Recent Advances in the Theory and Simulation Of Pellet Ablation and Fast Fuel Relocation In Tokamaks

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• Pellet Ablation

- New 2-D Eulerian code CAP treats multi-phase pellet-cloud dynamics.
 [Phys of Plasmas 11, 4064 (2004)]. Magnetic field interaction now partially included.
- Ionized cloud pushes B-field away forming a diamagnetic cocoon around pellet.
 Diverts plasma energy flux around pellet, prolonging its lifetime.
- ∇B induced cloud drift using **PRL** code (GA-A24807 submitted to Phys Rev Lett)
 - Included plasma pressure profile variations on drifting cloudlet
 - Showed that the M ~1 parallel ablation flow furthers cloudlet penetration
 - Elucidated the effect of rotational transform on cloudlet drift
 - Discovered how magnetic shear causes cloudlet "mass shedding" and dispersal
 - Generalized theory to include arbitrary poloidal angle of pellet entry
- \bullet 3-D MHD simulations of ∇B cloud drift using Adaptive Mesh Reduction code $\ensuremath{\mathsf{AMR}}$
 - Preliminary results verify that pellet cloud drifts in large-R direction



• Geometry

- Axisymmetric cylindrical coordinates (r, θ , z) with θ ignorable, and background (undisturbed) magnetic field B_{∞} parallel to z axis.
- Magnetic field \rightarrow B_r, B_z, flow velocity \rightarrow v_r, v_z, current J₀, electric field E₀

• Rectangular (r,z) Computational Box

- Ideally conducting wall boundary at $r_{wall} \approx 20 r_{pell}$
- Reflecting (outflow) boundary conditions at end faces $z_{end} \approx \pm$ 20 r_{pell}
- Some Physical Assumptions Made:
 - Solve magnetic flux equation with simple Ohms law, $E_{\theta} = -v \times B + J_{\theta} / \sigma_{\perp}$.
 - Use step-function heat flux to pellet (realistic pellet entry has gradual increase).
 - Artificially high electrical resistivity η_∞ in background plasma is needed to eliminate significant currents and J×B forces, which are "continually erased" by fast magnetosonic adjustments on the slow cloud evolution time scale.







 β ~1 Diamagnetic Cavity Can Divert Heat Flux Around Pellet



Hot ring can potentially maintain field-free cavity over the long skin time $\tau_{skin} = \mu_0 \sigma_\perp \Delta r_\perp \sim 60 \ \mu s > r_\perp / v \sim 1 \ \mu s$. Distorted field lines can potentially divert heat flux around pellet, reducing ablation rate.





• Ordering and scales - Cloudlet is like a strongly localized cylindrical pressure perturbation $\nabla_{\perp} \sim 1/r_c \ll 1/R, 1/a, \quad \delta \equiv r_c/qL(t) \sim v_c/qc_s \ll 1$ (key expansion parameter)

 $v_c = E/B$ drift velocity ~ 10³-10⁴ m/s, $c_s =$ cloudlet sound speed ~ 10⁴-10⁵ m/s,

Transverse force balance (no fast time-scale magnetosonic waves $v_c \ll c_{A\infty}$) $0 \simeq \nabla_{\perp}(\delta p + B_{\infty} \delta B/\mu_0) + O(r_c/R) \longrightarrow \delta p = -B_{\infty} \delta B/\mu_0$ $\delta p = p_c - p_\infty$ $\rho_c = cloud pressure, "\infty" = ambient plasma quantity$ "excess pressure" is source of curvature drive Parallel expansion flows relax pressure perturbation δp -Pressure Relaxation Lagrangian (PRL) code (Parks, PoP 2000) -End boundary condition applied on a drifting cloudet $p_c(z = L) = p_{\infty}(\rho = \rho_c)$ $\rho_c(t) =$ minor radius of cloud centroid **IAEA 2004 GENERAL ATOMICS**



A local positive pressure bump (δp > 0) is necessary to start the inward drift V_c. In doing so the pressure bump could change to a pressure hole (δp < 0), causing drift reversal. Pellet penetration well past edge pedestal region seems to be necessary for good cloud penetration.



