ARIES-CS PROJECT: DETERMINE FEASIBILITY OF COMPACT STELLARATOR CONCEPT AS A FUSION REACTOR

- Evaluate engineering options
  - Coil options
  - Auxiliary systems
  - Maintenance options
  - Power extraction and breeding

- Evaluate physics options
  - Equilibrium configuration
  - Stability limits
  - Confinement
  - Plasma heating

[Develop an integrated engineering and physics design for a feasible reactor system]

- Two configurations considered:
  - A scaled-up three field period NCSX configuration
  - A two field period quasi-axisymmetric MHH2 configuration
  - Fixed boundary equilibria computed from VMEC
MODULAR COIL CONFIGURATIONS FOR THREE AND TWO FIELD PERIOD CONFIGURATIONS

- Scaled NCSX: Eighteen coil design
- MHH2: Eight moderately twisted modular coil design

(Courtesy: T.K. Mau and T. Wang)

- Good quasi-axisymmetry:
  - Asymmetric $B_{mn}$ terms are same order as a typical tokamak with ripple from toroidal coils or helical excursion of magnetic axis from MHD instability

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COMPACT STELLARATOR EQUILIBRIUM AND LINEAR IDEAL MHD STABILITY ANALYSIS

- Scaled-up NCSX three-field period equilibrium from VMEC:
  - Reference equilibrium:
    - $\langle \beta \rangle = 4.1\%$

<table>
<thead>
<tr>
<th>V (m$^3$)</th>
<th>R(m)</th>
<th>a(m)</th>
<th>A</th>
<th>$&lt;B&gt;(T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>556.5</td>
<td>8.25</td>
<td>1.85</td>
<td>4.47</td>
<td>4.47</td>
</tr>
</tbody>
</table>

- MHH2 two-field period equilibrium from VMEC:
  - Reference equilibrium:
    - $\langle \beta \rangle = 4.52\%$

- Sequence of equilibria with increasing $\beta$ by uniformly scaling pressure profile:
  - Volume, Average major and minor radius and vacuum field held fixed
  - Shape of the outer plasma boundary fixed: no coils specified
  - $\psi$ profile adjusted to force average current density to vanish as $\beta$ increases
    - Bootstrap current not self consistently calculated
  - Magnetic axis ($R_m(0)$): major radius of magnetic axis at a single toroidal plane) shifts outward as $\beta$ increases

- Scaled NCSX sequence

<table>
<thead>
<tr>
<th>$&lt;\beta&gt;$ (%)</th>
<th>4.1</th>
<th>4.9</th>
<th>5.7</th>
<th>7.0</th>
<th>8.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_m(0)$ (m)</td>
<td>9.20</td>
<td>9.21</td>
<td>9.22</td>
<td>9.24</td>
<td>9.26</td>
</tr>
</tbody>
</table>

- MHH2 sequence

<table>
<thead>
<tr>
<th>$&lt;\beta&gt;$ (%)</th>
<th>4.52</th>
<th>5.71</th>
<th>6.92</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_m(0)$ (m)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
MAIN SHAPE DIFFERENCE BETWEEN TWO-FIELD PERIOD MHH2 AND THREE-FIELD PERIOD NCSX EQUILIBRIA IS
LOW ORDER SHAPING NEAR $N_{fp} \phi \sim 180!$
ROTATIONAL TRANSFORM PROFILES ARE VERY DIFFERENT BETWEEN SCALED NCSX AND MHH2

EQUILIBRIA TESTED FOR IDEAL STABILITY AT INTERMEDIATE WALL POSITIONS

- Stability results obtained for both reference equilibria using Terpsichore:
  - For the three and two field period stellarators mode coupling is confined to a single mode family represented by $n = 1$

- Stability results restricted to a range of moderately placed external conformal conducting walls
  - For Scaled NCSX: Wall between 1.7 and 2.7 times the minor radius
  - For MHH2: Wall inside 3.5 times the minor radius
  - Vacuum calculation fails outside these ranges

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**IDEAL KINK STABILITY LIMIT FOR BOTH CONFIGURATIONS: $\beta > 4\%$**

- Squared growth rate $\gamma^2$ versus equilibrium $\beta$:
  - $n=1$ mode family: Positive $\gamma^2 \Rightarrow$ stable / Negative $\gamma^2 \Rightarrow$ unstable
  - Normalized to toroidal Alfvén time

- Scaled NCSX:

  $\Rightarrow$ $\beta$ limit for a wall in the range 1.7 to 2.5 expected to be around 6% 

