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F.L. HINTON and M.N. ROSENBLUTH*

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*ITER EDA, San Diego Co-Center

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Dynamics of Axisymmetric ExB and Poloidal Flows in Tokamaks

F.L. Hinton

General Atomics, P.O. Box 85608,
San Diego, California 92186-5608, U.S.A.

M.N. Rosenbluth

ITER EDA, San Diego Co-Center,
11025 North Torrey Pines Road, La Jolla, California 92037, U.S.A.

Abstract

As a result of turbulence and finite Larmor radius effects, random radial currents are present in a tokamak plasma, and these drive sheared axisymmetric poloidal flows. We model these currents with a noise source with given statistical properties and calculate the linear kinetic response to this source. Without collisions, there is no long term damping of these flows; when collisions are included, poloidal flows are damped. The mean square potential associated with these flows is given in terms of the linear response function we calculate and a model correlation function for the current source. Without collisions, the mean square $E \times B$ flow increases linearly with time, but with collisions, it reaches a steady state. In the long correlation time limit, the collisionless residual flows are important in determining the mean square $E \times B$ flow.

I. Introduction

Sheared poloidal flows are driven by turbulence in tokamaks and are known to strongly influence the level of the turbulence and transport [1–4]. Both gyrofluid [1] and gyrokinetic [4] codes show that, with these flows artificially suppressed, the turbulent heat flux is increased by a factor of 5 to 10. The damping of these self-generated flows is therefore an important issue. In previous work [5], we have shown that linear collisionless kinetic mechanisms do not damp these flows; poloidal rotation started initially asymptoted to a finite value at large times. The predicted value was verified by gyrokinetic simulation [6], while gyrofluid models incorrectly predicted a total collisionless decay of the rotation [7]. We have suggested [5] that this gyrofluid overdamping may lead to overestimation of turbulent diffusion by gyrofluid codes. The gyrofluid equations were derived [1] from the gyrokinetic equation by taking moments and closing the moment hierarchy by approximations which model kinetic effects. These include linear damping terms which are correct for the nonaxisymmetric modes in the turbulence, but are incorrect for the axisymmetric driven poloidal flows.

We have extended our treatment to include the effects of collisions; in this case, the poloidal rotation does decay to zero. The radial wavelengths of the flows we calculate are assumed to be small compared with the equilibrium scales, and we do not consider the equilibrium flows due to the equilibrium radial electric field and pressure and temperature gradients [8].

Our approach to this problem is as follows. We divide the electrostatic potential fluctuations into two groups, (i) the nonaxisymmetric fluctuations which include those which are linearly unstable, which cause the transport, and (ii) the axisymmetric fluctuations which are nonlinearly driven by the first group, and which act to regulate them. Although a self consistent nonlinear theory of both groups would be desirable, it is beyond the scope of this paper. Here, we treat the axisymmetric fluctuations in detail, with the effect of the nonaxisymmetric fluctuations modeled as a noise source with known properties. We determine the linear kinetic response of the plasma to such a source. By considering a closely related initial value problem, we compare our results directly with predictions of gyrofluid and gyrokinetic codes.

The mean square potential associated with these flows is given in terms of the linear response function we calculate and a model current source correlation function. Without collisions, the mean square $E \times B$ flow increases linearly with time. With collisions, the mean square flow reaches a steady state. In the long correlation time limit, with reasonable assumptions about the numerical values of parameters, the collisionless residual flows are shown to be important in determining the mean square $E \times B$ flow. This implies that the effect of driven flow shear in suppressing and regulating the turbulence is not correctly calculated by the gyrofluid equations, which assume zero collisionless residual flows. Since the level of turbulence is known to be strongly regulated by these self-driven flows, our results imply that this level should be lower than determined by the solution of gyrofluid equations, which incorrectly predict long term collisionless damping of the flows.