• Evolution of plasma vorticity [Hazeltine, 1992]

$$\frac{\hat{b}}{B} \cdot \nabla \times mn \frac{d\vec{v}}{dt} = (\vec{B} \cdot \nabla) \frac{J_{\parallel}}{B} - \frac{(\hat{b} \times \vec{\kappa}) \cdot \nabla_{\perp} B^2}{\mu_0 B}$$

where $\hat{b} = \vec{B}/B$, $\kappa = \hat{b} \cdot \nabla \hat{b}$ (curvature), $B^2 = B_{\infty}^2 + 2B_{\infty}\delta B$, $J_{||} = J_{||\infty} + \delta J_{||}$.

• Perturbed result with substitution $-2\delta p = 2B_{\infty}\delta B/\mu_0$



Vorticity Equation Yields Electrostatic Potential Φ and Cross-Field Cloud and Plasma Drifts Near Cloud

$$\vec{v}_{\perp} = \hat{b} \times \frac{\nabla \Phi}{B} \longrightarrow \nabla_{\perp} \cdot \left(\frac{mn}{B^2} \frac{D \nabla_{\perp} \Phi}{Dt}\right) = (\vec{B} \cdot \nabla) \frac{\delta J_{\parallel}}{B} + \frac{(\hat{b} \times \vec{\kappa}) \cdot \nabla_{\perp} (2\delta p + mn v_{\parallel}^2)}{B}$$

Centrifugal force driving term operates, even after $\delta p \rightarrow 0$
• Tokamak coordinate system (ρ, χ, ϕ) for poloidal plane (ρ, χ) variations

$$\hat{b} \times \vec{\kappa} = -(\hat{\chi} \cos \chi + \hat{\rho} \sin \chi)/R$$

• Helical magnetic-field-line-following (MFLF) coordinate system (*x*,*y*) or (*r*, ϑ) for local cloud variables: $\nabla_{\perp}(2\delta p + mnv_{\parallel}^2) = \partial/\partial r(2\delta p + mnv_{\parallel}^2)\hat{r}$

$$(\hat{b} \times \vec{\kappa}) \cdot \hat{r} = \frac{-1}{R} (\cos \chi \sin \vartheta + \sin \chi \cos \vartheta)$$

$$\chi(z,t) = \chi_0 + \int_0^t \frac{v_{\chi}(t')}{\rho(t')} dt' \pm \frac{z}{qR}$$
How Toroidicity
modifies curvature drive

$$q = \text{safety factor}$$

$$\chi_0 = \text{pellet launch angle}$$

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Helical Magnetic-Field-Line Following (MFLF) Coordinates \vec{x} Suitable for Local Cloud Variables



Local cross-field coordinates (x, y) affixed to cross-section of cloud: x = const, y = const identify a field line. Due to rotational transform, the coordinates rotate counter-clockwise with longitudinal distance z

 $\nabla_{||} \Phi \approx 0$ so $\Phi(x,y)$ is frozen to B-field lines in cloud



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• Match parallel end-current $\delta J_{||}$ with the current carried off by shear Alfven wave excited by motion in ambient plasma $E_{||} = -ik_{||}\Phi + i\omega A_{||} = 0$ (Parks, PoP 2000) $\nabla_{\perp} \cdot \vec{\Gamma} = S(r, \vartheta) \equiv \frac{2}{BR} \delta(r - r_c) [\sin \vartheta \cos \chi_c(t) + \cos \vartheta \sin \chi_c(t)] \Psi(t)$ Assumed flat radial cloud profiles $\vec{\Gamma} = \frac{m\tau}{B^2} \frac{D\nabla_{\perp} \Phi}{Dt} + \frac{\nabla_{\perp} \Phi}{\mu_0 c_{A\infty}}$ Shear Alfven wave part Line-averaged inertial (polarization) drift current $\tau = \int_{-L_c}^{L_c} ndz, \ \tau_{cloud} \sim 10^3 \tau_{plasma}$

• Drive integral folds parallel (z) variations with changing direction of ∇ B-drift current

$$\Psi(t) = \int_0^{L_c(t)} \left[p_c(z,t) [1 + M_c^2(z,t)/2] - p_{\infty}(t) \right] \cos(z/qR) dz$$

Boundary condition on electrostatic potential at "infinity"

$$\Phi(\vec{x},t) \rightarrow -E_{\rho\infty}[\rho_c(t)]x$$

Radial (ρ) electric field in tokamak at cloud centroid



Nondimensional Drift Equations Use Parameters of 1-D Ablation Channel Flow Formed around Solid Pellet



