A Comprehensive Theory Based Transport Model

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A Comprehensive Theory Based Transport Model

comprehensive physics: shaped magnetic geometry, electron-ion collisions, fully electromagnetic, dynamic electrons, ions and impurity ions

Theory based: model fit to first principles gyro-kinetic theory

A more accurate transport model with comprehensive physics was needed

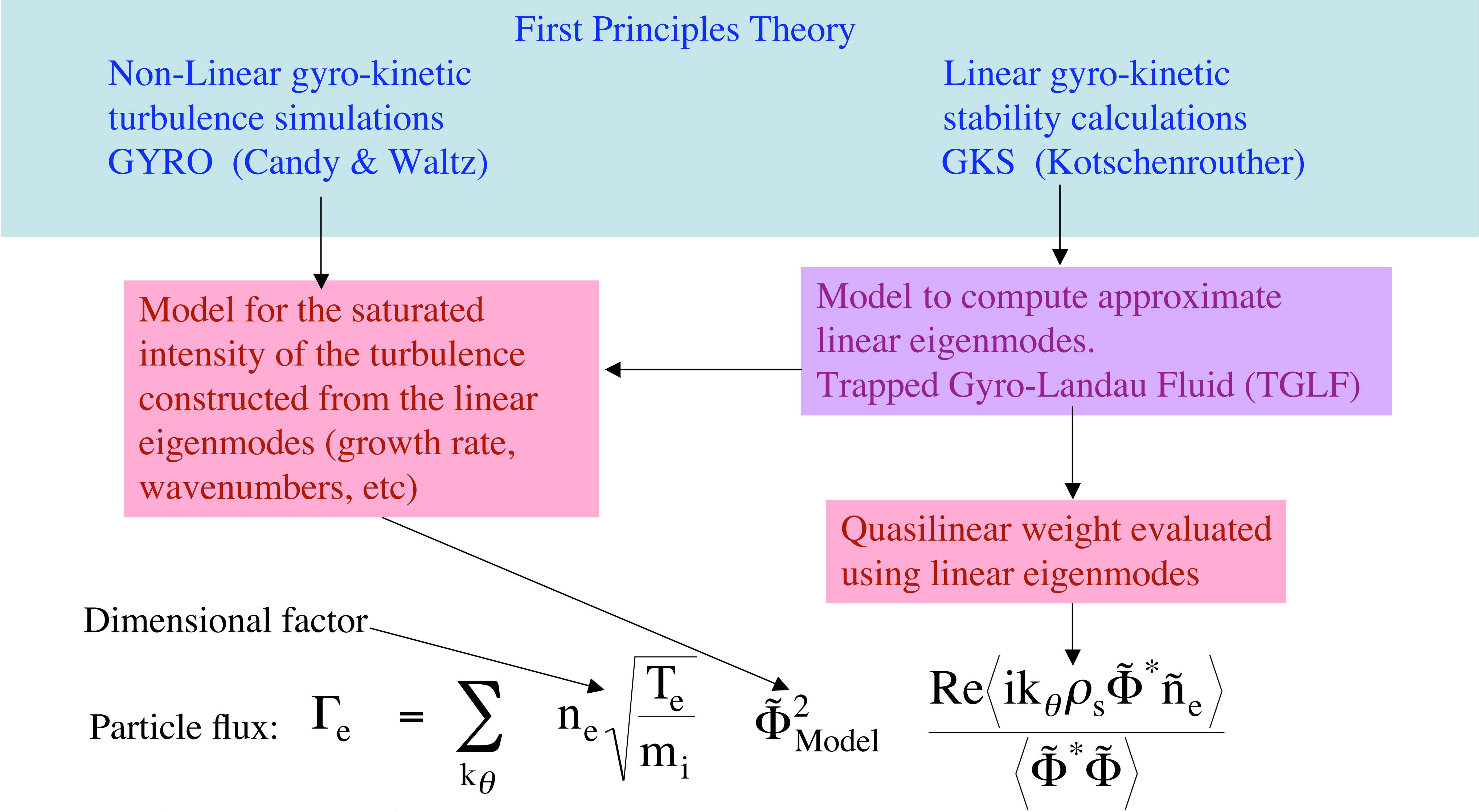
- The GLF23 transport model (Waltz, Staebler et.al.,1997) has been used successfully worldwide to predict core temperature profiles in tokamaks.

- A new transport model has been developed using the same methodology as GLF23: the Trapped Gyro-Landau Fluid (TGLF) model.
  - TGLF has particularly improved the treatment of trapped particles compared to GLF23.
  - TGLF includes the physics missing from GLF23.

- The TGLF linear stability code is being used for fast analysis of experiments.
  - growth rates agree very well with gyro-kinetic linear stability codes.
  - 100X faster for linear stability analysis of experimental discharges.

- The TGLF quasilinear transport model is a better fit to nonlinear gyro-kinetic turbulence simulations than GLF23.
  - 86 non-linear turbulence simulations over a wide range of parameters were used for the TGLF intensity model fit.

Anatomy of a quasilinear transport model



First Principles Theory

Non-Linear gyro-kinetic turbulence simulations
GYRO (Candy & Waltz)

Linear gyro-kinetic stability calculations
GKS (Kotschenrouther)

Model for the saturated intensity of the turbulence constructed from the linear eigenmodes (growth rate, wavenumbers, etc)

Model to compute approximate linear eigenmodes.
Trapped Gyro-Landau Fluid (TGLF)

Quasilinear weight evaluated using linear eigenmodes

Dimensional factor

Particle flux: $\Gamma_e = \sum_{k_\theta} n_e \sqrt{\frac{T_e}{m_i}} \; \tilde{\Phi}^2_{\mathrm{Model}} \; \frac{\mathrm{Re}\left\langle i k_\theta \rho_s \tilde{\Phi}^* \tilde{n}_e \right\rangle}{\left\langle \tilde{\Phi}^* \tilde{\Phi} \right\rangle}$

Similar models for electron and ion energy fluxes.

