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CALCULATION OF EDGE TOROIDAL CURRENT DENSITY DISTRIBUTIONS FROM DIII–D LITHIUM BEAM MEASUREMENTS USING AMPÈRE'S LAW

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ABSTRACT

The local edge current density j(r) is a parameter of basic importance in understanding the stability of high performance tokamaks as well as the dynamics of ELM behavior. On DIII–D the lithium beam polarimetry diagnostic provides precise measurements of the local magnetic field projection along the field of view at 32 radial locations in the plasma edge. Using these measurements, the known spatial calibration and a minimal amount of information about the magnetic field shape from equilibrium reconstructions, Ampères law may be used to provide a straightforward parameterization for the edge toroidal current density in terms of the measured magnetic field and its radial derivative. This approach is relatively insensitive to errors in the reconstruction and is simple to apply.

I. INTRODUCTION

In order to optimize the performance of magnetically confined fusion plasmas, it is important to understand the stability limits of the pedestal or edge region. This in turn requires an accurate knowledge of the current in this region. The recent development of a detailed model and associated magnetohydrodynamic (MHD) stability code (ELITE) based on coupled peeling-ballooning modes has succeeded in describing many aspects of the pedestal including edge localized modes (ELMs) [1,2]. Given a radial current distribution j(r) and pressure distribution P(r), one may efficiently calculate the stability and growth of the relevant modes for a broad range of plasma parameters [3]. However, determining the actual current distribution in the edge is quite difficult.

To provide a good set of measured values for j(r) on the DIII–D tokamak we have deployed the lithium beam polarimetry (LIBEAM) system [4–7], which provides a closely spaced array of magnetic field values in the edge region. This is done by examining the ratio of circular to linearly polarized light in one of the Zeeman components of the lithium $2^2S–2^2P$ resonance transition (λ_0 = 670.78 nm). The LIBEAM data may be combined with other magnetics measurements such as those derived from motional Stark Effect [8] and magnetic probe diagnostics and used as constraints on the equilibrium solver EFIT [9]. With the resulting equilibrium and the measured pressure profiles, a current distribution may be inferred from a bootstrap current model. To date, this approach has not been satisfactory because of the extreme sensitivity of the equilibria to the precise details of the current distribution and the coarseness of the grid spacing typically used in the reconstruction. Eventually, an iterative solution at a much higher resolution will be required to get the best possible estimate for j(r).

This paper discusses an alternative analytical technique for interpreting a value for j(r) directly from the LIBEAM measurements using Ampere's Law and an approximate knowledge of the flux surface shape in the region of the measurement.

II. DIAGNOSTIC GEOMETRY

Figure 1 shows the layout of the LIBEAM viewing geometry on DIII–D. The beam is injected radially at a position a few cm below the midplane, and viewed by a radial fan of views from below. The intersection of the viewchords with the beam provides 32 highly localized measurements of the magnetic field component parallel to the view chord, or B_{view}. (Note: because of our choice of geometry, B_{view} is approximately equal to the poloidal field B_{pol} although this is not required for the analysis.)

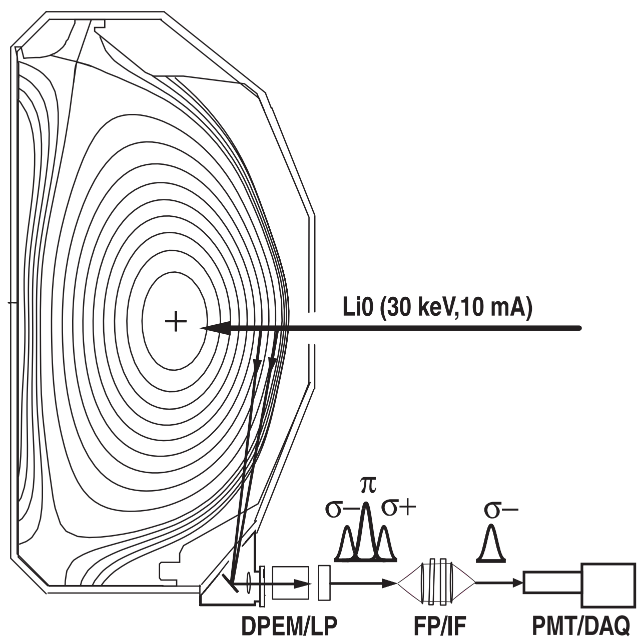

Fig. 1. Viewing geometry for the LIBEAM diagnostic on DIII-D, showing the flux surfaces from an EFIT equilibrium reconstruction for discharge 115099. DPEM/LP = dual photoelastic modulator/linear polarimeter combination; FP = Fabry-Perot; IP = interference filter; PMT = photomultiplier tube detector; DAQ = data acquisition system. Adapted from Ref. [5].

