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OF RESISTIVE INSTABILITIES IN DIII-D**

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Nonlinear Modeling of the Onset and Evolution of Resistive Instabilities in DIII-D

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1. Overview

The understanding of resistive neoclassical tearing mode (NTM) stability is crucial to all magnetically confined plasmas, and a major challenge for fusion. The onset mechanism for most neoclassical tearing modes can presently be identified and explained by accurate theoretical analyses. The detailed effects of linear instability, neoclassical drive and coupling drive in three standard scenario cases are studied for an overview of the physics involved in the onset of NTMs. Each of these processes dominate in turn under varying conditions. In the first case, a spontaneous NTM is destabilized, with a seed island due to linear instability. In the second, after a series of similar sawteeth the n^{th} sawtooth seeds a NTM, while classical linear destabilization plays a key role. In the third case a sawtooth crash directly seeds a NTM that is linearly stable. These are arranged in progressive order from dominant linear drive to dominant nonlinear drive for the initial onset, while three cases will have similar neoclassical characteristics for the saturated state.

The island evolution equation captures the essential physics necessary to describe the onset and early evolution of uncoupled NTMs, and includes the effects of linear drive and neoclassical effects, but not coupling drive.

$$\frac{dw}{dt} = \frac{\eta^*}{k_0} \left\{ \Delta^* + \frac{wD_{\text{tot}}}{w_d^2 + w^2} + \frac{D_{\text{pol}}}{w^3} \right\}, \quad D_{\text{tot}} \equiv D_{\text{nc}} + D_{\text{R}} / (\alpha_s - H), \quad \Delta^* \equiv \Delta' (w/2)^{-2\alpha_1} (-4D_{\text{I}})^{1/2}.$$

With weak coupling drive, and accurate time dependence of the various terms in this equation, the onset and early evolution of NTMs can be predicted. However, in cases with strong coupling drive from mode activity, nonlinear initial value codes are used to accurately study the onset mechanism.

In this paper, for DIII-D discharges with NTMs, nonlinear, neoclassical and classical effects are comprehensively analyzed showing the critical physics for onset. Theoretical predictions of the effects of time dependence in the equilibrium state are compared to DIII-D experiments with spontaneous and sawtooth seeded NTM onset

2. Linearly Destabilized NTMs

The basic physics of the time dependence of the linear tearing stability index Δ' can be expressed with a simple analytic model, although it has been shown that accurate treatment of the linear stability can only be achieved through advanced numerical methods, such as the PEST-III code [1]. In the simplest model of slab geometry with a constant current gradient $dj_0/d\psi$, the analytic solution for the resistive stability of this system gives:

$$\Delta'a = 2k_x a \frac{E \sin(k_x a) + \cos(k_x a)}{E \cos(k_x a) - \sin(k_x a)}, E \equiv \left[e^{k_x a} - e^{k_x (2b-a)} \right] \left/ \left[e^{k_x a} + e^{k_x (2b-a)} \right] \right., k_x = \sqrt{j'_0 - k_y^2}.$$

Here ψ is the y or poloidal flux, $j_0=0$ at the plasma edge at a , a conducting wall is outside the plasma at b , the equilibrium only depends on x , the rational surface is at $x=0$, and the equation reduces to the form $\Delta'a = -2k_x a \text{Cot}(k_x a)$ for $b=a$. This shows how the tearing stability depends on the location of the wall, and the ideal stability boundary in Fig. 1. Although Δ' is often assumed to be constant and moderately negative for NTM cases, analytic and numerical models of linear tearing stability show strong variation of Δ' near ideal instability boundaries, the regime where NTMs are most prevalent. In this regime, small changes in equilibrium dramatically affect Δ' due to a pole discontinuity at the ideal limit. Spontaneous NTMs have been shown to be destabilized by this mechanism alone [2].

A theoretical prediction was made that for spontaneous NTMs, the value of β_N when $w \sim w_d$ as a function of $d\beta_N/dt$ is specified. A DIII-D experiment was designed and performed to determine this effect. The experiment isolated the Δ' pole mechanism, avoided other modes such as sawteeth ($q_{\min} \sim 1.4$), and varied $d\beta_N/dt$ using neutral beam ramping on approach to onset of a 2/1 NTM. Agreement was found with the prediction as shown in Fig. 2. This effect is also seen in nonlinear simulations with heating, using the NIMROD 3-D MHD code [3], as seen in Fig. 3. The simulation shows a 2/1 mode where the growth rate increases as the pressure is increased mimicking the ramping of neutral beam heating. The pressure is increased much faster than the current relaxation time, and therefore the safety factor (q) profile is conserved. The growth rates increase with the pressure throughout the evolution. This confirms the validity of the effect of the linear drive at finite island size, and that the linear drive effects the nonlinear evolution.