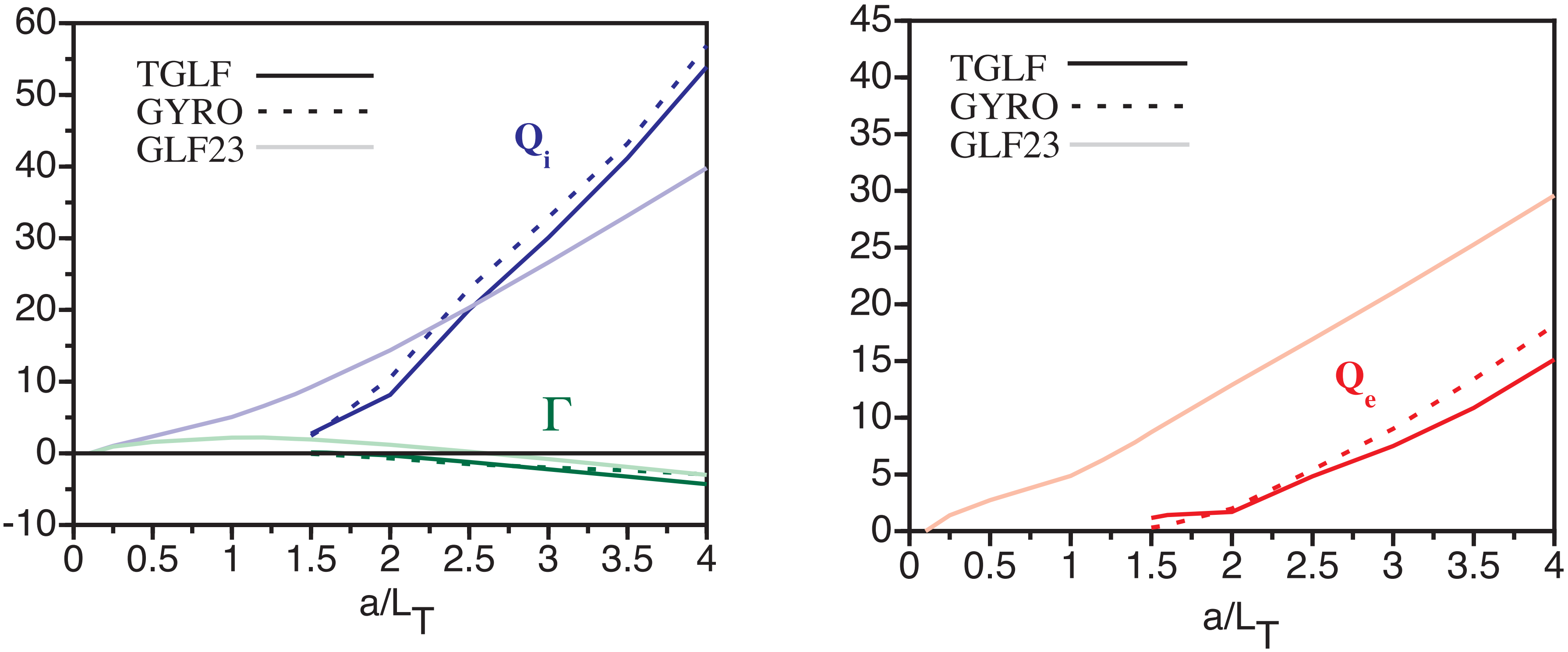
TGLF is a major upgrade from GLF23

TGLF	GLF23
TIM, ITG,TEM, ETG modes from a single set of equations	Different equations for low-k (ITG,TEM) and high-k (ETG)
Exact FLR integrals keep accuracy for high-k i.e. $k_\theta\rho_i>1$	FLR integrals used Pade approximation valid for low-k
Adaptive Hermite basis function solution method valid for the same range as the GK equations	Parameterized single Gaussian trial wavefunction valid for a limited range of conditions
All trapped fractions	Small trapped fraction required.
Shaped geometry (Miller model)	Shifted circle (s-alpha) geometry
Fully electromagnetic $\left(\tilde{B}_\perp,\tilde{B}_\parallel\right)$	Normally run electrostatic
New electron-ion collision model fit to pitch angle scattering	Inaccurate electron-ion collision model only for low-k equations
Transport model fit to 86 GYRO runs with kinetic electrons	Transport model fit to a few GLF non-linear turbulence runs
15 moment equations per species	4 moment equations per species
10-30 times slower than GLF23	Fast enough for 1997 computers!

The new TGLF transport model is a more accurate fit to gyro-kinetic turbulence than GLF23

- **TGLF fits GYRO well for all three channels.**
- **GLF23 electron energy flux is systematically high.**
- **GLF23 with trapped electrons misses critical temperature gradient.**
- **GLF23 was fit to this same scan with adiabatic electrons.**

