GA-A24671

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**MAY 2004** 



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# PHASE ERROR CORRECTION METHOD FOR A VIBRATION COMPENSATED INTERFEROMETER

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This is a preprint of a paper to be presented at the 15<sup>th</sup> High Temperature Plasma Diagnostics Conf., San Diego, California, April 19–22, 2004 and to be published in *Rev. Sci. Instrum.* 

Work supported by the U.S. Department of Energy under DE-FC02-04ER54698

GENERAL ATOMICS PROJECT 30200 MAY 2004



## Abstract

In order to compensate for vibrations while simultaneously measuring plasma lineaveraged density, the existing real-time interferometer on DIII-D uses both a HeNe and  $CO_2$  laser operating at 10.59 µm and 0.632 µm respectively. The line density resolution is  $10^{18} \text{ m}^{-2}$  which is predominantly determined by the measurement electronics. In particular, errors resulting from detection of the  $CO_2$  phase shift and finite bit size in the processing electronics are the major limiting factors. This paper describes a method for post-shot error correction generally applicable to two-color interferometers that utilize analog quadrature phase detection. The error correction scheme exploits the facts that error introduced by the phase comparator is periodic and that measurements can be made while the density is zero and only the errors are recorded. Application of this method will reduce unphysical density oscillations which are purely a result of vibration induced phase shifts. Application to DIII-D data shows a 25 percent reduction in noise amplitude on line density measurements.

#### I. INTRODUCTION

Measurement of the line-averaged electron density is a fundamental diagnostic need for tokamak plasmas. Interferometers operating from the visible to microwave range have been used with great success. Each end of the spectrum, however, has corresponding drawbacks. Microwave interferometers suffer from refraction off plasma density gradients as well as rapid fringe shifts from sudden density changes. Visible interferometers are severely affected by optical component vibration and, in order to separate plasma from mechanical effects, are typically required to operate simultaneously with a path sharing longer wavelength interferometer. While more complicated, this type of interferometer does not suffer from refraction, can follow rapid plasma density changes, and generally operates independent of X or O polarization. This technique is known as two color vibration compensated interferometery [1, 2].

A proven method for decoding the phase shift of each of the two co-propagating interferometer legs is quadrature heterodyne phase detection. For analog phase detection circuitry, the limit of this configuration often lies in the actual quadrature phase comparator itself, which sets the line-averaged density resolution [2–4]. This paper describes a method for post-shot correction of the errors introduced by analog quadrature phase compartors that are employed in two-color vibration compensated heterodyne interferometers. The correction scheme exploits the facts that errors introduced by phase comparators are typically periodic and that measurements can be made while the density is zero and only the errors are recorded.

#### II. PHASE ERROR

The principle of two-color interferometry is to measure the phase shift  $\phi$  experienced by two co-propagating interferometers. This phase shift is given by

$$\phi = A\lambda + B/\lambda \tag{1}$$

where the first term  $(A\lambda)$  is the plasma contribution and the second term  $(B/\lambda)$  is the phase shift due to vibrations. Measurement of  $\phi$  at two different wavelengths  $(\lambda)$  allows the determination of A and B and provides the desired plasma line-integrated density from

$$A = 2.82 \times 10^{-15} \int n_e dl = k n_{av} L$$
 (2)

and

$$B = 2\pi V \tag{3}$$

where  $k = 2.82 \times 10^{-15}$  m (the classical electron radius) and  $n_{av}$ , L, and V are the lineaveraged electron density, plasma dimension, and optics vibrations respectively [1]. For two different wavelengths,  $\lambda_c$  and  $\lambda_h$ , the density and vibration are given by

$$n_{av} = \frac{\lambda_c}{kL(\lambda_c^2 - \lambda_h^2)} \left(\phi_c - \phi_h \lambda_h / \lambda_c\right) \tag{4}$$

$$2\pi V = \lambda_h \phi_h - \frac{\lambda_h^2 \lambda_c}{k L (\lambda_c^2 - \lambda_h^2)} \left( \phi_c - \phi_h \lambda_h / \lambda_c \right) \quad . \tag{5}$$

