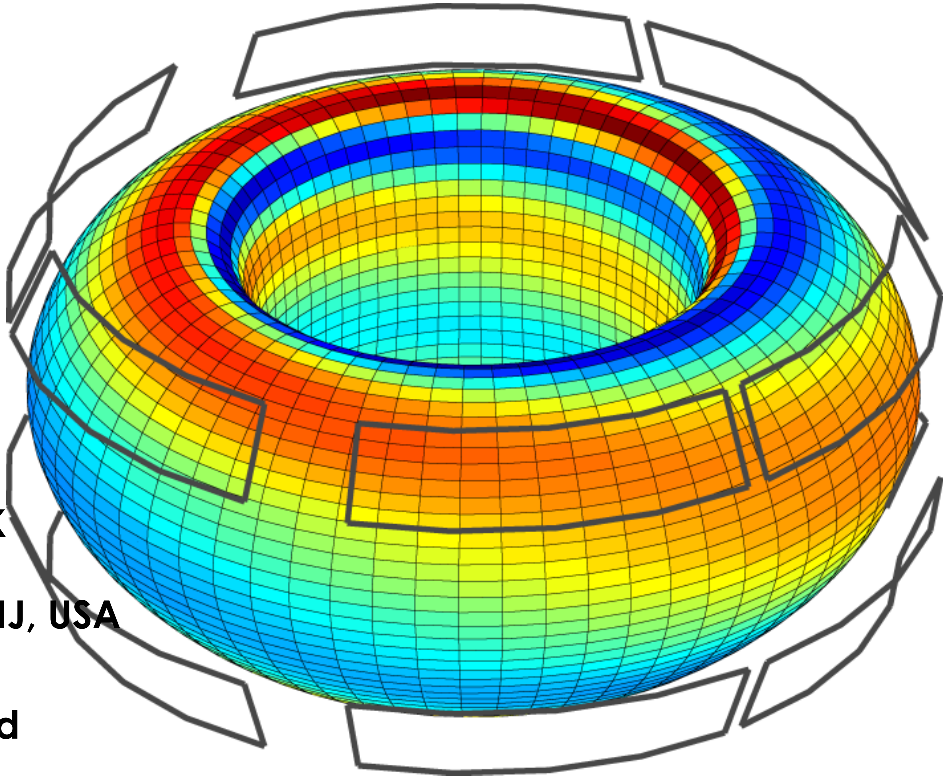
Non-ideal Modifications of 3D Equilibrium and Resistive Wall Mode Stability Models in DIII-D

By
H. Reimerdes*

In collaboration with
J.W. Berkery, M.J. Lanctot, R.J. Buttery[1],
M.S. Chu[1], A.M. Garofalo[1], J. Hanson, Y. In[2],
R.J. La Haye[1], Y.Q. Liu[3], G. Matsunaga[4],
G.A. Navratil, M. Okabayashi[5],
S.A. Sabbagh, O. Schmitz[6], and E.J. Strait[1]

[1]General Atomics, San Diego, CA, USA
[2]FAR-TECH, Inc., San Diego, CA, USA
[3]EURATOM/CCFE Fusion Association, Abingdon, UK
[4]Japan Atomic Energy Agency, Naka, Japan
[5]Princeton Plasma Physics Laboratory, Princeton, NJ, USA
[6]Forschungszentrum Jülich, Jülich, Germany

*Present address: CRPP-EPFL, Lausanne, Switzerland



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DIII-D
NATIONAL FUSION FACILITY

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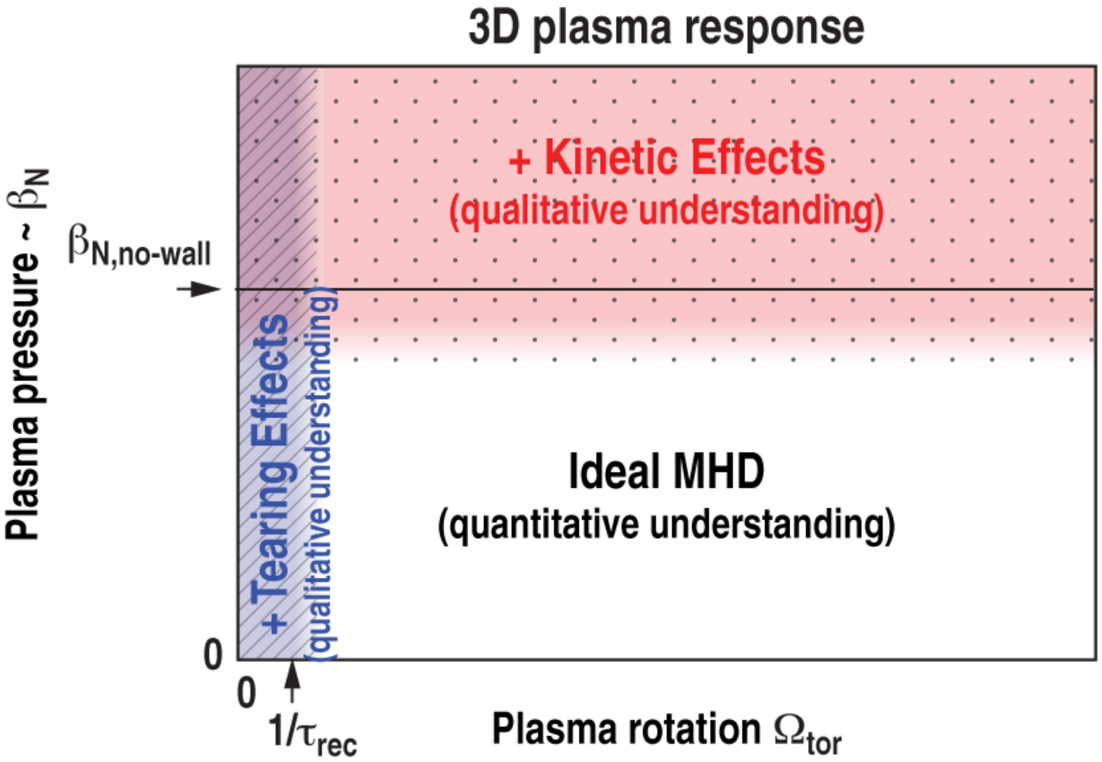
IN THE CITY OF NEW YORK

Experiments Show When Non-ideal Effects Modify 3D Equilibria and Deliver Evidence for Kinetic RWM Stabilization

Main results

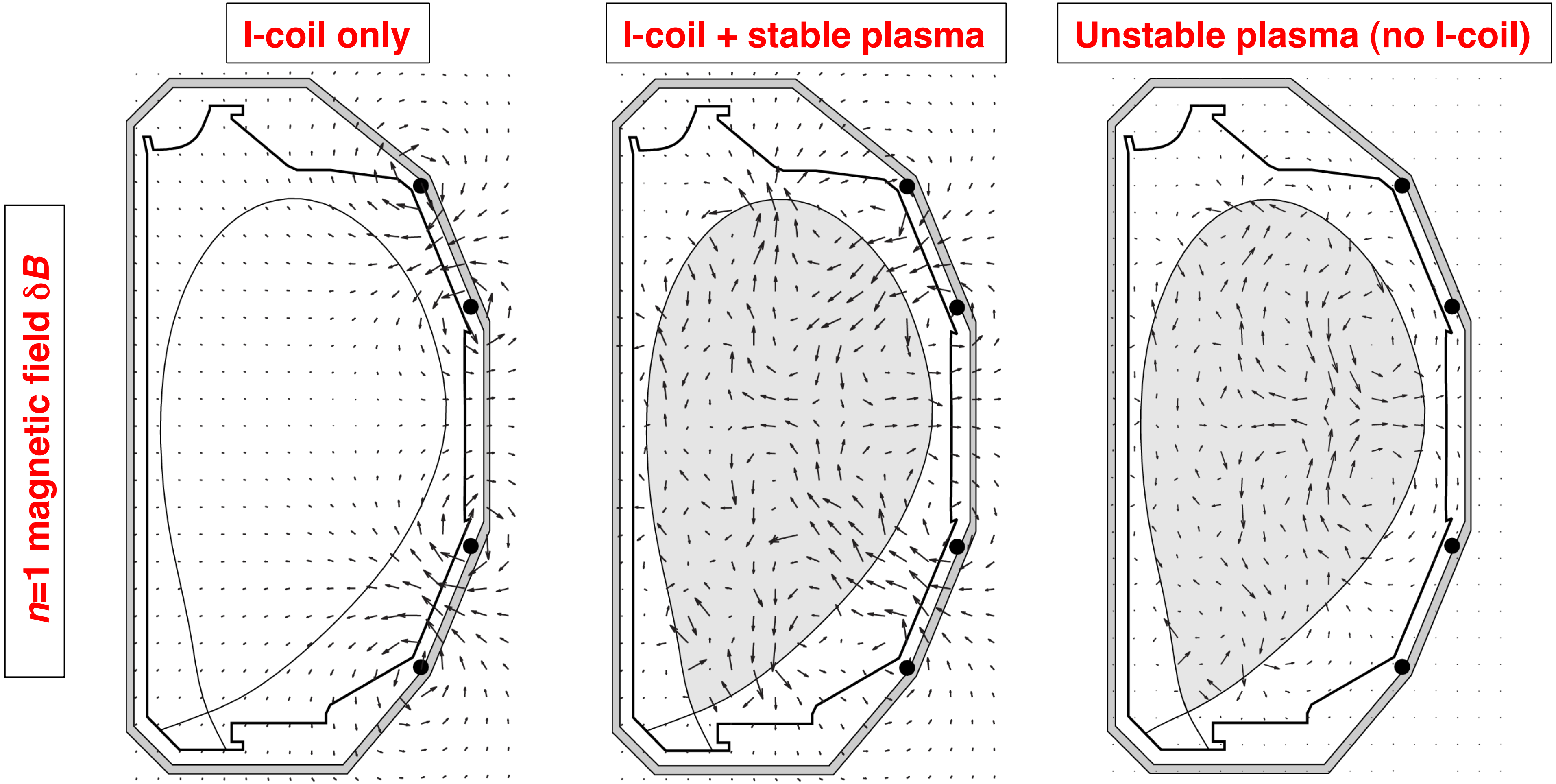
- **Linear ideal MHD describes <u>*n*=1 equilibria</u> as long as**
  - **Plasma rotation is sufficiently fast**
  - **Beta is sufficiently low**

- **Kinetic effects explain <u>resistive wall mode</u> (RWM) stability**



➜ Opens possibility of passive RWM stabilization even at low plasma rotation, i.e. under reactor conditions