Temperature gradient scan with trapped electrons



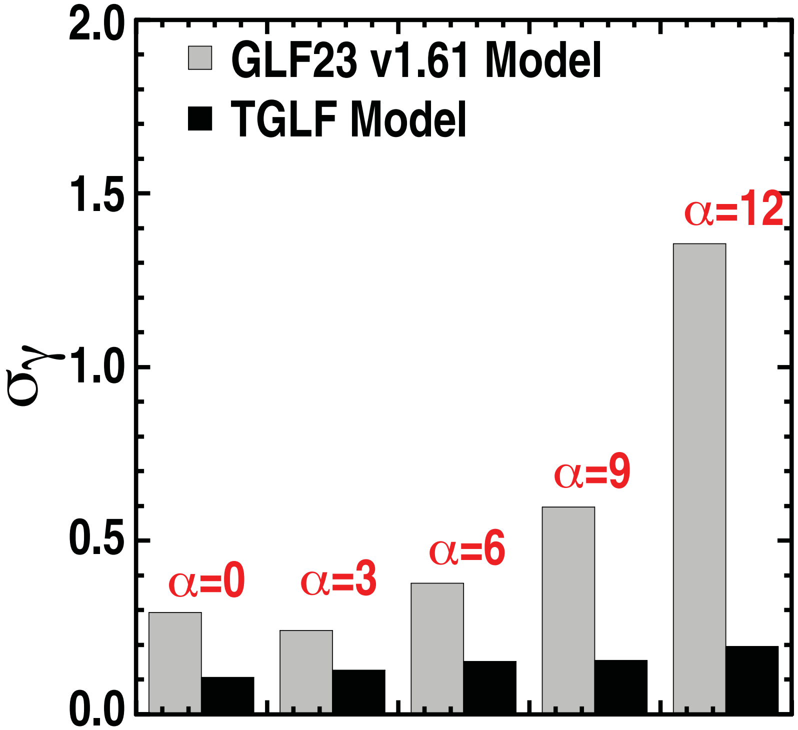
TGLF has a uniformly good accuracy over a wide range of plasma conditions

- **Linear stability benchmarks in s-alpha geometry show that TGLF is an accurate approximation to gyro-kinetic calculations with GKS.**

- **The average fractional deviation for the TGLF growth rate is 11.4% for the whole database of 1800 GKS runs.**

$$\sigma_\gamma = \sqrt{\sum_{\mathrm{i}} \left(\gamma_{\mathrm{i}}^{\mathrm{TGLF}} - \gamma_{\mathrm{i}}^{\mathrm{GKS}}\right)^2} \Big/ \sqrt{\sum_{\mathrm{i}} \left(\gamma_{\mathrm{i}}^{\mathrm{GKS}}\right)^2}$$

Each bar is the fractional deviation for a set of 80 pts varying shear and safety factor about the PED point over the range $1 \le \hat{s} \le 7, 3 \le q \le 7$

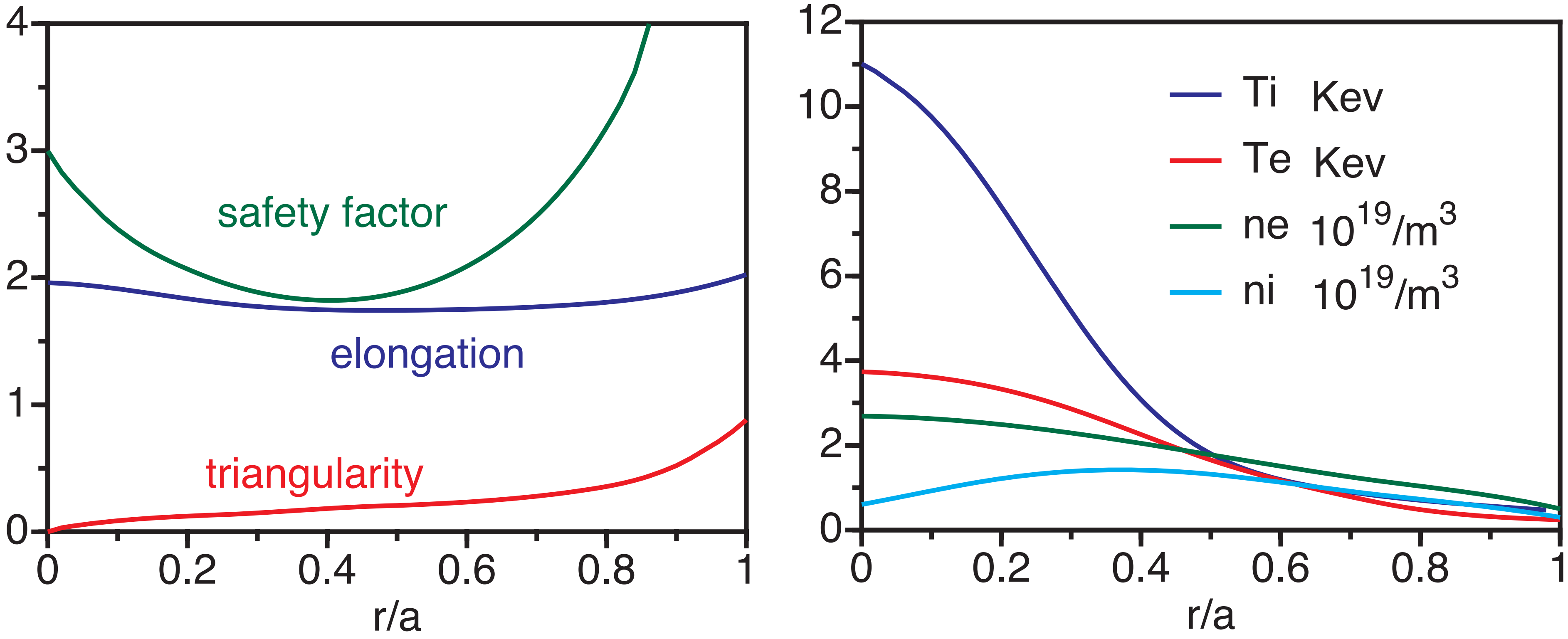


$$\mathrm{PED} = \left(a/L_{n_e} = a/L_{n_i} = 3,\ a/L_{T_e} = a/L_{T_i} = 10,\ T_i/T_e = 1,\ q = 4,\ \hat{s} = 3,\ \alpha = 5,\ k_y = 0.3,\ r/a = 0.5,\ R/a = 3.0\right)$$

