

COMPARISON OF IDEAL MHD STABILITY PREDICTIONS WITH MHD BEHAVIOUR IN DIII-D

A.D. Turnbull, L.L. Lao, E.J. Strait, M.S. Chu,
J.R. Ferron, T.H. Osborne, P.A. Politzer,
R.D. Stambaugh, T.S. Taylor, A.M. Garofalo,
E.A. Lazarus, J.D. Callen, and K. Comer

This work was supported by US DOE Grant DE-FG03-95ER54309 and Contract W-7405-ENG-48

COMPARISON OF IDEAL MHD STABILITY PREDICTIONS WITH MHD BEHAVIOR IN DIII-D¹ A.D. TURNBULL, L.L. LAO, E.J. STRAIT, M.S. CHU, J.R. FERRON, T.H. OSBORNE, P.A. POLITZER, R.D. STAMBAUGH, T.S. TAYLOR, *General Atomics*, A.M. GAROFALO, *Columbia U*, E.A. LAZARUS, *ORNL*, J.D. CALLEN, K. COMER, *UW-Madison*, B.W. RICE, *LLNL* — New diagnostics in DIII-D have greatly improved equilibrium reconstructions over the past decade. This, coupled to a corresponding improvement in ideal MHD stability code accuracy and capabilities, has resulted in a convergence between the predicted MHD stability limits and the observed limits. The comparisons have evolved beyond global scalings to detailed comparisons of the stability predictions of unstable mode structures and growth rates for individual discharges. These demonstrate that ideal MHD predictions are remarkably accurate --- to within a few percent --- for a wide range of discharges. Several prominent examples include infernal modes, resistive wall modes, and intermediate n ideal edge modes in H-Mode discharges, VH-Mode and NCS H-Mode discharges, and $n = 1$ ideal modes in L-Mode NCS discharges.

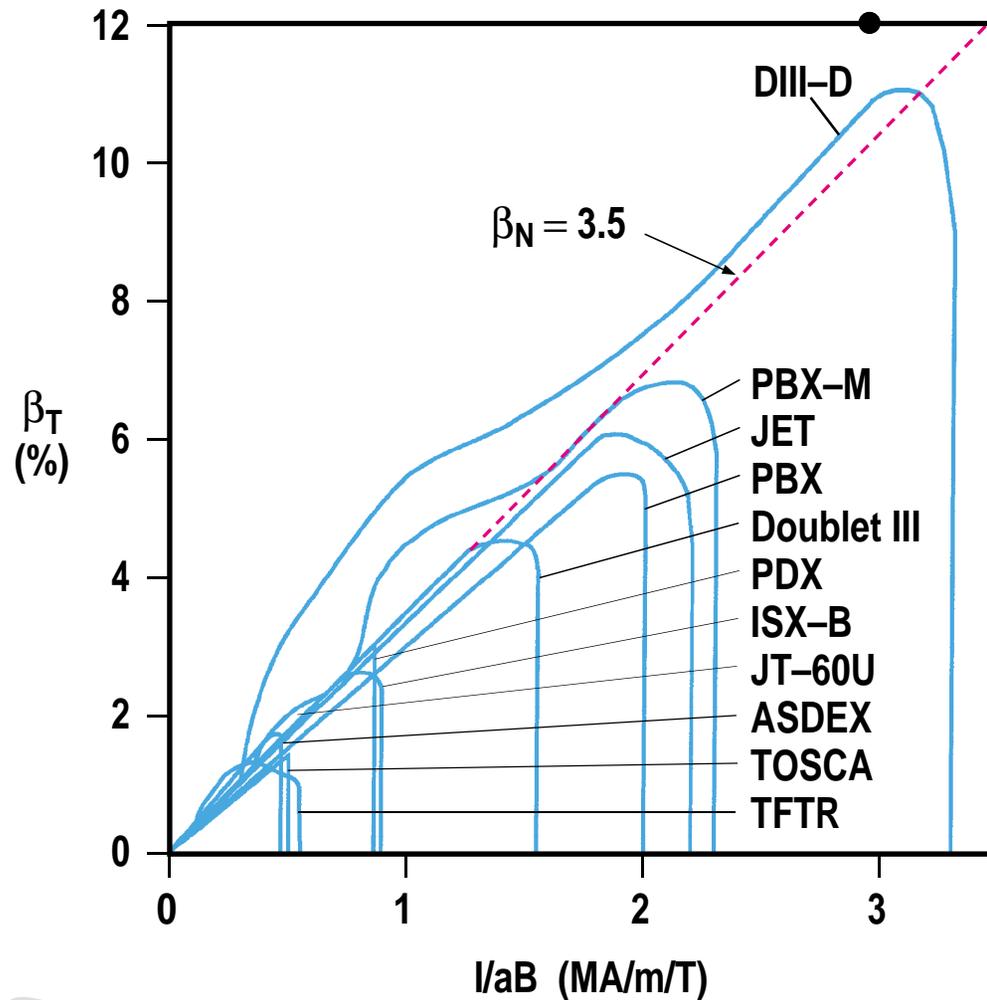
¹Work supported by U.S. DOE Grants DE-FG03-95ER54309, DE-FG02-89ER53297, DE-FG02-92ER54139, and Contracts DE-AC05-96OR22464 and W-7405-ENG-48.

IDEAL MHD CAN PREDICT MANY OF THE FEATURES OBSERVED IN TOKAMAK DISCHARGES

- Reliable prediction of b limit scaling
- In many cases ideal MHD predicts details of unstable modes when stability limits are violated
- Accurate prediction of stability of individual discharges requires
 - Comprehensive equilibrium diagnostics for accurate equilibrium reconstruction
 - Accurate reproduction of details of the equilibrium
 - ★ Cross section shape
 - ★ Profiles
 - ★ Stabilizing wall

IDEAL MHD HAS HISTORICALLY PROVIDED RELIABLE GUIDANCE FOR TOKAMAK CURRENT AND β STABILITY LIMITS

- Troyon limit $\beta^{\text{crit}} = \beta_N^{\text{crit}} I/aB$: $\beta_N^{\text{crit}} \sim 3$ from numerical optimization studies (1984)



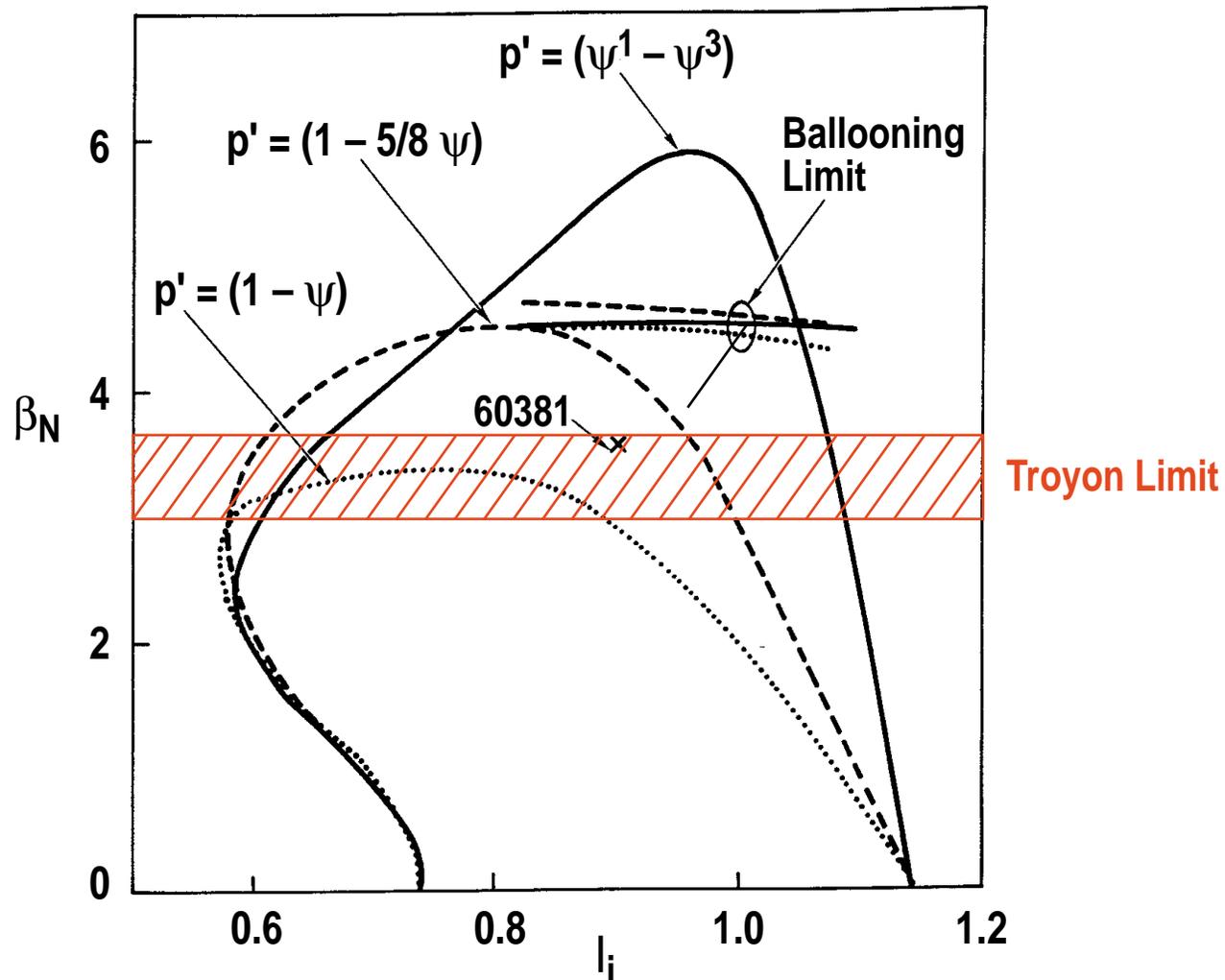
Experiments confirm
Troyon β limit scaling

