SCALING AND MODELING STUDIES OF HIGH-BOOTSTRAP-FRACTION TOKAMAKS

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ISSUES

- **Ultimate goal is a steady-state tokamak reactor with $Q \geq 10$.**
  - **Thermonuclear heating drives only bootstrap current**

- **Experiments to investigate 100% bootstrap tokamaks must use heating with *no current drive capabilities*:**

Gyrotrons , Minority ICRF

EXPERIMENTAL RECIPE

1. **Form initial low-density discharge with flat, high q-profile**

2. **Turn off transformer; $V_{loop} \rightarrow 0$**

3. **Start central ECH (P > 3 MW) ; increase density to $5 \cdot 10^{19} m^{-3}$**

4. **Wait for several current relaxation times**

5. **Plasma Response ??**

   - **Settle into 100% bootstrap discharge**
   - **Continues Current Decay**
   - **Disrupt**

SELF-CONSISTENT MODELING OF HIGH BOOTSTRAP DISCHARGES

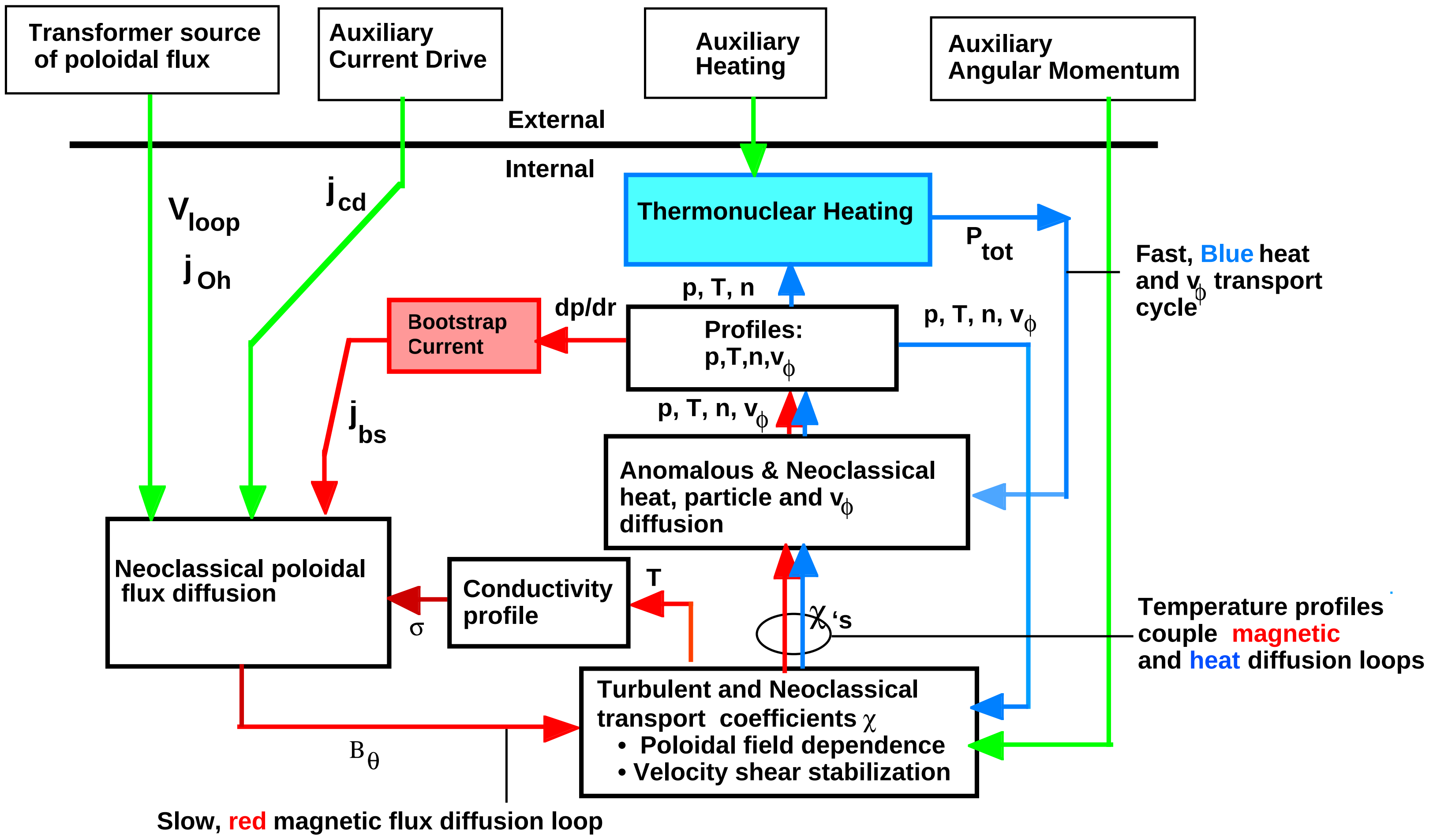
- **Model 100% bootstrap tokamak discharges with self- consistent transport.**