TGLF is now being used for linear stability analysis of experimental data

- **As an illustration of the accuracy of TGLF with comprehensive physics, an analysis of a DIII-D NCS discharge will be made.**
- **The geometry and plasma profiles are shown below.**
  - **L. Lao et al. Phys. Plasmas 3 (1996) 1951.**

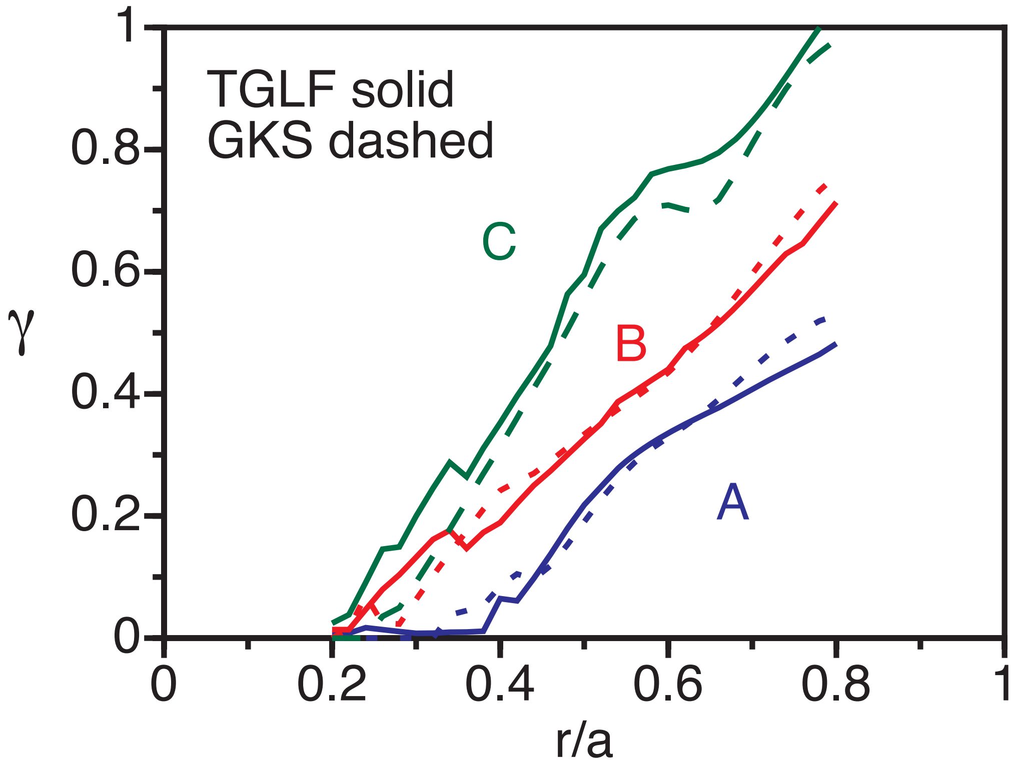
DIII-D NCS discharge 84736 at 1.3 sec



TGLF linear growth rates are accurate for real experimental conditions.

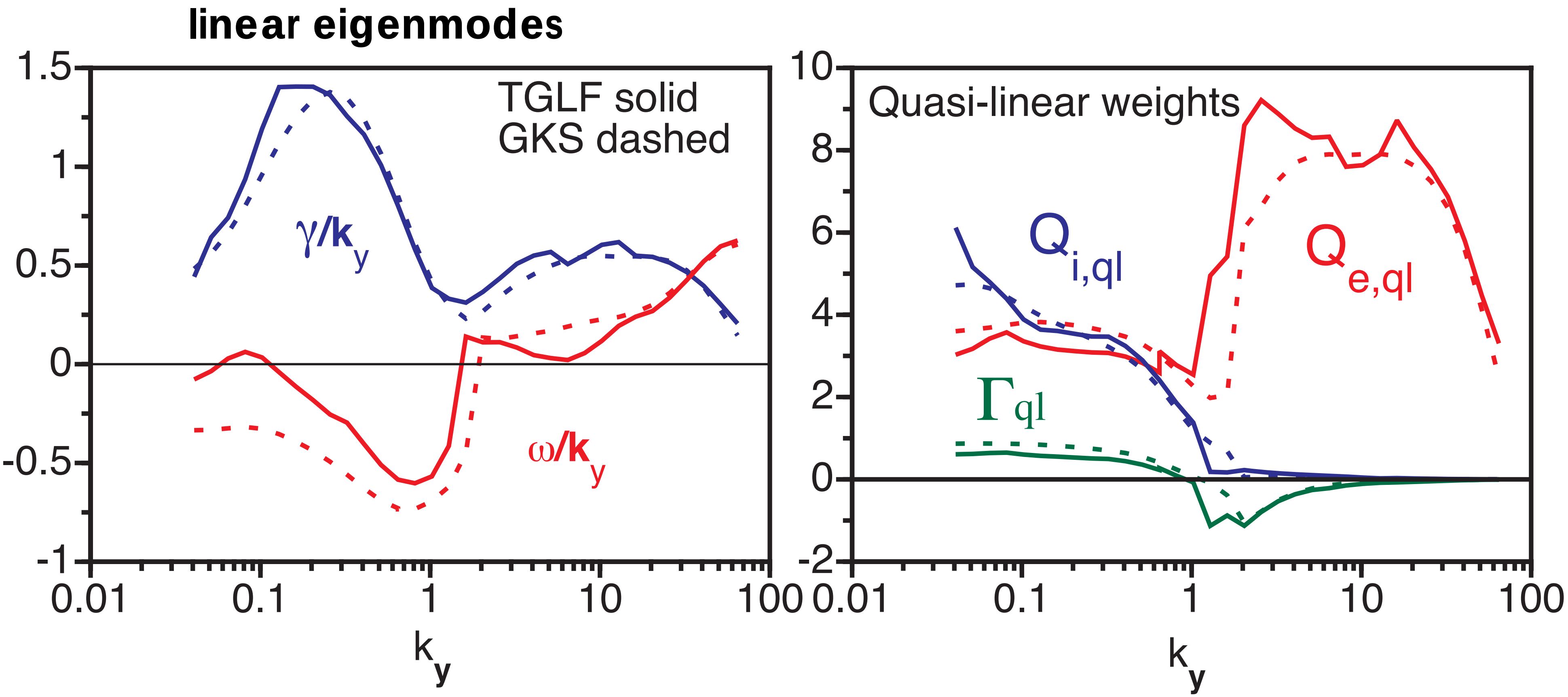
- The radial profile of the normalized linear growth rate for k_y=0.3 and three different physics settings is shown.
  - (A) comprehensive physics
  - (B) collisionless, electrostatic
  - (C) s-alpha geometry dillution, collisionless, electrostatic
- The TGLF code was 200X faster than GKS for (C) (2.4sec vs 8.9min)