$$\beta_N(\text{max}) = 3.5 \pm 0.5$$

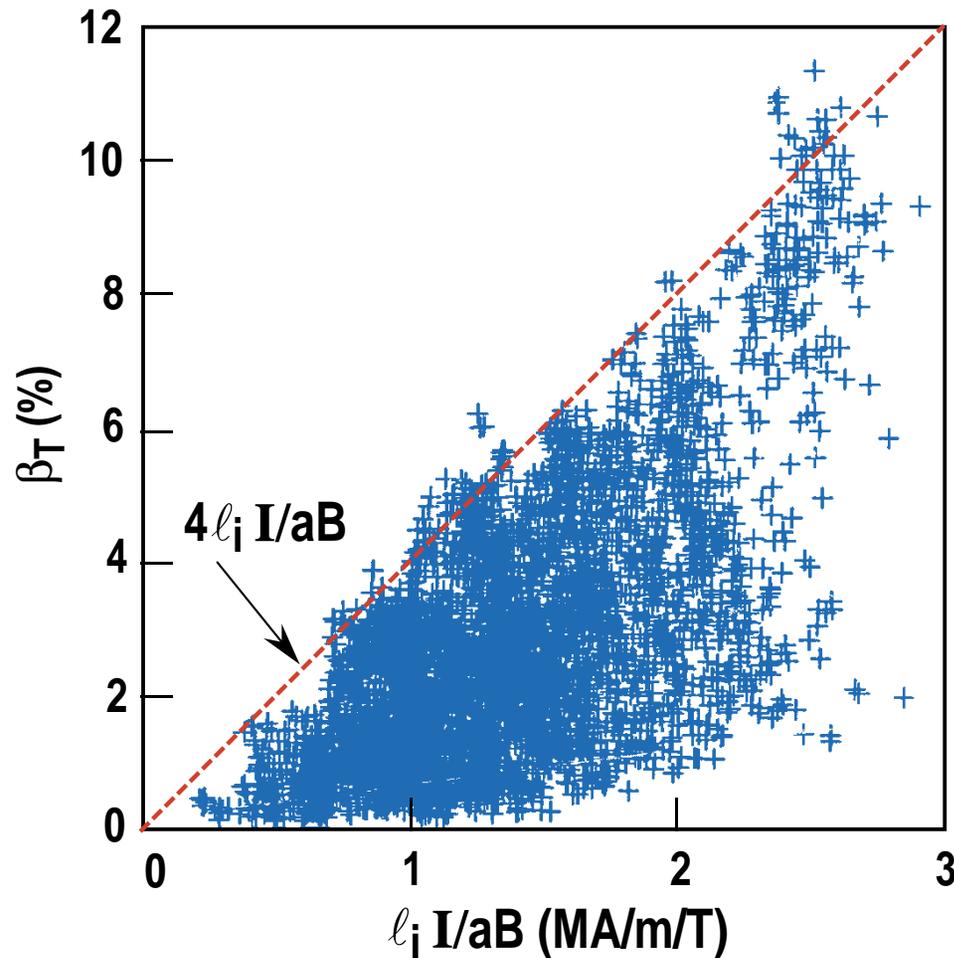
I/aB is limited by current-driven kink instabilities at $q \approx 2$

CALCULATIONS INDICATE β_N CAN SIGNIFICANTLY EXCEED TROYON LIMIT WITH HIGH I_i AND BROAD PRESSURE

- $n = 1$ kink limit exceeds the ballooning limit for optimized profiles (Howl et al. 1992)
 - Wall stabilization is negligible for the optimized profiles



EXPERIMENTS CONFIRM β_N SCALING WITH l_i

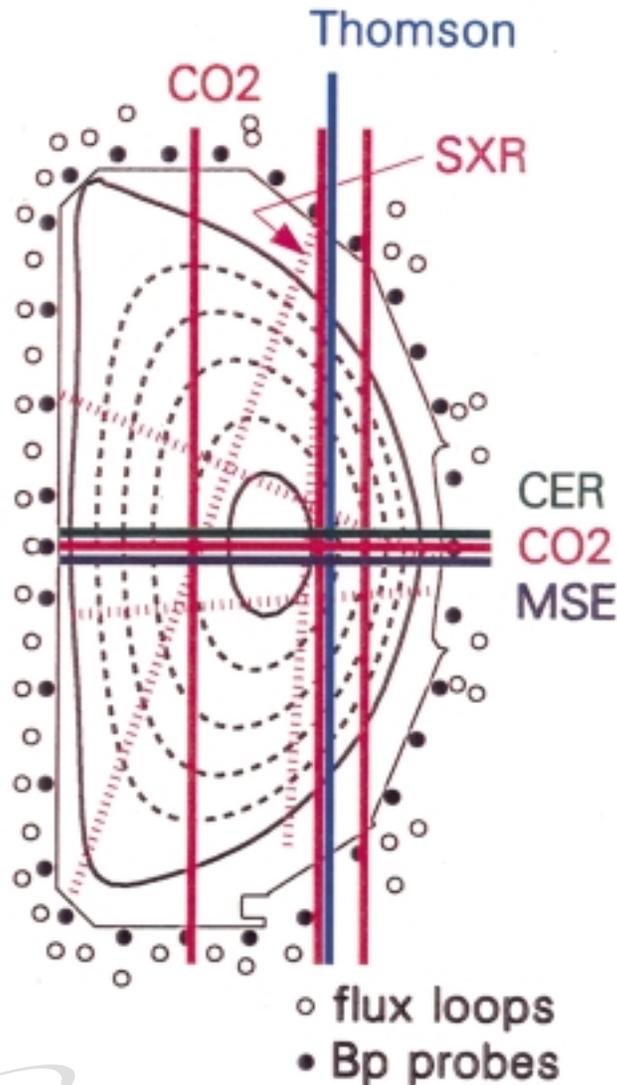


● $\beta_N^{\text{crit}} \sim 4 l_i$

● $4 l_i$ scaling provides better limit than simple Troyon scaling

— Exceptions can be explained by wall stabilization (1994)

ACCURATE PREDICTION OF STABILITY OF INDIVIDUAL DISCHARGES REQUIRES COMPREHENSIVE EQUILIBRIUM DIAGNOSTICS



Profile Data:

36 Channels MSE

Thomson T_e, n_e

CER T_i, Ω

ECE T_e

SXR axis shift

External Magnetic Data:

50 Poloidal B_p probes

40 Poloidal flux loops

Diamagnetic loop

Rogowski loop

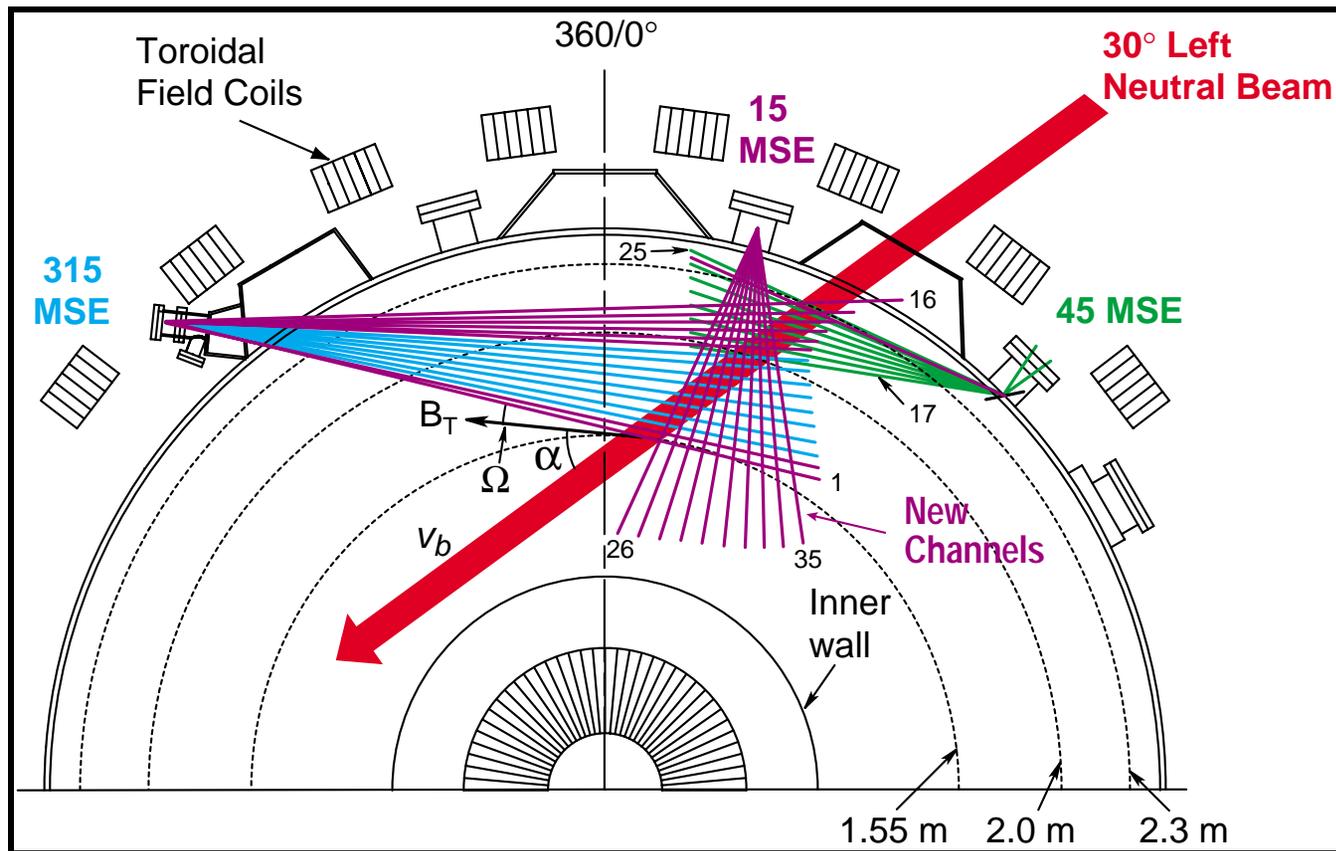
Other Constraints:

SXR rational surfaces

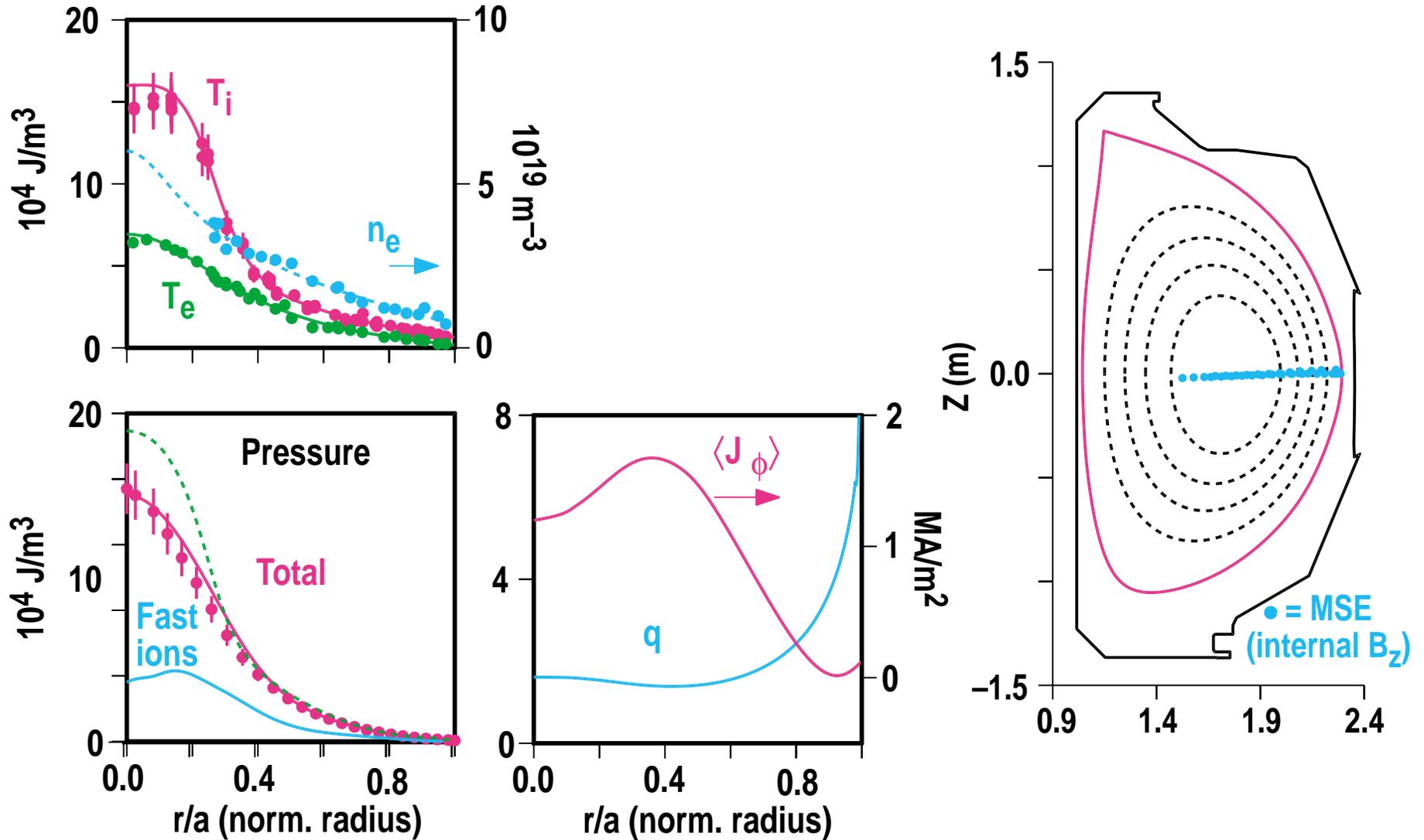
- MSE is crucial to obtaining well constrained current profile

