Progress in the Peeling-Ballooning Model of ELMs: Toroidal Rotation and 3D Nonlinear Dynamics

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Motivation and Background

- ELMs and the edge pedestal are key fusion plasma issues
  - “Pedestal Height” controls core confinement and therefore fusion performance (Q)
  - ELM heat pulses impact plasma facing materials

Predicted Impact of Pedestal Height

Observed Impact of Pedestal Height (DIII-D)
**Background: Extending the Peeling-Ballooning Model**

- Peeling-Ballooning Model of ELMs - significant successes
  - ELMs caused by intermediate wavelength (n~3-30) MHD instabilities
    - Both current and pressure gradient driven
    - Complex dependencies on \( \nu^* \), shape etc due to bootstrap current and “2nd stability”
  - Successful comparisons to experiment both directly and in database studies
- Need to understand sources and transport to get profile shapes (“pedestal width”)
- Rotation and non-ideal effects to precisely characterize P-B limits, nonlinear dynamics for ELM size
Outline

• Toroidal Flow Shear
  – Conventional ballooning theory (1D)
  – How toroidal rotation complicates the theory (1D⇒2D)
  – Eigenvalue formulation and resolution of ‘discontinuity’
  – Impact on peeling-ballooning modes in the tokamak edge region

• Nonlinear ELM Simulations
  – General challenges
  – 2 fluid reduced Braginskii (BOUT) simulation results
    • Expected peeling-balloonning characteristics in linear phase
    • Fast burst, radially propagating filaments

• Summary and Future Work
Conventional ballooning mode theory

We seek solutions $\xi \sim e^{-in\phi}$ and consider large toroidal mode number, $n$.

the largest operators in the ideal MHD equations are then those related to field line bending:

$$B \cdot \nabla = \frac{B_\theta}{r} \left( \frac{\partial}{\partial \theta} - inq \right)$$

In order to balance terms, we must ensure $(B.\nabla)\xi \sim 1$, ie

$$\xi = \hat{\xi}(\psi,\theta)e^{-in[\phi-q(\psi)(\theta-\theta_0)]}$$

$$\frac{\partial \hat{\xi}}{\partial \theta} \sim 1$$

We can then write the full ideal MHD equations (schematically) in the form:

$$L\left( \frac{\partial}{\partial \theta} - inq, \frac{-i}{n} \frac{\partial}{\partial \psi}, \frac{\partial}{\partial t} \right) \hat{\xi} e^{-in[\phi-q(\theta-\theta_0)]} = 0$$

For large $n$ and taking solutions $\sim e^{\gamma t}$, we then derive the ballooning equation:

$$L\left( \frac{\partial}{\partial \theta}, q'(\theta-\theta_0), \gamma(\theta_0) \right) \hat{\xi} = 0$$

A 2nd order ODE, 1D

Higher order theory $\Rightarrow$ choose $\theta_0$ to maximize $\gamma(\theta_0)$
Incorporate toroidal flow shear

• We introduce a toroidal flow: \[ v = R^2 \Omega(\psi) \nabla \phi \]

  — introduce the ordering:

\[ R \Omega / C_s \sim n^{-1} \ll 1 \quad \frac{1}{q'} \frac{\partial (R \Omega / C_s)}{\partial \psi} \sim 1 \]

• The principle effect is a Doppler-shift of the mode frequency, so we have

\[ L \left( \frac{\partial}{\partial \theta} - inq, -i \frac{\partial}{n \partial \psi} ; \frac{\partial}{\partial t} - in \Omega(\psi) \right) \xi = 0 \]

• To remove rapid radial variations, we introduce a time-dependent eikonal:

\[ \hat{\xi}(\psi, \theta, \phi; t) = \hat{\xi}(\psi, \theta; t) e^{-in[\phi-q(\theta-\theta_0)+\Omega t]} = 0 \]

Cooper, PPCF 30, 1805 (1988)

• Then the “ballooning” equation with flow becomes:

\[ L \left( \frac{\partial}{\partial \theta}, q'(\theta - \theta_0 + \Omega t/q'), \frac{\partial}{\partial t} \right) \hat{\xi} = 0 \]