DIII-D NCS discharge 84736 at 1.3sec



The full spectrum of drift wave eigenmodes can be computed with TGLF

- **k-spectrum with comprehensive physics at r/a=0.7 for DIII-D NCS discharge 84736 at 1.3 sec.**



TGLF flux model is fit to a large database of nonlinear gyro-kinetic turbulence simulations

The normalized TGLF fluxes are given by

$$\hat{Q}_i = c_{Q_i}\overline{\Phi}^2 Q_{i,ql} \qquad \hat{Q}_e = c_{Q_e}\overline{\Phi}^2 Q_{e,ql} \qquad \hat{\Gamma} = c_{\Gamma}\overline{\Phi}^2 \Gamma_{ql}$$

- **The coefficients are chosen to give zero offset for the whole database of 86 non-linear GYRO turbulence simulations.**
  - **s-alpha geometry, kinetic electrons and ions**
- **The coefficients would all be the same if the ratios of the fluxes were exactly the quasi-linear ratios.**

c_{Qi}	c_{Qe}	c_{Γ}
30.0	32.4	36.6

A model for the zonal flow shear is included to fit the spectrum of the turbulence

- A model for the reduction of the growth rate by the ExB shear due to"zonal flows" γ_{ZF} and the equilibrium electric field γ_{ExB} is used to obtain an effective net growth rate used in the saturated intensity model. $\overline{\Phi}^2(\overline{\gamma})$

$$\overline{\gamma} = \mathrm{Max}\left[\left(\hat{\gamma} - \alpha_{\mathrm{E}}\hat{\gamma}_{\mathrm{ExB}} - \alpha_{\mathrm{ZF}}\hat{\gamma}_{\mathrm{ZF}}\right)/\hat{\omega}_{\mathrm{d0}}\,, 0\right]$$

where $\hat{\omega}_{\mathrm{d0}} = \mathrm{k_y}(\mathrm{a}/\mathrm{R})$

and $$\hat{\gamma}_{\mathrm{ZF}} = \hat{\omega}_{\mathrm{d0}}\left(\mathrm{Max}\left[\hat{\gamma} - \alpha_{\mathrm{E}}\hat{\gamma}_{\mathrm{ExB}}, 0\right]/\hat{\omega}_{\mathrm{d0}}\right)^{\beta_{z\gamma}} \Big/ \left(\mathrm{k_y}^{\beta_{\mathrm{zk}}}\,\mathrm{q}^{\beta_{\mathrm{zq}}}\right)$$

α_{ZF}	$\beta_{z\gamma}$	β_{zk}	β_{zq}
0.369	0.906	0.420	0.317

α_E
0.35

A local model for the saturated intensity is sufficient for a good fit

The model for the saturated intensity of the potential fluctuations is local in both k_y and space. $\overline{\Phi}^2 = \tilde{\Phi}^2 \big/ (\rho_s / a)^2$

$$\overline{\Phi}^2 = \Delta_{ky} \frac{\hat{\omega}_{d0}^2}{k_y^4} \Lambda \qquad \Lambda = \frac{\overline{\gamma}^{\beta_\gamma}\left[\alpha_{d0} + \left(\alpha_d \mathrm{Max}[\overline{\omega}_d, 0]\right)^{\beta_d}\right]}{\left[1 + \left(\alpha_\gamma \overline{\gamma}\right)^{\beta_\gamma}\right]\left[1 + \left(\alpha_d |\overline{\omega}_d|\right)^{\beta_d}\right] k_y^{\beta_k}}$$

$\overline{\omega}_d = \langle \hat{\omega}_d \rangle \big/ \hat{\omega}_{d0}$ **is the wavefunction average of the curvature drift.**