Three Dimensional Tokamak Equilibria and RWM Stability Share the Same Physics Basis



External 3D field

3D equilibrium
(usually static)

Unstable RWM
(growth, rotation$\leq\tau_W^{-1}$)

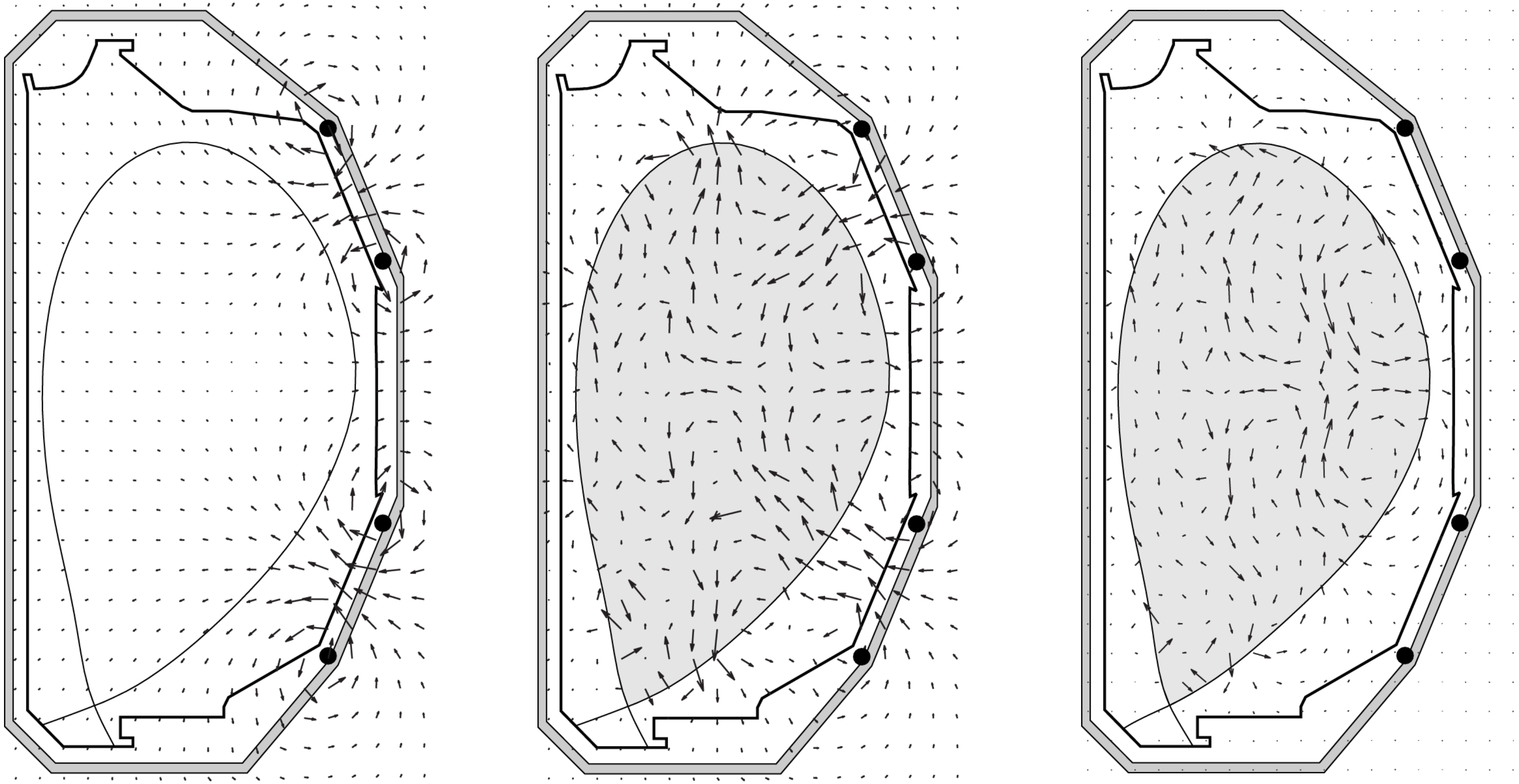
Three Dimensional Tokamak Equilibria and RWM Stability Share the Same Physics Basis

n=1 magnetic field δB

I-coil only

I-coil + stable plasma

Unstable plasma (no I-coil)



External 3D field

3D equilibrium
(usually static)

Unstable RWM
(growth, rotation$\leq\tau_W^{-1}$)

➜ Both are a quasi-static global perturbation

Extend Ideal MHD 2D Equilibrium Model to 3D

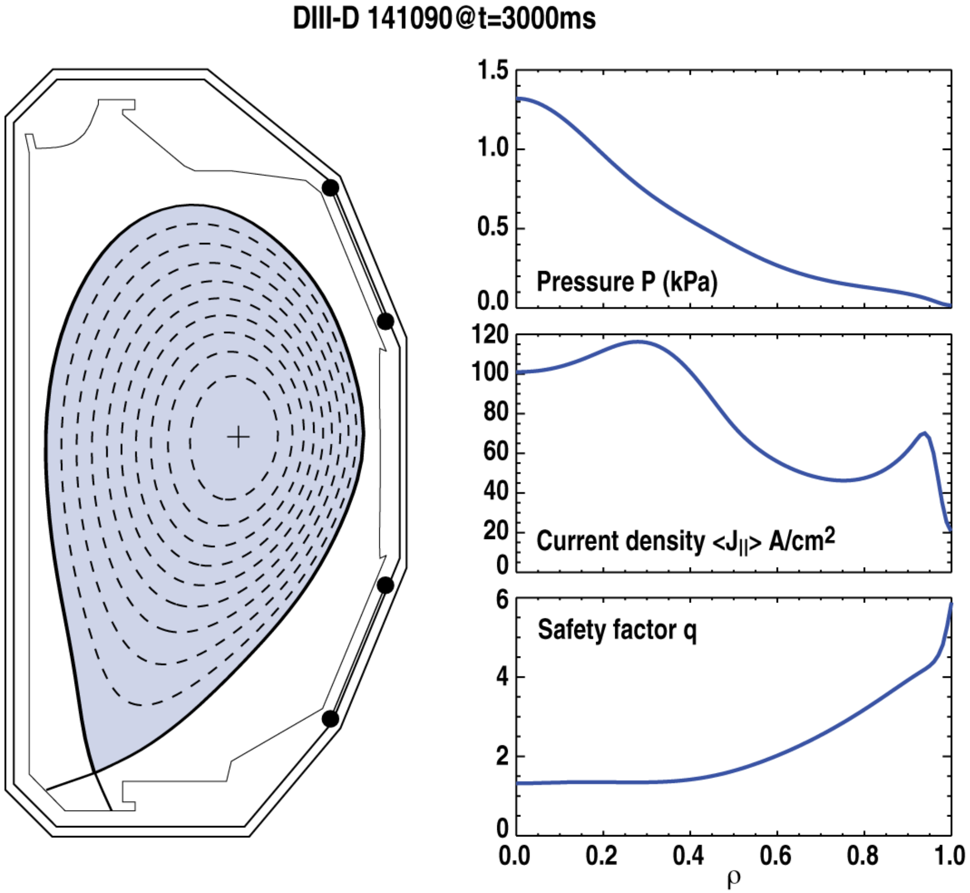
- **Ideal MHD force balance:**

$$\boxed{\vec{J} \times \vec{B} = \nabla P}$$

- **Axisymmetry (2D)**
  - Grad-Shafranov equation solved by various codes
- **Non-axisymmetric equilibrium (3D)**
  - ***VMEC*** [Hirshman, Betancourt, *J. Comput. Phys.* 1991]
  - **Linearize force balance**

$$\delta\vec{J} \times \vec{B} + \vec{J} \times \delta\vec{B} = \nabla \delta P$$

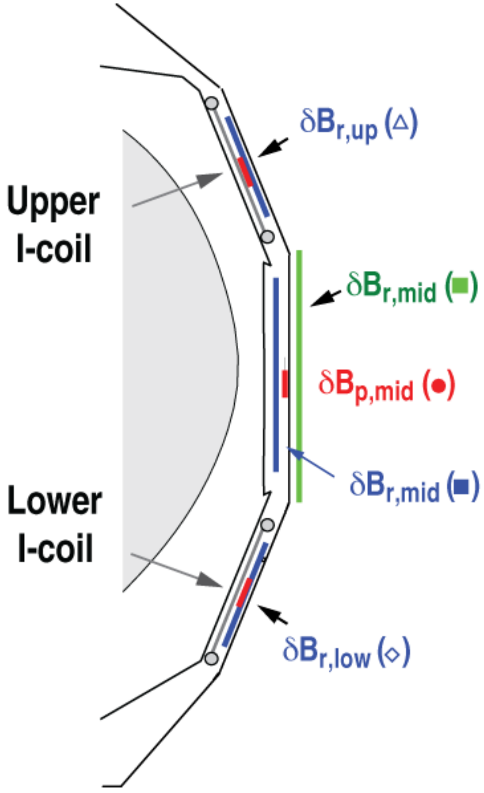
    + ***MARS-F*** [Liu et al., *Phys. Plasmas* 2000]
    + ***IPEC*** [Park, et al., *Phys. Plasmas* 2007]

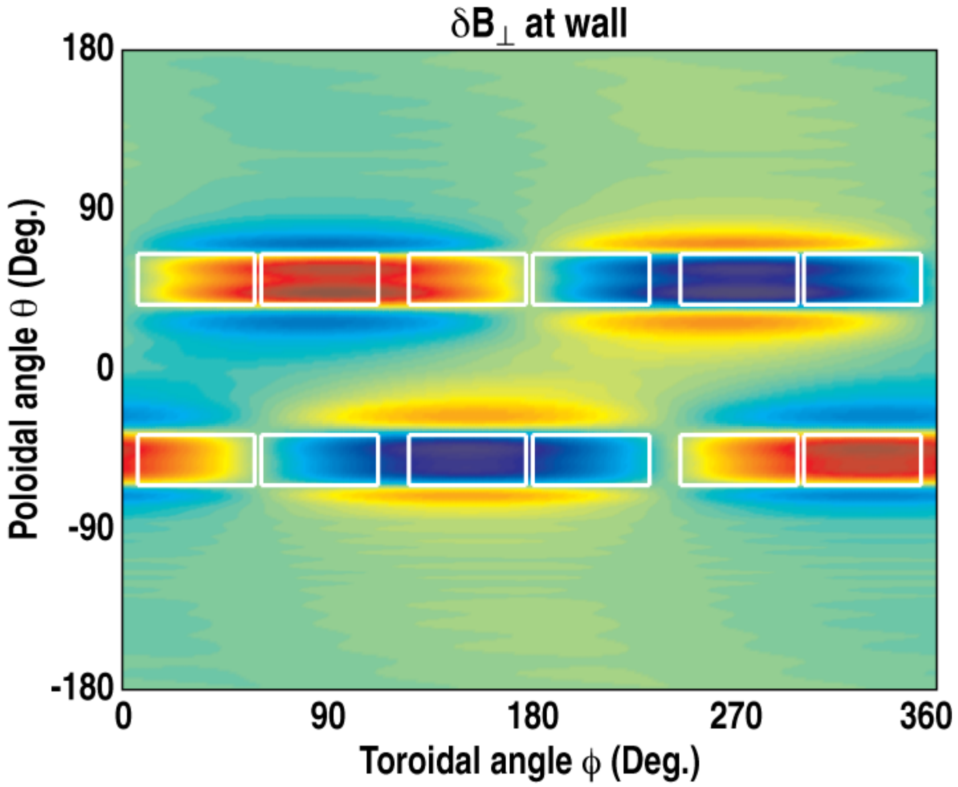


Comparison with Magnetic Measurements Shows that Ideal MHD Can Quantitatively Describe 3D Equilibria