Based on a spatial calibration for each of the viewing chords, we know the R and Z locations for each of the view chords as well as the view inclination angle θ_v defined with respect to the vertical (Fig. 2). This gives

$$B_{VIEW} = B_Z \cos\theta_V + B_R \sin\theta_V \quad . \tag{1}$$

where B_Z and B_R are the vertical and radial magnetic field components.

The relationship between B_R and B_Z is expressed in terms of the magnetic inclination angle θ_B:

$$\tan\theta_B = B_R / B_Z \quad . \tag{2}$$

The specific value of $\tan\theta_B$ in the region of interest is very insensitive to the specific details of the current distribution and thus may be obtained from any reasonably converged equilibrium. It is therefore a good parameterization to use in terms of analyzing edge currents from the field measurements directly.

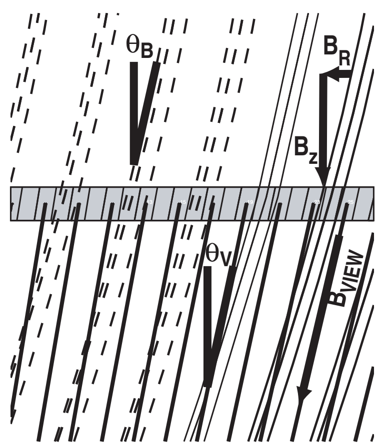

Fig. 2. Expanded viewing region from Fig. 1, showing the definition of viewing and magnetic inclination angles θ_v and θ_B used in the text. The beam trajectory (shaded region) intersection with the sightlines is shown. Viewchords (heavy lines) are drawn for every other one of the viewing locations. The three equilibria shown here (dashed lines) have different edge current distributions but essentially identical θ_B.

III. ANALYSIS

We start with Ampere's Law, in toroidal geometry

$$\mu_0 j = \nabla \times B => \mu_0 j_{\mathrm{TOR}} = \frac{\partial B_{\mathrm{R}}}{\partial z} - \frac{\partial B_{\mathrm{z}}}{\partial R} \quad . \tag{3}$$

In terms of the magnetic inclination angle this may be written as

$$\mu_0 j_{\mathrm{TOR}} = \frac{\partial B_{\mathrm{z}}}{\partial z} \tan\theta_{\mathrm{B}} + B_{\mathrm{z}} \frac{\partial \tan\theta_{\mathrm{B}}}{\partial z} - \frac{\partial B_{\mathrm{z}}}{\partial R} \quad . \tag{4}$$

Since from the definition of the poloidal magnetic flux φ in toroidal geometry we have

$$B_{\mathrm{R}} = -\frac{1}{R}\frac{\partial \varphi}{\partial z} \quad , \qquad B_{\mathrm{z}} = \frac{1}{R}\frac{\partial \varphi}{\partial R} \quad , \tag{5}$$

we can take the appropriate partials and are the definition of tan θ_{B} to obtain

$$\mu_0 j_{\mathrm{TOR}} = B_{\mathrm{z}} \left(\frac{\partial \tan\theta_{\mathrm{B}}}{\partial z} - \frac{\tan^2\theta_{\mathrm{B}}}{R} - \tan\theta_{\mathrm{B}} \frac{\partial \tan\theta_{\mathrm{B}}}{\partial R} \right) - \frac{\partial B_{\mathrm{z}}}{\partial R}\left(1 + \tan^2\theta_{\mathrm{B}}\right) \quad . \tag{6}$$

