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OF THE TOKAMAK EDGE REGION

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*EURATOM/UKAEA Fusion, Culham Science Center, Abingdon, United Kingdom.
†Lawrence Livermore National Laboratory, Livermore, California.

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P.B. Snyder, H.R. Wilson, X.Q. Xu, J.R. Ferron, L.L. Lao, M.S. Chu, E.J. Strait, and A.D. Turnbull

General Atomics, P.O. Box 85608, San Diego, California

1EURATOM/UKAEA Fusion, Culham Science Centre, Abingdon, United Kingdom.
2Lawrence Livermore National Laboratory, Livermore, California.

Tokamak performance is strongly tied to the stability and transport properties of the edge plasma. In many transport models, the height of the edge pressure “pedestal” essentially determines overall plasma confinement. Theoretical analysis of edge instabilities which may control the pedestal height and width is complex, in part because the sharp pressure gradients, and consequent large bootstrap currents, near the H–mode edge can destabilize kink, peeling, and ballooning modes over a wide range of toroidal mode numbers \( n \). Additional complications arise from separatrix geometries and non-ideal physics.

Ideal MHD stability studies of edge localized modes (ELMs) and experimentally observed ELM precursors suggest an important role for instabilities in the intermediate range of mode numbers \( 3 \lesssim n \lesssim 30 \). Continuing improvements in algorithms and computation speed have allowed low-\( n \) stability codes to treat a growing extent of the lower end of this range \( (n \lesssim 10) \) [1]. Higher values of \( n \) have traditionally been studied via ballooning theory. An important modification of classical ballooning theory has recently been developed, allowing proper treatment of the coupled system of edge ballooning and peeling modes [2,3]. This edge ballooning formalism has been studied in both shifted circle and shaped local equilibrium geometry [2,3,4], leading to the development of a model for the ELM cycle, and to insight about the role of second stability in the edge region.

A series of H–mode experiments on the DIII–D tokamak exhibits a strong dependence of edge localized mode (ELM) behavior on plasma shape [1,5]. Similar observations have been made on other tokamaks and are summarized in Ref. [6]. Here we focus on a set of experiments in which the squareness \( \delta_2 \) of the plasma cross section is modified. At moderate values of \( 0.05 \lesssim \delta_2 \lesssim 0.2 \), large infrequent ELMs are generally observed, together with a large value of the edge pressure gradient. In sharp contrast, high squareness \( (\delta_2 \sim 0.5) \) discharges generally produce low amplitude, high frequency ELMs, and smaller edge pressure gradients.

Low-\( n \) stability analysis has been carried out on a series of model equilibria designed to closely match the experiment [1], using a procedure outlined in [7]. A hyperbolic tangent pressure profile is employed in the edge region, with a maximum pressure gradient \( (p') \) at normalized poloidal flux \( \psi = 0.96 \). The edge current is set equal to the predicted bootstrap current. Low-\( n \) modes \( (n < 8) \) are found to be stable for the observed range of \( 2 \leq p'(10^5 \text{ Pa/Wb}) \lesssim 3 \) for the high squareness, high frequency ELM cases. However, the moderate squareness cases with large ELMs are observed to approach the low-\( n \) stability boundaries. Values of \( 5 \leq p' \leq 10 \) are observed, near the low-\( n \) stability boundaries for \( 5 \leq n \leq 10 \) [1].
Conventional infinite-\(n\) ballooning analysis for a typical case is shown in Fig. 1. While the high squareness edge pressure gradient is up against the ballooning limit, the moderate squareness edge has access to the second stability regime, and its pressure gradient is apparently not limited by ballooning modes.

At the plasma edge, coupling of ballooning and peeling modes can be important; in addition, finite-\(n\) effects may play an important role for the modes of interest (\(n \leq 40\)). The peeling mode\(^2\) is a current-driven instability localized near the plasma-vacuum interface. Its stability is a strong function of the proximity of the nearest vacuum rational surface (where the safety factor \(q = q_{0\ vac}\)) to the plasma edge (\(q = q_a\)), quantified by \(\Delta = n (q_{0\ vac} - q_a)\). At small values of \(\Delta\), the peeling mode is destabilized by an edge current density, and its coupling to ballooning modes can cut off access to the second stability regime. This coupling has been studied in an \(s-\alpha\) geometry with shaping modeled by a magnetic well parameter \(d_m = D_m s^2/\alpha\). With “poor” shaping (small negative \(d_m\)), second stability access is possible \(^3\). Peeling/ballooning coupling is also a function of \(n\), as shown in Fig. 2(a). Higher \(n\) modes decouple most easily, leading to the supposition that the most unstable mode is approximately the highest \(n\) without second stability access (provided that \(n\) is low enough that it is not strongly stabilized by finite Larmor radius effects). The coupling is a very strong function of shaping as illustrated in Fig. 2(b). Over a very small range of \(d_m\) values the plasma changes from having essentially no second regime access to having access for all \(n \geq 10\).