[M.J. Lanctot, et al., *Phys. Plasmas* 2010]

- **Perturb plasma with an externally applied *n*=1 field ($\delta B/B_T \leq 10^{-3}$)**





I-coil

Upper I-coil

Lower I-coil

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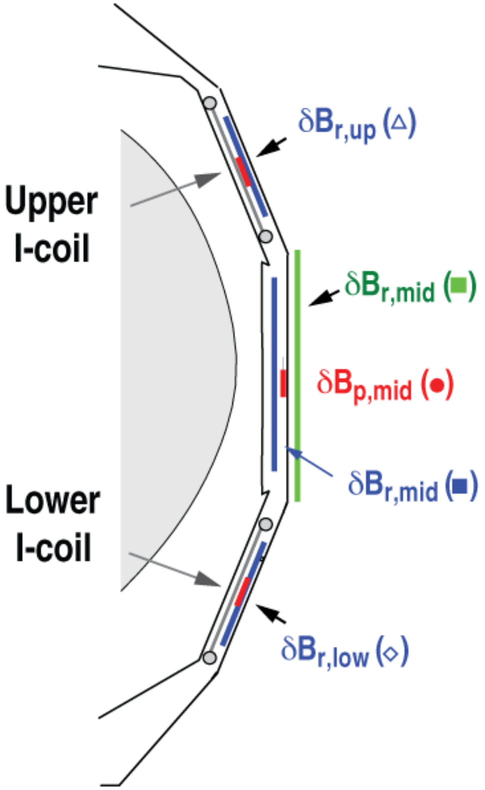

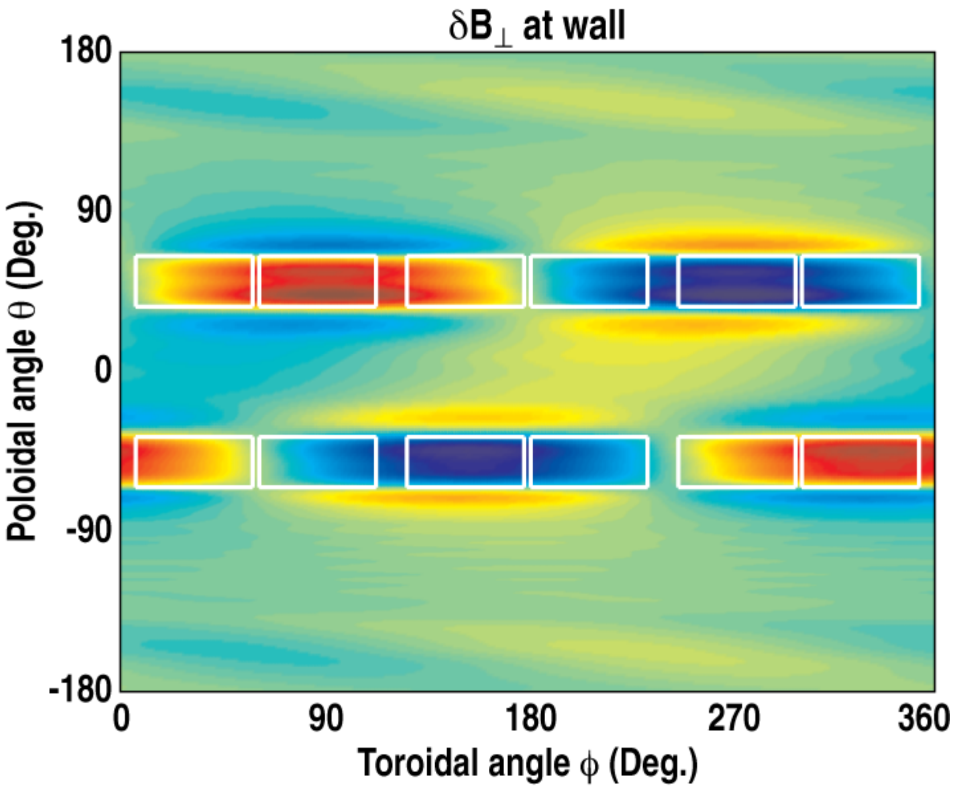

I-coil
+
Plasma

Upper I-coil

Lower I-coil

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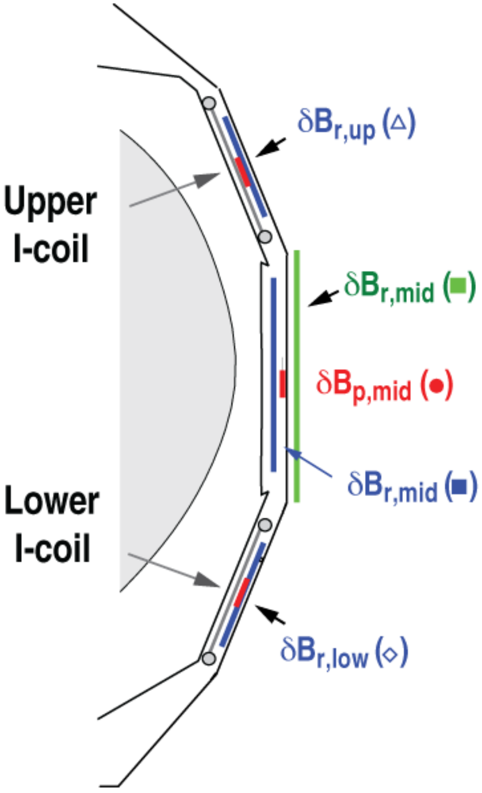

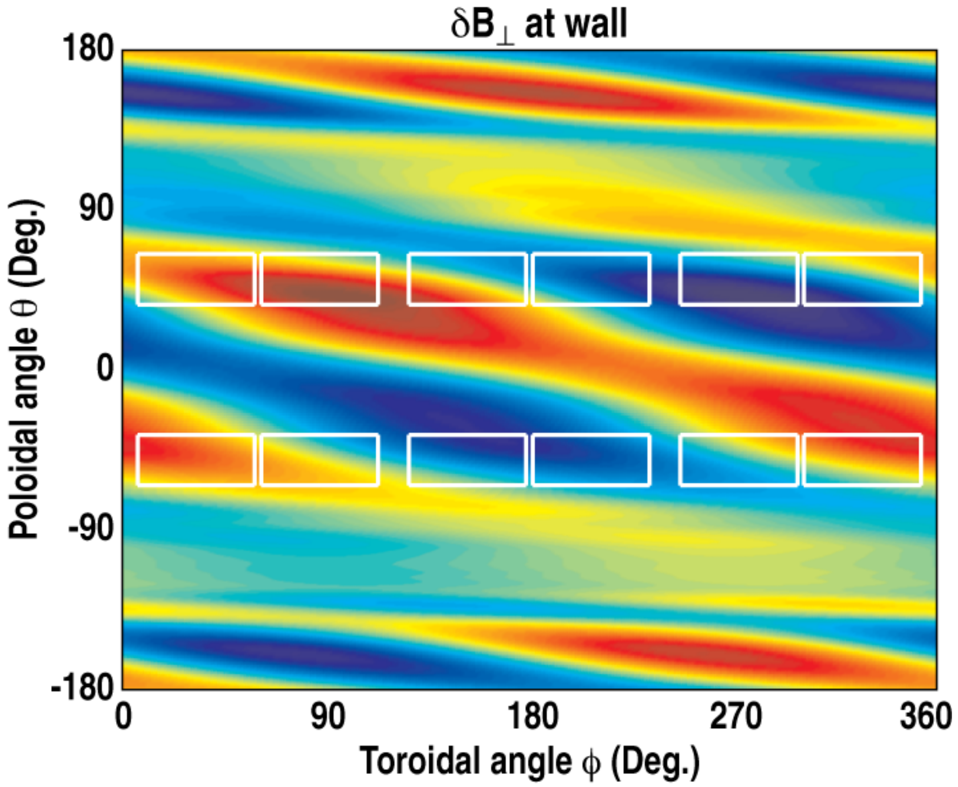

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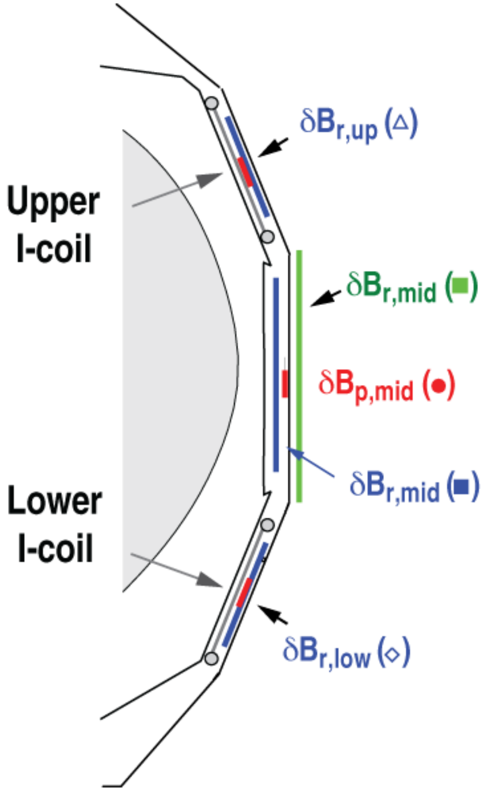