NEW 36 CHANNEL MSE DIAGNOSTIC PROVIDES DETAILED INTERNAL CURRENT PROFILE INFORMATION

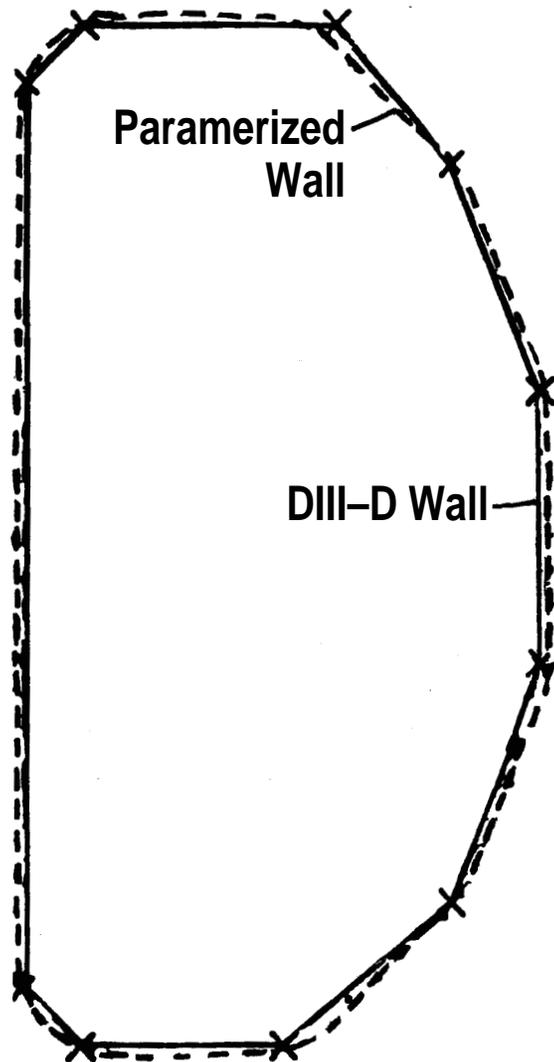
- Multiple viewing angles allow resolution of E_r and B_z :
 - Internal j is well resolved



DETAILED MEASUREMENTS ENABLE ACCURATE STABILITY SCIENCE



STABILITY CALCULATIONS REQUIRE ACCURATE REPRODUCTION OF DIII-D VACUUM VESSEL WALL FOR RELIABLE PREDICTIONS



- Wall parameterize by

$$R_W = R_0 + \alpha a_w \sum_{k=1}^N a_k \cos(k\theta)$$

$$Z_W = Z_0 + \alpha \kappa_w a_w \sum_{k=1}^N b_k \sin(k\theta)$$

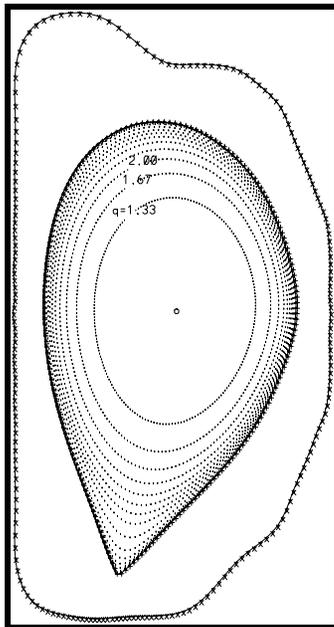
R_0 , Z_0 , a_w , κ_w , and the coefficients a_k , b_k , $k = 1 \dots N$, are kept fixed such that $\alpha = 1.0$ is the best fit to the real DIII-D wall ($N = 26$)

α is then used as an expansion parameter to vary the wall radius R_{wall} / R_{DIII-D}

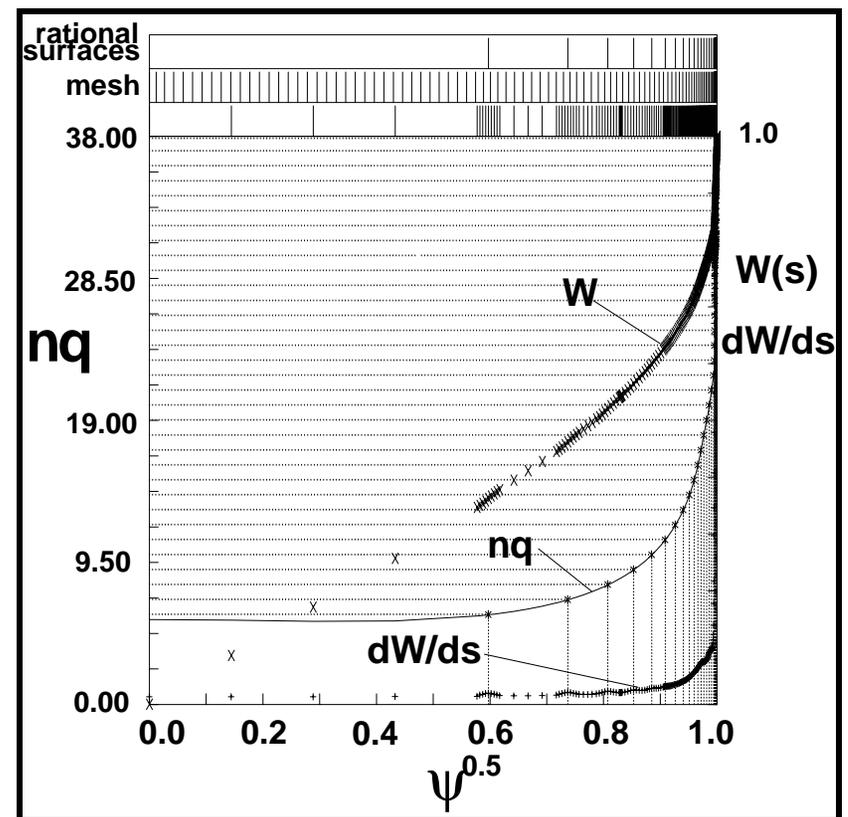
- 1997 upper divertor hardware reproduced by modified fit (up-down asymmetric)

STABILITY CALCULATIONS INCORPORATE ESSENTIAL DETAILS OF CURRENT PROFILE AND THE PLASMA AND WALL SHAPE

- Realistic DIII-D wall:
 - Plasma boundary set within 10^{-4} of separatrix
 - Up-down asymmetry

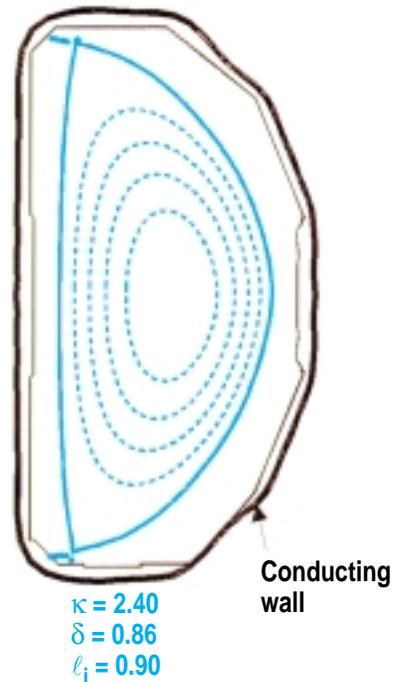


- Mesh packing used to resolve the edge

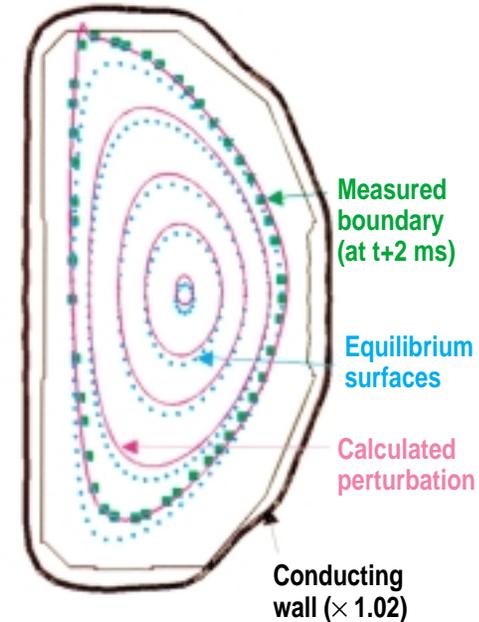


AXISYMMETRIC ($n = 0$) LIMITS ON ELONGATION AGREE WITH IDEAL MHD TO 2% ACCURACY WITH MODELED DIII-D WALL

Experimental
Equilibrium



Ideally unstable with
wall expanded 2%



- Ideal $n = 0$ stability calculation reproduces axisymmetric shift measured from magnetic data
- For broad current profiles and high triangularity, non-rigid effects are important
 - They limit elongation to about 80% of the rigid-shift prediction
 - Without $m = 3$ the comparison is poor

IDEAL MHD STABILITY PREDICTIONS FOR INDIVIDUAL CASE STUDIES DEMONSTRATE AGREEMENT WITH OBSERVATIONS AT SEVERAL DIFFERENT LEVELS

- Single equilibrium reconstruction
⇒ possible explanations of observed MHD behavior
- Parameter scan of nearby equilibria
⇒ plausible explanations of observed MHD behavior
- Complete sensitivity study with respect to all possible equilibrium reconstructions consistent with allowable variations in equilibrium data
⇒ Well constrained ideal MHD explanations of observed MHD behavior
(with caveats re: non MHD effects)

Case studies from DIII-D exhibit varying degrees of these levels over a wide range of discharge types and for a wide range of MHD phenomena

- Fast disruptions
- b collapses
- Edge instabilities
- Core instabilities
- Wall modes