• The presence of time in the coefficients \( \Rightarrow \) we can no longer assume eigenmode solutions \( \sim e^{\nu t} \): this becomes a 2-D initial value system to solve
The weak flow shear limit: relation to conventional ballooning theory

Let us define $\tau = \Omega' t / q' - \theta_0$; $\partial / \partial t \rightarrow (\Omega' / q') \partial / \partial \tau$ and seek Floquet-type solutions:

$$\xi(\theta, t) = e^{\gamma \tau} \hat{\xi}(\theta - \tau, \tau)$$

We then have

$$L\left( \frac{\partial}{\partial \theta}, q'(\theta - \theta_0 + \Omega' t / q'), \frac{\partial}{\partial t} \right) \hat{\xi} = 0$$

For low flow shear, this has a separable solution of the form

$$\xi = A(\tau) F(\theta - \tau)$$

where $F$ satisfies the conventional ballooning equation with $\theta_0 \rightarrow \tau$ and eigenvalue $\gamma(\tau)$

The boundary condition that $A$ be periodic provides:

$$\gamma = \frac{1}{2\pi} \oint \gamma(\theta_0) d\theta_0$$

Waelbroeck and Chen Phys Fluids B3 601 (1991)
Flow shear and the Eigenmode Formalism

• Previous studies of the effect of flow shear on ballooning modes have been based on time-dependent eikonal analysis [eg 11, 12]
  — valid in the limit $n \to \infty$
  — Then, in the absence of flow we choose $\theta_0$ to maximize $\gamma(\theta_0)$
  — For infinitesimally small flow, we average $\gamma(\theta_0)$ over $\theta_0$
  — There is a discontinuity in the theory, which we would like to understand
  — Suggests that flow shear could in principle have a big effect on ballooning modes

• We would like to develop an eigenmode formalism for the effect of flow shear on ballooning modes for a number of reasons:
  — Working with an eigenmode formalism allows us to smoothly connect to the conventional ballooning modes as $\Omega' \to 0$ and understand this ‘discontinuity’
  — It allows us to calculate the radial eigenmode structure
  — Provides an eigenmode frequency
  — Enables consideration of finite $n$ corrections
  — Permits flow shear to be incorporated into ELITE (an eigenmode code)
    • Test impact on P-B modes in experimental equilibria
The $n \to \infty$ eigenmode growth rates agree with Miller et al; The radial mode width reduces with increasing flow shear.
Increasing $n$, growth rates tend to Miller result at lower $\Omega'$. 
\Rightarrow \text{for } n \to \infty, \text{ rapid change in } \gamma \text{ for infinitesimally small flow}

$s=1.0 \quad \alpha=1.7$

\text{Conventional Ballooning}
\[
\text{Max } \gamma(\theta_0) = 0.68
\]

\text{Rotation Ballooning}
\[
\frac{1}{2\pi} \int \gamma(\theta_0) d\theta_0 = 0.28
\]

\text{Discontinuity resolved, transition physics similar to [Waltz 98] case}
**ELITE** is a Highly Efficient 2D MHD Code for $n>\sim 5$

ELITE is a 2D eigenvalue code, based on ideal MHD (amenable to extensions):

- Generalization of ballooning theory:
  1) incorporate surface terms which drive peeling modes
  2) retain first two orders in $1/n$ (treats intermediate $n>\sim 5$)

- Makes use of poloidal harmonic localization for efficiency

- Successfully benchmarked against GATO, MISHKA and MARS

- Formalism and code extended to include leading order ($n\Omega \sim 1$, $\Omega' \sim 1$) sheared toroidal flow and compression
Flow Shear Effect on Growth Rates is Modest in Experimental Equilibria, Mode Structure Does Change

Rotation Shear on P-B Modes:
- Stabilizing near marginality
- Finite n and large $\gamma$ dramatically reduce effect
- Does not measurably change expected ELM onset time

Mode structure strongly altered
- Narrowing and phase changes
- May impact dynamics, ELM size

Measured (DIII-D 113207)

$\Omega_{\text{ped}} = 0$

$\Omega_{\text{ped}} = 10 \text{ kHz}$
Calculated Mode Rotation Agrees with Observation during ELM

