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ABSTRACT. A theoretical formulation of the feedback stabilization of the resistive wall mode (RWM) for an ideal plasma in a general equilibrium configuration is reported. It is described in terms of two problems that correspond to two different experimental operation scenarios: the open loop and the closed loop operations. Detailed formulation and numerical solution of the open and closed loop problems and its application to DIII-D are presented.

INTRODUCTION. In many magnetic fusion configurations, such as the high β advanced tokamak and the RFP, the configurations are unstable without a nearby perfect conducting wall but stable with the conducting wall present. When the resistivity of the conducting wall is taken into account, the configuration is found to be unstable to the RWM which is driven unstable by the plasma but with its growth rate determined by the diffusion of the perturbed magnetic flux through the external resistive wall. Therefore, to achieve long term stability of the plasma, it is necessary to stabilize the plasma against the RWM. At the present moment, the most promising method for stabilization of the RWM is feedback stabilization by using a set of sensor loops and feedback coils placed external to the plasma. In this work, we present a general formulation for the modeling of the feedback stabilization of the RWM for an ideal plasma. We show that conceptually, this issue consists of two related problems, each of which corresponds to a different operation of the system in the experiment; i.e. the open loop operation and closed (feedback) loop operation. When the feedback loop is open, the feedback system acts passively. With the plasma obeying ideal MHD, the dynamics of the plasma together with a thin resistive wall can be cast into a self–adjoint form. It describes the very low frequency ideal MHD response of the plasma with respect to an arbitrary skin current distribution pattern on the external resistive wall. The dynamics of the system can be described as a set of (both stable and unstable) spatially distributed L/R circuits with a set of associated eigenmodes. The inverse of the L/R times of these circuits are the growth (decay) rates; the amplitudes of these eigenmodes determine the amplitudes of the resistive wall mode. The magnetic fluxes induced by these eigenmodes in the sensor loops form a sensor matrix. Similarly, the excitation of these open loop eigenmodes by the external coils forms an excitation matrix. By utilizing the matrices defined above, together with the feedback logic, the closed feedback loop problem is reduced to a small set of coupled lumped circuit equations. This set of equations is, in general, non-self-adjoint and determines the complete dynamic behavior of the system during feedback. Solution of the characteristic equations [1] gives the stability of the system during feedback. These two problems have been implemented numerically and applied to the DIII-D tokamak. We found that in general, installing the sensor loops inside of the resistive wall is superior to installing them outside. When the unstable mode structure is identified and filtered out as input to the feedback, inside located sensor loops do not give rise to a theoretical upper limit to the gain factor; whereas, there is a theoretical upper limit to the gain factor if the sensors are located outside.

GENERAL CONSIDERATIONS. We divide the region under consideration into the plasma (P), the inner vacuum (IV) (between the plasma and the resistive wall), the resistive wall (W), the outer vacuum (OV) (between the resistive wall and the feedback coil), the coil (C), and the external vacuum (EV) (outside of the resistive wall) regions. We consider only

slowly evolving plasma motions, such that the plasma kinetic energy is negligible, and the plasma response is described by the ideal MHD equations. In the vacuum regions, the perturbed magnetic fields may be represented by $\delta B = \nabla\chi$, with the magnetic potential χ satisfying the Poisson's equation $\nabla\chi^2 = 0$. Starting from these equations, it is then easy to show that during the perturbed plasma motion, the following bilinear energy equation is satisfied:

$$\delta W_p + \delta W_{IV} + \delta W_{OV} + \delta W_{EV} + D_W + \delta E_c = 0 \quad . \tag{1}$$

In Eq. (1), δW_p is the plasma potential energy, δW_{IV} , δW_{OV} , and δW_{EV} are the vacuum potential energies in the various vacuum regions, D_W is the energy flux into the resistive wall and δE_c is the energy injected by the feedback coils. The explicit form of δW_p in terms of the plasma displacement ξ, and the forms of the vacuum energies in terms of χ are well known. The expression for D_W is

$$D_W = \frac{1}{2\mu_0}\int_W dS[(\chi_+^+ - \chi_-^+)\frac{\partial\chi}{\partial n}] \quad . \tag{2}$$

Here the superscript (+) stands for the adjoint function and the subscript (–,+) indicates the value of on the (inner, outer) side of the resistive wall. A thin resistive wall with normal magnetic field continuous across it has been assumed. δE_c is given by an expression similar to Eq. (2), except integrating over the coil surface.

