The Role of Shaping in the Sawtooth Instability


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Core Stability of **Bean** and **Oval** Shapes

(D_{ij} & D_f are essentially the same for monatonic q profiles.)

**Safety Factor**

- q(1)=3.2, q(0)=1.1
- q(1)=3.2, q(0)=0.95

**Unstable Region**

**Stable Region**

**Shaping can move the interchange stability boundary (Mercier Criterion) across q=1, separating the role of interchange and internal kink instabilities in the sawtooth collapse.**
Plasma boundary shape and shape at sawtooth inversion radius

Principal Diagnostics:
The magenta solid circles are Thomson Scattering \( (n_e, T_e) \). The upside-down red daggers are CER \( (T_i) \) locations. The blue bar is the MSE (\( B_Z/B_T \)) range of locations. The brown diamonds are ECE (\( T_e \)) locations.
Discharge Evolution

We always use an early beam and grow the plasma as the current rises to promote early sawteeth. Two levels of plasma current have been used in each shape.

Resulting beans have a central magnetic well. The ovals have a central magnetic hill.
Some Plasma Parameters

<table>
<thead>
<tr>
<th>Parameters of Interest</th>
<th>bean 1</th>
<th>bean 2</th>
<th>oval 1</th>
<th>oval 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shot</td>
<td>113920</td>
<td>118162</td>
<td>113915</td>
<td>118164</td>
</tr>
<tr>
<td>$I_P$ (MA)</td>
<td>1.38</td>
<td>1.52</td>
<td>0.88</td>
<td>1.18</td>
</tr>
<tr>
<td>$B_T$ (T)</td>
<td>1.79</td>
<td>1.85</td>
<td>1.80</td>
<td>1.85</td>
</tr>
<tr>
<td>$V$ (m$^3$)</td>
<td>17.5</td>
<td>17.6</td>
<td>20.1</td>
<td>20.1</td>
</tr>
<tr>
<td>$&lt;n_e&gt;$ ($x10^{19}$ m$^3$)</td>
<td>2.0</td>
<td>2.5</td>
<td>1.9</td>
<td>2.75</td>
</tr>
<tr>
<td>$\beta_P$</td>
<td>0.22</td>
<td>0.24</td>
<td>0.45</td>
<td>0.38</td>
</tr>
<tr>
<td>$\beta_{P1}$</td>
<td>0.16</td>
<td>0.19</td>
<td>0.41</td>
<td>0.33</td>
</tr>
<tr>
<td>$\ell_1$</td>
<td>1.24</td>
<td>1.15</td>
<td>1.41</td>
<td>1.29</td>
</tr>
<tr>
<td>$W_{mhd}$ (MJ)</td>
<td>0.25</td>
<td>0.34</td>
<td>0.24</td>
<td>0.37</td>
</tr>
<tr>
<td>$\tau_e$ (ms), $\tau_{th}$ (ms)</td>
<td>93</td>
<td>127,75</td>
<td>90</td>
<td>140,75</td>
</tr>
<tr>
<td>$\tau_S$ (ms)</td>
<td>90</td>
<td>154</td>
<td>59</td>
<td>80</td>
</tr>
<tr>
<td>$q_{95}$</td>
<td>3.9</td>
<td>3.6</td>
<td>5.4</td>
<td>4.2</td>
</tr>
<tr>
<td>$r_i$ (normalized inversion radius)</td>
<td>0.38</td>
<td>0.44</td>
<td>0.28</td>
<td>0.35</td>
</tr>
<tr>
<td>$W_{fast}/W_{mhd}$</td>
<td>0.27</td>
<td>0.13</td>
<td>0.25</td>
<td>0.11</td>
</tr>
<tr>
<td>reconnection $\delta B_6$ (G)</td>
<td>219</td>
<td>293</td>
<td>85</td>
<td>129</td>
</tr>
</tbody>
</table>

In the bean plasma current is limited by coil current limits. In the oval the limitation is our requirement that the two plasmas are made with the same poloidal power supply configuration so that the shape can be switched on a shot-to-shot basis. This allows us to minimize systematic errors.
The Sawteeth - In Beans and Ovals
Outline

The Crash
1. ideal or resistive
2. interchange or kink

The Sawtooth Ramp
1. equilibrium evolution
2. ion behavior
3. transport rates
4. response to ECH
Reconnection - average over 7 sawteeth and do a linear fit. Reconnection larger in beans.

Chose MSE channel with largest change. Reconnection is seen as an identifiable event in the MSE signal in the bean, but not in the oval. Results inside/outside magnetic axis differ only a few percent.
The crash as an ECE color map.

- No Structure (Hot Core) => Crash
- Helical State
- Crash time ~ 40 µs

- Saturated State => Mixing
- The End is Near
- ~Axisymmetric State.