For detailed comparisons to experiment, the edge ballooning equations must be solved in realistic, non-local geometry. Here the same equilibria employed in the low-\(n\) study \(^1\) are evaluated for edge ballooning/peeling stability with the ELITE code. For the moderate squareness case, no extended ballooning type instabilities are seen, even when \(p'\) exceeds the low-\(n\) threshold. A highly localized [Fig. 3(a)] peeling instability is seen only at very low values of \(\Delta\). The critical \(\Delta\) for this instability is only a very weak function of \(n\) and, as expected, it scales with the normalized edge current (\(\hat{j||}/\hat{q}\)) (Fig. 4). High-\(n\) modes are thus unstable in this case only when \(nq_a\) passes very near a rational value, and even then the mode is so strongly localized to the edge that it is unlikely to have a significant impact.

For the high squareness case, a coupled peeling/ballooning instability is seen. The mode has a predominant ballooning character, and extends over the entire high gradient region (\(\psi \geq 0.94\)), as shown in Fig. 3(b).

These results support the following hypothesis regarding ELM character (similar to that suggested in Ref. \(^1\)). When the H–mode edge is unstable to extended ballooning-like modes
across the steep gradient region, turbulent transport driven by these modes can, in many cases, hold $p'$ below the low-$n$ limit. The high-$n$ mode turbulence results in the relatively steady fluctuations identified as small, high-frequency ELMs. However, if the equilibrium provides access to the second stability regime for coupled peeling/ballooning modes, the edge pressure gradient can rise to large values. High-$n$ peeling modes may go unstable at small $\Delta$, but these do not extend far enough into the plasma to relax $p'$, which will continue to rise until a relatively low-$n$ kink mode, with a mode structure extending across the pedestal, is driven unstable. This radially extended, fast-growing mode rapidly destroys edge confinement, resulting in a large ELM.

Fig. 2. (a) Stability boundaries of $s-\alpha$ equilibria for coupled peeling/ballooning modes at $n=10,20,40$, $d_m = -0.645$. (b) Calculated marginal value of $n$ for second stability access, as a function of the magnetic well parameter ($d_m$).

Fig. 3. Radial mode displacement ($u$) in: (a) the moderate squareness case ($\delta_2 = 0.05$), $\Delta = 0.05$, (b) the high squareness case ($\delta_2 = 0.5$), $\Delta = 0.05$. $x = m_{0_yq} - nq$ is a measure of radial location which increases from zero at the innermost vacuum rational surface to ~40 at the top of the H-mode pedestal.

Fig. 4. Critical value of $\Delta$ for instability to peeling modes, as a function of the pressure gradient, using model equilibria with squareness $\delta_2 = 0.05$, $n = 15,20$. Also shown is the normalized edge current ($j_\parallel / \langle j \rangle$).
This picture is consistent with nonlinear simulations of turbulence driven by ballooning modes and their kinetic and resistive analogs [9–10] which find that these modes can evolve into a saturated, turbulent state with large transport. However, the evolution of the current profile is not well understood. It is possible that ballooning turbulence does not allow sufficient current relaxation, resulting in the current build-up and crash model of ELMs proposed in Ref. [2].

A number of other factors, including non-ideal effects and separatrix geometry, may impact ELMs, and nonlinear simulations are ultimately needed to understand the detailed evolution of ELMs and profiles in different regimes. These issues are beginning to be explored via an extension of the BOUT boundary turbulence code[10]. BOUT evolves the nonlinear electromagnetic Braginskii equations in separatrix geometry, modeling both the plasma edge and scrape-off layer. Parallel current terms, which are likely to be important for a full understanding of ELMs, have recently been added to the code, and preliminary simulations with edge current have been undertaken. Figure 5 shows the structure of the electrostatic potential ($\phi$) in the linear phase of evolution, for a ballooning unstable equilibrium. The peaks in $\phi$ near the top and bottom of the machine highlight the impact of X–point geometry on these modes.

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