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**AN EXPLICIT NONPERIODIC, NONLINEAR
EULERIAN GYROKINETIC SOLVER FOR
MICROTURBULENCE SIMULATION**

by

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A Nonperiodic, Nonlinear Eulerian Gyrokinetic Solver for Microturbulence Simulation ¹

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Introduction The gyrokinetic-Maxwell (GKM) equations lay a firm but computationally challenging foundation for the investigation of microinstabilities and anomalous transport in fusion plasmas. Our attempts to solve these equations led to **gyro** – a parallel Eulerian code with gyrokinetic ions, collisional drift-kinetic (or adiabatic) electrons, radial profile variation, and a fully time-explicit method for the collisionless part of the problem. This approach grew out of earlier attempts to solve the equations with a semi-implicit method [1]. We found the high-time-accuracy explicit approach with advective Riemann solvers to be a more efficient and robust approach than semi-implicit discretization schemes which split linear and nonlinear terms.

The decision to pursue a full-radius code was made in order to capture enough physics to treat the ρ_* -dependence of turbulent transport in tokamaks, since the relative gyroradius, $\rho_* \equiv \rho_s/a$, is a critical parameter for scaling to reactors (ρ_s is the local ion gyroradius). While similar in many respects to the proven Eulerian (continuum) code **gs2** [4], which assumes vanishingly small ρ_* inside a periodic flux tube, **gyro** can operate in a radially nonperiodic tube with finite ρ_* . We are hopeful that thermal diffusivities obtained from **gyro** can eventually be compared directly with experiment. Because of the great complexity of a full-radius electromagnetic solver, we have aimed for simplicity, and report here results from a restricted version of **gyro** suitable for collisional but electrostatic turbulence. The more general electromagnetic, finite- β version will be described elsewhere.

Numerical Approach Here we give a very brief overview of the numerical approach used in **gyro**. A complete description can be found in [2]. We decompose fluctuating quantities into field-line harmonics according to $h_s = \sum_n h_{sn}(r, \theta, \lambda, \epsilon) \exp[in(\varphi - q\theta)]$, and solve the coupled gyrokinetic-Maxwell equations for the electron and ion gyrocenter distributions h_{sn} . Here, s is a species label; (r, θ) are the radial and poloidal coordinates; (λ, ϵ) the pitch-angle and energy coordinates; n the toroidal mode number; and φ the toroidal angle. After attempting a variety of different solution methods, we recognized that discretization of the parallel advection operator, $v_{\parallel}(\theta)\partial_{\theta}$, in the gyrokinetic equations must be treated with care – particularly for trapped particles and passing particles close to the trapped-passing boundary. While the poloidal angle, θ , is convenient for the description of equilibrium quantities, it is a **poor** choice of variable for numerical solution of the GKE. At bounce points θ_b , where $v_{\parallel}(\theta_b) = 0$, $h_{sn}(\theta)$ develops cusps. However, we can remove these cusps analytically by changing variable to the orbit time: $\tau(\lambda, \theta) \equiv \int_{-\theta_b}^{\theta} d\theta' / \sqrt{1 - \lambda B(\theta')}$, where B is the magnetic field strength, normalized to unity on-axis. For trapped particles, $B(\theta_b) = 1/\lambda$, while for passing particles, we set $\theta_b = \pi$. The equations are then discretized using a third-order upwind scheme on

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an equally-spaced τ -grid. Fig. 1 shows the results of a numerical calculation of the θ -dependence of h_{en} . The latter remains well-behaved across bounce points, even for coarse grids. Note, however, that the τ -grid method creates a new problem: the distribution functions h_{sn} will be known at a different set of points θ_j for each discrete value of the pitch angle, λ . Consequently, the Poisson equation cannot be solved directly on a fixed θ -grid. Instead, we represent the θ -dependence of velocity-space integrals (appearing in the Maxwell equations) by polynomial *blending functions*. This is convenient because of the way θ -projections transform velocity integrals into orbit integrals, and also because the resultant θ -dependence of these integrals is smooth even when velocity space is very coarse-grained. This approach, which we have found to be far more efficient than poloidal decomposition by classical orthogonal polynomials or Fourier series, is described in detail in [2]. There are a variety of options for discretization of radial derivatives and gyroaverages. For linear benchmarking with adiabatic electrons, fully pseudospectral schemes for both are most efficient. For non-linear runs, however, we have found upwind methods to be best for radial advection (at least 7-point), and truncated pseudospectral schemes (typically 17-point) for gyroaverages. Velocity-space integrations in both dimensions are done with generalized Gauss-Legendre rules. While the full collisionless problem is advanced in time using explicit 4th-order Runge-Kutta methods, typical grids prohibit the use of explicit methods for electron pitch angle scattering due to huge viscous Courant numbers near the trapped-passing boundary. Instead, we use operator-splitting (as in `gks` and `gs2`), and so take a separate, implicit collisional step. We also point out that the addition of collisions to the GKE using the τ -grid method is complicated because λ -derivatives at constant θ must be taken using a two-dimensional stencil. A direct sparse solver (UMFPACK) is used for the associated matrix problem.