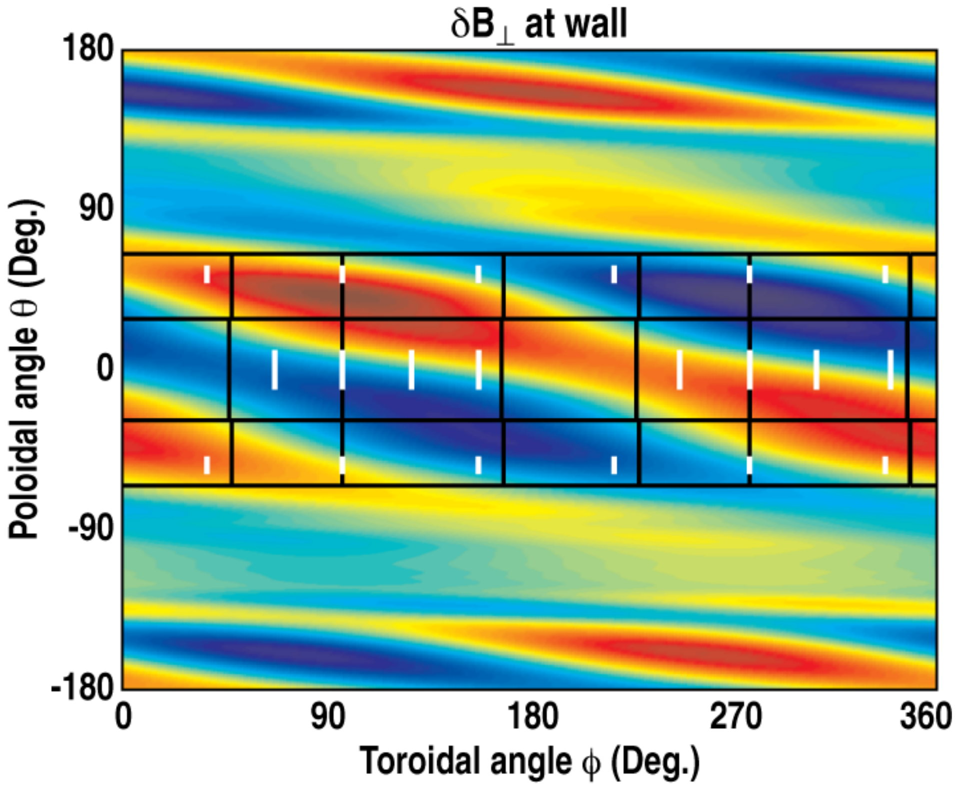

Plasma only

B_p probes

B_r loops

- **Toroidal arrays of B_p and B_r sensors measure <u>amplitude</u> and <u>toroidal phase</u> of the n>0 plasma response**

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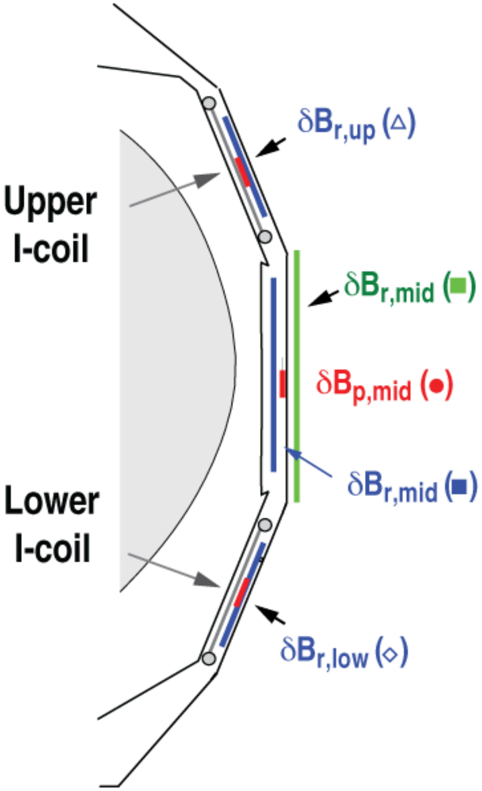

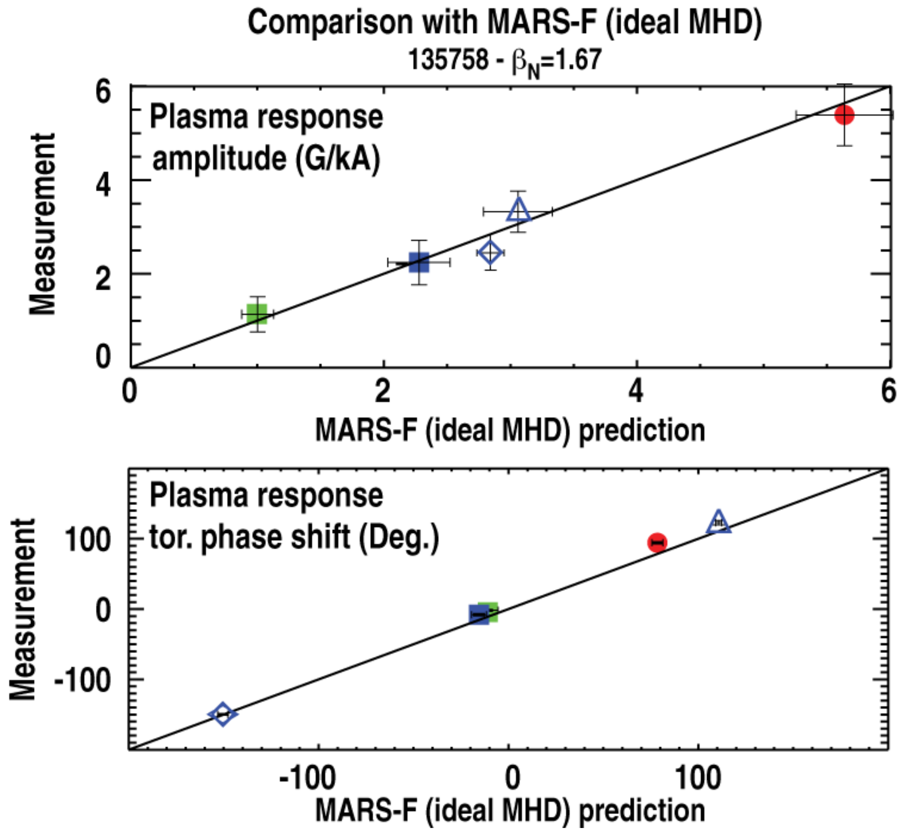

Plasma only

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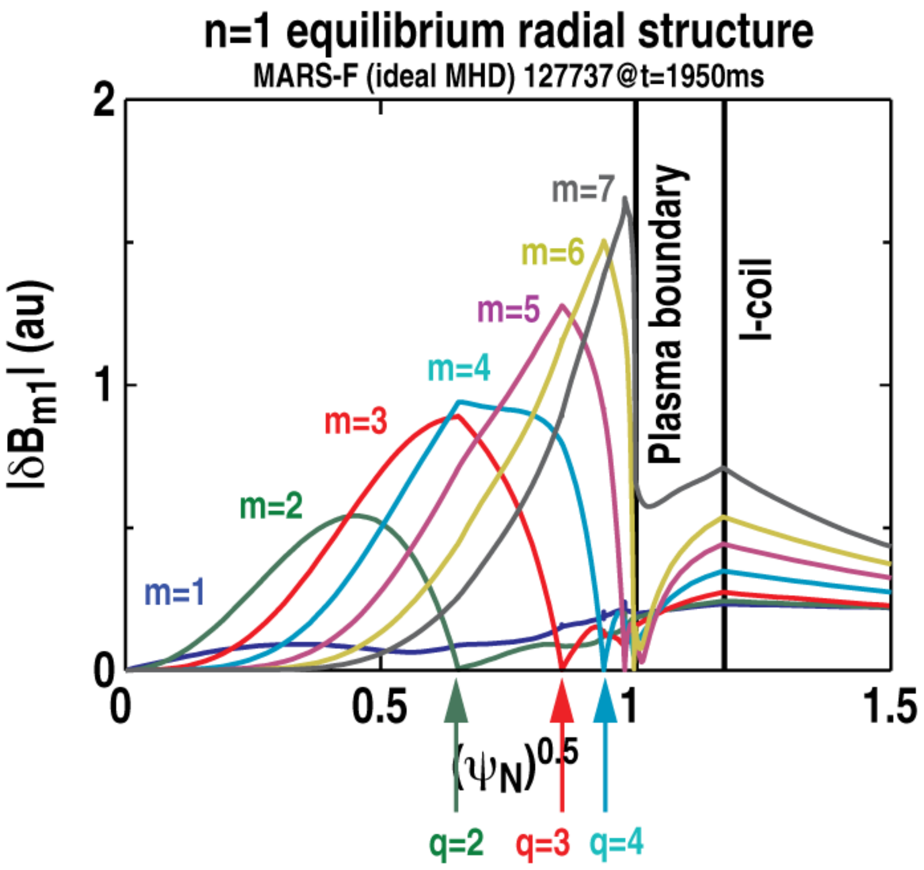
Ideal MHD 3D Equilibrium Assumes Perfect Shielding of Resonant Fields



- **Resonant components**

  δB_{mn} **with** $m = nq$

  of the perturbed field are zero
  - A finite resonant component would lead to an island

➜ Magnetic topology of nested flux surfaces is preserved

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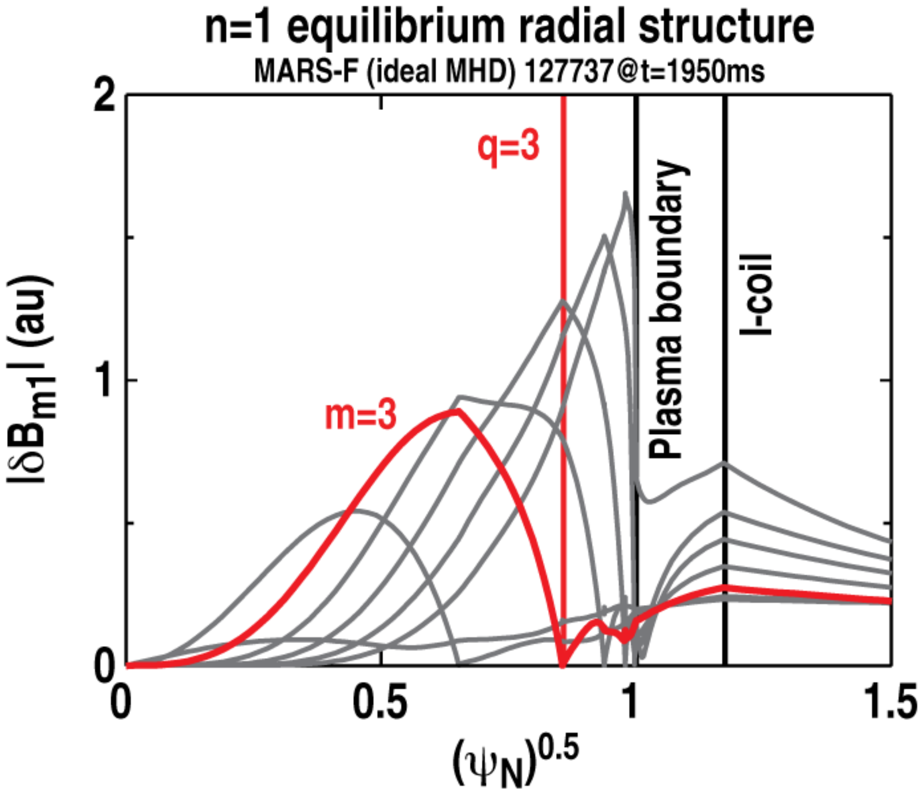