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Wall position</th>
<th>1.7</th>
<th>2.0</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1%</td>
<td></td>
<td>$+8.65 \times 10^{-5}$</td>
<td>$+8.65 \times 10^{-5}$</td>
<td>$+8.65 \times 10^{-5}$</td>
</tr>
<tr>
<td>5.7%</td>
<td></td>
<td>$+1.81 \times 10^{-4}$</td>
<td>$+1.81 \times 10^{-4}$</td>
<td>$+1.81 \times 10^{-4}$</td>
</tr>
</tbody>
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</tr>
</thead>
<tbody>
<tr>
<td>7.0%</td>
<td></td>
<td>$-8.66 \times 10^{-4}$</td>
<td>$-9.47 \times 10^{-3}$</td>
<td>$-1.62 \times 10^{-2}$</td>
</tr>
<tr>
<td>8.3%</td>
<td></td>
<td>$-7.25 \times 10^{2}$</td>
<td>$-7.75 \times 10^{2}$</td>
<td>$-8.06 \times 10^{2}$</td>
</tr>
</tbody>
</table>

- **MHH2:**
  - Marginally unstable growth rates
  $\Rightarrow$ $\beta$ limit for a wall in the range 1.0 to 2.5 expected to be 4% or more

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Wall position</th>
<th>1.7</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.25%</td>
<td></td>
<td>$-3.59 \times 10^{-5}$</td>
<td>$-3.87 \times 10^{-5}$</td>
<td>$-3.94 \times 10^{-5}$</td>
<td>$-3.92 \times 10^{-5}$</td>
</tr>
<tr>
<td>5.71%</td>
<td></td>
<td>$-1.49 \times 10^{-5}$</td>
<td>$-1.49 \times 10^{-5}$</td>
<td>$-1.49 \times 10^{-5}$</td>
<td>$-1.49 \times 10^{-5}$</td>
</tr>
<tr>
<td>6.92%</td>
<td></td>
<td>$-1.91 \times 10^{-5}$</td>
<td>$-1.91 \times 10^{-5}$</td>
<td>$-1.91 \times 10^{-5}$</td>
<td>$-1.91 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

- **Mercier criterion indicates stability up to $\beta = 5\%$:**

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RELEVANCE OF IDEAL MHD $\beta$ LIMITS IN STELLARATORS IS NOT WELL UNDERSTOOD

- Historically, tokamaks and stellarators designed using ideal MHD stability criteria:
  - Ideal localized Mercier and ballooning criteria plus global stability
- In tokamaks these limits are considered well understood:
  - Ideal MHD appears to predict not just tokamak stability limits but also growth rates and mode structures in many situations
  - Fast, global instabilities identified with disruptions and $b$ collapse
  - Localized and weakly growing instabilities identified with benign MHD activity: Edge Localized Modes (ELMs), Sawteeth, etc.
- Modern large stellarators however appear to violate these limits:
  - $\beta$ appears to be limited by a soft limit of degrading confinement:
    $\beta$ limits in the tokamak sense have not yet been observed
  - Predicted localized MHD limits are grossly violated in many cases:
    LHD and W7AS have exceeded predicted $\beta$ limits by a factor two
  - Global limits also appear to be exceeded in more recent experiments

But stellarators and tokamaks have the same underlying physics based on Maxwell’s Equations and Newtonian mechanics!
TOKAMAK STABILITY PROVIDES A CONTEXT FOR RESOLVING THE CONFLICT IN THE CASE OF GLOBAL MODES

- Local MHD stability limits appear to be irrelevant in Stellarators:
  - Localized modes predicted to be unstable for $\beta$ well below the global MHD limits should be stabilized by kinetic effects

- Tokamaks also routinely violate some MHD stability limits:
  - MHD limits are open to interpretation and are not always hard limits
  - MHD limits can be sensitive to details in the equilibrium

- There are also some important distinctions between tokamaks and stellarators that may produce superficially different behavior:
  - MHD theory in both, assumes the existence of nested flux surfaces:

  $\Rightarrow$ In tokamaks this is normally an accurate assumption
  In stellarators surfaces may not exist! They may exist but be non-nested!