CASE STUDY #1

HIGH $\epsilon\beta_p$

DIII-D DISCHARGE

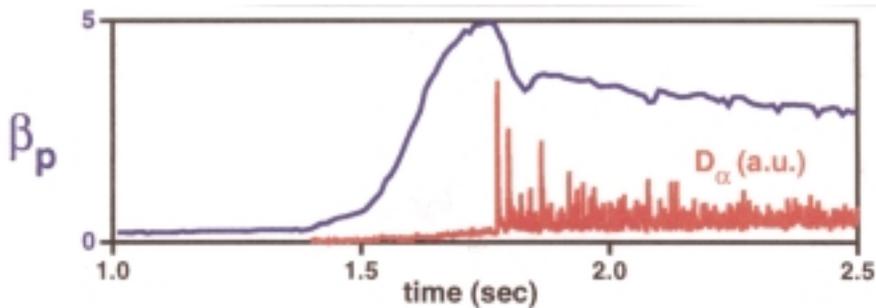
67700

Case Study #1

High $\epsilon\beta_p$
#67700

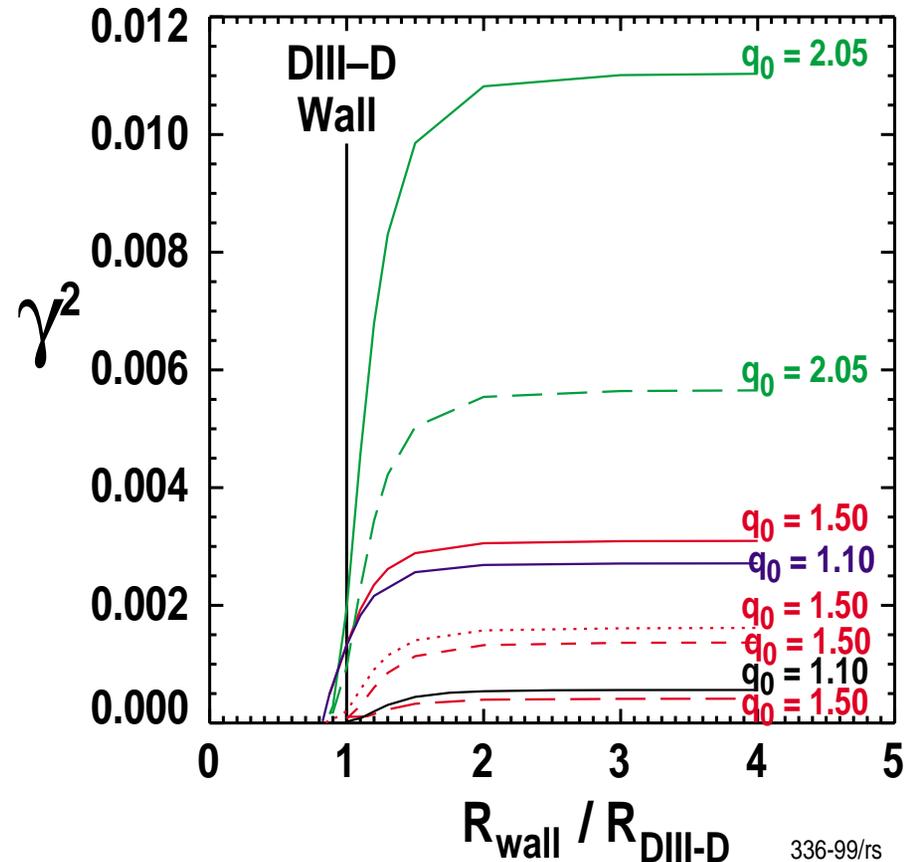
HIGH $\epsilon\beta_p$ DISCHARGE #67700 β COLLAPSE IS DUE TO $n = 1$ IDEAL KINK INSTABILITY

- A fast (μs) β collapse occurred at 1765 ms



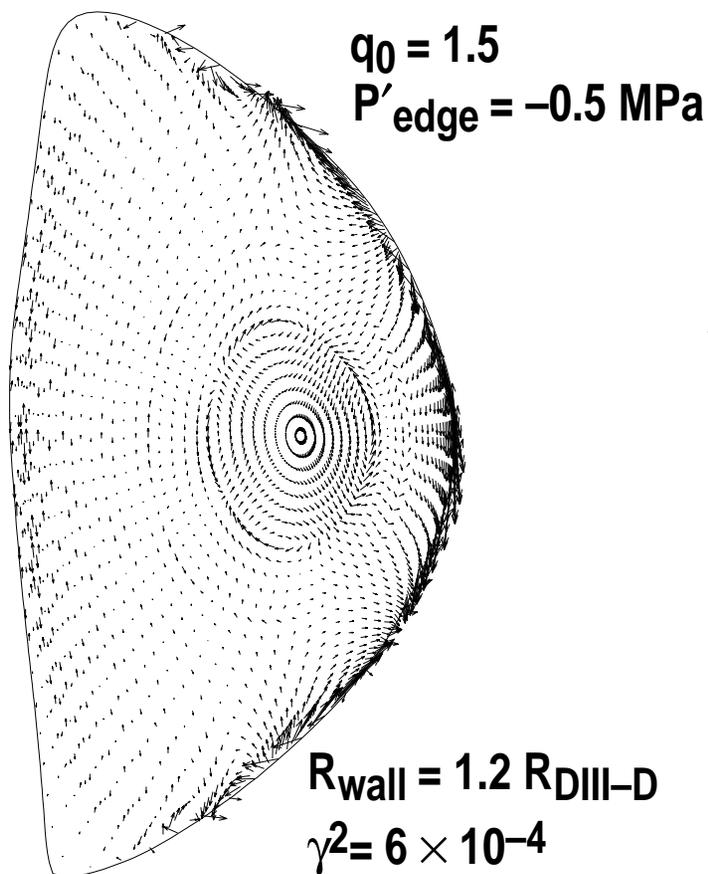
- No MSE data available at 1750 ms
 - Sensitivity to $1 \cdot 1 \leq q_0 \leq 2.0$ and $0.4 \text{ MPa} \leq P'_{\text{edge}} \leq 0.7 \text{ MPa}$ in equilibrium reconstructions

- Calculations show external $n = 1$ kink marginally unstable with DIII-D wall at 1750 ms
- Insensitive to variations in equilibrium parameters