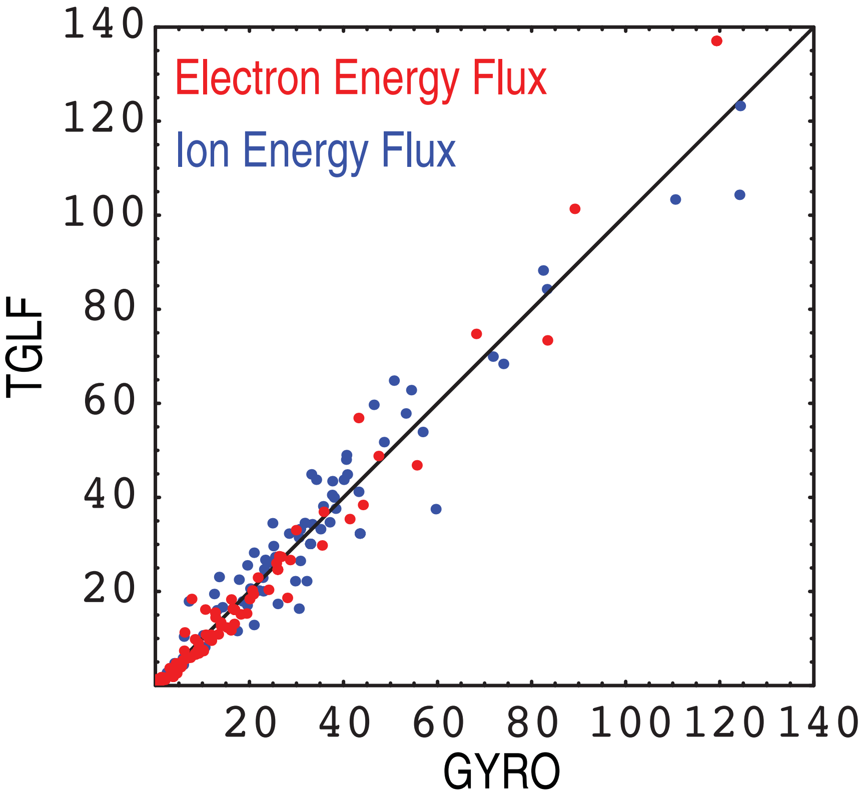
The intensity model has the weak turbulence limit: $\overline{\Phi}^2 \propto \overline{\gamma}^2$ for $\overline{\gamma} << 1$
and the strong turbulence limit: $\overline{\Phi}^2 \propto \hat{\omega}_{d0}^2$ for $\overline{\gamma} >> 1$

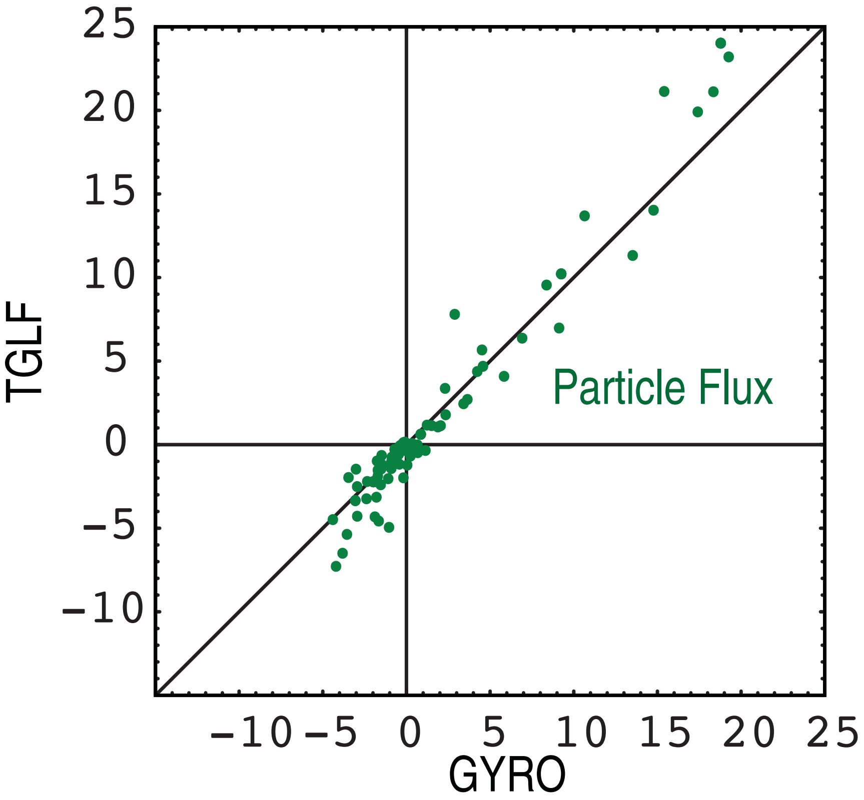
α_γ	β_γ	α_{d0}	α_d	β_d	β_k
0.893	**1.98**	**0.072**	**2.55**	**1.94**	**0.933**

Quasi-linear TGLF fits nonlinear GYRO total fluxes well

- **The total fluxes for TGLF fit the database of 86 GYRO runs with fractional deviations of :** $\sigma_{Q_i} = 16\%,\ \sigma_{Q_e} = 15\%,\ \sigma_{\Gamma} = 28\%$

- **GLF23 is a much poorer fit:** $\sigma_{Q_i} = 42\%,\ \sigma_{Q_e} = 78\%,\ \sigma_{\Gamma} = 78\%$

