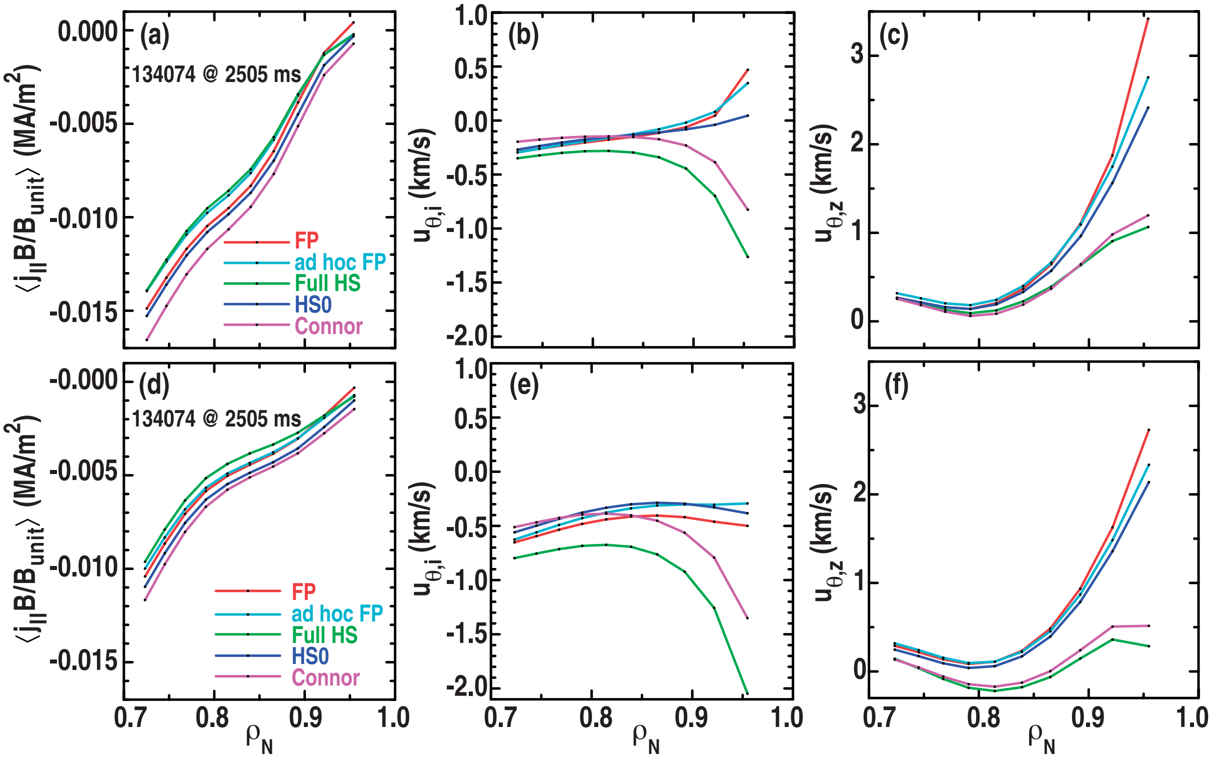


*FIG. 2. Bootstrap current and deuterium and carbon poloidal flows for DIII-D #1374074 @ 2505 ms (top plots) and DIII-D #134046 @ 2505 ms (bottom plots) computed with NEO. Results from various model collision operators are compared with results from the full Fokker-Planck collision operator (FP).*

While most NEO work focuses on the local neoclassical transport dynamics, non-local effects due to finite-orbit width may be important in the plasma edge. These effects were previously studied with neoclassical simulations which purport to compute the total distribution, $f$, more accurately than in the standard local $\mathcal{O}(\rho_{*i})$ ordering by retaining some nonlinear terms related to finite-orbit width, while simultaneously reusing some form of the linearized collision operator. However, we have previously shown that non-local corrections to the distribution function are not generally valid if the nonlinear correction to the collision operator is ignored.[14]

## III. FULL-F SIMULATIONS, EXTENSIONS TO OPEN FIELD LINES, AND ANOMALOUS TRANSPORT

While NEO is limited to the closed magnetic field line region and is a $\delta f$ code, extensions of these studies further into the edge, including the transition from the pedestal to the scrape-off-layer region, are explored using COGENT and UEDGE.