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Linear Ideal MHD Can Describe 3D Equilibria as Long as the Plasma Rotation is Sufficiently Large

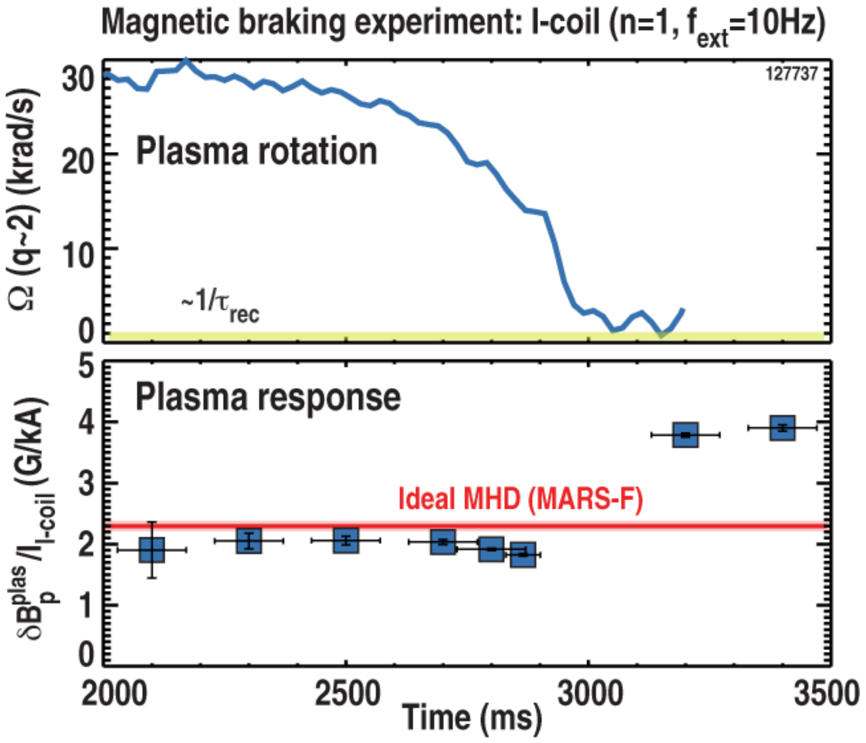
- **Measure response to *n*=1 I-coil field in magnetic braking experiment**

- **For "large" rotation**
  - δB^{plas} *is* independent of rotation
  - δB^{plas} *is* consistent with ideal MHD

- **After the rotation has collapsed**
  - δB^{plas} deviates from ideal MHD
  - A magnetic island forms

- **Consistent with shielding as long as** $\Omega\tau_{rec}>>1$ [Fitzpatrick, *Nucl. Fusion* 1993]

- **Resonant braking torque indicates a local deviation from ideal MHD**



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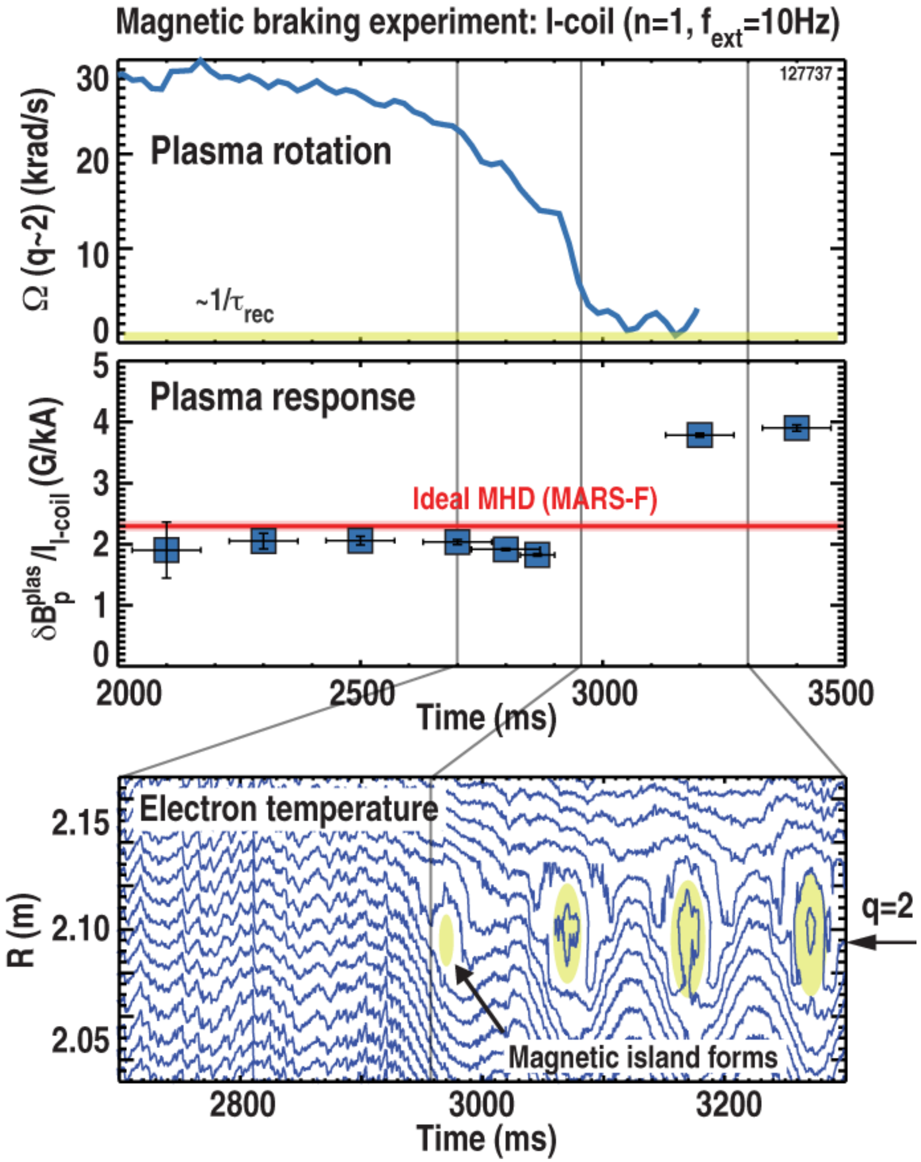
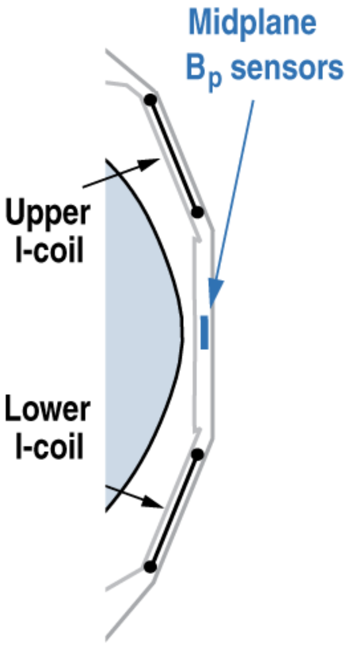
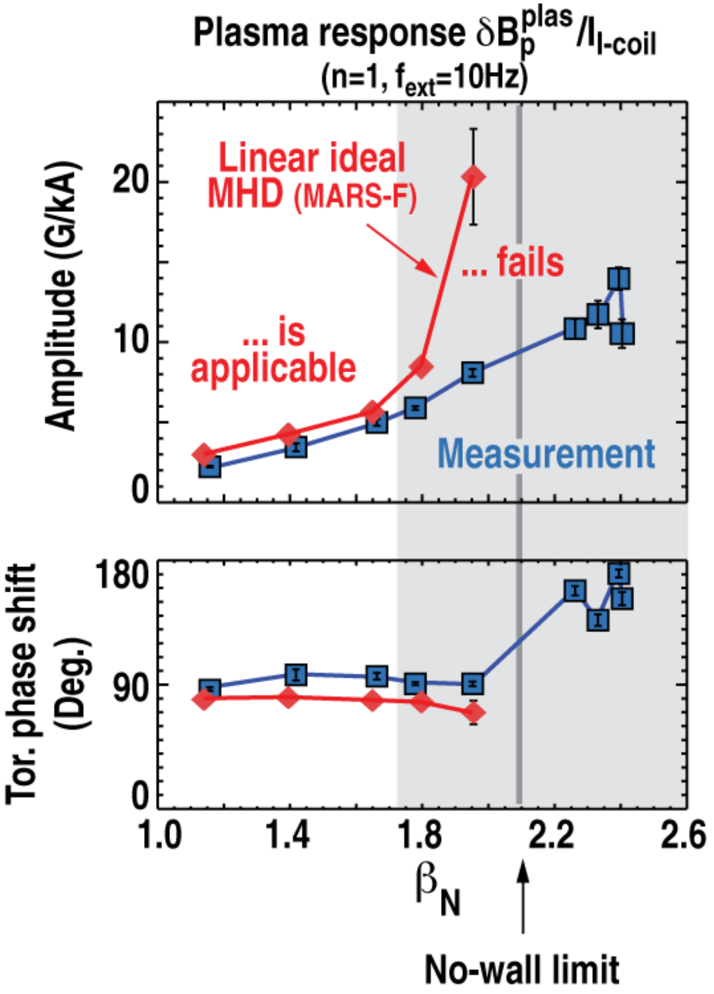


Linear Ideal MHD Can Describe 3D Equilibria as Long as Beta is Well Below the Ideal MHD No-wall Limit

[M.J. Lanctot, et al., *Phys. Plasmas* 2010]