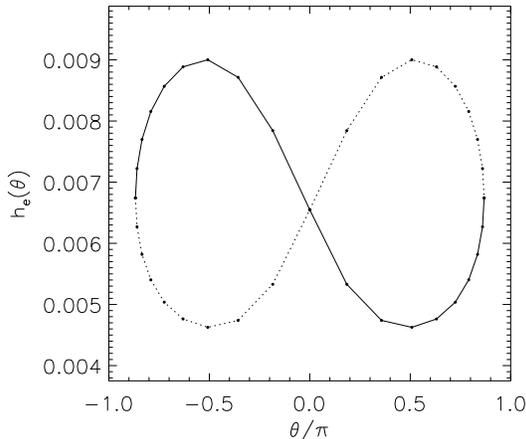


Figure 1: Numerical calculation of h_e using an equally spaced τ -grid. Solid (dotted) line indicates $\sigma = 1(-1)$

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Linear Results For linear benchmarking we use the “CYCLONE DIII-D base case parameter set” [$R/a = 2.78, r/a = 0.5, q = 1.4, s = 0.8, R/L_n = 2.2, R/L_T = 6.9$]. These are also defined in [3]. For adiabatic electrons, complete agreement with the `gs2` code [4] is obtained over the entire unstable range of $k_\theta \rho_s$, as shown in Fig. 2. Preliminary testing of electron dynamics has been done against a GA version of `gks`, with satisfactory agreement for collisionless drift waves, trapped electron modes, and collisional drift waves up to and including the resistive ballooning branch.

Nonlinear results Without profile variation, linear growth rates are independent of ρ_* and mode widths scale as ρ_s . This leads inexorably to gyroBohm scaling (in the infinite-radial-domain limit) of the diffusivity $\chi_{gB} = \rho_* \chi_{Bohm}$, where χ_{Bohm} is the Bohm-scaled diffusivity. When the equilibrium profiles vary within the simulation domain, a shearing in the local mode phase velocity occurs, causing a reduction in turbulence levels. When shear rates become comparable to linear growth rates, we also expect gyroBohm scaling to be violated. Since the shear rates in the tokamak core are typically weak,

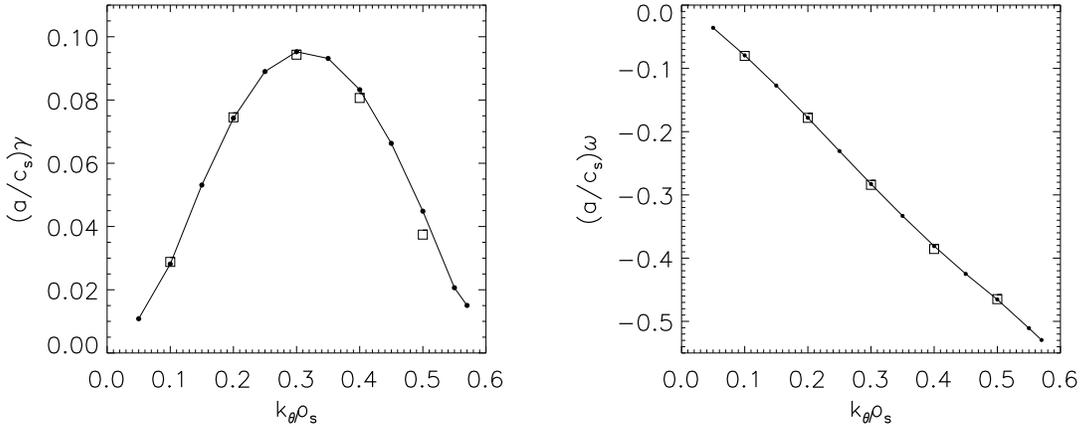


Figure 2: Linear growth rates (left) and frequencies (right) for CYCLONE base case. Solid curve shows gyro results, boxes show gs2 values.

breaking of gyroBohm scaling is expected only sufficiently close to the linear instability

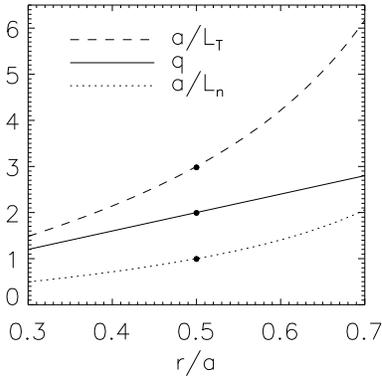


Figure 3: Radial profiles used to demonstrate breaking of gyroBohm scaling.