2. Drift Kinetic Equation

The fluctuating $E \times B$ flows are determined by the potential. We consider potential fluctuations which are small but rapidly varying across the magnetic field, with wavelengths much smaller than the equilibrium gradient lengths. The rapid spatial variation of the potential across the magnetic field is assumed to be contained in an eikonal factor, $\phi(\vec{x},t) = \phi_k \exp[i\mathcal{S}(\vec{x}_\perp)]$, where ϕ_k may depend on position along a field line. The wave vector label is defined by $\vec{k}_\perp = \nabla \mathcal{S}$. For axisymmetric perturbations, $\vec{k}_\perp = \mathcal{S}'(\psi)\nabla\psi$ where ψ is the poloidal flux function, used as the radial coordinate. We assume $k_\perp a_i << 1$, where $a_i = (T_i / m_i)^{1/2} / \Omega_i$ is a thermal ion gyroradius, with $\Omega_i = eB / m_i c$. The potential is determined by quasineutrality:

$$- n_0 \frac{e}{T_i} k_\perp^2 a_i^2 \phi_k + n_{ik} = n_{ek} \quad . \tag{1}$$

The first term is the polarization density, the second term is the ion guiding center density, $n_{ik} = \int d^3v\, f_{ik}$ and the third term is the electron density, $n_{ek} = \int d^3v\, f_{ek}$.

The ion guiding center distribution function satisfies the drift kinetic equation:

$$\frac{\partial f_{ik}}{\partial t} + v_\parallel \hat{b}\cdot\nabla f_{ik} + i\omega_D f_{ik} + \frac{e}{T_i} F_0 \left(v_\parallel \hat{b}\cdot\nabla\phi_k + i\omega_D \phi_k \right) - C_{ii} f_{ik} = S_{ik} \quad , \tag{2}$$

where $\hat{b} = \vec{B} / B$, and the drift frequency is $\omega_D = \vec{k}_\perp \cdot \vec{v}_d$, with $\vec{v}_d$ the guiding center drift, or

$$\omega_D = \vec{v}_d \cdot \nabla\psi\, \mathcal{S}'(\psi) = v_\parallel \hat{b}\cdot\nabla Q \quad , \tag{3}$$

where $Q = I\mathcal{S}' v_\parallel / \Omega$, with $v_\parallel = [2(\mathcal{E} - \mu B)]^{1/2}$ and $I = RB_\phi$. Also F_0 is the equilibrium Maxwellian and C_{ii} is the linearized ion-ion collision operator. The independent velocity variables used are the energy $\mathcal{E} = v^2 / 2$, magnetic moment $\mu = v_\perp^2 / 2B$, and the sign of the parallel velocity, $\sigma = \mathrm{sgn}(v_\parallel)$. The velocity integration needed in Eq.(1) is defined in terms of these by $\int d^3v = \pi \int v^2 dv\, \Sigma_\sigma \int B d\lambda / |\xi|$, with $\lambda = \mu / \mathcal{E}$ and $\xi = (1 - \lambda B)^{1/2}$.

The source term S_{ik} comes from the $E \times B$ nonlinearity in the gyrokinetic equation [9]:

$$S_{ik} = \frac{c}{B}\sum_{\vec{k}'}\left(\hat{b}\cdot\vec{k}'\times\vec{k}''\right) J_0\left(k'_\perp\rho\right) J_0\left(k''_\perp\rho\right)\phi_{k'} f_{ik''} \ , \tag{4}$$

where $\vec{k}'' = \vec{k} - \vec{k}'$, J_0 is a Bessel function, and $\rho = v_\perp / \Omega_i$. Axisymmetric perturbations make no contribution to S_{ik} (since $\vec{k}'$ and $\vec{k}''$ would be in the same direction). Therefore it is reasonable to take S_{ik} as given, in solving for the axisymmetric potential ϕ_k.

The drift kinetic equation for electrons is similar, but the drift terms in Eq.(2) can be neglected, and $J_0 \simeq 1$ in Eq.(4).