  - Coupled fast heat transport system and slow poloidal field diffusion.

  - Use nondimensional approach

  - Heat diffusion coefficent is an analytic formula with physics properties given by construction

ADVANCED TOKAMAK NONLINEAR TRANSPORT COUPLINGS



Transformer source of poloidal flux

Auxiliary Current Drive

Auxiliary Heating

Auxiliary Angular Momentum

External

Internal

V_{loop}

j_{Oh}

j_{cd}

Thermonuclear Heating

P_{tot}

Fast, Blue heat and v_ϕ transport cycle

p, T, n

dp/dr

Bootstrap Current

Profiles: p, T, n, v_ϕ

p, T, n, v_ϕ

j_{bs}

p, T, n, v_ϕ

Anomalous & Neoclassical heat, particle and v_ϕ diffusion

Neoclassical poloidal flux diffusion

σ

Conductivity profile

T

χ's

Temperature profiles couple magnetic and heat diffusion loops

Turbulent and Neoclassical transport coefficients χ

- Poloidal field dependence
- Velocity shear stabilization

B_θ

Slow, red magnetic flux diffusion loop

STEADY - STATE COUPLED DIFFUSION LOOPS: CYLINDRICAL MODEL

- Assume circular tokamak, fixed density and heating profile, $j=j_{bs}$
- Model will give scaling properties vs. B, I_p , n_e , P_{aux} , etc.

$$\boxed{j = \left(\frac{r}{R}\right)^{1/2} C_{bs} \frac{n_e}{B_\theta}\left(\frac{dT}{dr}\right)}$$

$$\boxed{\frac{\partial}{r\partial r}\left(rB_\theta\right) = \mu_o j}$$

$$\boxed{\frac{\partial}{r\partial r}\left(r\, n\, \chi \frac{\partial T}{\partial r}\right) = Q = \frac{P}{2\pi^2 R\Delta^2}\left\{\frac{\Delta^2}{\Delta^2 + r^2}\right\}^2}$$

$$\boxed{\chi = C_\chi \frac{T^{3/2} M^{1/2}}{e^2 B^2 L_T} q^2 = C_\chi \frac{T^{1/2} r^2 M^{1/2}}{e^2 B_\theta^2 R^2}\left(\frac{\partial T}{\partial r} - \frac{T}{\Delta}\right)}$$

GYROBOHM DIFFUSIVITY

- **Diffusivity constructed to have properties observed in tokamak experiments**

$$\boxed{\chi = C_\chi \frac{T^{3/2} M^{1/2}}{e^2 B^2 L_T} q^2 = C_\chi \frac{T^{1/2} r^2 M^{1/2}}{e^2 B_\theta{}^2 R^2} \left(\frac{\partial T}{\partial r} - \frac{T}{\Delta} \right)}$$