D_W , the energy influx into the resistive wall, is dissipated by its resistivity. The resistive wall mode arises because $D_W \neq 0$. For a thin wall, the current $j = (1/\mu_0)\nabla\times B$ can be represented by $j = \nabla z\times\nabla K$, here z is a coordinate perpendicular to the wall and K is the current flux. With a wall thickness δ, and wall resistivity η, the current flux K on the resistive wall and the magnetic potential χ are connected through

$$\nabla_s \bullet \left[\eta\,|\nabla z|^2\,\nabla_s \bullet K\right] = -\,|\nabla z|^2\,\frac{\partial}{\partial t}\frac{\partial\chi}{\partial z} \quad . \tag{3}$$

Here the subscript s in ∇_s indicates the operator operates only along the surface of the resistive wall. The operator in Eq. (3) is self-adjoint. This means that K may be solved in terms of χ through the utilization of a set of orthonormal tank-eigenfunctions $\{K_i\}$ defined by $\nabla_s \bullet\nabla_s K_i = -\omega_i K_i$. For simplicity, we have taken the wall property (δ,η) to be uniform. In terms of K_i, then

$$D_W = \frac{1}{2}\sum_i \frac{\partial a_i^+}{\partial t} a_i \quad , \tag{4}$$

and the quantity a_i is given by $a_i = \frac{1}{\sqrt{\omega_i}}\int_w dV\left[K_i\frac{\partial\chi}{\partial n}\right]$.

The expression for δE_c , however, has to be left in its general form. It can not be determined until the currents in the coils (determined by the feedback scheme) are given. δE_c is generally non-self-adjoint. In special cases, δE_c is also self-adjoint. The general solution of the feedback problem in this case is given in Ref. [2].

OPEN LOOP RESISTIVE WALL MODES. We first consider the open loop case. Without closing the feedback loop, δE_c =0. With $\gamma = (\partial/\partial t)$,utilizing Eq. (4), Eq. (1) has been shown to be a self-adjoint expression and can be solved by modifications of the MHD codes. It gives us a set of ortho-normal eigenfunctions, with $\xi = \xi_i$, and $\chi = \chi_i$,, that satisfies

$$\delta W_p(i,j) + \delta W_{IV}(i,j) + \delta W_{OV}(i,j) + \delta W_{EV}(i,j) = -D_w(i,j) = -\frac{\gamma_i}{2}\delta_{ij} \quad , \tag{5}$$

with the normalization $\sum a_i^+ a_i = 1$. We note that Eq. (5) gives us a set of resistive wall eigenmodes $O_{pi} = (\xi_i, \chi_i)$ with growth (or damping) rate γ_i. Each eigenmode has a distinct

eddy current pattern on the resistive wall. This set $\{O_{pi}\}$ is complete for arbitrary eddy current distribution on the resistive wall. They are the open loop eigenfunctions. Conceptually, these $\{O_{pi}\}$ describes a set of interwoven circuits on the resistive wall driven by the plasma, with growth (or damping) rates $\{\gamma_i\}$.

The above formulation has been implemented numerically by utilizing the DCON [3] and the VACUUM [4] codes and applied to the DIII-D geometry shown in Fig. 1. Examples of an unstable n=1 O_{pi} with its eddy current pattern on the resistive wall is shown Fig. 2. The pattern remains essentially intact when the plasma β is increased across the stability boundary given by the wall positioned at infinity. The stable mode that is least stable is interesting. Its eddy current pattern is found to have helicity opposite to that of the unstable mode. It has the (m/n=1/1) pattern. It is expected that an external perturbation with the (1/1) signature will easily deform the plasma.

CLOSED LOOP FEEDBACK STABILIZATION. We next consider the case of closed loop operation. Because $\{O_{pi}\}$ is complete; the normal magnetic field on the resistive wall can be expanded in terms of this set of eigenfunctions. From the boundary conditions, the perturbation inside of the inner surface of the resistive wall is completely determined if the value of the magnetic field is given on the resistive wall. The magnetic potential needs to deviate from a superposition of $\{O_{pi}\}$ only in the region outside of the outer surface of the resistive wall. Or in the OV and EV regions,

$$\chi = \sum \alpha_i(t)\chi_i + I_c\chi^c_{out} \quad . \qquad (6)$$

In Eq. (6), a_i's are the expansion coefficients, and are the amplitudes of the resistive wall modes that are excited. We note that the open loop eigenfunctions are assumed to be constant in the present study or we implicitly assumed that the plasma has reached steady state with time constant longer than the resistive wall time, whereas the amplitudes of them, the a_i 's vary on the time scale of the resistive wall mode. χ^c_{out} is the extra solution that exists outside of the outer wall. The subscript out is used here specifically to remind us that χ^c_{out} is present only outside of the resistive wall. χ^c_{out} has the special property that it not only satisfies the Poisson's equation but also has no perpendicular magnetic field at the resistive wall. In Eq. (6), we also utilized the fact that this extra solution is proportional to the currents in the coils. We substitute Eq. (6) into Eq. (1), and noting Eq. (5),

to obtain $\dfrac{\partial\alpha_i}{\partial t} - \gamma_i\alpha_i = \sum_c E^c_i\ I^c \quad .$

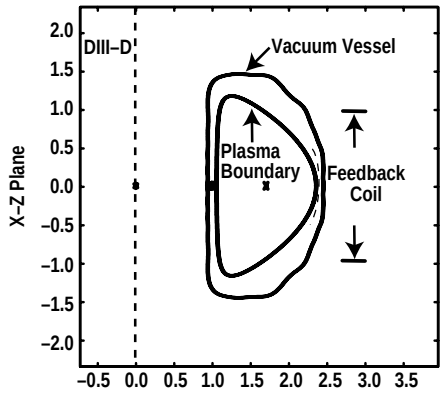

Fig. 1. DIII–D geometry used for the study location of the sensor loops is marked by the dotted line.