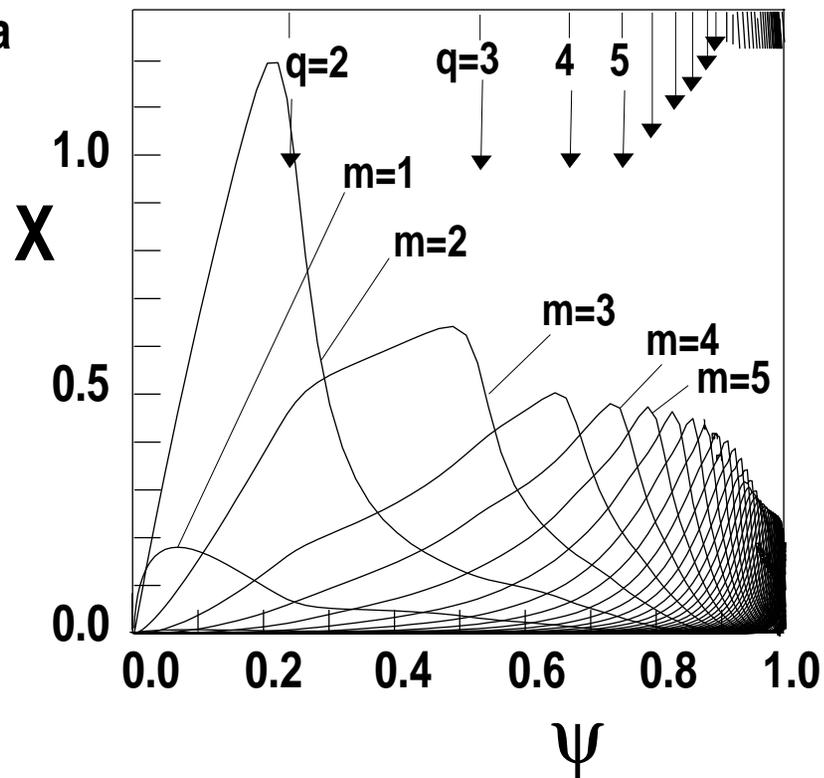


PREDICTED $n = 1$ INSTABILITY AT 1750 ms IS A PRESSURE DRIVEN KINK BALLOONING MODE

● Mode Displacement



● Fourier Decomposition



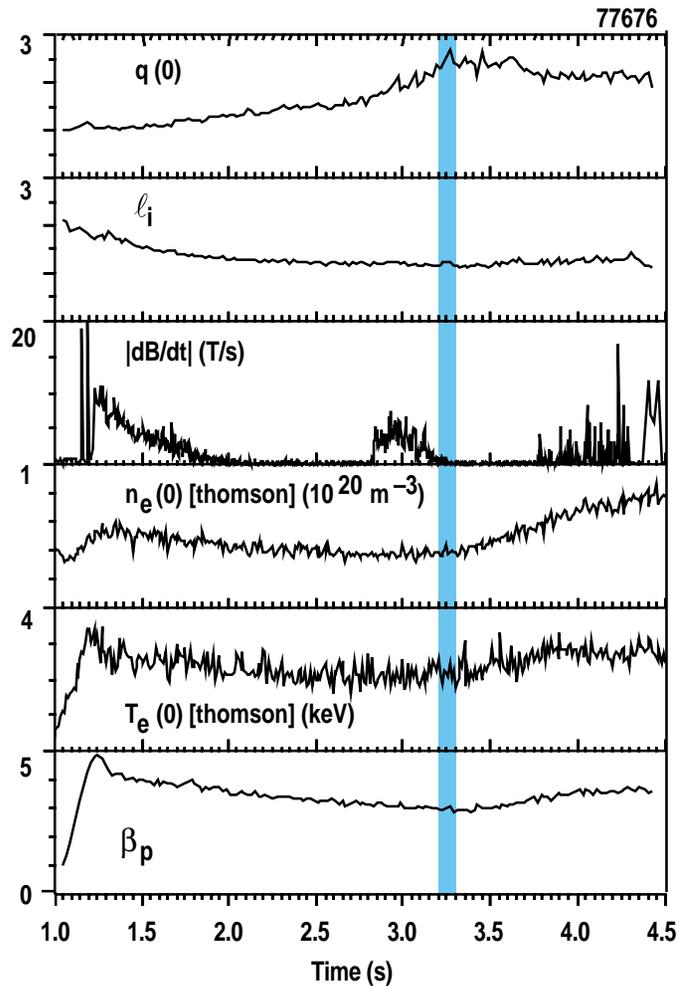
CASE STUDY #2

HIGH $\epsilon\beta_p$

DIII-D DISCHARGE

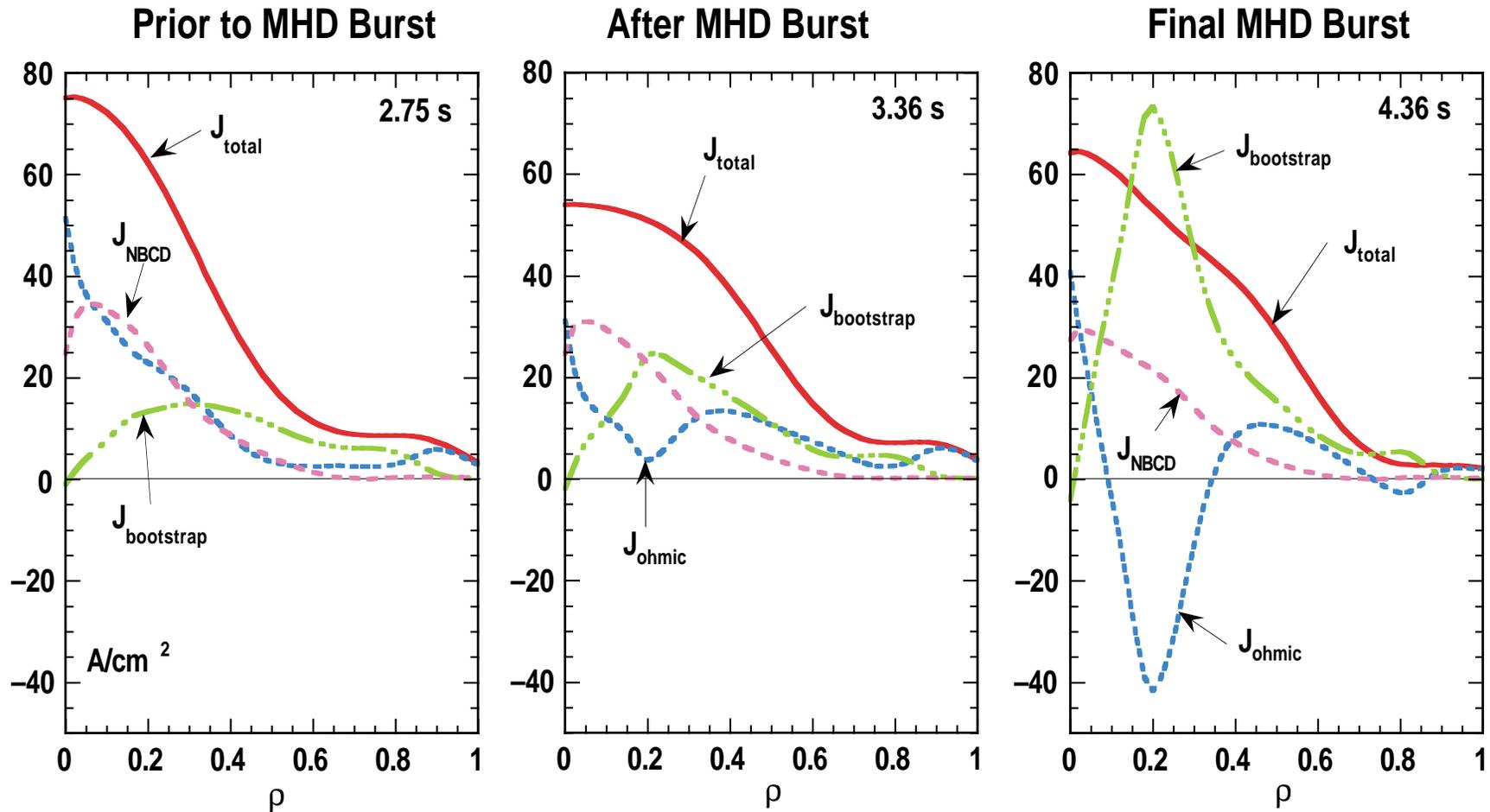
77676

DISCHARGE #77676 REACHED HIGH $\epsilon\beta_p$ AND MAINTAINED QUASI-STEADY STATE FROM NEUTRAL BEAM AND BOOTSTRAP CURRENT DRIVE



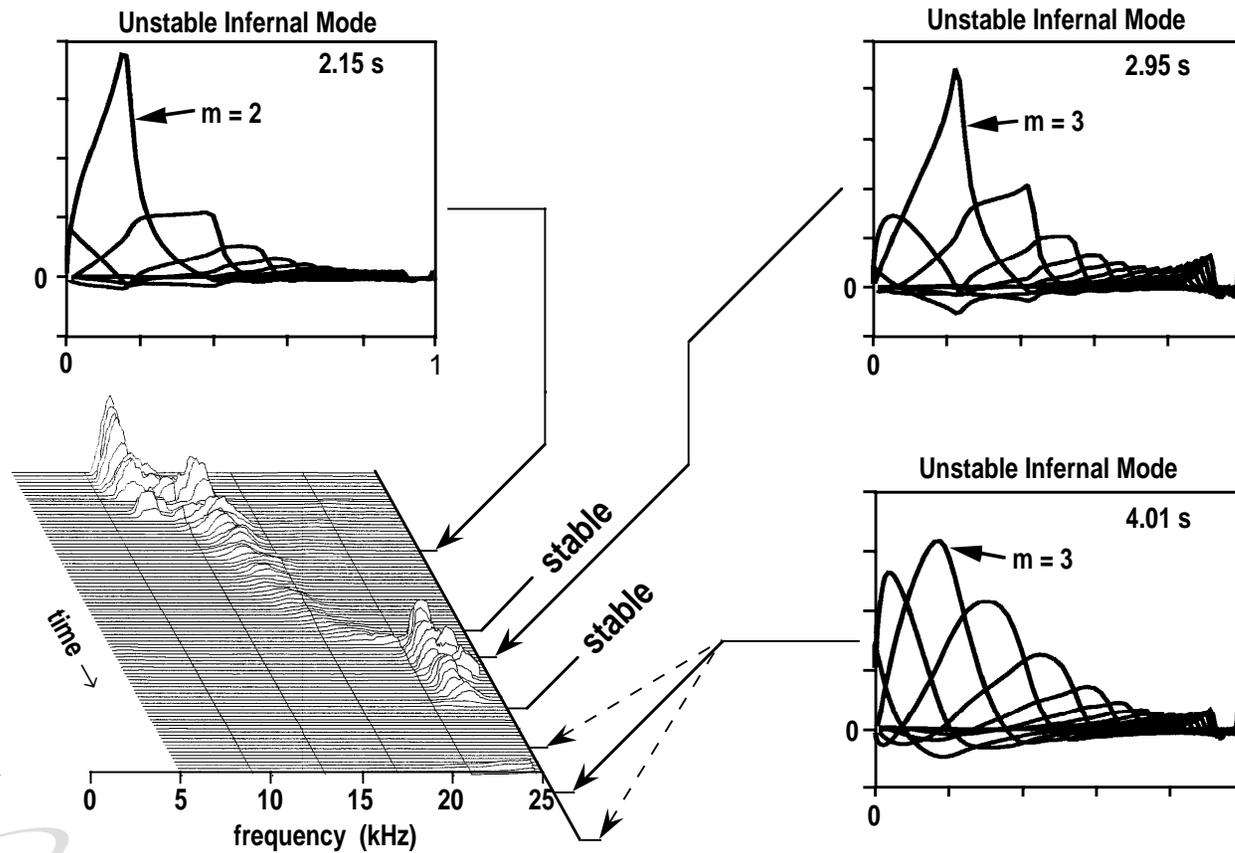
- MHD activity appeared
 - Before 2 s, in bursts
 - Between 2.8 s and 3.2 s and
 - After 3.8 s
- Transport barrier formed between 3.2 s and 3.8 s while $q_0 > 2$

MHD BURSTS IN HIGH $\epsilon\beta_p$ DISCHARGE #77676 RESULT FROM OHMIC CURRENT EVOLUTION



STABILITY COMPUTED FOR EQUILIBRIA RECONSTRUCTED FROM DISCHARGE DATA AT SEVERAL TIMES IS IN AGREEMENT WITH OBSERVED MHD ACTIVITY

- 8 Channel MSE and Thomson \rightarrow profiles
- Equilibrium reconstructed at 5 times slices



CASE STUDY #3

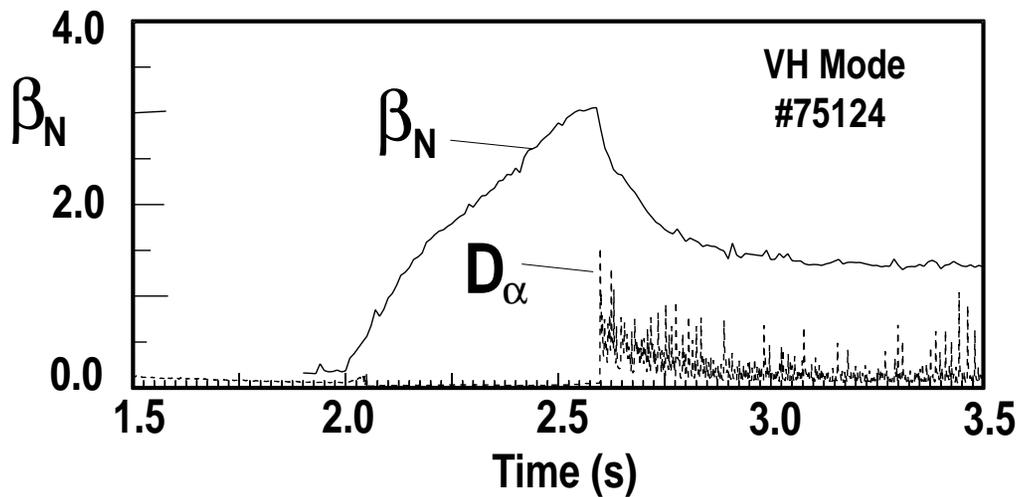
VH-MODE

DIII-D DISCHARGE

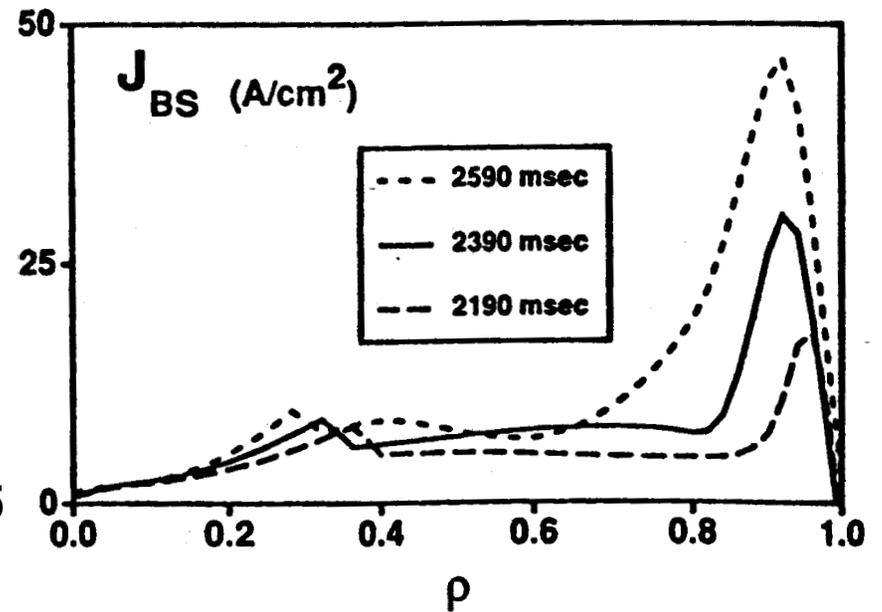
75121

VH-MODE IS TYPICALLY TERMINATION BY AN $n \sim 3-5$ MHD BALLOONING-LIKE KINK MODE

- VH-mode confinement lost at ELM-like X event discharge reverts to ELMing H-mode