We consider the solution of Eq.(2) for times longer than an ion bounce time, in order to determine the evolution of the potential after the Geodesic Acoustic Modes [10] (GAMs) have damped. We take $\hat{b}\cdot\nabla\phi_k = 0$, assuming also that the ion acoustic modes have Landau damped. It is sufficient to obtain f_{ik} to first order in ion banana width divided by radial wavelength, in order to find the ion density to second order. The first order equation is obtained from Eq. (2) by neglecting the $i\omega_D f_{ik}$ term. Expanding in $1/(\omega_b t)$ and using Eq.(3), we obtain $f_{ik} = -(e/T_i)F_0\, iQ\phi_k + h_k$, where $\hat{b}\cdot\nabla h_k = 0$, and h_k satisfies the bounce averaged drift kinetic equation:

$$\frac{\partial h_k}{\partial t} - \overline{\left(C_{ii}h_k\right)} = \frac{e}{T_i}\, F_0\, i\overline{Q}\, \frac{\partial \phi_k}{\partial t} + \overline{S_{ik}} \ , \tag{5}$$

where the bar indicates a bounce average, defined by $\overline{A} = \oint (dl/v_\|)A / \oint (dl/v_\|)$ with $dl = Bdl_p / B_p$. For trapped particles, the integral goes over a closed orbit, while for untrapped particles, it goes once around the poloidal circumference. The potential is determined by the magnetic surface average of Eq. (1), where the magnetic surface average is defined by $\langle A\rangle = \oint (d\ell / B_p)A / \oint (d\ell / B_p)$.

3. Dielectric Susceptibility and Response Kernel

The Laplace transform of Eq. (5) without the source term, suitably normalized, is

$$G - \frac{1}{p}\,\overline{\left(C_{ii}G\right)} = \overline{\left(\frac{v_\|}{B}\right)}F_0 \ . \tag{6}$$

We define the dielectric susceptibility $\tilde{\chi}_p(p)$ in terms of the solution of this equation:

$$\tilde{\chi}_k(p) = \left\langle k_\perp^2 a_i^2 \right\rangle + \frac{(m_i c I S')^2}{n_0 e^2}\left\langle \int d^3v\, \frac{v_\|}{B}\left[\frac{v_\|}{B}\, F_0 - G\right]\right\rangle \ . \tag{7}$$

The two terms in the integral come from the guiding center drift contribution to the radial current, to second order in the ion banana width.

Quasineutrality is used to solve for the Laplace transformed potential:

$$n_0 \frac{e^2}{T_i} \tilde{\phi}_k(p) = \frac{\langle \tilde{\rho}_k^{(S)} \rangle}{\tilde{\chi}_k(p)} \ , \tag{8}$$

where the Laplace transformed source charge density is obtained from the gyrokinetic source terms:

$$p\left\langle \tilde{\rho}_k^{(S)} \right\rangle = -i\left\langle \vec{k} \cdot \tilde{\vec{j}}_k^{(S)} \right\rangle = e\left\langle \int d^3v\, \tilde{S}_{ik} \right\rangle - e\left\langle \int d^3v\, \tilde{S}_{ek} \right\rangle \ . \tag{9}$$

The time dependent potential can then be written as

$$\phi_k(t) = \int_0^t dt'\, \mathcal{K}_k(t-t')\, R_k(t') \ , \tag{10}$$

where R_k is a suitably normalized source current,

$$R_k(t) = \frac{-i\langle \vec{k} \cdot \vec{j}_k^{(S)} \rangle}{n_0\,(e^2 / T_i)\langle k_\perp^2 a_i^2 \rangle} \ , \tag{11}$$

and $\mathcal{K}_k$ is the response kernel, defined in terms of the susceptibility by the Laplace inversion integral

$$\mathcal{K}_k(t) \Big/ \left\langle k_\perp^2 a_i^2 \right\rangle = \frac{1}{2\pi i} \int \frac{dp\, e^{pt}}{p\, \tilde{\chi}_k(p)} \ . \tag{12}$$