From Eqn. (1) and (2) we can express the local toroidal current distribution solely as a function of the measurement, the magnetic inclination angle, and their respective radial derivatives:

$$\begin{aligned} \mu_0 j_{\mathrm{TOR}} = {} & B_{\mathrm{VIEW}} \frac{\left[\frac{\partial \tan\theta_{\mathrm{B}}}{\partial z} - \frac{\tan^2\theta_{\mathrm{B}}}{R} - \tan\theta_{\mathrm{B}} \frac{\partial \tan\theta_{\mathrm{B}}}{\partial R} \right]}{\left[\cos\theta_{\mathrm{V}} + \sin\theta_{\mathrm{V}}(\tan\theta_{\mathrm{B}})\right]} \\ & + B_{\mathrm{VIEW}} \frac{\left[\frac{\partial \cos\theta_{\mathrm{V}}}{\partial R} + \frac{\partial \sin\theta_{\mathrm{V}}}{\partial R} \tan\theta_{\mathrm{B}} + \sin\theta_{\mathrm{V}} \frac{\partial \tan\theta_{\mathrm{B}}}{\partial R} \right]\left[1 + \tan^2\theta_{\mathrm{B}}\right]}{\left[\cos\theta_{\mathrm{V}} + \sin\theta_{\mathrm{V}}(\tan\theta_{\mathrm{B}})\right]^2} \\ & - \frac{\partial B_{\mathrm{VIEW}}}{\partial R} \left[\frac{1 + \tan^2\theta_{\mathrm{B}}}{\cos\theta_{\mathrm{V}} + \sin\theta_{\mathrm{V}}(\tan\theta_{\mathrm{B}})} \right] \quad . \end{aligned} \tag{7}$$

IV. APPLICATION TO DIII–D DATA

Each of the terms in Eq. (7) can be quickly evaluated within an IDL [10] routine to provide a local value for the current distribution. The only inputs required are the analyzed B_{VIEW} profile, the spatial calibration data, and an EFIT eqdisk (output file) to determine the value of $\tan\theta_B$ along the beam trajectory. Examples are shown in Fig. 3 for L–mode and QH–mode phases of DIII–D shot 115099. The QH–mode phase is of particular interest because it possesses an appreciable edge pressure gradient in the absence of ELMs. In this case one might expect to see an edge current structure due to the edge pressure gradient (bootstrap current effect) and the data tend to confirm this [7].

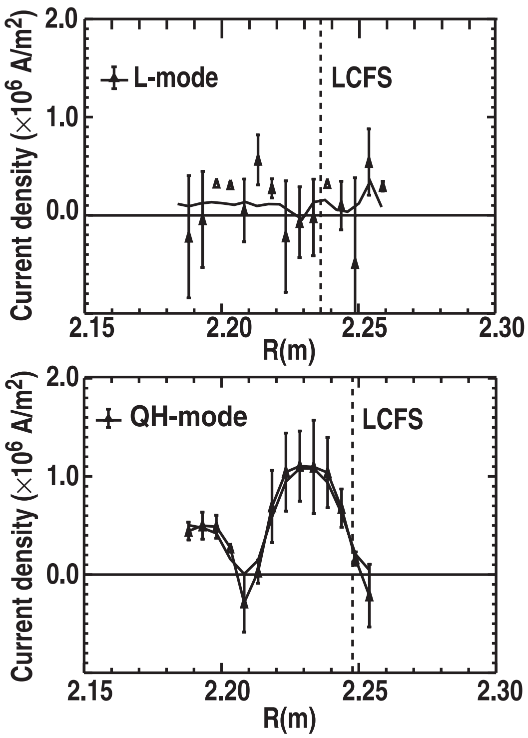

Fig. 3. Calculation of edge toroidal current density (solid line) using Eq. (11) for the L-mode and QH-mode phases of DIII-D shot 115099, using the measured B_{VIEW} from LIBEAM. The last closed flux surface (LCFS) as calculated by EFIT is shown by the dotted line. Adapted from Ref. [7].

V. DISCUSSION

The major contribution to the error bars on j(r) from this technique comes from the difficulty in taking an accurate derivative of the noisy B_{VIEW} data, as well as some uncertainty in the value of the derivative of $\tan\theta_B$ due to the large grid spacing in EFIT. We are examining the best way of reducing this error. One possibility is to fit the measured B_{VIEW} to a given functional form. The choice of form, while ad hoc, can be based on physics arguments to a certain point. For example, the edge pressure gradient is typically parameterized in terms of a modified hyperbolic tangent [11], and some variation of this may prove to be a good fitting function for B_{VIEW} in the future.

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