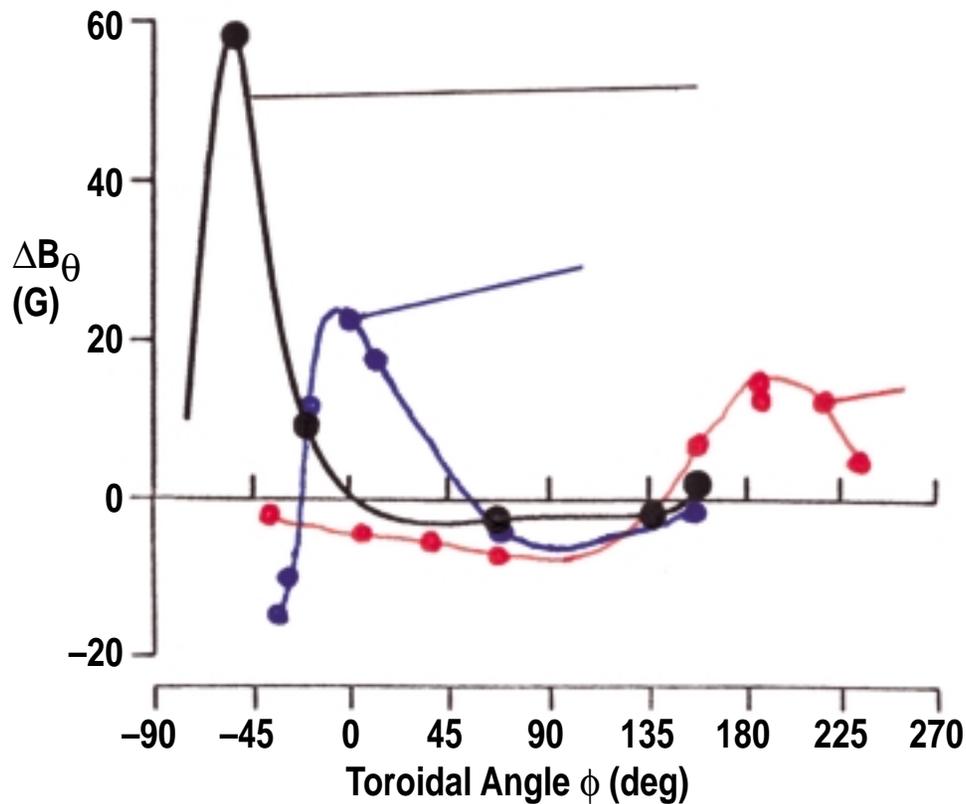


- Large bootstrap current is associated with large edge pressure gradient in VH-mode

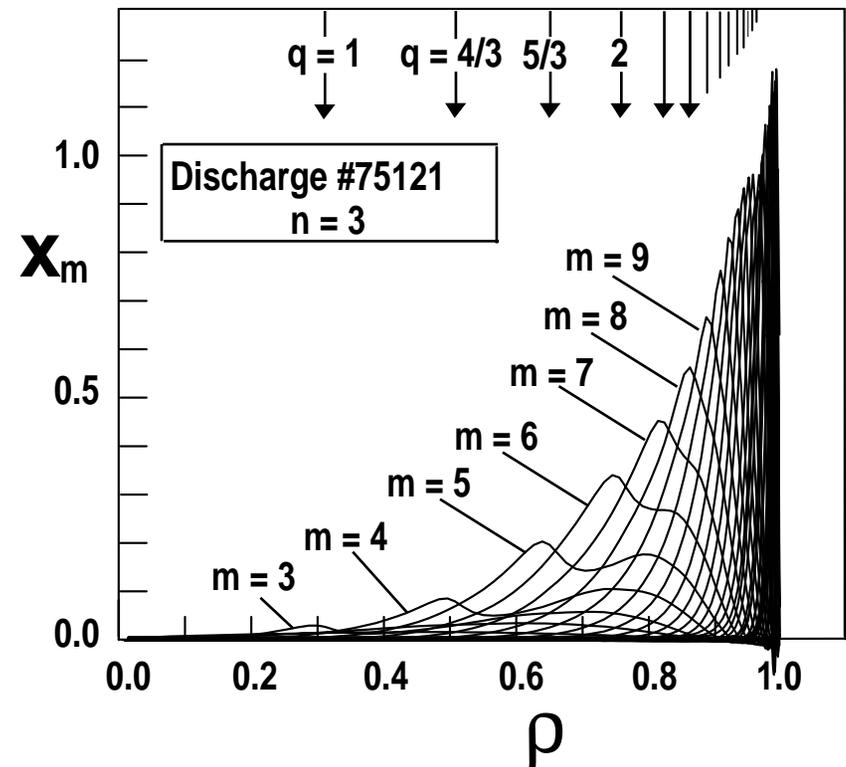


IDEAL MHD STABILITY CALCULATIONS ARE CONSISTENT WITH THE OBSERVED X EVENT

- Observed mode is strongly poloidally localized on outboard midplane (ballooning)

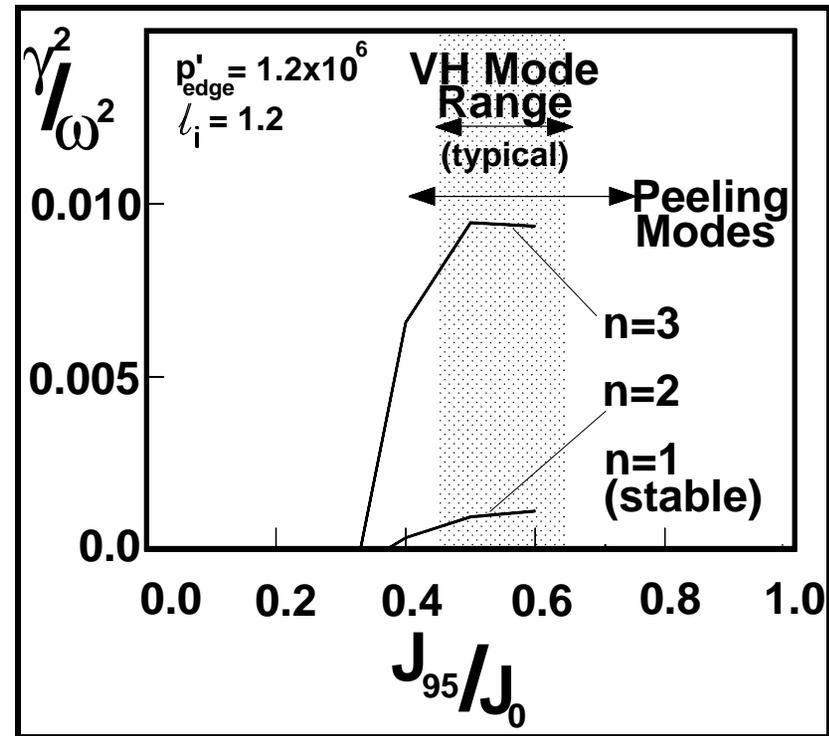
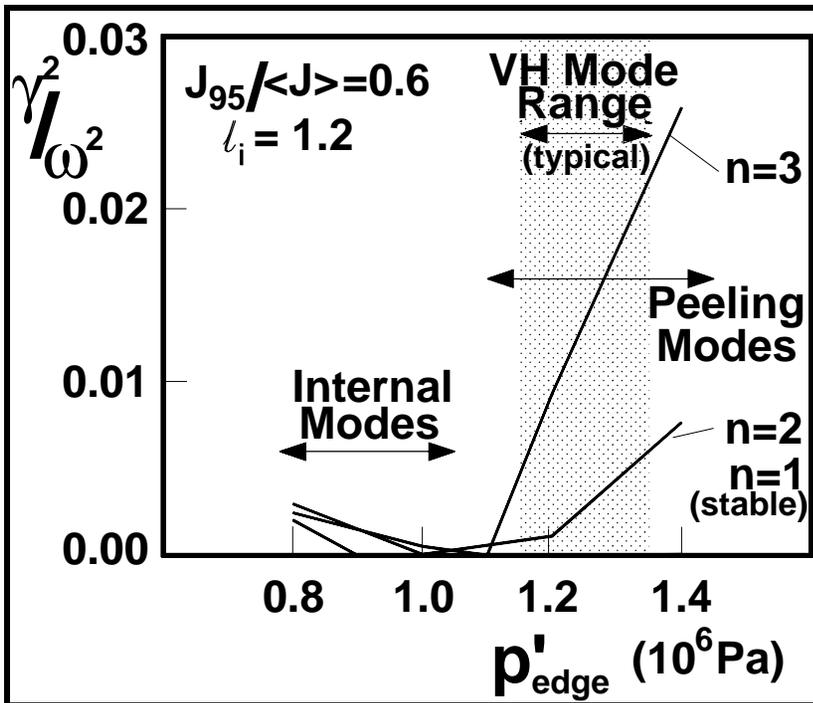


- Unstable $n = 2, 3, 4$ modes computed with edge localized "peeling" structure but strongly ballooning on outboard side



VH MODE TERMINATION EVENT IS DRIVEN BY COMBINATION OF p'_{edge} and j_{edge}

- Parameter scans of nearby equilibria show stability threshold in p'_{edge} , j_{edge}



- Typically higher n are unstable first:
 — $n = 1$ is usually stable unless $q_0 \rightarrow 1$