With this definition, $\mathcal{K}_k(t)$ is simply related to the solution of an initial value problem, as follows. If we use a source $S_{ik}(t) = f_{ik}(t=0)\,\delta(t)$, then we will obtain the solution of an initial value problem, with $f_{ik}(t=0)$ the kinetic initial condition. We assume that an initial charge is established in a time larger than a gyroperiod, although much smaller than a bounce time. The equivalent source charge density is $\langle \rho_k^{(S)} \rangle = e\langle \delta n_k(0) \rangle$, where $\delta n_k(0)$ is the initial ion density perturbation. This is equivalent to an external charged grid which is placed in the plasma at the initial time. Through polarization current, quasineutrality is satisfied during the time this density is established, so we have an initial potential perturbation given by $n_0\,(e / T_i)\langle k_\perp^2 a_i^2 \rangle \phi_k(0) = \langle \delta n_k(0) \rangle$. Therefore the Laplace transformed source charge density for this problem is $\langle \tilde{\rho}_k^{(S)} \rangle = n_0\,(e^2 / T_i)\langle k_\perp^2 a_i^2 \rangle \phi_k(0) / p$. It then follows that the time dependence of the potential is given by the response kernel:

$$\phi_k(t) = \phi_k(0)\, \mathcal{K}_k(t) \tag{13}$$

4. The Collisionless Limit

Neglecting collisions in Eq. (6), the solution is $G = \overline{(v_\parallel / B)}\, F_0$ and after substituting this into Eq. (7), the susceptibility is given by

$$\tilde{\chi}_k(p) = \left\langle k_\perp^2 a_i^2 \right\rangle + \frac{1}{n_0}\left\langle \int d^3v\ F_0 Q\left(Q - \overline{Q}\right)\right\rangle \ , \tag{14}$$

where $Q = IS' v_\parallel / \Omega$. In the special case of large aspect ratio circular geometry, this becomes [5]

$$\tilde{\chi}_k \simeq \left\langle k_\perp^2 a_i^2 \right\rangle \left(1 + 1.6\, q^2 / \varepsilon^{1/2}\right) \ . \tag{15}$$

The collisionless residual value of the potential is therefore

$$\phi_k(t)/\phi_k(0) = K_k(t) = \frac{1}{1 + 1.6\, q^2 / \varepsilon^{1/2}} \ . \tag{16}$$

This result agrees with gyrokinetic simulations [6] but not with gyrofluid simulations [7], which give a zero residual potential after many bounce times.

5. Collisional Decay of Poloidal Flow

We now consider the initial value problem, including collisions. We must distinguish the poloidal component of the flow velocity from the $E \times B$ flow. The flow velocity is given by the sum of parallel and $E \times B$ contributions (the perturbed pressure gradient contributes a flow which is smaller by a factor of $k_\perp^2 a_i^2$ and is neglected). The poloidal flow is determined by a moment of G.

We use a variational principle for Eq. (6): $\delta V = 0$, where $V = N/|A|^2$ with

$$N = \left\langle \int d^3v\, \frac{G^*}{F_0}\left[G - \frac{1}{p} C_{ii} G \right]\right\rangle \ , \tag{17}$$

and

$$A = \left\langle \int d^3v\, \frac{v_\parallel}{B} G \right\rangle \ . \tag{18}$$

With the constraint $\hat{b} \cdot \nabla G = 0$, this is equivalent to Eq. (6). The susceptibility is given in terms of A by Eq. (12) or, using $B_p = |\nabla \psi| / R$,

$$\tilde{\chi}_k(p) = \left\langle k_\perp^2 a_{ip}^2 \right\rangle \left[1 + \frac{m_i I^2}{n_0 T_i \langle R^2 \rangle} A \right] , \tag{19}$$

where $a_{ip} = (T_i / m_i)^{1/2} / \Omega_{ip}$, the poloidal ion gyroradius, with $\Omega_{ip} = eB_p / m_i c$. In Eq. (17) we have assumed that p is real, so the result will have to be analytically continued for use in Eq. (12). We consider separately the limits of large and small $|p|\tau_{ii}$, corresponding to short and long times compared with the macroscopic ion-ion collision time τ_{ii}, defined in Ref. 11.