• Radial and Poloidal velocities and coordinates $\rho_c(\tilde{t}), \chi_c(\tilde{t})$ are coupled



Magnetic Shear Leads to "Mass Shedding"



- Elliptical compression and rotation re-orients polarization charges
- Leads to small differential poloidal drift increasing with z
- Limits axial extent of drift response
- "Mass shedding" results

 $L_s = qR/\hat{s}$ magnetic shear length $\hat{s} = \frac{\rho}{q} \frac{dq}{d\rho}$ shear parameter



• Natural "twisted basis" coordinates for a flux tube cloud filament, where $x_* = const$, $y_* = const$ identifies a field line r_7

$$x_* = x$$
, $y_* = y - \frac{x_2}{L_s}$

• Transform internal cloud electrostatic potential solution without shear $\Phi^0(x,y)$ to an approximate one with shear by simple mapping $\Phi_{in}^0(x,y) \rightarrow \Phi_{in}(x_*,y_*)$

• For this representation $\frac{\partial \Phi_{in}}{\partial x} = \frac{\partial \Phi_{in}}{\partial x_*} - \frac{\partial \Phi_{in}}{\partial y_*} \frac{z}{L_s}$, $\frac{\partial \Phi_{in}}{\partial y} = \frac{\partial \Phi_{in}}{\partial y_*}$ $\rightarrow \vec{v}_{in} = v_{\rho}^0 \hat{x} + v_{\chi}^0 \hat{y} + v_{\rho}^0 \frac{z}{L_s} \hat{y}$ Cloud acquires small differential poloidal drift ($z < L(t) << L_s$)

• The coherent cloud drift is effectively limited in longitudinal extent when

$$2r_{c} = \int_{0}^{t_{S}(z_{0})} \frac{v_{\rho}^{0}(t)z(z_{0},t)dt}{L_{s}[\rho_{c}(t)]}$$
 Shedding time $t_{S}(z_{0})$ for fluid element with
Lagrangian coordinate z_{0}
• Mass loss rate $\frac{d\tilde{M}}{dt} = \frac{d\tilde{M}}{dz_{0}} \frac{dz_{0}}{dt_{S}}$ decreases with weaker shear (larger L_{s})



Pressure Relaxation Lagrangian (PRL) Code Solves Coupled Drift and Parallel Dynamics for a Series of Cloudlets

- The PRL code uses the pellet size and plasma parameters at each point along the ablation track determined by PELLET code [Houlberg, 1988] to initialize the cloudlet parameters using model of Parks et al PoP 2000.
- The experimental plasma profiles are used by PRL to calculate the subsequent cloudlet pressure relaxation and drift velocity.
- The deposition profiles from each cloudlet are summed, yielding a net Δn profile.
- Parallel Mach number M for a single cloudlet (DIII-D 98796) is shown as a function of normalized time and Lagrangian coordinate \tilde{z} .





Simulation of Temperature and Pressure Inside Cloudlet with Mass Shedding



Temperature and pressure evolution for typical cloudlet in DIII-D 98796. The cloudlet mass shedding can be seen by the reduced number of cells that starts at $t_{cycle} = 150$ (3 µsec). The cloudlet pressure builds quickly then decays as the density decays due to expansion along the field lines.





Theory and DIII-D Experiments Agree



Vertical arrows indicate pellet burnout point.

• Fueling efficiency for inside launch is much better (even with slower pellets) outside launch $\eta_{\text{theory}} = 66\%$, $\eta_{\text{exp}} = 46\%$ (discrepancy due to strong ELM) inside launch $\eta_{\text{theory}} = 100\%$, $\eta_{\text{exp}} = 92\%$ (discrepancy due to weak ELM)









• In ITER, pellets ablate near plasma edge, forming ~ 10-20 cloudlets. We find that the cloudlet drift distance is sensitive to L_s near edge













 $\Delta_{\text{ped}} = 8 \text{ cm}$ $T_0 = 20 \text{ keV}$ $6 \text{ mm } D_2 \text{ pellet}, \text{ v}_{\text{pell}} = 300 \text{ m/s},$ 45 deg below midplane entryq-profile with k = 5

• As T_{ped} increases, the temperature where pellet burns out T_{burn} increases as $T_{burn} \sim T_{ped}^{3/8}$, forming higher cloudlet pressures...However penetration is more shallow, so the cloudlets have to drift further up the pedestal. These two effects almost cancel. \longrightarrow Deposition remains fairly constant.