CASE STUDY #4

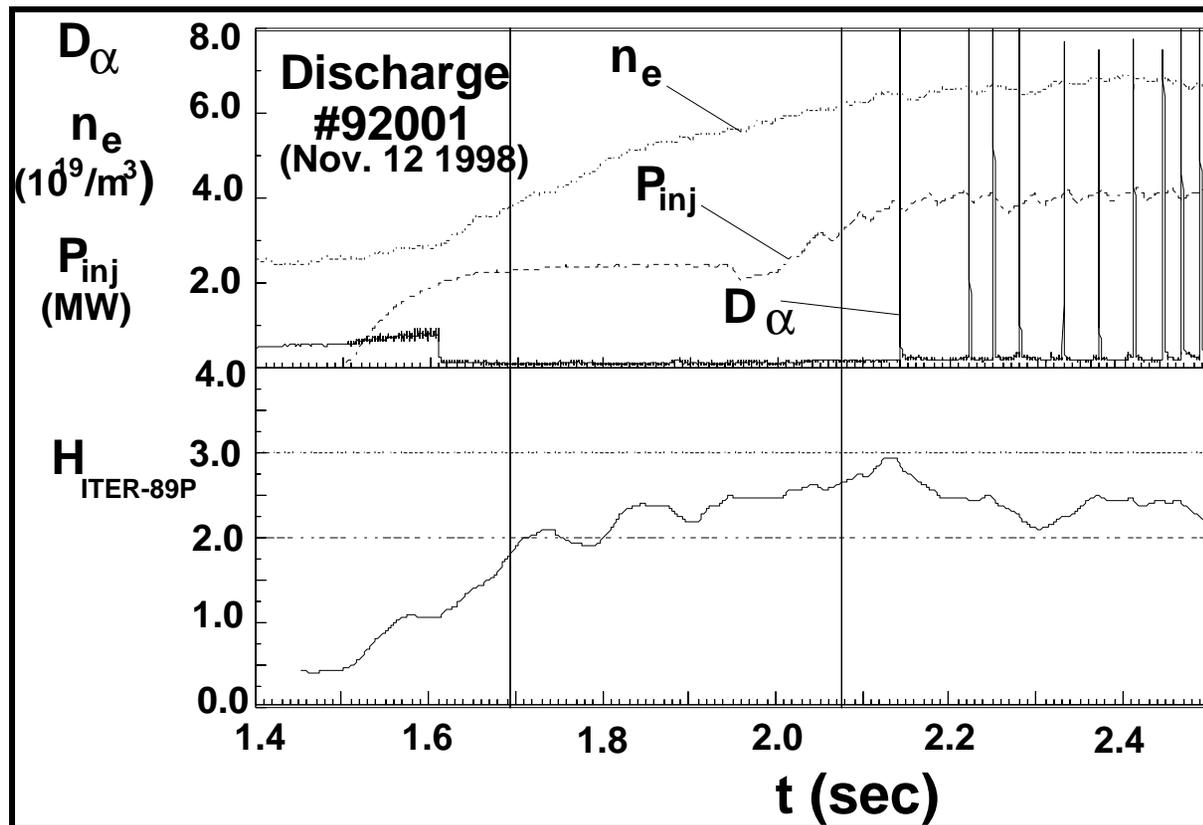
H-MODE

DIII-D DISCHARGE

92001

PLASMA IS PREDICTED UNSTABLE TO LOW n KINK - PEELING MODE BEFORE TYPE I ELM FOR DISCHARGE #92001

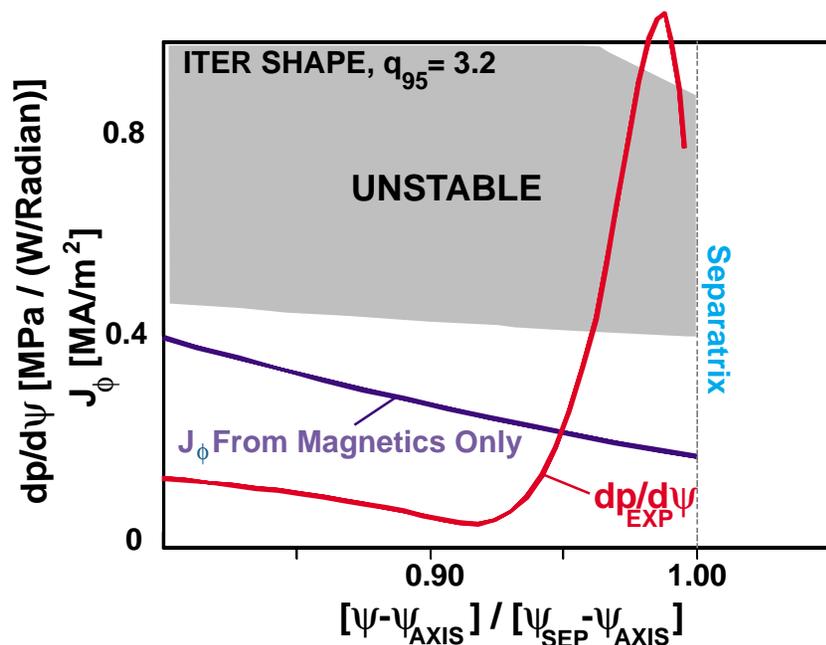
- Equilibrium and low n ($n = 1$ through 5) stability analysis performed at 1693 msec and 2075 msec



ACCURATE EQUILIBRIUM RECONSTRUCTION PROCEDURE FOR EDGE PRESSURE AND CURRENT IS CRUCIAL

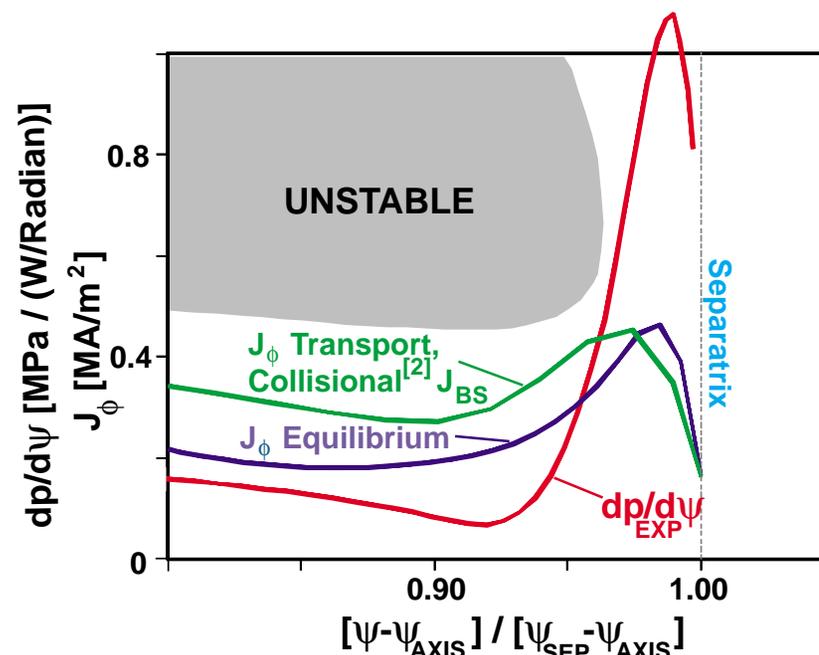
WITHOUT BOOTSTRAP CURRENT

- Measured pressure gradient exceeds first regime limit by factor ~ 2



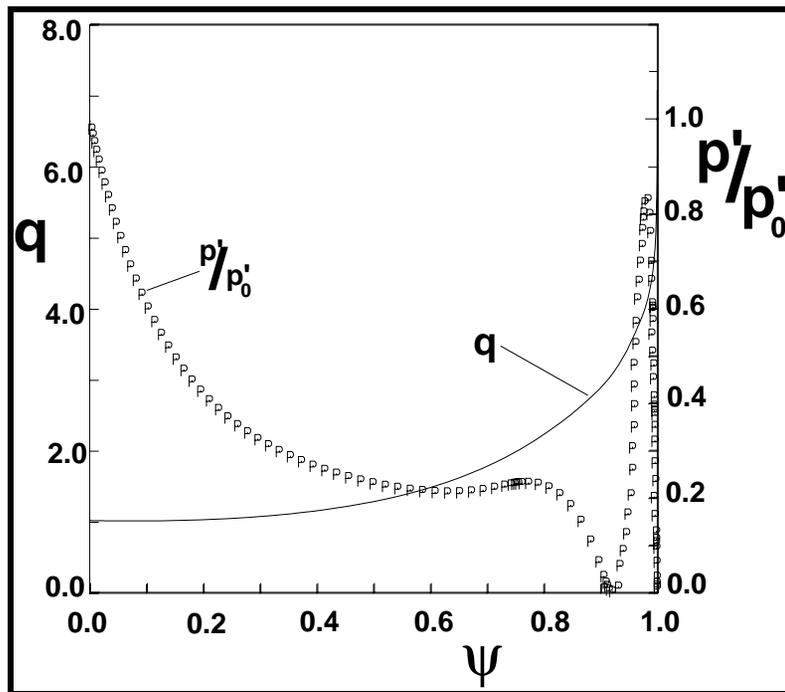
WITH BOOTSTRAP CURRENT

- Restored consistency with with edge ballooning stability

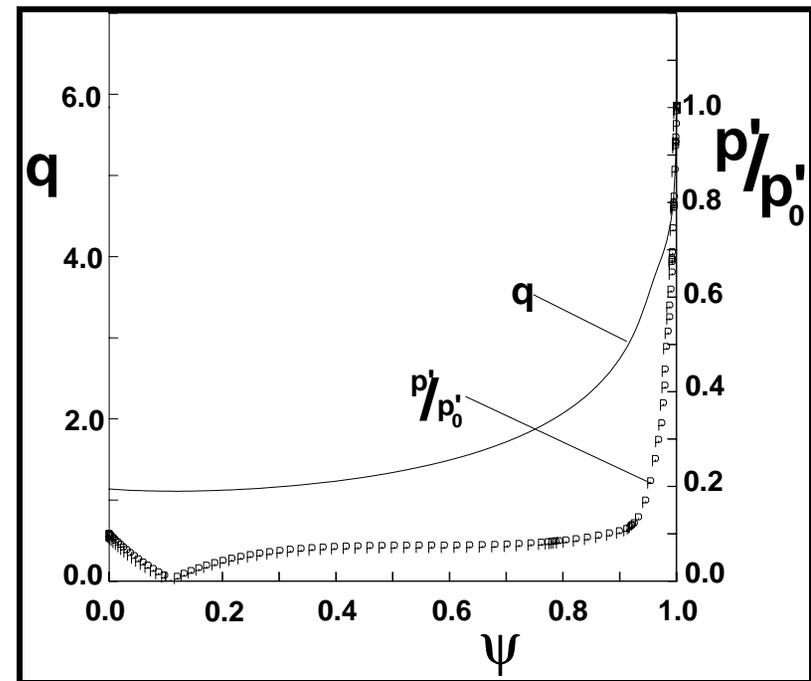


PROFILES FOR DISCHARGE #92001 SHOW A LARGE P'_{edge} IN ELM FREE PERIOD AND JUST BEFORE THE ELM

- ELM Free Period



- Just before ELM



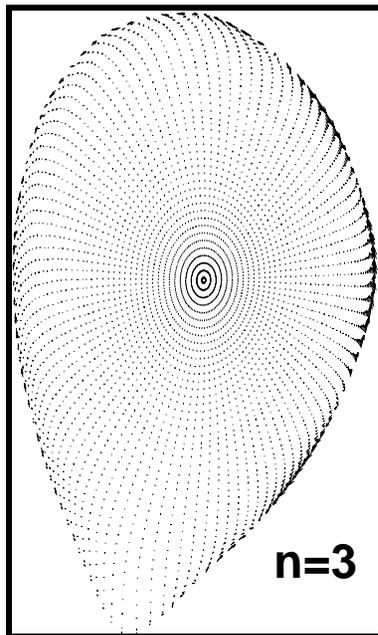
STABILITY CALCULATIONS FOR $n=1$ AND $n=2$ SHOW COMPLETE STABILITY

- Best fit to equilibrium data at 1693 msec yields $q_0 = 1.02 \pm 0.1$
 - **Unstable to $n = 1$ quasi-interchange mode**
 - » increasing q_0 to 1.05 \Rightarrow stable to $n = 1$ with or without wall
 - **Stable to $n = 2$ with or without a wall**