6. Short Time Collisional Decay

To find the time dependence of the potential and the poloidal flow for short times, we need the solution of Eq. (6) for $|p|\tau_{ii} >> 1$. The collisionless solution gives the result to lowest order, except near the trapping boundary. Since this is defined by the trapping value of the pitch angle, the collision operator can be approximated by pitch angle scattering only [12]. For large aspect ratio circular geometry, with $B = B_0 / (1 + \varepsilon \cos\theta)$, the bounce averages in Eq. (6) can be expressed in terms of elliptic integrals $K(k^{1/2})$ and $E(k^{1/2})$, where $k = 2\varepsilon / (1 + \varepsilon - \lambda B_0)$.

We solve this equation using the variational principle. Collisions are important in a boundary layer near $k = 1$. Following Ref. 12, we use a trial function

$$G = \frac{\pi}{2} \frac{v F_0}{B_0} \frac{(2\varepsilon / k)^{1/2}}{K(k^{1/2})} \left\{ 1 - \exp\left[-\gamma (1-k)/k \right] \right\} , \tag{20}$$

which satisfies the boundary condition that $G \to 0$ at the trapping boundary $k = 1$. Using the variational principle, we find $\gamma = (2\varepsilon p \Lambda / \nu_{ii})^{1/2}$ where $\Lambda = \ln[16(\varepsilon p / \nu_{ii})^{1/2}]$, and $\nu_{ii}(v)$ is the ion-ion collision frequency. The susceptibility can be obtained from the variational quantity V. The result is

$$\tilde{\chi}_k(p) = k_\perp^2 a_i^2 \left\{ 1 + \left[1.6 + 3\pi (\bar{\nu} / \varepsilon p)^{1/2} \Lambda^{-3/2} \right] q^2 / \varepsilon^{1/2} \right\} , \tag{21}$$

where $\bar{\nu}$ is an average ion-ion collision frequency, $\bar{\nu} = 0.61 / \tau_{ii}$ and Λ has been evaluated at $\nu_{ii} = \bar{\nu}$.

By deforming the integration path in Eq. (12) to pass around the branch line along the negative real p axis, we obtain the response kernel and the time dependence of the potential for short times:

$$\phi_k(t)/\phi_k(0) = K_k(t) \simeq \frac{1}{\alpha} \exp\left(\frac{\beta^2}{\alpha^2} t \right) \operatorname{erfc}\left(\frac{\beta^2}{\alpha^2} t^{1/2} \right) , \tag{22}$$

where erfc is the complementary error function,

$$\alpha = 1 + 1.6\, q / \varepsilon^{1/2} \ , \qquad \beta = 3\pi q^2\, \bar{\nu}^{1/2} \big/ \left(\varepsilon \Lambda^{3/2}\right) \ , \tag{23}$$

and $\Lambda \simeq \ln[16(\varepsilon / \bar{\nu} t)^{1/2}]$.

The poloidal flow for short times is given by

$$u_{pk}(t) \simeq i k_\perp \frac{c}{B}\, \phi_k(t) \ . \tag{24}$$

The initial decay of the poloidal flow is thus more rapid than a simple exponential decay because of the collisional boundary layer, in agreement with Ref. 13.

7. Long Time Collisional Decay

For longer times, we need the solution of Eq. (6) for $|p|\,\tau_{ii} \ll 1$, so the lowest order equation is obtained by neglecting the first term, *i.e.*,

$$\overline{\left(C_{ii} G\right)} = -p\, \overline{\left(\frac{v_\parallel}{B}\right)} F_0 \ . \tag{25}$$

where $\hat{b} \cdot \nabla G = 0$. We use the variational principle, with the approximation

$$N \simeq -\frac{1}{p} \left\langle \int d^3 v\, \frac{G^*}{F_0}\, C_{ii} G \right\rangle \ . \tag{26}$$

We again consider the case of large aspect ratio circular geometry. We make use of the fact that the trapped particle region of velocity space is narrow, so the dominant collision process is pitch-angle scattering. Using the method of Ref. 14, we find the dominant contribution to the variational expression V, and after carrying out the integrals, obtain the susceptibility:

$$\tilde{\chi}_k(p) = k_\perp^2 a_{ip}^2 \left(1 + p\tau_d\right) \ , \tag{27}$$

where $\tau_d = 0.91\, \tau_{ii} / \varepsilon^{1/2}$. The response kernel is then given by using Eq. (12).