1. **GyroBohm scaling; transport independent of β**
2. **Flat density profile**
3. **Experimental confinement time depends only poloidal field (ie only on plasma current)**

- **Poloidal field enters diffusivity; transport *independent* of toroidal field.**

SIMPLIFIED NONDIMENSIONAL EQUATION

- **Introduce nondimensional variables; $\tau = T/T(0)$, $u = r/a$, $u_o = \Delta/a$**

- **Second order eigenvalue equation**

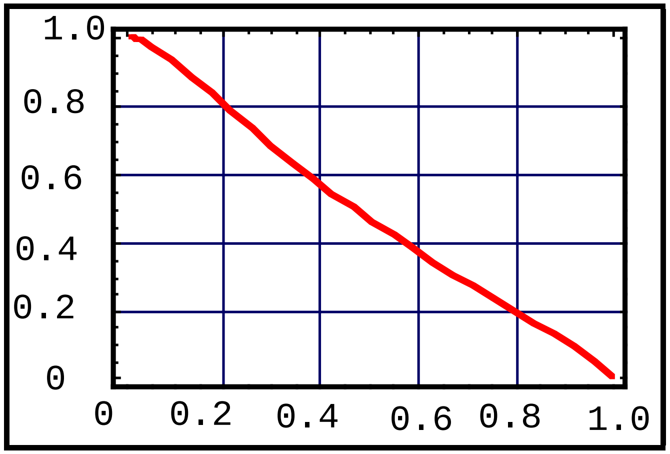
$$\boxed{\frac{d\tau}{du} = -\frac{\tau}{\Delta} - \sqrt{\left(\frac{\tau}{\Delta}\right)^2 + \frac{\psi^2 G}{u^5 \tau^{0.5}}} \qquad \frac{d\psi}{du} = -\lambda\, u^{2.5}\left(\frac{d\tau}{du}\right)}$$

$$\boxed{\lambda = \frac{C_{bs}}{C_\chi}\left(\frac{R}{a}\right)^{1/2} \frac{e^2 \mu_o P}{(2\pi)^2 M^{1/2} T_o^{3/2}} = \lambda(u_o) = 0.47}$$

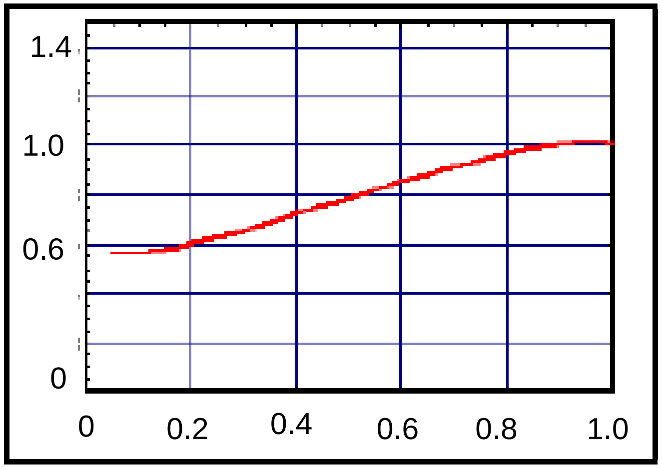
- **Solution determines value of λ and hence the relation between P and T_o**

SOLUTIONS

- **Nondimensional profiles for no critical gradient**



1. Relative temperature profile vs normalized radius



Relative q profile: q/q(a) vs normalized radius

EIGENVALUE DETERMINES CENTRAL TEMPERATURE

1. Numerical Solution determines that eigenvalue $\lambda = 0.47$

- **Normalizing C_χ to DIII-D Discharges yields $C_\chi = 0.05$**
- **Evaluation of central temperature is**

$$T(0)^{3/2} = 0.60 \cdot P_{MW} (R/a)^{1/2} C_{bs}$$

- **Temperature depends only on $P^{2/3} (R/a)^{1/3}$**
  - **No dependence on density , size, or toroidal field.**
  - **Special case of gyroBohm scaling at constant β_p (or β)**