3-D AMR code: Numerical Approach and Assumptions

- Detailed 3D AMR simulations of pellet injection using the MHD equations
 – pellet treated as moving density source
 - Ratio of pellet size to device size is $\sim O(10^{-3})$
- Phased approach to understand the basic physics of mass redistribution with varying degrees of complexity
 - Simple Cartesian geometry (Samtaney, Jardin, Colella and Martin, Sherwood Fusion Theory Conference 2003)
 - Toroidal geometry (ICNSP 2003 Invited talk. To appear in Comput. Phys. Comm.)
- Physical assumptions
 - Pellet ablation rate uses a semi-analytical model
 - Instantaneous heating of ablated mass by plasma electrons
 - Finite rate heating using kinetic model (Parks, PoP 2004) is in progress
 - Single fluid MHD equations describe plasma
 - Plasma pressure and B-field initialized by a Grad-Shafranov equilibrium solution $\rightarrow p_{\infty}(\Psi)$, $T_{\infty}(\Psi)$, $n_{\infty} = const$





Equations and Mathematical Model

Equations in conservation form + source terms

$$\frac{\partial U}{\partial t} + \frac{\partial F_j(U)}{\partial x_j} = \frac{\partial F_{v,j}(U)}{\partial x_j} + S_T(U) + S_{\nabla \cdot \mathbf{B}}(U) + S_{pellet}(U)$$

- Flux vector
 - $-B_{T}$ is the toroidal component of the equilibrium magnetic field

$$F_{j}(U) = \begin{cases} \rho u_{j} \\ \rho u_{i}u_{j} + p_{t}\delta_{ij} - B_{i}B_{j} + B_{T}B_{3}\delta_{ij} - B_{i}B_{T}\delta_{3j} - B_{j}B_{T}\delta_{i3} \\ u_{j}B_{i} - u_{i}B_{j} + B_{T}\delta_{i3}u_{j} - B_{T}\delta_{3j}u_{i} \\ (e + p + \frac{1}{2}B_{k}B_{k})u_{j} - B_{j}(B_{k}u_{k}) + B_{T}B_{3}u_{j} - (B_{k}u_{k})B_{T}\delta_{3j} \end{cases}$$

• Equation of state

$$e = \frac{p}{\gamma - 1} + \frac{\rho}{2}u_ku_k + \frac{1}{2}B_kB_k$$





 $-n = 1.5 \times 10^{13} \text{ cm}^{-3}$, B = 0.23 T, a = 0.26 m, $r_p = 1 \text{ mm}$, $v_{pell} = 3200 \text{ m/s}$

Density iso-surface shows pellet cloud expanding along B-field



Adaptive Mesh Tracks Elongating Cloud





Inner Wall vs Outer Wall Pellet Injection (Midplane Entry)



• Density contours of pellet cloud (at cloud midplane cross section) indicate significant large-*R* displacement relative to instantaneous pellet position.





Summary and Conclusions

- Pellet Ablation used 2-D CAP code, the most sophisticated ablation code which is now including the interaction of the ablation flow with the magnetic field
 - Showed that a $\beta \sim 1$ cavity can be formed around pellet by the high ablation pressure. This will reduce heat flux to pellet, but it contradicts previous quasi-steady ablation models in which *B*-field is only slightly distorted.

• However CAP used a "switch-on" heat flux, described just the initial transient period to 0.4µs, and kept heat flux || to z, NOT DIVERTED AROUND B (which would reduce p, T and σ . "Switch-on" heat flux might apply to step-function high-T pedestal.

The theoretical ∇B drift model has been reformulated with new physics

- **PRL** code was validated by inner and outer wall pellet injection experiments on the DIII-D tokamak. Good agreement was found!
- For ITER, **PRL** showed much better penetration of pellet material for inner wall injection, but penetration is sensitive to L_s , T_{ped} , and Δ_{ped} .
- 3-D numerical simulations of ∇B pellet cloud drift using AMR code are underway, verifying drift in the large-R direction for both inner- and outer-wall injection. **IAEA 2004**