In addition to the bulk ion response, there is a contribution from high energy ions which have small collision frequencies. Although such ions are few in number, their contribution to the poloidal flow decays more slowly than exponential and will dominate for sufficiently long times. For these, we use a collision term in Eq. (6) appropriate for tail ions with speeds v satisfying $v \gg v_i$, where v_i is the ion thermal speed. We also assume the speed is small enough that electron drag can be neglected,

$\nu << (m_i / m_e)^{1/6} \nu_i$. We further neglect energy scattering because it is smaller than the ion drag term by v_i^2 / v^2. We expand in $\varepsilon^{1/2}$ keeping only the zeroth order term. The distribution function is then easily obtained, and the integrals carried out to give the response kernel.

After combining this with the bulk ion contribution, we obtain

$$\phi_k(t)/\phi_k(0) \simeq K_k^{(l)}(t) \ , \tag{28}$$

where the long time collisional response is

$$K_k^{(l)}(t) = \frac{B_p^2}{B^2} \left\{ 1 - \exp\left(- t / \tau_d\right) + a_t \left(3\nu_0 t\right)^{5/9} \exp\left[- \left(3\nu_0 t\right)^{2/3} \right] \right\} \ , \tag{29}$$

where $a_t \simeq 0.98$ and $\nu_0 = 1.9 / \tau_{ii}$. The poloidal flow is proportional to $K_k^{(l)}(t) - B_p^2 / B^2$ and is damped. The damping is slower than exponential for long times because of ions in the tail of the distribution function.

The potential approaches a nonzero value for long times:

$$\phi_k(t)/\phi_k(0) \to B_p^2 \big/ B^2 \ , \tag{30}$$

for $t \to \infty$. This steady state value can be obtained from toroidal angular momentum conservation, assuming the initial flow was perpendicular and the poloidal flow has damped. This is because collisional damping of the toroidal flow is a higher order process [15].

8. Complete Time Dependence

We now combine the results for the short and long time collisional decay by including in the Laplace inversion integral all of the results we have obtained for the susceptibility in different parts of the complex p plane. We include also a contribution from the fast GAM oscillations, defined so that $K_k(0) = 1$, and obtain

$$K_k(t) = K_k^{(s)}(t) + K_k^{(l)}(t) \ , \tag{31}$$

where $K_k^{(s)}(t)$ is the short time response

$$K_k^{(s)}(t) = \left(1 - \frac{1}{\alpha}\right) \exp\left(- \nu_f t\right) \cos\left(\omega_f t\right) + \frac{1}{\alpha} \exp\left(\frac{\beta^2}{\alpha^2} t\right) \operatorname{erfc}\left(\frac{\beta}{\alpha} t^{1/2}\right) \ , \tag{32}$$

where $\omega_f = v_i / R$ and $\nu_f = \omega_f \exp(-q^2)$ are estimates of the frequency and damping rate of the GAMs, and α and β are defined in Eq. (23). The ratio of the fast oscillation frequency to the macroscopic collision rate is $\omega_f \tau_{ii} = q / (\varepsilon^{2/3} \nu_{i*})$, where q is the tokamak safety factor, ε is the inverse aspect ratio, and ν_{i*} is the neoclassical ion collisionality [16]. This response function is plotted in Fig. 1, along with the collisional part (without the GAM contribution). The parameter values used are $\varepsilon = 0.18$, $q = 1.4$, and $\nu_{i*} = 0.04$.

