SCALING AND MODELING STUDIES OF HIGH-BOOTSTRAP-FRACTION TOKAMAKS

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ISSUES

• Ultimate goal is a steady-state tokamak reactor with $Q \geq 10$.
  
  - Thermonuclear heating drives only bootstrap current

• Experiments to investigate 100% bootstrap tokamaks must use heating with no current drive capabilities:
  
  Gyrotrons, Minority ICRF
EXPERIMENTAL RECIPE

1. Form initial low-density discharge with flat, high q-profile

2. Turn off transformer; $V_{\text{loop}} \rightarrow 0$

3. Start central ECH (P > 3 MW); increase density to $5 \times 10^{19} \text{m}^{-3}$

4. Wait for several current relaxation times

5. Plasma Response ??
   - Settle into 100% bootstrap discharge
   - Continues Current Decay
   - Disrupt
SELF-CONSISTENT MODELING OF HIGH BOOTSTRAP DISCHARGES

• Model 100% bootstrap tokamak discharges with self-consistent transport.
  
  - Coupled fast heat transport system and slow poloidal field diffusion.
  
  - Use nondimensional approach
  
  - Heat diffusion coefficient is an analytic formula with physics properties given by construction
ADVANCED TOKAMAK NONLINEAR TRANSPORT COUPLINGS

Transformer source of poloidal flux

Auxiliary Current Drive

Auxiliary Heating

Auxiliary Angular Momentum

External

Internal

$V_{\text{loop}}$

$j_{\text{cd}}$

$j_{\text{Oh}}$

Bootstrap Current

Neoclassical poloidal flux diffusion

Profiles:

$p, T, n, v_{\phi}$

Conductivity profile

Temperature profiles couple magnetic and heat diffusion loops

$P_{\text{tot}}$

Fast, Blue heat and $v_{\phi}$ transport cycle

Anomalous & Neoclassical heat, particle and $v_{\phi}$ diffusion

Thermonuclear Heating

Fast, Blue heat and $v_{\phi}$ transport cycle

Slow, red magnetic flux diffusion loop

Turbulent and Neoclassical transport coefficients $\chi$

- Poloidal field dependence
- Velocity shear stabilization

T

$B_{\theta}$

$\sigma$

$\frac{dp}{dr}$
STEADY - STATE COUPLED DIFFUSION LOOPS: CYLINDRICAL MODEL

- Assume circular tokamak, fixed density and heating profile, $j = j_{bs}$

- Model will give scaling properties vs. B, $I_p$, $n_e$, $P_{aux}$, etc.

\[
\begin{align*}
  j &= \left(\frac{r}{R}\right)^{1/2} C_{bs} \frac{n_e}{B_\theta} \left(\frac{dT}{dr}\right) \\
  \frac{\partial}{\partial r} \left(r n \chi \frac{\partial T}{\partial r}\right) &= Q = \frac{P}{2\pi^2 R \Delta^2} \left(\frac{\Delta^2}{\Delta^2 + r^2}\right)^2 \\
  \chi &= C_\chi \frac{T^{3/2} M^{1/2}}{e^2 B^2 L_T} q^2 = C_\chi \frac{T^{1/2} r^2 M^{1/2}}{e^2 B_\theta^2 R^2} \left(\frac{\partial T}{\partial r} - \frac{T}{\Delta}\right)
\end{align*}
\]
GYROBOHM DIFFUSIVITY

• Diffusivity constructed to have properties observed in tokamak experiments

\[
\chi = C_x \frac{T^{3/2} M^{1/2}}{e^2 B^2 L_T} q^2 = C_x \frac{T^{1/2} r^2 M^{1/2}}{e^2 B_\theta^2 R^2} \left( \frac{\partial T}{\partial r} - \frac{T}{\Lambda} \right)
\]

1. GyroBohm scaling; transport independent of \( \beta \)

2. Flat density profile

3. Experimental confinement time depends only poloidal field (ie only on plasma current)

• Poloidal field enters diffusivity; transport *independent* of toroidal field.
SIMPLIFIED NONDIMENSIONAL EQUATION

• Introduce nondimensional variables; $\tau = T/T(0)$, $u = r/a$, $u_0 = \Delta/a$

• Second order eigenvalue equation

\[
\frac{d\tau}{du} = -\frac{\tau}{\Delta} - \sqrt{\left(\frac{\tau}{\Delta}\right)^2 + \frac{\psi^2 G}{u^5 \tau^{0.5}}} \quad \frac{d\psi}{du} = -\lambda u^{2.5} \left(\frac{d\tau}{du}\right)
\]

\[
\lambda = \frac{C_{ts}}{C_\chi} \left(\frac{R}{a}\right)^{1/2} \frac{e^2 \mu_0 P}{(2\pi)^2 M^{1/2} T_o^{3/2}} = \lambda(u_0) = 0.47
\]

• Solution determines value of $\lambda$ and hence the relation between $P$ and $T_o$
SOLUTIONS

- Nondimensional profiles for no critical gradient

1. Relative temperature profile vs normalized radius

Relative q profile: q/q(a) vs normalized radius
EIGENVALUE DETERMINES CENTRAL TEMPERATURE

1. Numerical Solution determines that eigenvalue $\lambda = 0.47$

   - Normalizing $C_\chi$ to DIII-D Discharges yields $C_\chi = 0.05$

   - Evaluation of central temperature is

     $$T(0)^{3/2} = 0.60 \cdot P_{MW} (R/a)^{1/2} C_{bs}$$

• Temperature depends only on $P^{2/3} (R/a)^{1/3}$

   - No dependence on density, size, or toroidal field.