RESULTS AND SCALING

- **GyroBohm Confinement, ≈100%Bootstrap no critical gradient**

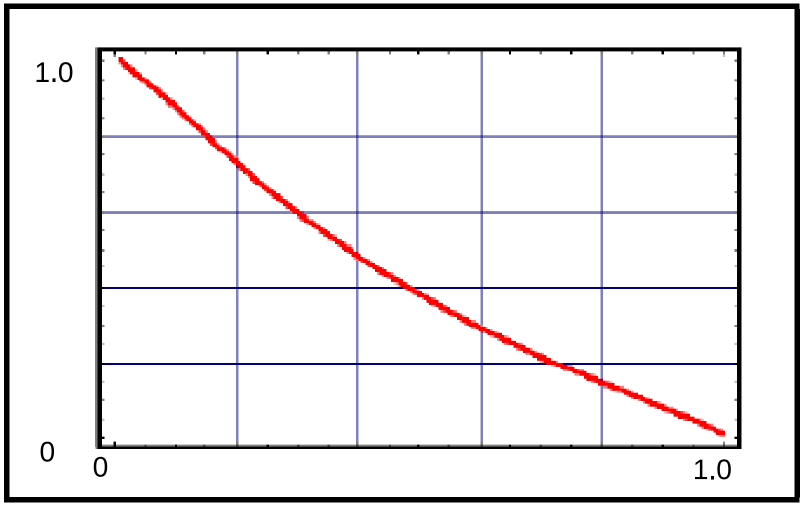
$$T_{keV}(0)^{3/2} = 0.60 \cdot P_{MW} \left(\frac{R}{a}\right)^{1/2} C$$

$$I_{p,MA} = (0.15)\,(C_{bs})^{5/6}\,(n_{19})^{1/2}\,(P_{MW})^{1/3} a_{r}$$

SOLUTIONS WITH CRITICAL GRADIENT

- **Critical scale length equal to minor radius.**
- **Eigenvalue doesn't change much ($\lambda = 0.45$)**

Relative Temperature Profile



q profile

FULL TORIDAL MODEL AND COMPATIBILIY WITH NEGATIVE SHEAR

- **2- Dimensional model couples solution to 2D Grad - Shafranov equilibrium with 1D heat transport equation.**

  - **Equations define the model.**

1. Non dimensional Grad-Shafranov equation

$$u\frac{\partial}{\partial u}\frac{1}{u}\frac{\partial\tilde{\psi}}{\partial u} + \frac{\partial^2\tilde{\psi}}{\partial v^2} = -\lambda_1\frac{dp}{d\tilde{\psi}}\left\{\left[\frac{u^2}{u_o^2} - \frac{1}{\left\langle\frac{u_o^2}{u^2}\right\rangle}\right] + \frac{f_t C_{bs}}{\left\langle\frac{u_o^2}{u^2}\right\rangle}\right\}$$

MORE EQUATIONS

2. Heat transport equation has many physics processes

$$\boxed{\frac{P_{\mathrm{heat}}}{S} = C\, e^{\alpha s} \frac{n\, M^{0.5}\, A^{*}\, T^{0.5}}{(2\pi)^{3}\, e^{2}} \left(\frac{\partial T}{\partial \psi}\right) \left(\frac{\partial T}{\partial \psi} - \frac{T}{\delta}\right)}$$

- **Critical temperature gradient**
- **GyroBohm heat diffusion**
- **Gradients only with respect flux functions**
- **Confinement depends only on plasma current; no TF**
- **No dependence of diffusivity on β**
- **Overall magnetic shear dependence improves confinement**