For a given error  $\delta\phi$  in the phase, there will be a corresponding error in the line-averaged density. The magnitude of this error, however, is weighted by the wavelength ratio. Thus, for two color interferometers operating in the visible and infrared, such as those on DIII-D, the ratio of the introduced error is  $\lambda_h/\lambda_c = 0.632/10.59 = 1/16.8$  – indicating that phase error measurements of the longer wavelength beam will be the dominant term. This paper refers to  $\lambda_c$  as the longer wavelength corresponding to the 10.59  $\mu$ m CO<sub>2</sub> line and  $\lambda_h$  as the shorter 0.632  $\mu$ m HeNe line. Any wavelength ratio can be used, however, the method for error correction outlined in the next section becomes increasingly valid as  $\lambda_c/\lambda_h \to \infty$ .

The technique used to measure  $\phi$  is described in detail elsewhere[3], but briefly, relies on frequency shifting the reference beam by a small amount  $\Delta f$  and deriving the phase from the beat signal of the plasma traversing leg and the reference signal – quadrature heterodyne detection [3]. The accuracy with which this phase detection can be carried out then limits the resolution of the line-averaged density. Most commercial analog quadrature phase comparators have a phase error of  $\geq 3$  degrees, as well as an appreciable amplitude imbalance and dc offset [3, 4]. To get an idea of the form density errors resulting from the phase comparator will take, consider the most general case for the I (sin) and Q (cos) output of the comparator itself.

$$I = A_1 \sin(\phi + d\phi_1) + \delta A_1 \tag{6}$$

$$Q = A_2 \cos(\phi + d\phi_2) + \delta A_2 \quad . \tag{7}$$

 $A_i, d\phi_i$ , and  $\delta A_i$  are the amplitude, phase, and dc offsets respectively. Direct calculation of  $\phi$  by  $\phi' = tan^{-1}(I/Q)$  will result in a measured phase of  $\phi' = \phi + \delta \phi$ , where  $\delta \phi$  is the phase error and for small  $d\phi_i$  is given by

$$D = \frac{A_1[\sin(\phi) + d\phi_1 \cos(\phi)] + \delta A_1}{A_2[\cos(\phi) - d\phi_2 \sin(\phi)] + \delta A_2} \quad .$$
(8)

$$\delta\phi = \frac{D\cos(\phi) - \sin(\phi)}{D\sin(\phi) + \cos(\phi)} \tag{9}$$

The major points to observe are: (1) The resulting  $\delta \phi$  is oscillatory with period  $2\pi$  (as one would expect), and (2)  $\delta \phi$  is not simply the phase imbalance  $d\phi_i$ . Point 1 indicates that even for zero density, vibration induced fringes may have the appearance of an apparent density oscillation with a frequency that depends on the rate of vibration itself. Point 2 indicates that estimates of the error given in terms of  $d\phi_i$  may be low.

#### III. PHASE CORRECTION AND APPLICATION TO DIII-D DATA

For most interferometers of this type, the actual stored data consists of line-averaged density and vibration measurements. Shown in Fig. 1 is a sample density and vibration time series from the DIII-D real-time density computer described elsewhere [1]. Looking at Fig. 1, the density is recorded long after the plasma has decayed (t > 5.5 s) during which time significant vibration is still occurring. Any recorded density at this late time ( $\Delta n$ ) is coming directly from phase measurement errors. The density error ( $\Delta n$ ) is given by

$$\Delta n = \frac{\lambda_c}{kL(\lambda_c^2 - \lambda_h^2)} [\delta \phi_c(\phi_c) - \delta \phi_h(\phi_h) \lambda_h / \lambda_c]$$
(10)

where  $\delta \phi_h$  and  $\delta \phi_c$  are the phase errors for the HeNe and CO<sub>2</sub> beams respectively, which themselves depend on  $\phi_h$  and  $\phi_c$  as in Eq. (9). Neglecting the contribution from the HeNe term we get an approximation for the CO<sub>2</sub> phase error in terms of a stored quantity  $\Delta n$ 

$$\delta\phi_c(\phi_c) = \Delta n \frac{kL(\lambda_c^2 - \lambda_h^2)}{\lambda_c} \quad . \tag{11}$$

It is expected that this error will be periodic in CO<sub>2</sub> fringes. Figure 2 shows  $\delta \phi_c$  derived from Eq. (11) as a function of  $\phi_c$  where  $\phi_c \approx 2\pi V/\lambda_c$  (valid for n = 0).