Resolution requires testing predictions using discharge equilibria
  $\Rightarrow$ Detailed measurements of stellarator $\phi$ profiles are needed

Compact Stellarators are probably more tokamak like than conventional stellarators!
TOKAMAKS ALSO ROUTINELY VIOLATE SOME MHD STABILITY LIMITS

- The most well known example is the internal kink instability:
  - Tokamaks routinely operate with \( q < 1 \) ⇒ Sawteeth
- Tokamak ballooning modes can have consequences near ‘the \( \beta \) limit’ But:
  - Consequences are not always devastating ⇒ Soft \( \beta \) limit
- In H-mode Tokamaks also routinely reach intermediate \( n \) stability limits:
  - ELMs appear to be the result ⇒ Generally benign

STABILITY LIMITS DEPEND SENSITIVELY ON THE EQUILIBRIUM

- It is not normally sufficient to fit the equilibrium to just the global characteristics of tokamak discharges:
  - Stability depends quite sensitively on the details of both the current density (or safety factor) and pressure profiles
  - In stellarators the \( \imath \) profile is not normally measured in the discharge and may be different at finite \( \beta \)
EQUILIBRIUM CODES CAN PROVIDE SOME LIMITED GUARANTEES OF STABILITY

- Existence of equilibria can be considered as either an equilibrium or stability problem:
  - Unstable equilibria will evolve to a nearby lower energy state if physically possible

- Equilibrium codes can be considered stability codes:
  - An equilibrium computed under certain constraints must be stable unless those constraints can be avoided by a physically valid motion

- Different equilibrium codes guarantee varying degrees of stability:
  - VMEC: $\Rightarrow$ Equilibria should be stable to all topology preserving and profile preserving (i.e. fixed $p(\psi)$ and $j$) MHD instabilities
  - Free boundary direct equilibrium codes $\Rightarrow$ Equilibria should be stable to profile preserving (i.e. fixed $p(\psi)$ and $j$) MHD instabilities

- NSTAB code exploits this by searching for bifurcated 3-D equilibria:
  - Assumes nested flux surfaces but:
  - Resolves discontinuities accurately by using conservative difference equations $\Rightarrow$ Equilibria should be stable to profile preserving (i.e. fixed $p(\psi)$

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GENERAL ATOMICS
GUARANTEE OF STABILITY SUBJECT TO IMPORTANT CAVEATS

- Claim is that convergence to physically unstable equilibria is not possible unless constraints are imposed on the numerical procedure that prevent either:
  - Equilibrium states without specific symmetries (e.g., axisymmetric), or
  - Symmetry breaking perturbations away from force balance

  \[ \Rightarrow \text{Lack of convergence does not imply lack of stable equilibrium!} \]
  Only the converse is claimed: that lack of stability will prevent convergence unless constraints are imposed!

- Free boundary direct equilibrium codes assume \( p = \text{constant} \) for flux surfaces inside islands:
  - Pressure is a different function of flux in separate simply connected regions
    \[ \Rightarrow p \text{ is not a single valued function of } \psi \]
  - States with different prescriptions for the multiple values for \( p \) and \( j \) in different simply connected regions (islands etc.) are possible and may be physically accessible
  - The actual profiles will be determined by transport and the topology of the region

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LINEAR STABILITY PREDICTIONS WITH NESTED FLUX SURFACES CAN BE RELEVANT IF INTERPRETED PROPERLY

- Local stability criteria should probably be ignored:
  - There is little reason that infinite n should provide a physical limit
  - Finite n corrections appear to be large given the difference between the global code limits and the infinite n localized limits

- Global MHD stability must be tested using reconstructed equilibria:
  - Need to use the measured equilibrium profiles
  - May need to construct a non-nested flux surface equilibrium (with islands)
  - Flux surfaces might not even exist

- Nonlinear consequences are crucial in interpreting stability calculations:
  - Generally it might be expected that internal modes surrounded by a fairly robust and stable outer shell might be benign
  - If nested surfaces are not valid the stability problem can be reformulated in terms of finding nonlinearly stable equilibria

Require: program to decide when linear instability of nested flux surface equilibria result in benign nonlinear evolution to ‘nearby’ states by:
Direct comparison with experiments and nonlinear stability calculations
NSTAB EXPLOITS RELATION BETWEEN EQUILIBRIUM AND STABILITY CODE BY SEARCHING FOR BIFURCATED EQUILIBRIA

- NSTAB equilibrium and nonlinear stability code:
  - Computes fixed boundary 3-D equilibria inside a torus
  - Applies MHD variational principle to compute weak solutions of the conservation form of the dynamical equations
  - Solutions represented in terms of Clebsch Potentials
  - Spectral representation for dependence on poloidal and toroidal angles
  - Utilizes a finite difference scheme that captures discontinuities with unusually fine resolution

- Existence of discontinuities implies current sheet within nested flux surface approximation: \( \Rightarrow \) Resolved in reality by formation of islands

Nonlinear stability evaluated by employing a mountain pass theorem with the search for bifurcated equilibria

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NSTAB UTILIZES MHD EQUATIONS IN CONSERVATION FORM TO ENSURE HIGH ACCURACY FOR FORCE BALANCE

For linearly unstable global modes, the nonlinear consequences need to be examined on a case-by-case basis. Nonlinear stability can be analyzed by computing weak solutions for fixed boundary 3-D equilibria, with highly resolved discontinuities to effectively simulate current sheets and island chains. The key issue is to accurately resolve the discontinuities. Comparable problems in fluid dynamics are treated by finite difference schemes that capture shock waves and contact discontinuities by putting the equations of motion in conservation form.