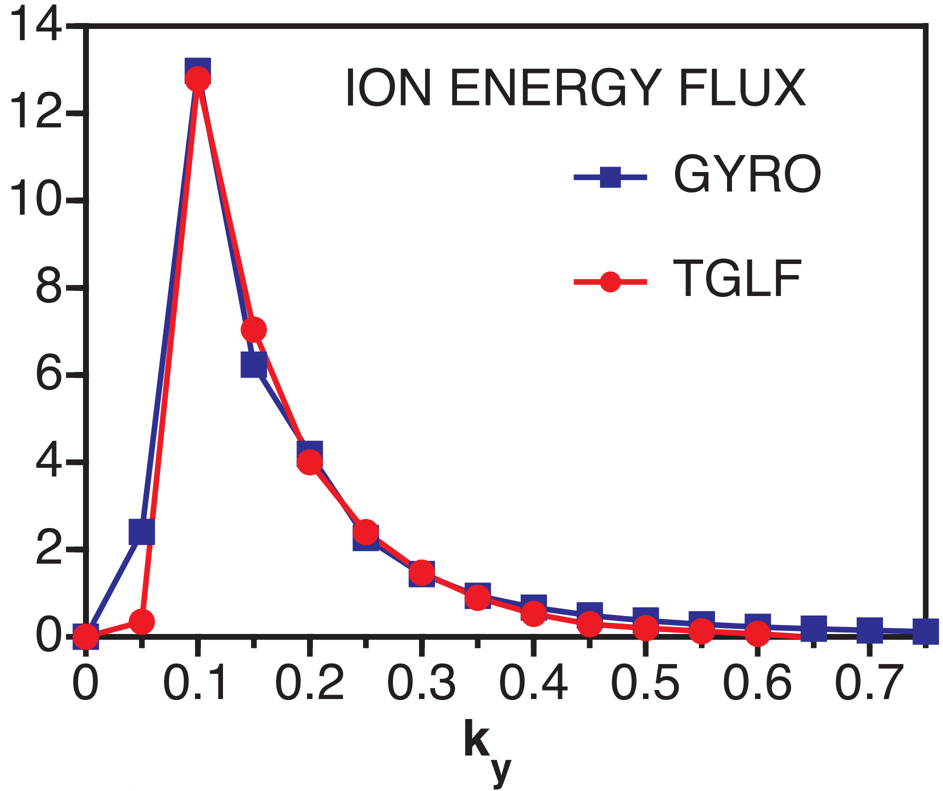


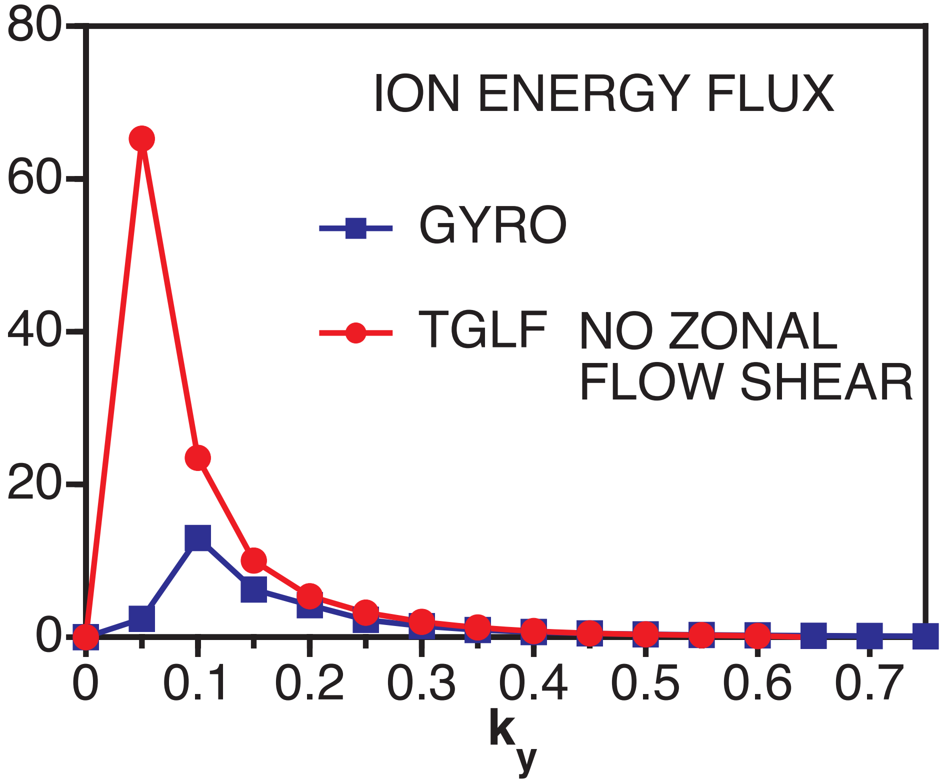


Quasi-linear TGLF fits the GYRO flux spectrum well

- **The detailed shape of the ion energy flux spectrum is well fit by the TGLF model.**

- **The lowest k_y modes are suppressed by the "zonal flow" shear model as can be seen below with α_{ZF}=0.**