   - Special case of gyroBohm scaling at constant $\beta_p$ (or $\beta$)
RESULTS AND SCALING

• GyroBohm Confinement, \( \approx 100\% \) Bootstrap no critical gradient

\[
T_{\text{keV}}(0)^{3/2} = 0.60 \cdot P_{\text{MW}} \left( \frac{R}{a} \right)^{1/2} C
\]

\[
I_{p,\text{MA}} = (0.15) \left( C_{bs} \right)^{5/6} (n_{19})^{1/2} (P_{\text{MW}})^{1/3} a_r
\]
SOLUTIONS WITH CRITICAL GRADIENT

- Critical scale length equal to minor radius.
- Eigenvalue doesn’t change much (\( \lambda = 0.45 \))

**Relative Temperature Profile**

**q profile**
FULL TORIDAL MODEL AND COMPATIBILITY WITH NEGATIVE SHEAR

• 2- Dimensional model couples solution to 2D Grad - Shafranov equilibrium with 1D heat transport equation.

• Equations define the model.

1. Non dimensional Grad-Shafranov equation

\[ u \frac{\partial}{\partial u} \frac{1}{u} \frac{\partial \tilde{\psi}}{\partial u} + \frac{\partial^2 \tilde{\psi}}{\partial v^2} = -\lambda \frac{dp}{d\tilde{\psi}} \left( \frac{u^2}{u_o^2} - \frac{1}{\left\langle \frac{u_o^2}{u^2} \right\rangle} \right) + \frac{f_t C_{bs}}{\left\langle \frac{u_o^2}{u^2} \right\rangle} \]
MORE EQUATIONS

2. Heat transport equation has many physics processes

\[
\frac{P_{\text{heat}}}{S} = C \ e^{\alpha s} \ \frac{n \ M^{0.5} \ A^* \ T^{0.5}}{(2\pi)^{3/2} \ e^2} \left( \frac{\partial T}{\partial \psi} \right) \left( \frac{\partial T}{\partial \psi} - \frac{T}{\delta} \right)
\]

- Critical temperature gradient
- GyroBohm heat diffusion
- Gradients only with respect flux functions
- Confinement depends only on plasma current; no TF
- No dependence of diffusivity on \( \beta \)
- Overall magnetic shear dependence improves confinement
COMPATIBILITY

- Combined heat and plasma current sources lead to compatibility criterion

\[
\left( \frac{2i^2}{\lambda_3 A^* f_i} \right) U = -p' = \frac{(p / \delta) + \sqrt{(p / \delta)^2 + \lambda_2 \left( \frac{\Pi}{\Sigma} \right) 8\pi p^{(3\gamma-1)/2}}}{2 (1 - \gamma) e^{-\alpha U}}
\]

- \( U = I' / I \) is related to magnetic shear

- Implicit equation for magnetic shear

- For negative magnetic shear, a solution may not be available
LONG DURATION PULSE IN DIII-D

- Thermal diffusivity normalized to this shot

Fig 1. Nearly stationary discharge for over 2 s at high beta (107736). (a) Plasma current, NB power (200 ms average; the power is modulated to maintain constant stored energy), and EC power. The transformer current is fixed from 2.0 s onward. (b) $\beta_N$, $\beta_p$, and $l_i$. $\beta_N$ is held constant by the NB feedback control. There is a very slow broadening of the current profile indicated by the decreasing $l_i$. 
MODELING OF PROPOSED DIII-D DISCHARGES

• Normalize diffusivity to low-current, ECCD supported plasmas.

• Compare evolution with (ECCD) and without (ECH) electron cyclotron current drive

5 MW ECCD at 0.5 sec

5 MW ECH at 0.5 SEC
CONCLUSIONS:1

1. A framework has been devised to generate plasma equilibria having profiles consistent with both transport physics and a 100% bootstrap source for plasma current.

2. Such steady tokamak discharges will exist provided thermal diffusivity depends weakly on magnetic shear

   • Profiles satisfy coupled heat-diffusion flux-diffusion system.
   • Plasma current proportional $I_p \propto n^{1/2} P^{1/3} (a/R)^{1/12}$, $T(0) \propto P^{2/3}$
3. But, steady-state discharges can be incompatible with 100% bootstrap current if diffusivity decreases strongly with increasingly negative magnetic shear.

4. By construction, toroidal field enters through $\beta_{\text{tor}}$ limit and $q(0)$
   - It remains for stability limits to be evaluated for plasmas with transport-determined profiles
   - Shape will be a parameter

5. In present tokamaks, such as DIII-D, NBI current drive competes with bootstrap current.