The Maxwell stress tensor: \[ T = B B - (B^2/2 + p) I \]

enables us to put the MHD equations describing force balance in conservation form:

\[ \nabla \cdot T = 0, \nabla \cdot B = 0. \]

Finite difference equations that respect the conservation form have the advantage that when they are summed over a test volume they telescope into an approximate statement of force balance over the boundary. :

\[ \int T \cdot dS = 0 \]

In a simplified example using Burgers equation as a model problem:

\[ (\Psi_x^2)_x = \eta \Psi_{xxx} \]

subject to boundary conditions

\[ \Psi(-1) = \Psi(-1) = 0, \quad \Psi_x(-1) = 0 \]

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NSTAB DIFFERENCE EQUATIONS MAINTAIN TOTAL FORCE BALANCE BY RESPECTING CONSERVATION FORM

A conservative difference scheme:

\[(\Psi_{n+1} - \Psi_n)^2 - (\Psi_n - \Psi_{n-1})^2 = \eta (\Psi_{n+2} - 3\Psi_{n+1} + 3\Psi_n - \Psi_{n-1})\]

computes jumps across discontinuities correctly and therefore imposes force balance across the sharp boundary which occurs at \(x = 0\) in the limiting case where \(\eta = 0\) so that the solution reduces to \(\Psi = 1 - |x|\).

However, a non-conservative difference scheme with an additional diffusive term

\[(\Psi_{n+1} - \Psi_n)^2 - (\Psi_n - \Psi_{n-1})^2 = \eta (\Psi_{n+2} - 3\Psi_{n+1} + 3\Psi_n - \Psi_{n-1}) + \varepsilon (\Psi_{n+1} - 2\Psi_n + \Psi_{n-1})^2\]

diverges when \(\eta\) is much smaller than the mesh size \(\varepsilon\).

When applied to a simple one-dimensional example that models a reversed field pinch (RFP) in slab geometry depending on only one coordinate \(x\), where \(\Psi\) is the flux, \(\Psi_x\) is the principal component of the magnetic field, \(\Psi_{xx}\) is the current, and \(\varepsilon\) is an artificial resistivity, the slopes on the two sides of the limiting solution \((\Psi = 1 - |x|)\) characterize the magnetic field on opposite sides of a current sheet, which is like a chain of magnetic islands. Using a finite difference scheme that is not in conservation form, a poor solution is obtained for \(\Psi(x)\) where the two slopes are unequal.
POINCARE MAP OF FLUX SURFACES FOR LHD AT $\beta = 4\%$

- Magnetic axis shifted inward to $R = 3.6\ m$
  - Broad pressure profile: $p = p_0(1 - s^2)$
  - Magnetic surface ripple of equilibrium solution
  $\Rightarrow$ Probably marginally stable

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POINCARE MAP FOR DIII-D - LIKE EQUILIBRIUM AT $\beta = 5\%$

- **Nonaxisymmetric solution**

  - Broad pressure profile: $p = p_0(1 - s^{1.1})^{1.1}$
  
  - Rotational transform profile: $0.9 > \iota > 0.3$
    
    $(1.1 < q < 3.3)$

  - Large $m/n = 3/2$ mode in solution

  $\Rightarrow$ Large 3/2 island predicted in experiment

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POINCARE MAP FOR NCSX EQUILIBRIUM AT $\beta = 6\%$

- Broad pressure profile:
  \[ p = p_0 (1 - s^{1.2})^{1.8} \]

- Rotational transform profile: $0.40 < \iota < 0.70$

- Low order ballooning mode in solution

$\Rightarrow$ Probably at or exceeded $\beta$ limit

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POINCARE MAP FOR MHH2 EQUILIBRIUM AT $\beta = 6\%$

- Broad pressure profile:
  \[ p = p_0 (1 - s^{1.1})^{1.1} \]

- Rotational transform profile: $0.45 < \nu < 0.65$

- Low order ballooning mode in solution

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