### A. COGENT studies

Development of the full-$f$ code COGENT is on-going. COGENT has been verified in neoclassical simulations for the case of magnetic geometry with concentric circular flux surfaces.[15] The results of COGENT simulations obtained with the Lorentz collision operator are found to be in good agreement with the analytical theory developed in Ref. [16] as shown in figure 3. The COGENT geometry has recently been extended to include the open-field-line region outside the magnetic separatrix.

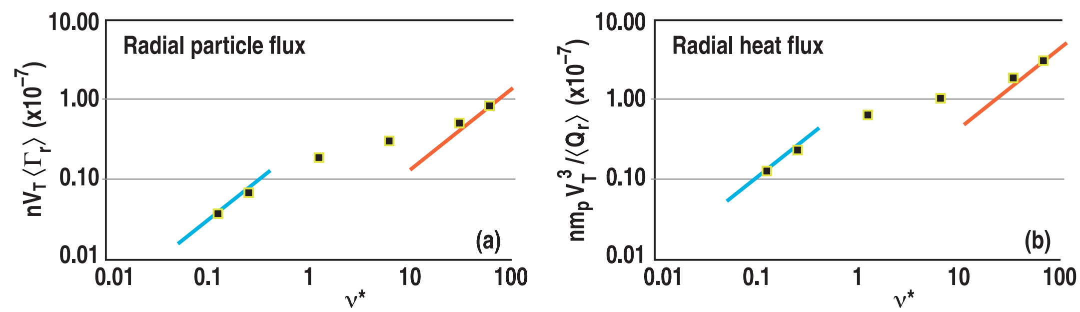


*FIG. 3. COGENT simulations (dots) of the radial neoclassical ion particle flux and energy flux versus the normalized collision frequency for the case of the Lorentz collision operator. The red and blue lines correspond to analytical calculations[16] in the banana and Pfirsch-Schluter regimes, correspondingly. The parameters of the simulation correspond to circular geometry with q=3, $\epsilon$=0.1, $R_0$=45.6 m, $B_0$=7.5T, $T_i$=3 keV, and $R_0/L_n$=$R_0/L_T$=10.*

### B. UEDGE studies

UEDGE simulations of the combined closed/open field line regions are used to examine the consequences of a generalized fluid parallel viscosity model[9] and turbulence transport as modeled by anomalous diffusivities. Initial comparisons of the old and new parallel viscosity terms in UEDGE use an old standard test single-null MHD equilibrium for DIII-D (#66832) with 2 MW of injected core power. The electron density at the core boundary is $3 \times 10^{13}$ cm$^{-3}$, and a radial slip condition ($du_\parallel/dr$) is used on the core boundary. Figure 4 shows the change in the outer mid-plane hydrogen ion parallel velocity profile as various new effects are included. The solid curve includes all terms, although the poloidal drift velocities are at 40% of full strength owing to a numerical instability issue that is being investigated.

Calculation of the carbon $C^{+6}$ parallel velocity is in progress. The corresponding radial electric field profiles for the old model and the new model with 40% drifts are also shown in figure 4.

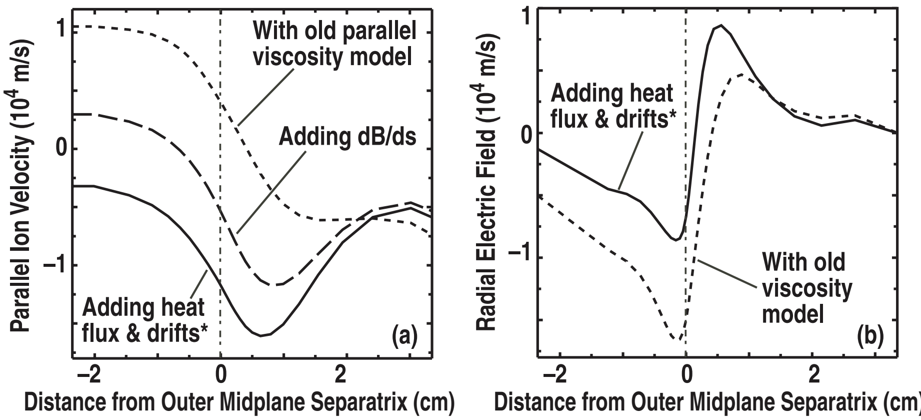


*FIG. 4. Hydrogen ion parallel velocity from UEDGE for different viscosity models and the radial electric field profiles corresponding to two cases.*