By an equivalent definition
Crash time ~ 70 µs
Examine FFT of Cross Correlation of ECE signals with magnetic n=1 signal.

No coherent structure before crash. Double island structure after crash (inboard / outboard 180° phase jumps).

ECE at z=1.5 cm => amplitude ≠ 0 at R=R₀

Single inboard phase jump => unique signature of quasi-interchange.
FFT Model => Quasi-Interchange

Components of model:
n=1 eigenfunction and a temperature profile

\[ T_e(r) \]

Signal will be
\[ \xi(r, \theta, t) \cdot T_e'(r, t) \]

\[ \xi(r, \theta, t) = \xi_1(r) \cos(\theta - t) + \xi_2(r) \cos(2\theta - t) \]

Phase jump from Cos terms and jump back is from Sin terms. The separation depends on slope of the m=1 component as it intersects the m=2 component of \( \xi \). If the m=1 component is kink-like (top hat) the terms cancel and the phase jump does not occur.
ECE Profiles and End of Sawtooth

In the bean the profile just caves in from the low field side. No precursor detected.
ECE Profiles and End of Sawtooth

Rotating structure is altered about 600 ms before the crash is completed. Probably not tearing until after $t_c - 420 \mu s$. 

[Diagram showing time evolution of ECE profile with labeled channels and time markers]
ECE-constrained equilibrium using MSE and outer pressure
Oval equilibrium does not develop shear
Bean equilibrium does develop shear
Characteristic Behaviors during Sawtooth Ramp

**Bean**

\[ T_i \approx T_e \]

\( T_i, v_\phi \) roll over

\( dB_\theta/dt \) crashes

**Oval**

\[ T_i > T_e \]

\( T_i \) does not roll over (relative to \( T_e \))

\( dB_\theta/dt \) meanders
Response to Central ECH

**Bean:** Sawtooth amplitude increased, baseline preserved. Sawteeth remain regular.

**Oval:** Sawtooth amplitude not increased, baseline rises. \( m=1 \) activity greatly increased, period and partial reconnections irregular.
Sustained ECH attenuate ion sawteeth and aggravates rollover in beans
Impulse Response

Electrons respond locally.

However $\nabla T_i$ is just fine.
$L_{Ti} \sim a$ inside $r_i$ before, during, & after ECH pulse
Central Thermal Diffusivities

**Oval:** $\chi_e$ is as large as needed to eliminate $\nabla T e$. $\chi_i$ at the neoclassical level **Bean:** $\chi_e$ and $\chi_i$ are typical of values seen in core of tokamak plasmas. $\chi_e$ rises late in the period when $T_i$ rolls over.

Transport analysis averages over a few sawteeth to create repeated identical sawtooth periods.

Note: when ECH is added, central power density to electrons is increased by a factor of 25 so $\chi_e$ is $\sim 100$ m$^2$/s.
Relaxation Event Appears To Be Key In The Evolution

Prior to relaxation event electron confinement in the bean is as bad as it is in the oval. Prior to the relaxation event, both shapes have little or no shear.

Bean
Ions recover before electrons.

Oval
Ions recover before electrons but even earlier than in the bean.
Shaping causes shear when $J_\phi$ is flat.

- We have been considering shear as a secondary quantity in a causal sense, because the bean develops shear but the oval does not.
- Assume the relaxation event has the current adjusting to the new $T_e$ and at the end of this event the current profile is perfectly flat inside $r_i$.
- Because of the triangularity of the inner surfaces in the bean, there will be considerably more shear than in the oval. If $\chi_e$ depends strongly on shear, $T_e$ may begin to peak in the bean leading to increased shear => a bootstrap process.

(VMEC model equilibrium with $p'\neq0$)
Conclusions

• In both the bean and oval, reconnection occurs. After the sawtooth crash both have reconnected to have a central region with $q \approx 1$ and little shear.
• The Mercier criterion is not satisfied in the oval plasmas. (Something akin to the Mercier criterion seems to be satisfied for electron pressure.)
• The oval plasma develops a quasi-interchange $n=1$ instability. By contrast, the bean instability is an internal kink.
• From ECH results, the bean is crash is not a result of pressure ($\beta_{p1}$). In the oval, adding central pressure makes the behavior chaotic. This same chaotic behavior is seen in the bean prior to the relaxation event.
• The rollover in $T_i$ seen in the bean is aggravated when ECH is added. The rollover may simply be an inverse dependence of $\chi_i$ on $T_e$.
• $\omega \ast i \ll \gamma_{mhd}$ in the oval -- unimportant; $\gamma_{mhd}^{-1} \sim 4 \mu$s
• $\omega \ast i \sim 1/2 -- 1$ ($\gamma_{mhd}$) in the bean ?? important; $\gamma_{mhd}^{-1} \sim 30 \mu$s
• || current evolution appears neoclassical (preliminary) during ramp
  ▪ (anomalous during relaxation event)