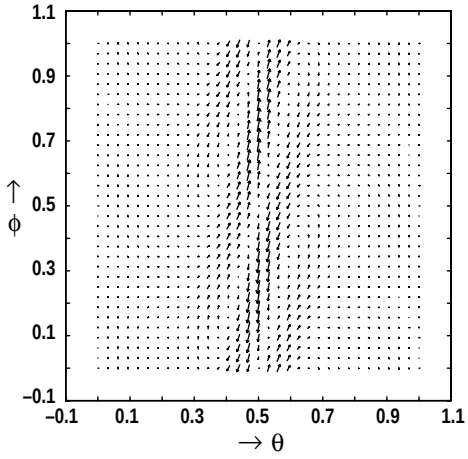

Fig. 2. The eddy current on the vacuum vessel in the (θ-ϕ) plane, θ = 0.5 is the outboard midplane.

We call E^c_i the excitation matrix for the resistive wall mode. It is given by

$$E^c_i = \frac{1}{\mu_0}\int_w dS\chi^c\frac{\partial\chi_i}{\partial n} \quad .$$

The system for describing the feedback operation is completed by specifying the circuit equations for the I^c's. We may represent them as

$$\frac{\partial I^c}{\partial t} + \frac{I^c}{\tau_c} = \sum_s G_s^c F_s(\{\alpha_i\},\{I^{c'}\}) \quad , \tag{7}$$

with F_S being the flux measured in the sensor loops.

CHARACTERISTIC EQUATIONS. If we let $s = (\partial/\partial t)$, and linearized Eq. (7), then the linear response of the feedback system is related to the following finite dimensional eigenvalue problem

$$\overleftrightarrow{R}\,\vec{V} = s\,\overleftrightarrow{I}\,\vec{V} \quad . \tag{8}$$

In Eq. (8), $\vec{V} = (\vec{\alpha}, \vec{I})$ is the vector describing the state of the feedback system. $\overleftrightarrow{R}$ is the response matrix, with the structure

$$\overleftrightarrow{R} = \begin{vmatrix} \Gamma & E \\ GF & L \end{vmatrix} \quad , \tag{9}$$

$\Gamma_{ij} = \gamma\delta_{ij}$ is a diagonal resistive wall growth matrix, $E_{ic} = E_i^c$ is the excitation matrix. The sensor loop coupling matrix GF is given by

$$GF_{ci} = \sum_s G_s^c \frac{\partial F_s}{\partial \alpha_i}$$

and the self-coupling matrix L of the feedback coils is given by $L_{cc'} = -\frac{\delta_{cc'}}{\tau_c} + \sum_s G_s^c \frac{\partial F_s}{\partial I_{c'}}$.

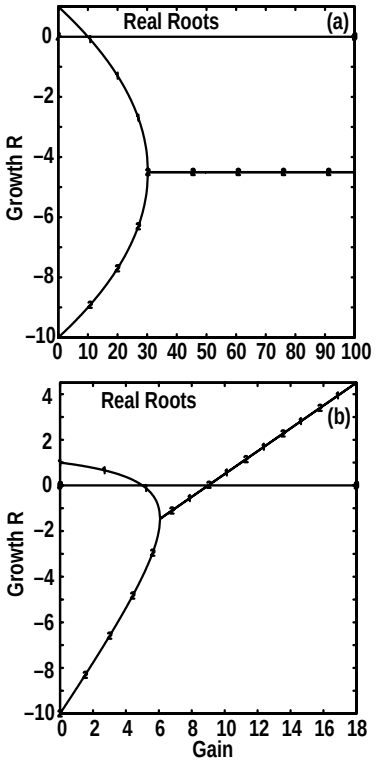

Fig. 3. The growth rate as a function of gain factor for (a) inside and (b) outside sensor loops.

Note that as formulated here, all the quantities in Eq. (9) are explicitly specified when the feedback system is specified. The characteristics determinant $D(s)$ of the system is given by

$$D(s) = \left| s\,\overleftrightarrow{I} - \overleftrightarrow{R} \right| \quad . \tag{10}$$

The solution of Eq. (10) gives us the growth rates of the whole system. The coil current is assumed to be sinusoidally varying in the toroidal direction. The sensor placement is also shown in Fig. 1. We have found that for stabilization the sensors have to be more effective in sensing the unstable mode than the other modes. The general characteristics of the mode behavior as a function of gain is shown in Fig. 3. It is seen that as the gain is increased, the mode is stabilized in Fig. 3(a) if the gain is large enough; whereas in Fig. 3(b), stability is obtained only over a bracketed range of gains.

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