The role of anomalous transport often used in fluid transport codes to model turbulence can have a strong effect in changing the coupling between SOL flow and that in the core. This effect is modeled by reducing the anomalous radial ion viscosity coefficient from 0.5 $m^2/s$ to 0.1 $m^2/s$ as shown in figure 5. Therefore, comparisons between purely neoclassical models and others having some representation of turbulence transport must be done with caution. Finally, it is found that the carbon $C^{6+}$ ion has a parallel velocity within $\sim$ 10% that of the $D^+$ ion across the simulated domain, even with the generalized parallel viscosity model, owing to strong frictional drag between the species. The reason for this difference with the NEO simulations is being investigated.

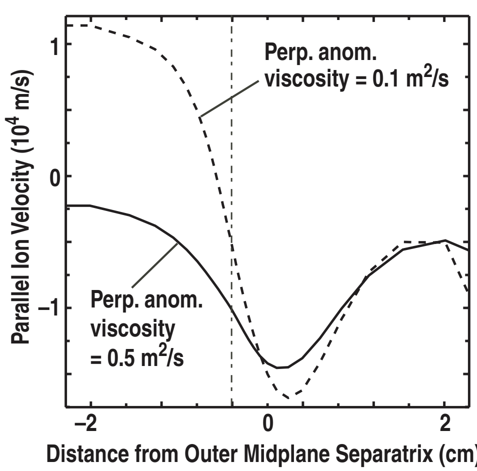


*FIG. 5. The deuterium parallel velocity from UEDGE at the outer midplane for the initial case comparing anomalous parallel viscous diffusivity of 0.5 $m^2/s$ and 0.1 $m^2/s$.*

## ACKNOWLEDGMENT

This work was supported in part by the US DOE under DE-FG02-95ER54309, DE-FG02-07ER54917, DE-AC52-07NA27344, DE-AC03-76SF00098, and DE-FC02-66ER54873.

## REFERENCES


[1]BELLI, E.A., and CANDY, J., Plasma Phys. Control. Fusion **50** (2008) 095010; Plasma Phys. Control. Fusion **51** (2009) 075018.

[2]BELLI, E.A., and CANDY, J., Plasma Phys. Control. Fusion **54** (2012) 015015.

[3]HOULBERG, W.A., *et al.*, Phys. Plasmas **4** (1997) 3230.

[4]COLELLA, P., DORR, M.R., HITTINGER, J.A.F, and MARTIN, D.F., J. Comput. Phys. **230** (2011) 2952.

[5]DORR, M.R., *et al.*, "Numerical Simulation of Phase Space Advection in Gyrokinetic Models of Fusion Plasma," Proc. 2010 Scientific Discovery through Advanced Computing Conf., Chattanooga, Tennessee (Oak Ridge National Laboratory, 2010) pp. 42-52.

[6]COLELLA, P., and SEKORA, M.D., J. Comput. Phys. **227** (2008) 7069.

[7]ROGNLIEN, T.D., *et al.*, J. Nucl. Mater. **196-198** (1992) 347.

[8]ROGNLIEN, T.D., *et al.*, Phys. Plasmas **6** (1999) 1851.

[9]ROZHANSKY, V., *et al.*, Nucl. Fusion **29** (2009) 025007.

[10]BOEDO, J.A., *et al.*, Phys. Plasmas **18** (2011) 032510.

[11]HIRSHMAN, S.P., and SIGMAR, D.J., Phys. Fluids **19** (1976) 1532.

[12]HIRSHMAN, S.P., SIGMAR, D.J., and CLARKE, J.F., Phys. Fluids **19** (1976) 656.

[13]CONNOR, J.W., Plasma Phys. **15** (1973) 765.

[14]CANDY, J., and BELLI, E.A., Phys. Rev. Lett. **106** (2011) 235003.

[15]DORF, M.A., *et al.*, Control. Plasma Phys. **52** (2012) 518.

[16]LIN, Z., TANG, W.M., and LEE, W.W., Phys. Plasmas **2** (1995) 2975.