Two things are apparent in this figure, a definite trend as well as the minimum bit spacing  $(\approx 1.5 \text{ degrees} = 9 \times 10^{17} \text{ m}^{-2})$  from data acquisition. The DAQ digitizer used to digitize the analog line-density information is 12 bits with the equivalent line-density of 16  $CO_2$  fringes full scale. The bit spacing places a limit on the accuracy that we can hope to achieve with this method. The trend is approximated by a moving average, shown as a dashed line in Fig. 2. In practice any type of fit could be used including a least squares fit to Eq. (9). In order to check the validity of this fit, the I and Q outputs of the  $CO_2$  quadrature phase detector were digitized directly at 10 MS/s (high speed digitization is essential in order to reliably follow rapid fringe shifts). Knowledge of the actual I and Q signals gives the amplitude imbalance as well as offset and permits determination of the phase error introduced by comparator imperfections. This high-speed digitization was carried out for 1 s intervals at various times througout the discharge. Little or no variation in the calculated error was observed from interval to interval. The corresponding periodic phase error is shown by a solid line in Fig. 2. The thickness of this line is representative of the degree to which the phase error is constant over the digitized interval. As a note, this method has also been used to acquire the lineaveraged density with a several MHz bandwidth in order to investigate internal MHD modes as well as disruption mitigation.

Using this fit, the density is re-evaluated for the entire time-series according to

$$n' = n - \frac{\lambda_c}{kL(\lambda_c^2 - \lambda_h^2)} \delta\phi_c(\phi_c)$$
(12)



Fig. 1. Line density and vibration, DIII-D shot 117092.



Fig. 2.  $CO_2$  phase error  $(\delta \phi_c)$  vs.  $CO_2$  phase  $(\phi_c)$  for the points between 5.5 and 7.0 s, shot 117092. Triangles are phase error inferred from actual density measurements and Eq. (11). Dashed curve is the moving average of the data points. Solid curve is the measured error from direct digitization of the *I* and *Q* outputs of the CO<sub>2</sub> quadrature phase detector.

where  $\phi_c$  now must include both the plasma and the vibration and is given by Eq. (1). The results of application of this process to shot 117092 is shown in Fig. 3 for t > 5.3 s, where a small offset apparent in Fig. 1 has been subtracted.

Before application of the error correction, the standard deviation of the line density fluctuations in the observed interval is  $2.24 \times 10^{18}$  m<sup>-2</sup> after correction this is reduced to  $1.69 \times 10^{18}$  m<sup>-2</sup> corresponding to a noise improvement of roughly 25 percent. It is expected that the two major contributing factors to the limit of this method, as applied here, are the resolution with which the vibration data is stored (within  $\lambda_h/4$ ) and DAQ digitizer bit noise. The finite bit noise corresponds to approximately 3 degrees=  $1.8 \times 10^{18}$  m<sup>-2</sup>. For systems with finer bit resolution this method has the potential to reduce phase measurement errors of existing datasets to less than 1 degree or equivalent line density values of better than  $6 \times 10^{17}$  m<sup>-2</sup> for a CO<sub>2</sub> system. Extensions of this method to arbitrary values of  $\lambda_c/\lambda_h$  are currently in progress.



Fig. 3. Density data for shot 117092 after plasma has decayed. Top: Stored line density measurement, Bottom: Corrected line density.

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#### IV. ACKNOWLEDGMENTS

This work was sponsored by the U.S. Department of Energy under DE-FC02-04ER54698. The help of Dr. G. Wang in obtaining fast digitization of the quadrature phase comaprator outputs is gratefully acknowledged. His measurement provided validation of this method. Interesting and helpful discussions with Dr. J.C. DeBoo are also appreciated.