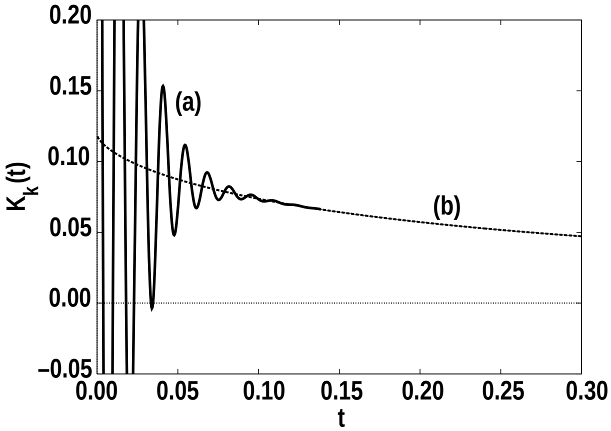

Fig. 1. (a) The short time response function $K_k^{(s)}$ versus time, normalized to τ_{ii}. (b) The collisional part of $K_k^{(s)}$ (*i.e.*, without the GAM contribution).

Since the decay of the poloidal flow is not a simple exponential, we define the total decay time τ by

$$\tau = \int_0^\infty dt \, \frac{u_{pk}(t)}{u_{pk}(0)} \quad . \tag{33}$$

The integral is equal to the Laplace transform at $p = 0$, and we find $\tau \simeq 1.5\, \tau_{ii} \varepsilon$. For small ε, this is much smaller than the time constant τ_d found for the long time decay. Most of the decay occurs in the faster than exponential boundary layer decay phase, which lasts a time of order $\tau_{ii}\varepsilon$. The numerical value of τ obtained here agrees with Ref. 13 for the value of ε used there.

9. Response to a Noise Source

We assume that the source current is random and statistically stationary. Using Eq. (10), the mean square potential can be written as

$$\left\langle\left\langle \left|\phi_k(t)\right|^2 \right\rangle\right\rangle = \left\langle\left\langle \left|R_k\right|^2 \right\rangle\right\rangle \int_0^t dt_1 \int_0^t dt_2 \, K_k(t_1) K_k(t_2) C(t_1 - t_2) \ , \tag{34}$$

where the double brackets indicate a statistical average. The source current correlation function $C(\tau)$ is defined by

$$\left\langle\left\langle R_k^*(t_1)\,R_k(t_2)\right\rangle\right\rangle = \left\langle\left\langle |R_k|^2 \right\rangle\right\rangle C(t_1 - t_2) \ . \tag{35}$$

We shall model the noise source correlation function by $C(\tau) = \exp(-\tau^2 / \tau_c^2)$ where τ_c is the correlation time.

Neglecting collisions, we have

$$\frac{d}{dt}\left\langle\left\langle |\phi_k(t)|^2 \right\rangle\right\rangle \to 2\left\langle\left\langle |R_k|^2 \right\rangle\right\rangle |K_k(\infty)|^2 \int_0^\infty d\tau\, C(\tau) \ , \tag{36}$$

for $t \to \infty$, which is a nonzero constant. Therefore the mean square potential increases linearly with time for long times, *i.e.*, the $E \times B$ flow executes a random walk.

Because of collisions, the response kernel decreases, starting from the collisionless residual value, Eq. (16), to the value B_p^2 / B^2, which is generally much less than unity. For the purpose of discussing the response to a noise source, we neglect this long time residual value. Then the response kernel is zero for long times, and the mean square potential reaches a steady state for $t \to \infty$.

In order to facilitate the evaluation of Eq. (34), we replace the error function expression in Eqs. (31) and (32) with an exponential and neglect the second term in Eq. (31). The response function is approximated by

$$K_k(t) = (1 - A_R)\, e^{-\nu_f t} \cos\omega_f t + A_R\, e^{-\nu_d t} \ , \tag{37}$$

where $A_R = 1/(1 + 1.6 q^2 / \varepsilon^{1/2})$ is the collisionless residual value, and $\nu_d = 0.67 / (\tau_{ii}\varepsilon)$ using the total decay time defined in Section 8.