- Measured rotation profile changes from strongly sheared just before the ELM to ~flat at ~45 km/s across pedestal region at ELM onset
- Study with ELITE finds peeling-ballooning unstable just before ELM - most unstable mode (max $\gamma/\omega_*$) is $n=9$
- Calculated frequency for this $n=9$ mode is $\omega/\omega_A=0.0082$, $V_{\text{rot}}=45$ km/s
- Suggests “locking” of pedestal region to the mode during initial phase of ELM crash
Nonlinear Edge/Pedestal Simulations

• Many challenges for nonlinear simulations of the edge region
  – Broad range of overlapping scales and physics (L-H transition, sources and transport, ELMs, density limit..)
  – Many techniques and formulations used to simplify core simulations are not applicable in edge
  – Long term goal is to unite full set of physics into massive scale simulations

• Here we focus on the fast timescales of the ELM crash event itself
  – Goal is to understand physics determining ELM size
  – Initialize with P-B unstable equilibria, evolve dynamics on fast timescales

• Reduced Braginskii 2 fluid simulations with the 3D BOUT code
- BOUT incorporates 2 fluid/diamagnetic physics and uses field line following coordinates
  - Bundle of lines (left) wraps around $2\pi$ poloidally
  - A group of such bundles (right) spans the flux surface
  - For ELM simulations, generally go 1/5 (or 1/2) of the way around the torus, ie treat $\Delta n=5$ (or $\Delta n=2$), $n=0,5,10…\sim 105$, $0.9 \leq \Psi \leq 1.1$
Fast ELM-like Burst Seen in BOUT Simulations

- High density (small ELM), DIII-D LSN case, $0.9 < \psi < 1.1$
- Initial linear growth phase, then fast radial burst begins at $t \sim 2000$, can see positive density (light) moving into SOL and negative density perturbations near pedestal top
- Radial burst has filamentary structure, extended along B field
Expected Peeling-Ballooning Character in Early Phase

- Plots show projections of bundles of field lines onto the RZ plane - field lines extend into and out of page (radial vs parallel)
- Linear phase: Mode has ~expected characteristics of linear mode, radial and poloidal extent, n~20, γ/ω_A~0.15
  - Reducing gradients slightly stabilizes the mode- abrupt onset near P-B boundary
- Fast Burst: Filaments extended along the field, but irregular
Fast ELM Burst Shows Toroidal Localization, “finger”

- $R, \phi$ plots on outer midplane. Linear phase, $n=20$. Burst occurs asymmetrically at a particular toroidal location. “Finger” is an extended filament along the field - similarities to observations (MAST, DIII-D) and nonlinear ballooning theory [16]
Summary

• Peeling-ballooning model has achieved a degree of success in explaining ELM onset and a number of ELM characteristics
  — Extend to include rotation and nonlinear, non-ideal dynamics

• Toroidal rotation shear included in ELITE
  — Discontinuity in previous studies removed via eigenmode formulation
  — Small effect on predicted ELM onset, but significant modification of mode structure (narrowing and phase)
    • Real frequency of mode matches plasma rotation near center of mode
  — Encouraging comparisons with fast CER observations

• 3D nonlinear ELM simulations carried out with BOUT
  — Early structure and growth similar to expectations from linear Peeling-Ballooning
  — Radially propagating filamentary structures
    • similar to observations (eg MAST and DIII-D), and nonlinear ballooning theory
  — Explosive burst propagates outward, negative density and T bursts in to psi~0.9, significant toroidal localization and finger-like filamentary structure
Future Work

• Initial set of simulations provide insight into linear and early nonlinear phases - comparisons to experiment underway
  - Improved numerical techniques and BC’s to extend duration of simulations

• Move on to larger problems:
  1) Toroidal scales – For some types of ELMs, need full torus (n=1 to ~ρ_i)
  2) Radial scales – extend to wall and further into core
  3) Time scale – Include sources and drive pedestal slowly across P-B boundary

• Scale overlap and close coupling with pedestal formation (L-H) physics, inter-ELM transport and source (including atomic) physics

• Need optimal formulations, efficient numerics and large computational resources
References