threshold. To tackle the problem of profile variation, we have devised “two-domain” nonperiodic BCs which allow code operation in a radial annulus with arbitrary profile variation. The distribution functions are forced to be strongly evanescent outside domain 1, and the fields strongly evanescent outside domain 2. Typically, domain 2 must be up to ten gyroradii wider than domain 1, which means that the potential is solved on a larger radial grid than the distributions. Raleigh damping of axisymmetric flows ($n = 0$) near the domain 1 boundary is also used to inhibit the development of shear layers. We emphasize that, in the absence of profile variation, simulations with these BCs give the same thermal diffusivity as periodic flux-tube BCs. First, a flux-tube run with $[R/a=3.0, r/a=0.5, q=2, s=1, R/L_n=3.0, R/L_T=9.0]$ (high above threshold) was made. We find $\chi \simeq 3.2$ so long as the domain size is greater than about 80 ion gyroradii. Introducing the radially-varying profiles of Fig. 4 (flux-tube values are indicated by dots), we see profile-shear-induced transport reduction but with scaling quite close to gyroBohm (Fig. 4a): $\chi/(\rho_s^2 c_s/a) \sim 1$. Moving closer to the instability threshold, conversely, we find Bohm-like scaling (Fig. 4b): $\chi/(\rho_s^2 c_s/a) \sim a/\rho_s$. These results are discussed further in [5]

Finally, we have done preliminary simulations with kinetic electrons. For a fixed grid size, the computational time is about a factor of 10 greater for kinetic electrons than for adiabatic ones. Fig. 5 shows that kinetic but highly collisional electrons, with $(a/c_s)\nu_{ei} = 2.0$, give an ion thermal diffusivity not much higher than the CYCLONE base case. The simulation box for these cases is 100 ion gyroradii wide. We also remark that (due to time constraints) all three cases were run at relatively small numerical grid density, and so the absolute diffusivities are slightly smaller than the fully-converged values.

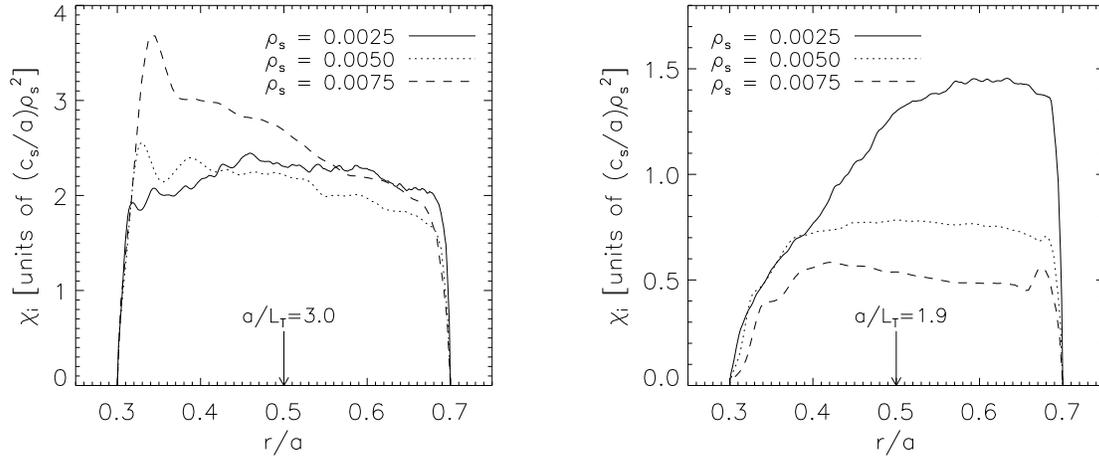


Figure 4: Simulations with profile variation showing (left) gyroBohm scaling far above threshold, (right) Bohm-like scaling closer to threshold.

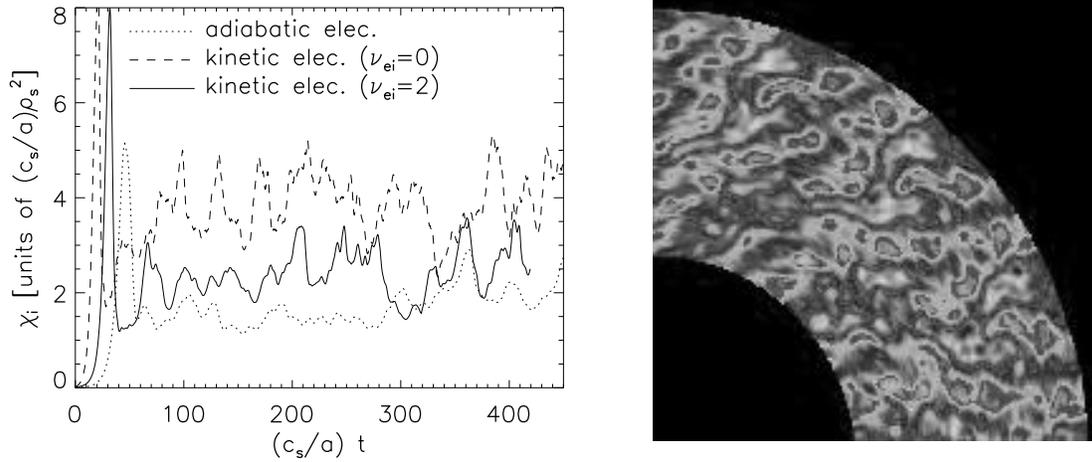


Figure 5: Comparison of ion thermal diffusivities (left) for adiabatic, collisionless and collisional electrons, and poloidal snapshot of electron density fluctuations (right) with collisions

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