3. Sawtooth Seeded NTMs

If we consider a series of sawteeth with increasing pressure, the nonlinear coupling drive will create seed islands which respond to the changing linear stability drive. Near the ideal

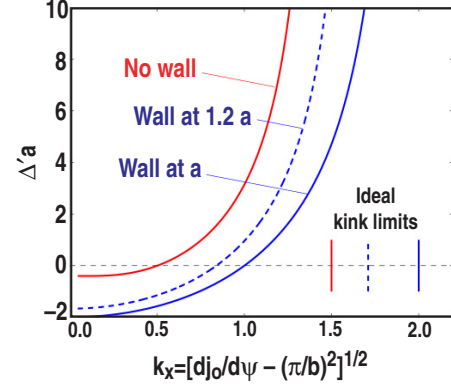


Fig 1. The linear stability index of an analytically unstable slab model, showing the form of the pole discontinuity and the effect of geometry.

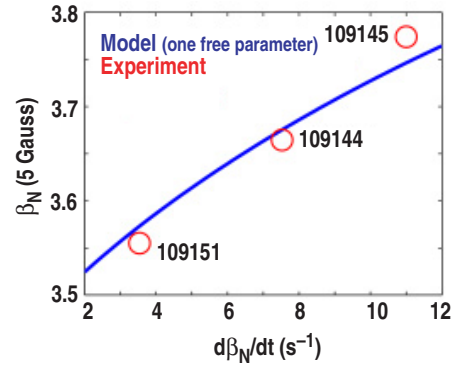


Fig 2. The β_N values reached at a specified small island size in three discharges as a function of $d\beta_N/dt$ for spontaneous 2/1 NTMs, showing agreement with the model.

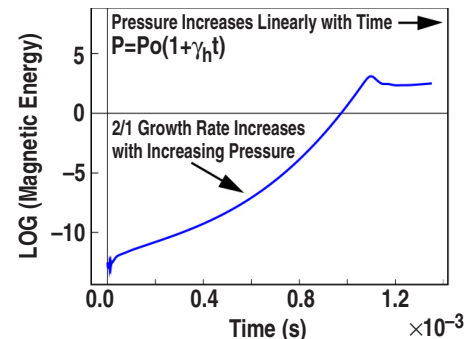


Fig. 3. The magnetic energy vs. a 2/1 tearing mode in a NIMROD simulation with heating, showing the time dependence of the growth rate due to increasing pressure.

stability limit an increase in pressure can destabilize a seed island and cause the onset of a 3/2 NTM. In a DIII-D discharge (Fig. 4) equilibrium reconstructions between sawteeth were analyzed for stability, and show that although neoclassical terms remain constant, a slight increase in pressure causes a sharp increase in Δ' due to the proximity of the ideal $n=2$ limit [2] and causes the 3/2 seed island to cross the neoclassical threshold and transition to the NTM state. The dw/dt from the island evolution equation agrees with dw/dt from experimental data, and gives a correct answer even without nonlinear coupling and with axisymmetric Δ' .

To characterize the linear stability evolution, the pressure profile at an early point in the discharge, stable to the 3/2 NTM, was gradually modified to approximate the profile at the NTM onset, showing the transition to instability in Fig. 5. Nonlinear coupling between $n=1$ and $n=2$ modes and the effects of finite island width on the linear drive were not included in this model. How these will affect the stability and evolution must be addressed.

Nonlinear simulations of these two discharge times show unstable $n=1$ and driven $n=2$ modes, in agreement with experiment, in Fig. 6. In the simulation of the later time, when the NTM is destabilized, the $m=3$ component of $n=2$ is dominant and growing after saturation and decay of $n=1$. The 3/2 decays, after the $n=1$ saturates and decays, in the simulation of the earlier time when the NTM was not destabilized, as in Fig. 7. The equilibria differ in these two simulations in the pressure profile and the linear instability drive, described above. The dynamics of these simulations indicate that the linear Δ' drive affects islands of finite width even in the presence of coupling, and the pole in Δ' is, in some cases, responsible for the onset of NTMs in sawtooth discharges.

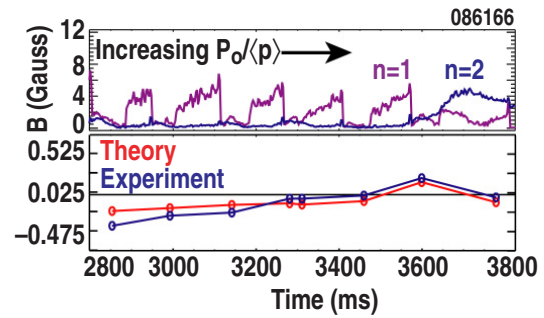


Fig. 4. The magnetic $n=1$ and 2 signals of a DIII-D sawtooth discharge where a 3/2 NTM is eventually destabilized, and the results of a stability analysis explaining the onset.