- **Ideal MHD starts to overestimate δB at ~80% of the no-wall limit $\beta_{N,nw}$**
  - Diverges for $\beta_N=\beta_{N,nw}$
  - Predicts instability for $\beta_N>\beta_{N,nw}$

Observed RWM Stability Above the No-wall Limit has Long Shown the Importance of Non-ideal Effects

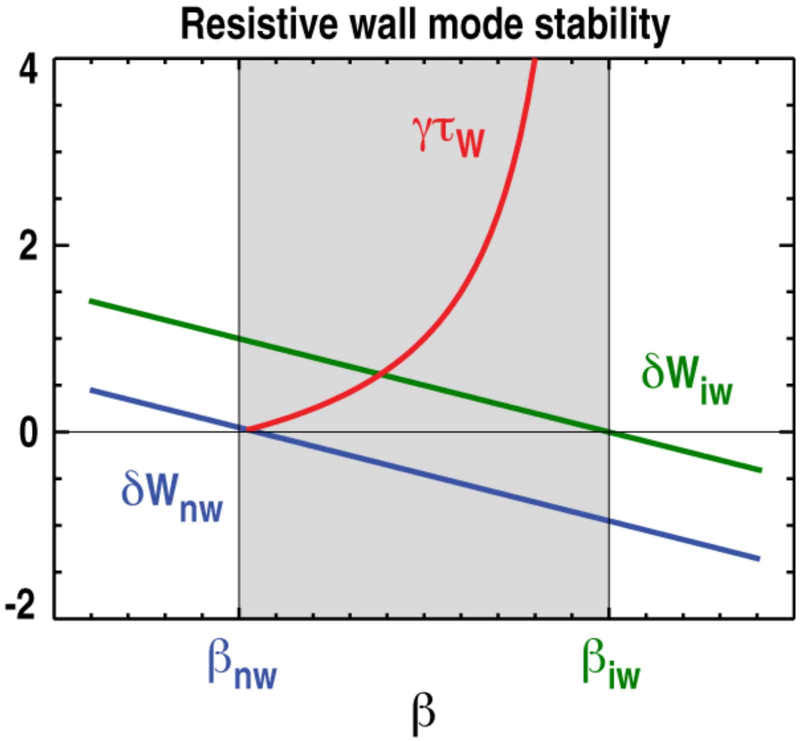
- **Ideal MHD RWM unstable when $\beta > \beta_{nw}$**
  - Energy principle [Haney, Freidberg, *Phys. Fluids* B 1989]

$$\gamma\tau_W = -\frac{\delta W_{nw}}{\delta W_{iw}}$$

Perturbed energy assuming no wall

RWM growth rate normalized with inverse wall time

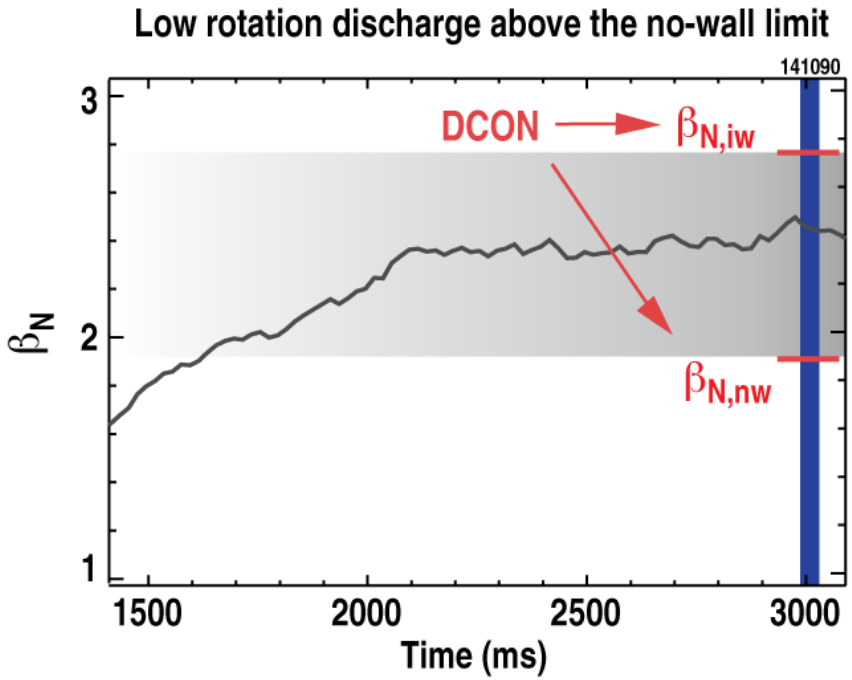
Perturbed energy assuming an ideal wall

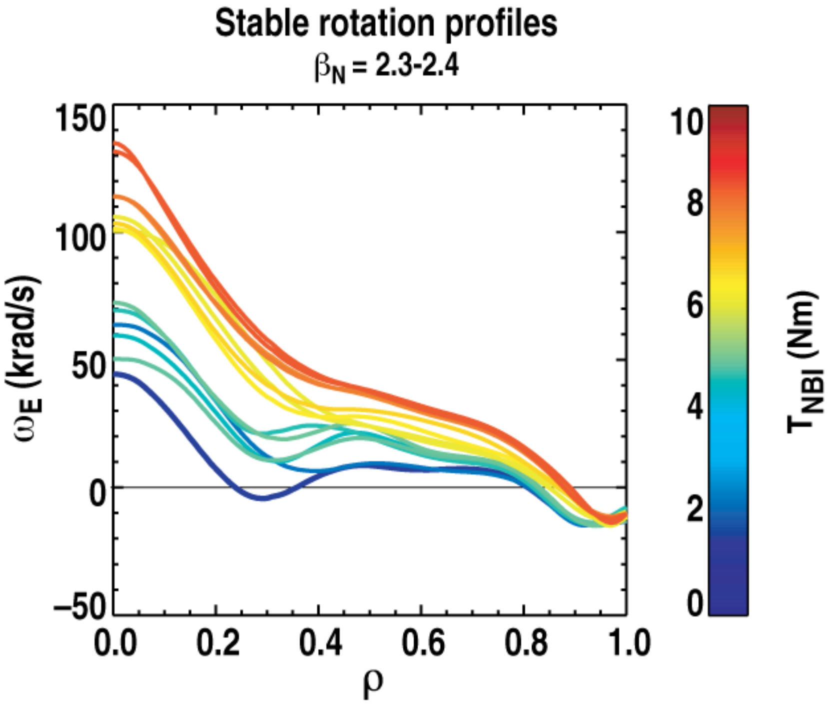

Resistive wall mode stability

- **Tokamaks routinely exceed the ideal MHD no-wall stability limit**
  - Originally associated with fast toroidal plasma rotation

DIII-D Discharges Exceed the No-wall Limit with a Wide Range of Rotation Profiles

- **Vary neutral beam torque T_{NBI} from 1.5 to 8.0 Nm while keeping $\beta_N \approx 2.3$ ($>\beta_{N,nw}$)**





ω_E: Toroidal rotation of the $E_r=0$ reference frame

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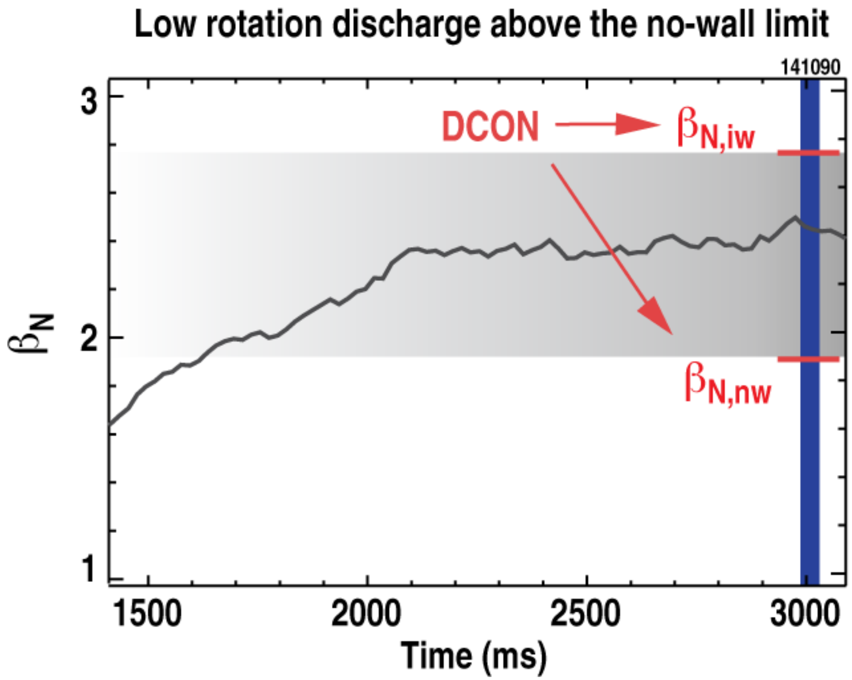