COMPATIBILITY

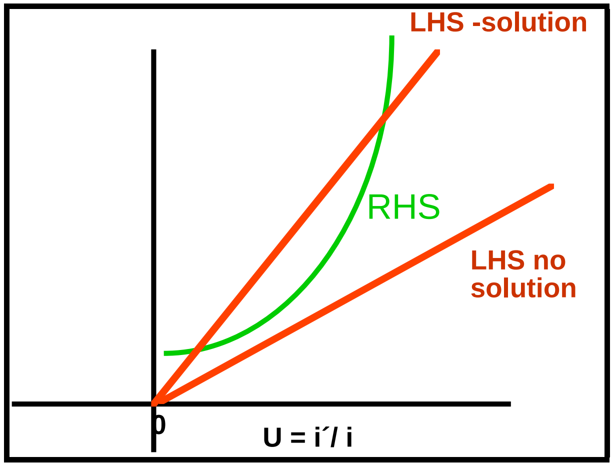
- **Combined heat and and plasma current sources lead to compatibility criterion**

$$\left(\frac{2i^2}{\lambda_3\,\tilde{A}^* f_t}\right)\mathbf{U} = -p' = \frac{(p/\delta) + \sqrt{(p/\delta)^2 + \lambda_2\left(\frac{\Pi}{\Sigma}\right)\frac{8\pi\, p^{(3\gamma-1)/2}}{\tilde{A}\;\; e^{-\alpha\mathbf{U}}}}}{2\,(1-\gamma)}$$

- **U = I´/I is related to magnetic shear**

- **Implicit equation for magnetic shear**

- **For negative magnetic shear, a solution may not be available**



LONG DURATION PULSE IN DIII-D

- **Thermal diffusivity normalized to this shot**

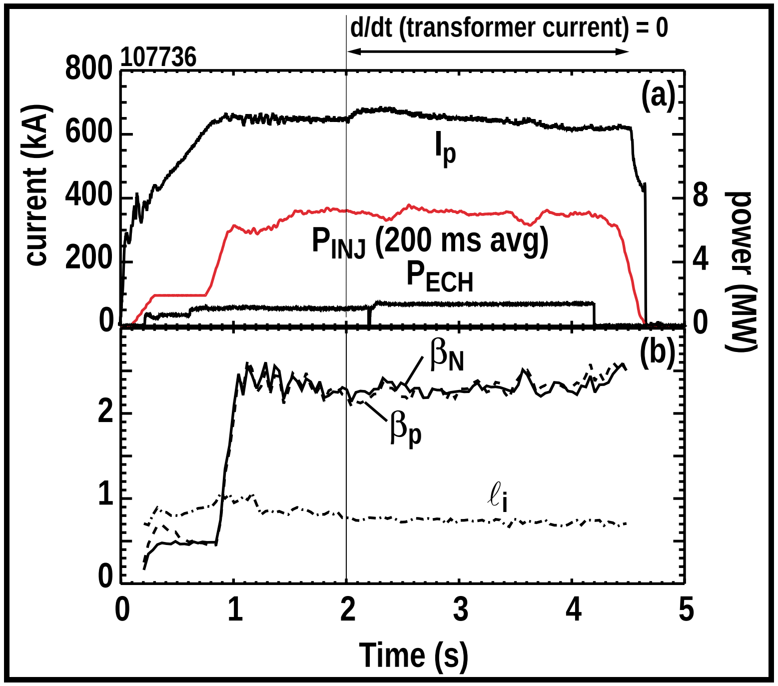
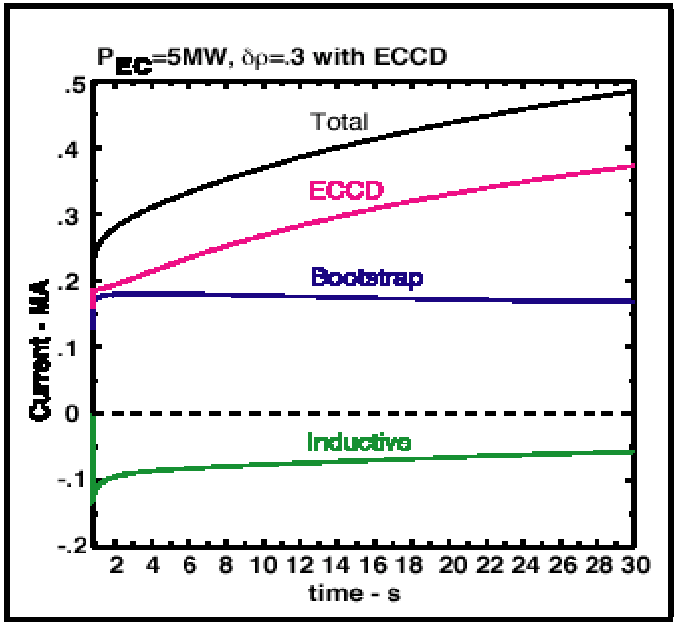