Gyrofluid codes [1] compute the fast time response approximately correctly, but incorrectly give a long time residual potential of zero. A condition for the validity of these codes is therefore that the residual value A_R makes a negligible contribution to the steady state mean square potential. We evaluate Eq.(34) for $t \to \infty$, assuming the correlation time of the turbulence is much longer than the GAM oscillation period, $\omega_f \tau_c \gg 1$, and that the GAMs are weakly damped, $\omega_f \gg \nu_f$. In order to obtain a simple estimate, we also take $\nu_d / \nu_f \sim \nu_d \tau_c \sim 1$ and obtain

$$\left\langle\left\langle |\phi_k(\infty)|^2 \right\rangle\right\rangle \Big/ \left\langle\left\langle |R_k|^2 \right\rangle\right\rangle \sim \frac{\nu_f^2}{\omega_f^4} + A_R^2\,\frac{\tau_c}{\nu_d} \ . \tag{38}$$

Using $\nu_d / \omega_f = (\varepsilon^{1/2} / q)\, \nu_{i*}$, the condition for neglecting the residual flow becomes

$$\left(\frac{1}{\nu_f \tau_c}\right)\left(\frac{\nu_f}{\omega_f}\right)^3 \left(\frac{\varepsilon^{1/2}}{q}\right)\left(1+1.6\,q^2/\varepsilon^{1/2}\right)^2 \nu_{i*} >> 1 \ . \tag{39}$$

The small factor $(\nu_f / \omega_f)^3$ occurs because the rapidly oscillating terms in the response function lead to near cancellations in the integrals in Eq. (34). For typical interesting values of q (>1) and ε (not extremely small), this is not satisfied except for very large values of ν_{i*}, *i.e.*, not in the banana regime, where $\nu_{i*} < 1$, as we have assumed. The gyrofluid closures are therefore not justified in the banana regime, for typical interesting values of q and ε.

11. Conclusions

We have extended our previous work [5] to include ion-ion collisions. We have formulated the problem of calculating the driven flows as the linear kinetic response to a noise source with given statistical properties. We have shown the relation of this formulation to an initial value problem for the potential. When this initial value problem is considered, we find that the plasma response occurs in five distinct phases, involving different physical processes, as follows.

(1) For times longer than a few gyroperiods, the classical polarization shields the radial electric field. (We have assumed that this has already occurred.)

(2) For times longer than a few ion bounce times, or after Geodesic Acoustic Modes have damped, the potential has a nonzero residual value given by Eq. (16).

(3) For times of the order of $\varepsilon\tau_{ii}$, where τ_{ii} is the macroscopic ion-ion collision time, the potential and poloidal flow damp more rapidly than exponentially, as collisions first have an effect in a boundary layer. Most of the poloidal rotation decay occurs in this phase. The results are given by Eqs. (22) and (24).

(4) For times comparable to $\tau_{ii} / \varepsilon^{1/2}$, the potential approaches a nonzero steady state value and the poloidal rotation decays approximately exponentially with the time constant $\tau_d = 0.91\,\tau_{ii} / \varepsilon^{1/2}$.

(5) For times much longer than τ_{ii}, the damping of poloidal rotation is due to energetic ions with small collision rates, resulting in a slow non-exponential decay, given by the last term in Eq. (29).

We have shown that a condition for the validity of the gyrofluid codes [1] is not satisfied for typical parameters. We conclude that, at least near marginal stability where nonlinear damping of the flows should be negligible, and in the sufficiently collisionless regimes of interest (in the banana regime), the level of $E \times B$ flows should be larger than predictions made by gyrofluid simulations which entail linear collisionless damping. Consequently, the turbulence level and transport should be smaller than the gyrofluid predictions.

A rough measure of the effective shear suppression of turbulence is given by Eq. (38). This is approximately proportional to $1/\nu_{i*}$, which increases with ion temperature. Thus, we may expect better confinement at higher temperatures because of the larger mean square poloidal flows and the increased shear suppression of turbulence.

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