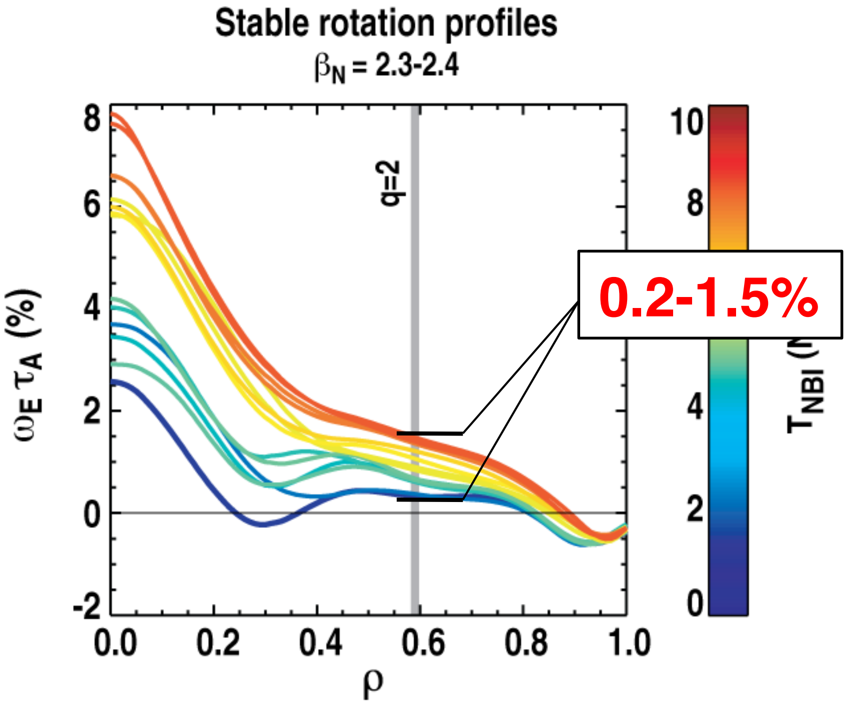

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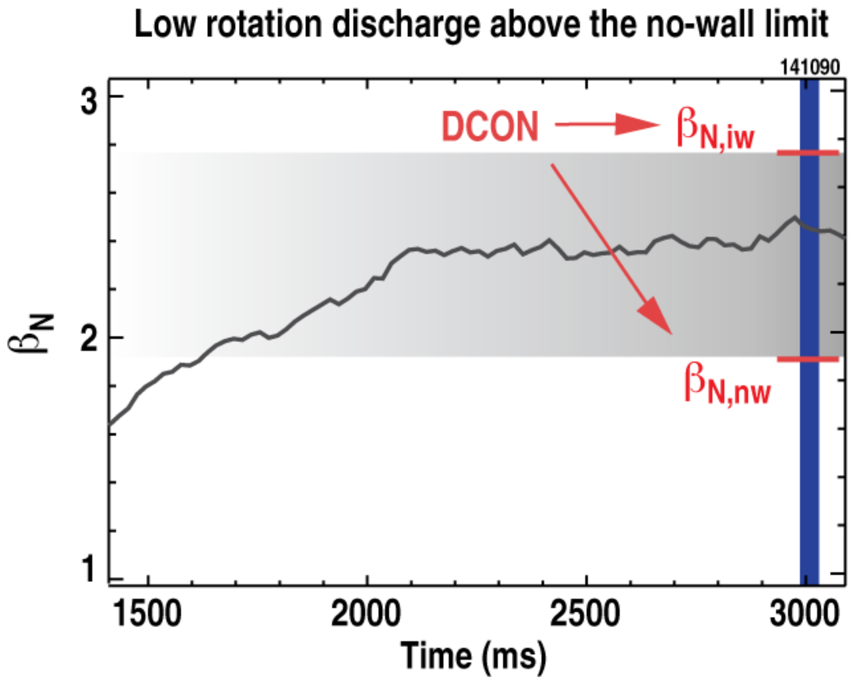
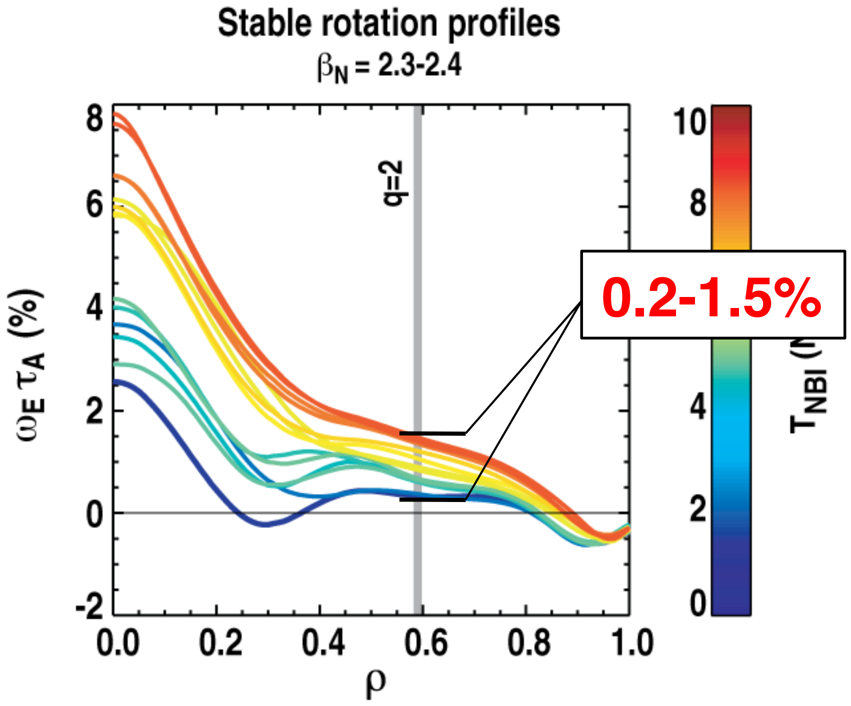




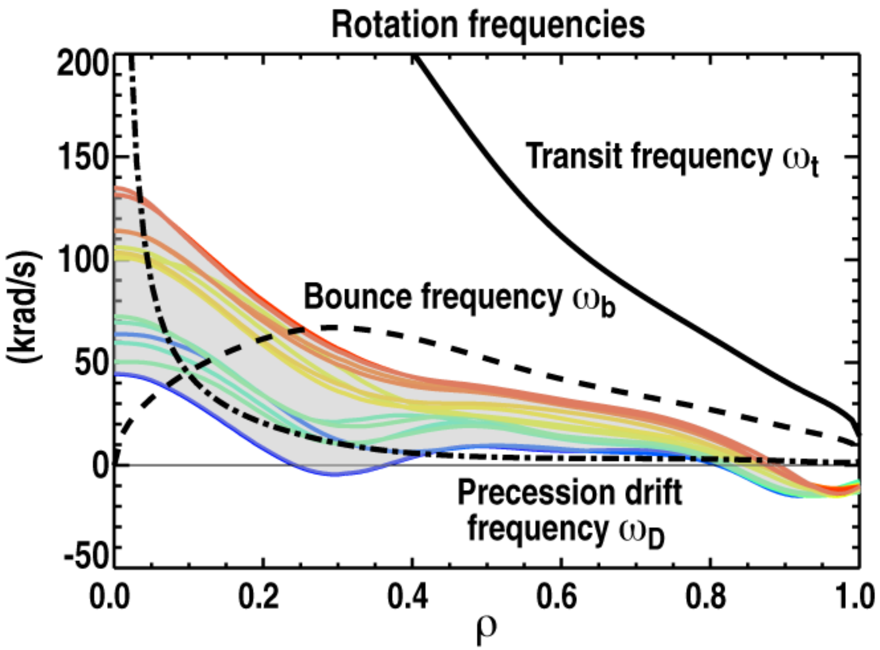
ω_E: Toroidal rotation of the $E_r=0$ reference frame

- **In NSTX the RWM becomes unstable at "intermediate" rotation values**

➜ S.A. Sabbagh, et al, next talk

Wave-particle Interaction Can Lead to an Exchange of Energy Between the RWM and Particles

- **Important particle frequencies are**
  - Transit frequency of passing particles: $\omega_{\mathrm{t}} \sim \frac{V_{th}}{qR}$
    [Bondeson, Chu, *Phys. Plasmas* 1996]
  - Bounce frequency of trapped particles: $\omega_{\mathrm{b}} \sim \sqrt{\frac{r}{2R}}\frac{V_{th}}{qR} < \omega_{\mathrm{t}}$
    [Bondeson, Chu, *Phys. Plasmas* 1996]
  - Precession drift frequency of trapped particles: $\omega_{\mathrm{D}} \sim \frac{qr_L}{r}\frac{V_{th}}{R} << \omega_{\mathrm{b}}$
    [Hu, Betti, *Phys. Rev. Lett.* 2004]



Perturbed Kinetic Energy Can Be Calculated with the MISK Code*

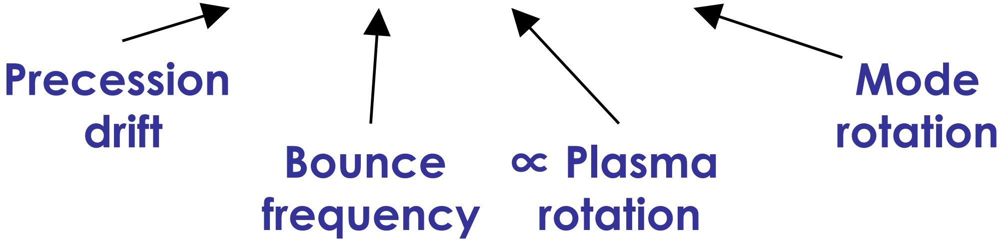
*[Berkery, et al., *Phys. Plasmas* 2010]

- **Energy principle has been extended to include kinetic effects** [Hu, Betti, *Phys. Rev. Lett.* 2004]

$$\gamma\tau_W = -\frac{\delta W_{nw} + \delta W_K}{\delta W_{iw} + \delta W_K}$$

- **The perturbed kinetic energy δW_K has the form (for trapped particles)**

$$\delta W_K^T \propto \sum_{l=-\infty}^{+\infty} \frac{\omega_{*N} + (\hat{\varepsilon} - 3/2)\omega_{*T} + \omega_E - \omega_{RWM}}{\langle\omega_D\rangle + l\omega_b + \omega_E - \omega_{RWM}}$$