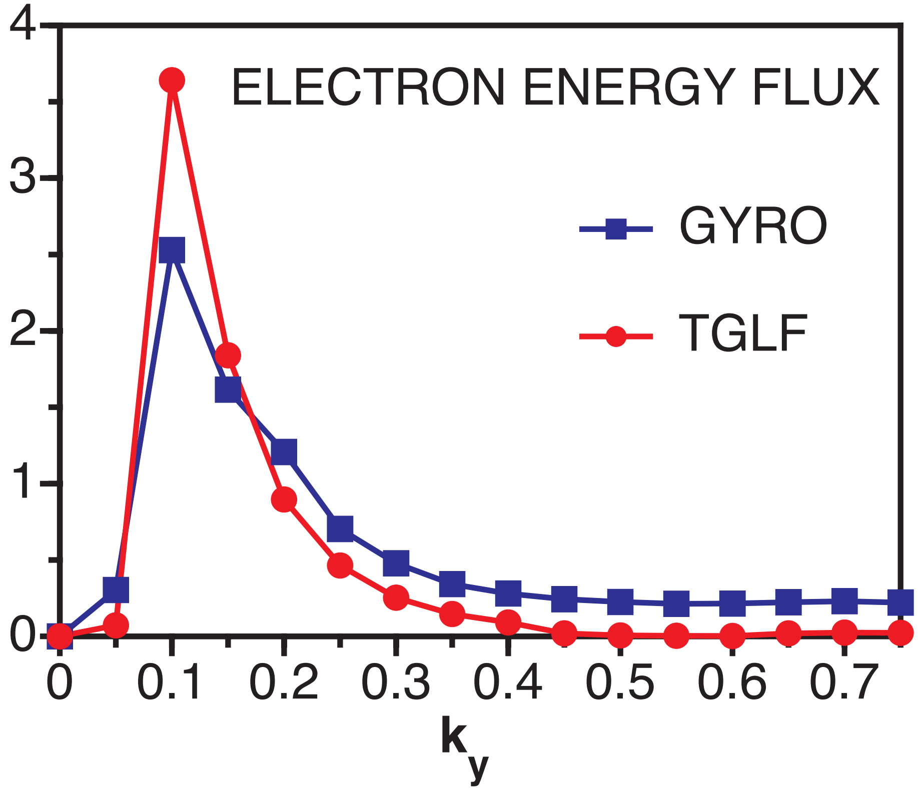




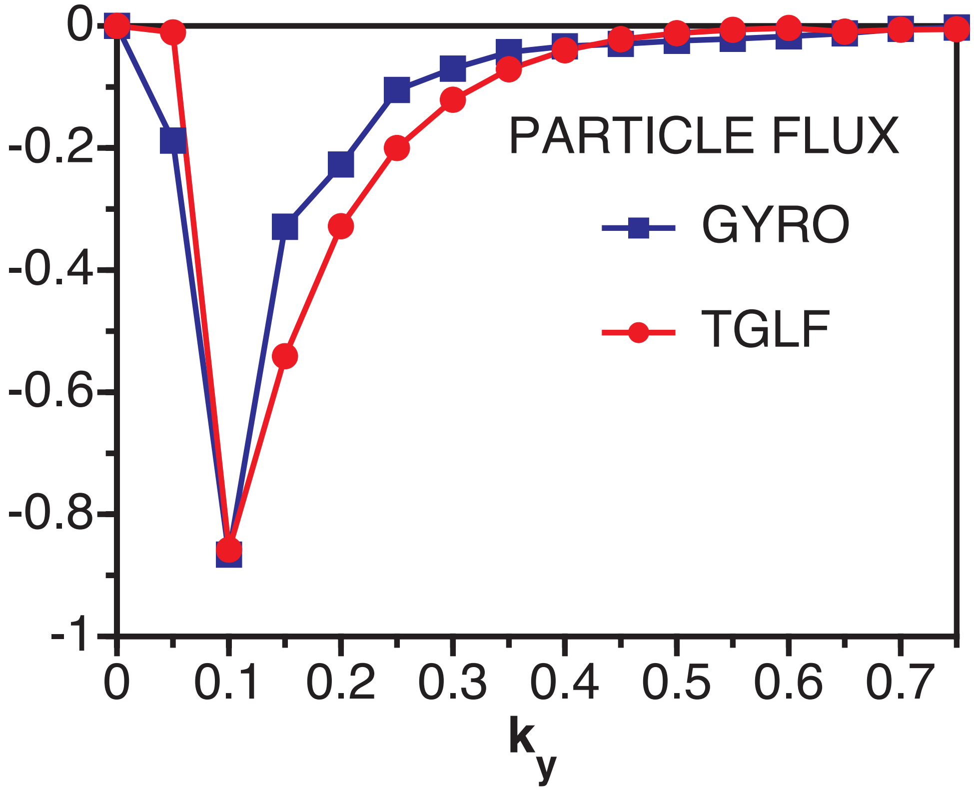
$$\mathrm{STD} = \left(a/L_{n_e} = a/L_{n_i} = 1,\ a/L_{T_e} = a/L_{T_i} = 3,\ T_i/T_e = 1,\ q = 2,\ \hat{s} = 1,\ \alpha = 0,\ r/a = 0.5,\ R/a = 3.0\right)$$

Spectral fits are good for all channels

- **The electron energy flux spectrum is generally fit about as well as the ion energy flux spectrum.**



- **The particle flux spectrum is best fit when the particle flux is not close to zero.**



TGLF is an accurate model of gyro-kinetic theory

- A new Trapped Gyro-Landau Fluid (TGLF) system of equations has been developed that yields a fast, accurate approximation to the linear eigenmodes of gyro-kinetic driftwave instabilities with comprehensive physics.
  - A TGLF eigenmode code is available for stability analysis of experimental discharges.

- A quasi-linear transport model using TGLF eigenmodes and a local model for the saturated fluctuation intensity achieves an excellent fit to a large database of nonlinear gyro-kinetic turbulence simulations using the GYRO code.
  - Extension of the intensity model to high-k ETG modes will use the latest coupled ITG/TEM-ETG GYRO simulations. (Candy & Waltz)

Will TGLF predict the transport in experiments?

- Transport predictions of the TGLF transport model will be tested in conventional tokamaks, low aspect ratio spherical tori and the near separatrix region.
    - This will be a true test of the first principles gyro-kinetic theory foundation of the TGLF model
    - Prediction of the pedestal width in H-mode is a high priority
    - Prediction of transport in ITER

- Planned extensions of TGLF include:
    - General geometry from numerical MHD equilibrium instead of the Miller model which is needed for the pedestal
    - Intensity models that include mode coupling (non-local in wavenumber) and turbulence spreading (non-local in space)
    - Inclusion of equilibrium parallel and ExB velocity shear in the linear eigenmodes (beyond the quench rule)