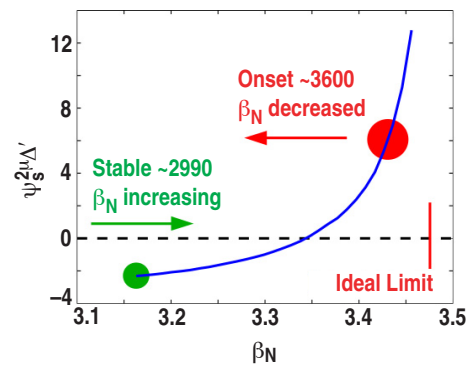


Fig. 5. The effect of morphing the pressure profile from the stable earlier time (green) to the later unstable time, showing the transition to instability due the approach of a pole.

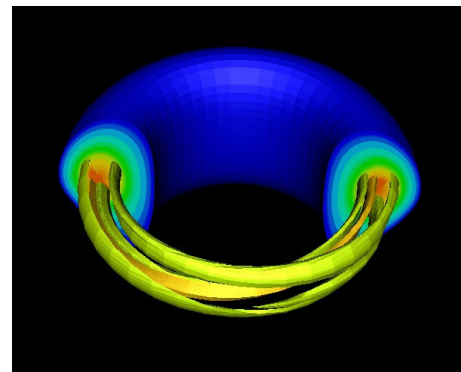


Fig. 6. Isosurfaces of the perturbed $n=1$ pressure in gold, surrounded by the perturbed $n=2$ toroidal current in green during simulation of a DIII-D sawtooth discharge that destabilizes a 3/2 NTM. The $n=1$ is dominated by the 1/1 perturbed pressure while the $n=2$ is dominated by the 3/2 perturbed current.

In the next case a sawtooth crash directly seeds a NTM and the mode is linearly stable. Here $m/n=1/1$ is the dominant mode and $m/n=3/2$ is linearly stable throughout the discharge. Nonlinear coupling from the $1/1$ mode to the $3/2$ mode generates a seed island larger than the neo-classical limit, which subsequently grows neoclassically. The equilibrium is far from the ideal β limit in this case.

With no linear drive the neoclassical drive must be accurately represented to qualitatively simulate the instability growth after the $n=1$ has saturated and begins decaying. As in experiment, the $n=1$ must drive a $n=2$ island beyond the neoclassical threshold. The heat flow dynamics within the small seed island [4] must be handled correctly and the calculations be done in extremely realistic parameter space ($S \sim 10^8$) to get a qualitatively correct answer. Simulations of a DIII-D discharge where a sawtooth crash directly seeds a NTM are shown in Fig. 8. Driven $3/2$ islands are too small to account for experimental observations. The accurate inclusion of neoclassical closures is challenging but shows promise of agreement in the limit of experimental parameters.

4. Summary

The mechanism for destabilizing NTMs can be deciphered with comprehensive stability analyses. For spontaneous NTMs nonlinear simulations confirm that axisymmetric Δ' is meaningful and are in agreement with experiment. In the case of a sawtooth seeding a NTM which is linearly unstable, eventual onset was caused by sharply increased Δ' due to approach of a pole in the presence of coupling drive. For a sawtooth seeding an NTM which is linearly stable, the seed island is nonlinearly driven directly past the neoclassical threshold and efforts continue to achieve an accurate and comprehensive simulation.

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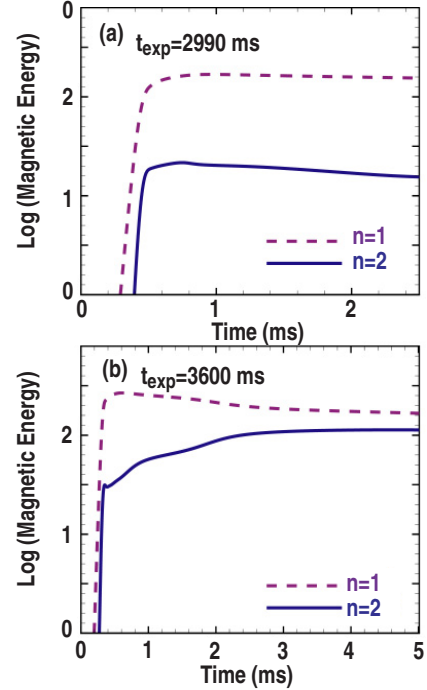


Fig. 7. The magnetic energy of the $n=1$ and $n=2$ modes in a simulation of a DIII-D sawtooth discharge that destabilizes a $3/2$ NTM. The later time (3600 ms) agrees with the mode being linearly unstable in addition to being driven by coupling.

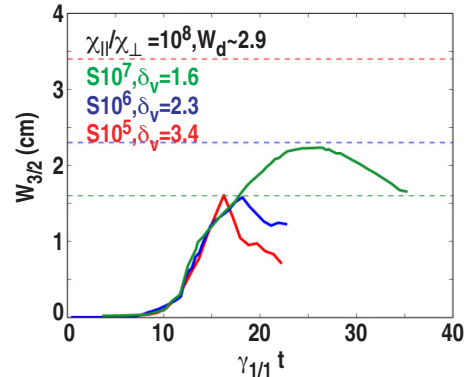


Fig. 8. The $3/2$ island width as driven by a $1/1$ mode for a series of simulations with increasing S .