Precession drift

Bounce frequency

∝ Plasma rotation

Mode rotation

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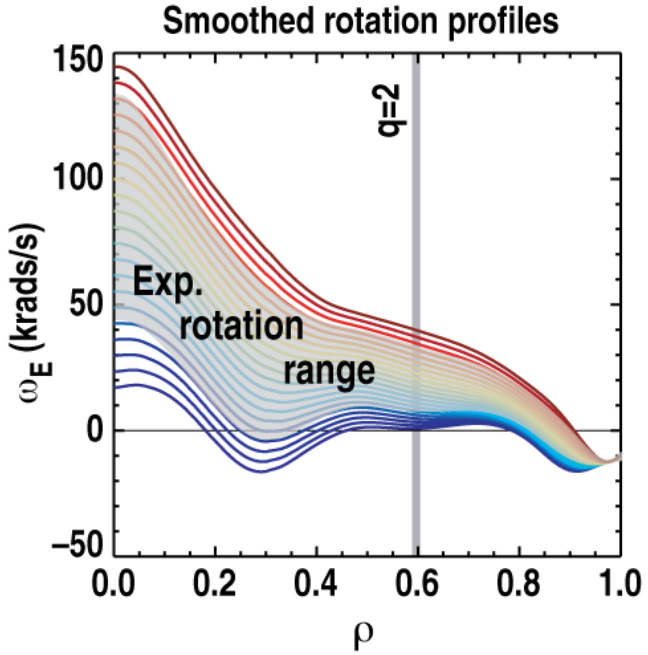
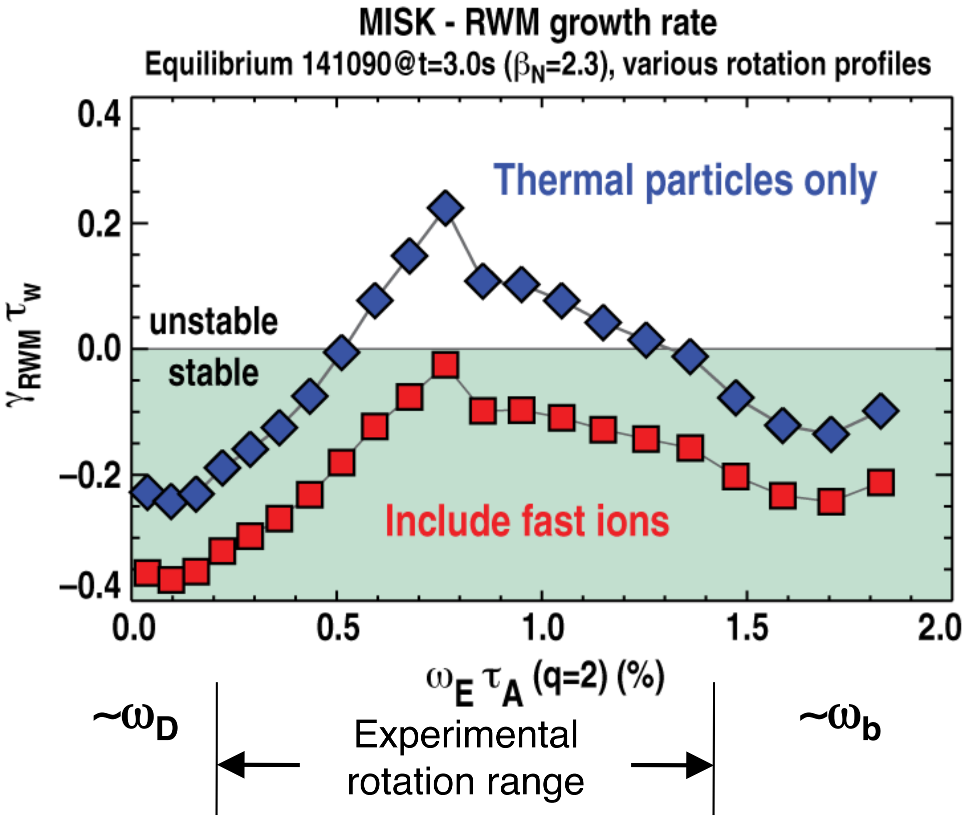
$\omega_{RWM} \approx 0$

Small when $\omega_E = -\langle\omega_D\rangle$ **or** $\omega_E = -l\omega_b$

- **MISK assumes structure of a marginally stable RWM (perturbative approach)**
  - Self-consistent approach implemented in MARS-K code [Liu, et al., *Phys. Plasmas* 2008]

Kinetic Stability Model Can Explain the Stability Over the Entire Range of Rotation Profiles





- Thermal particles alone are not sufficient to explain RWM stability
- Kinetic model has to include fast ions from the NBI heating to be consistent with the experiment
  - Fast ions constitute ~20% of the kinetic energy

Use Plasma Response to an External *n*=1 field, i.e. 3D Equilibrium, to Probe the Damping Rate

- **<u>Amplitude</u> of plasma response largest at intermediate plasma rotation**

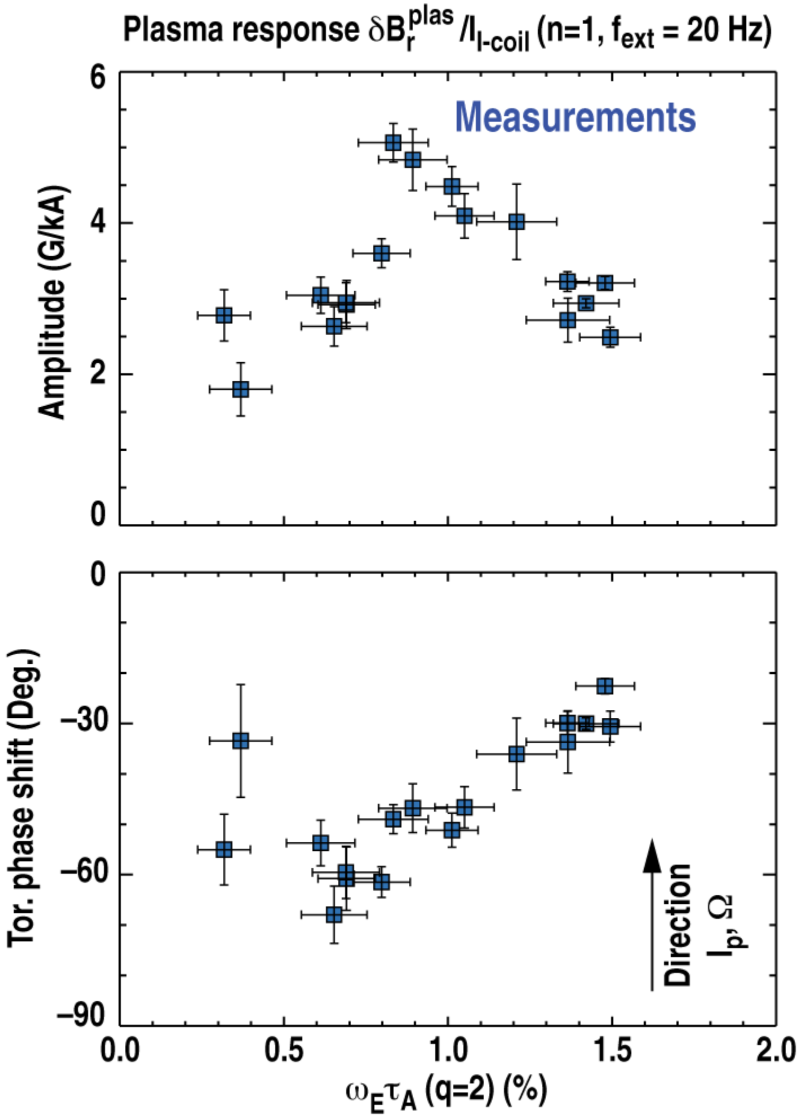
$$\omega_E \tau_A (q=2) \approx 0.9\%$$

- **<u>Phase shift</u> of plasma response with respect to external field largest at**

$$\omega_E \tau_A (q=2) \approx 0.6\%$$

- **Single mode model links γ_{RWM} and ω_{RWM} (e.g. from MISK) to amplitude and phase of δB^{plas}**

  [Reimerdes, et al., Phys. Rev. Lett. 2004]



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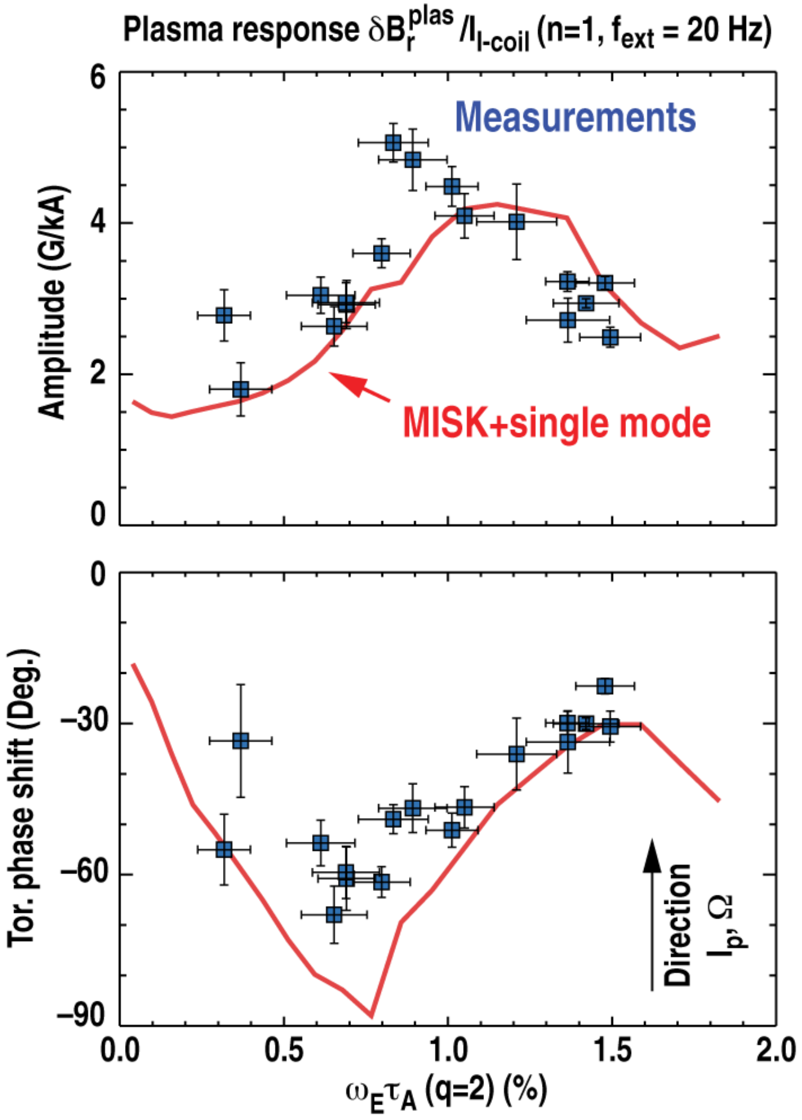
Measured Plasma Response Reveals the Characteristics of Kinetic Stabilization

- **MISK modeling reproduces the <u>characteristics</u> of the measured dependence of δB^{plas} on plasma rotation**
  - Uncertainty in the single mode coupling can lead to systematic shift of amplitude and phase shift

➜ Increased stability at low rotation is a direct effect of resonance with the precession drift of trapped ions



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Recent Results are an Important Step Towards a Quantitative Understanding of 3D Equilibria and RWM Stabilization

- **A linear ideal model is adequate to describe 3D equilibria resulting from externally applied 3D fields ($\delta B/B_T \leq 10^{-3}$) as long as**
  - **Plasma rotation maintains the shielding currents at resonant surfaces**
  - **Beta is well below the ideal MHD no-wall stability limit**

- **Kinetic models explain the observed RWM stability above the ideal MHD no-wall limit provided that fast ions are taken into account**

- **Measured rotation dependence of the *n*=1 plasma response reveals the interaction of a quasi-static perturbation with the precession and bounce frequencies of trapped thermal ions**

  ➜ Direct evidence for the relevance of kinetic effects for RWM stability

- **Quantitative validation of stability models is needed before relying on predictions of passive RWM stabilization in ITER**