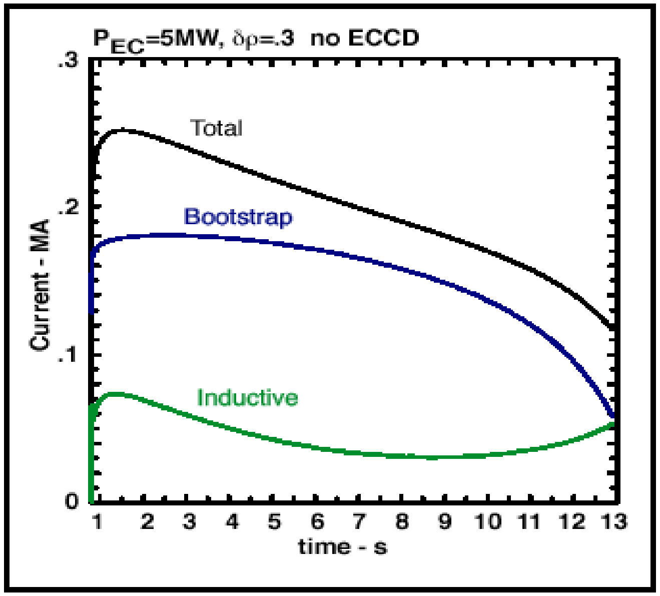
Fig 1. Nearly stationary discharge for over 2 s at high beta (107736). (a) Plasma current, NB power (200 ms average; the power is modulated to maintain constant stored energy), and EC power. The transformer current is fixed from 2.0 s onward. (b) β_N, β_p, and l_i. β_N is held constant by the NB feedback control. There is a very slow broadening of the current profile indicated by the decreasing l_i.

MODELING OF PROPOSED DIII-D DISCHARGES

- Normalize diffusivity to low-current, ECCD supported plasmas.
- Compare evolution with (ECCD) and without (ECH) electron cyclotron current drive



5 MW ECCD at 0.5 sec



5 MW ECH at 0.5 SEC

CONCLUSIONS:1

1. A framework has been devised to generate plasma equilbria having profiles consistent with both transport physics and a 100% bootstrap source for plasma current.

2. Such steady tokamak discharges will exist provided thermal diffusivity depends weakly on magnetic shear

- Profiles satisfy coupled heat-diffusion flux-diffusion system.
- Plasma current proportional $I_p \propto n^{1/2} P^{1/3} (a/R)^{1/12}$, $T(0) \propto P^{2/3}$

CONCLUSIONS: 2

3. But, steady -state discharges can be incompatible with 100% bootstrap current if diffusivity decreases strongly with increasingly negative magnetic shear.

4. By construction, toroidal field enters through β_{tor} limit and q(0)

   - It remains for stability limits to be evaluated for plasmas with transport-determined profiles

   - Shape will be a parameter

5. In present tokamaks, such as DIII-D, NBI current drive competes with bootstrap current.