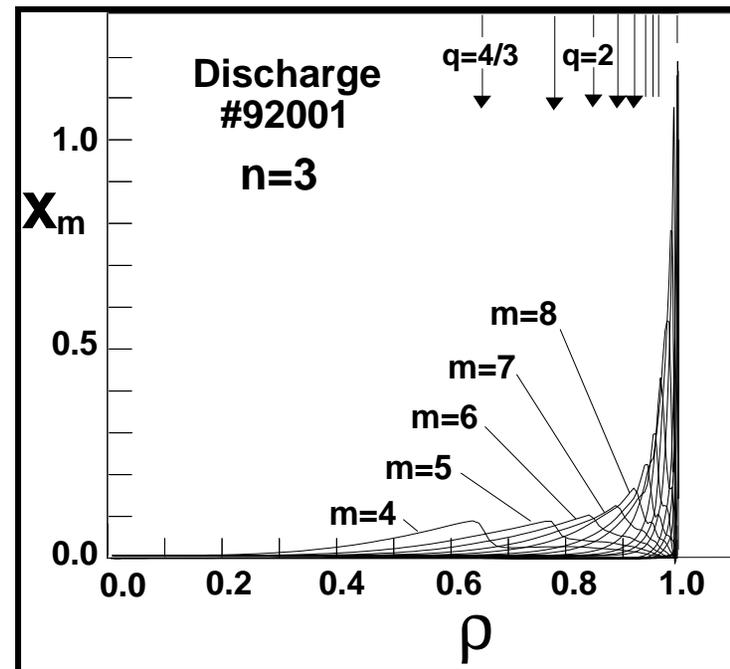
- Best fit to equilibrium data at 2075 msec yields $q_0 = 1.13 \pm 0.1$
 - **Stable to $n = 1$ with or without a wall**
 - **Stable to $n = 2$ with or without a wall**

UNSTABLE IDEAL $n=3$ MODE JUST BEFORE TYPE I ELM IS A KINK-PEELING MODE

- Mode displacement

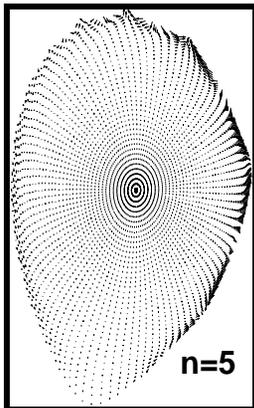
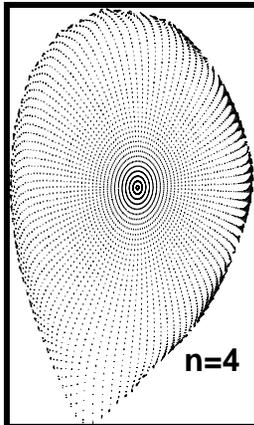


- Fourier decomposition

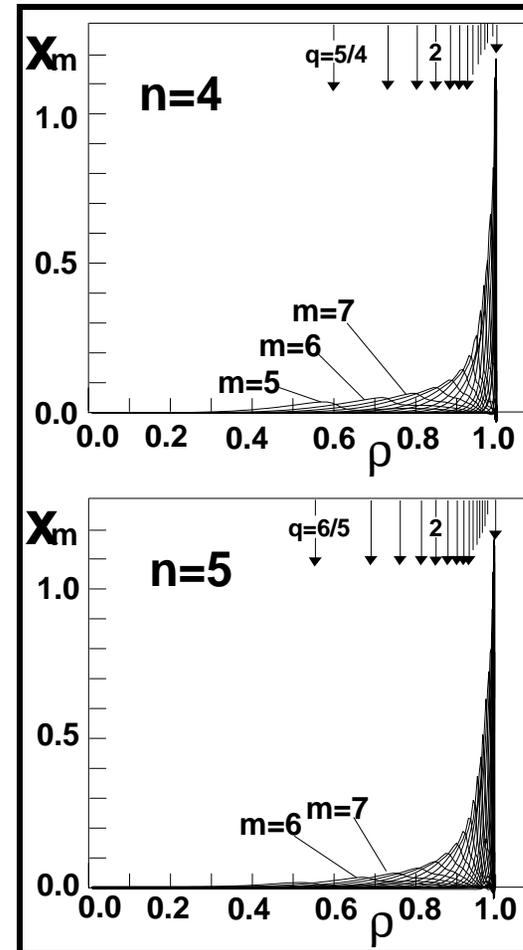


UNSTABLE IDEAL $n=4$ AND $n=5$ MODES ARE ALSO KINK-PEELING MODES

- Mode displacement



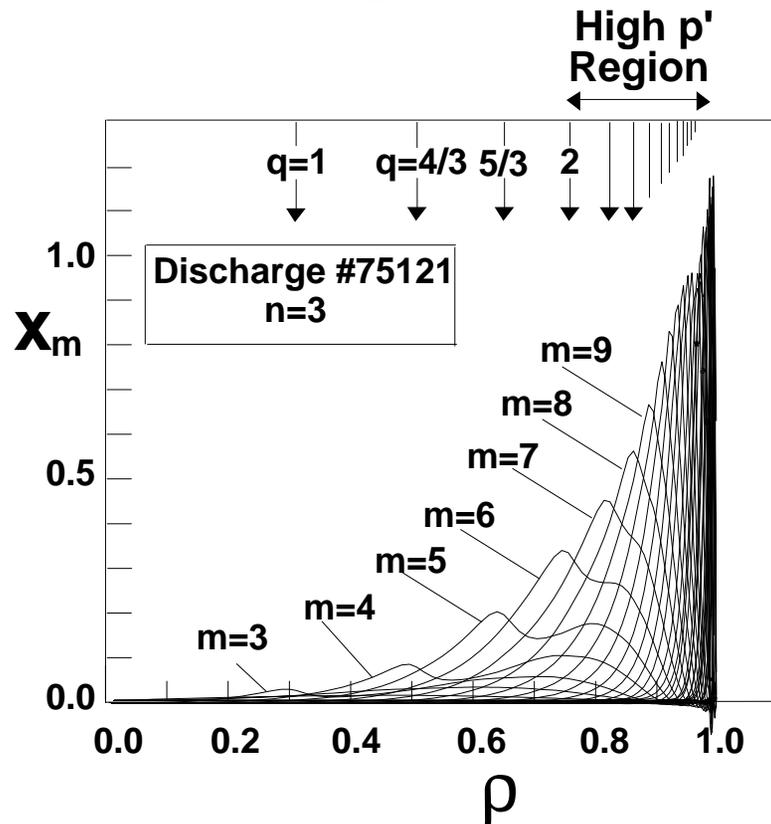
- Fourier decomposition



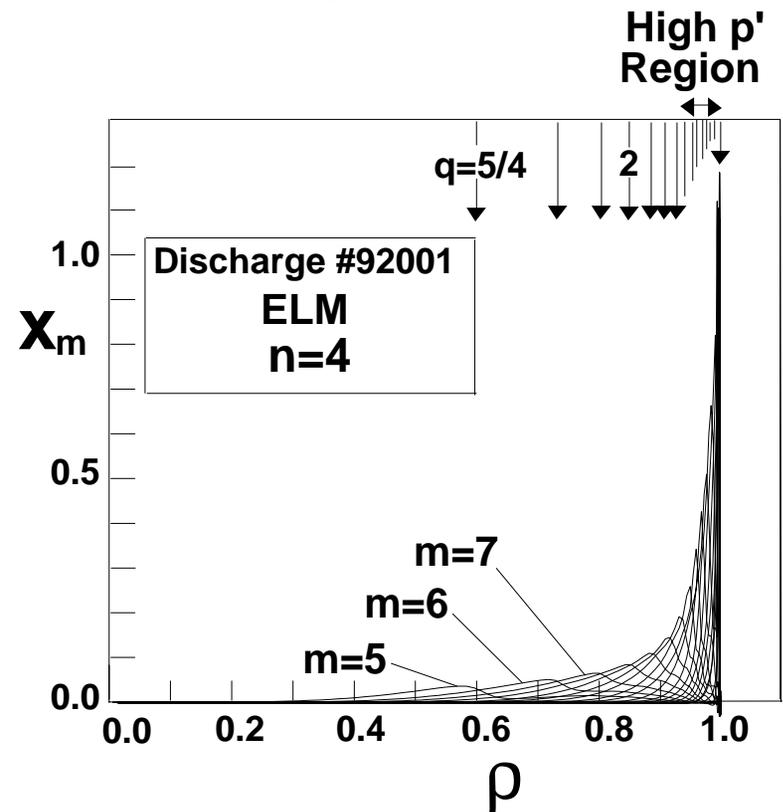
ELM IS MORE RADIALLY LOCALIZED THAN VH MODE TERMINATION INSTABILITY

- Mode width appears to correlate with width of high p' region

- **VH Termination:**
Discharge #75121 $n=3$



- **ELM:**
Discharge #92001 $n=3$



CAUTIONARY REMARKS

- **Ordinary Type I ELM instability is not always found from calculations:**
 - **Stability depends sensitively on j_{edge}**
 - **Requirement of 'reasonable' alignment of j_{BS} and j_{edge} in order to impose consistency with ballooning stability does not fully constrain j_{edge}**
 - » **j_{edge} depends on the fraction of the full collisionless bootstrap current j_{BS} that is used**
 - » **j_{BS} is reduced by collisional effects**
 - » **full alignment of j_{BS} and j_{edge} may not always be realized in individual discharges**
- **Limited comparison of predicted mode with experimental data is possible in this case:**
 - **Experimental identification of ELM toroidal mode number, real frequency, or growth, is difficult**
 - **In the few available cases, n has been measured between 2 and 9**

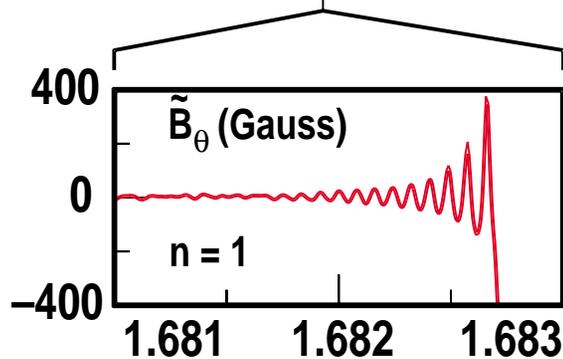
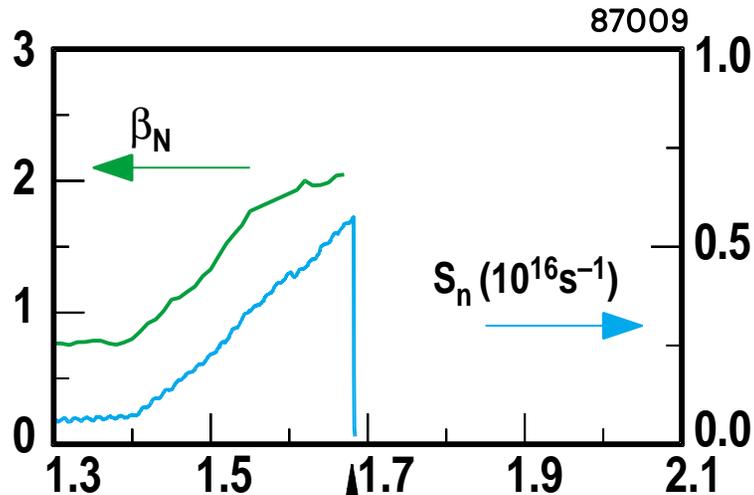
CASE STUDY #5

L-MODE

DIII-D DISCHARGE

87009

PERFORMANCE IN STRONGLY PEAKED PRESSURE NCS DISCHARGES IS LIMITED BY FAST GROWING MHD INSTABILITIES AT $\beta_N \leq 2$

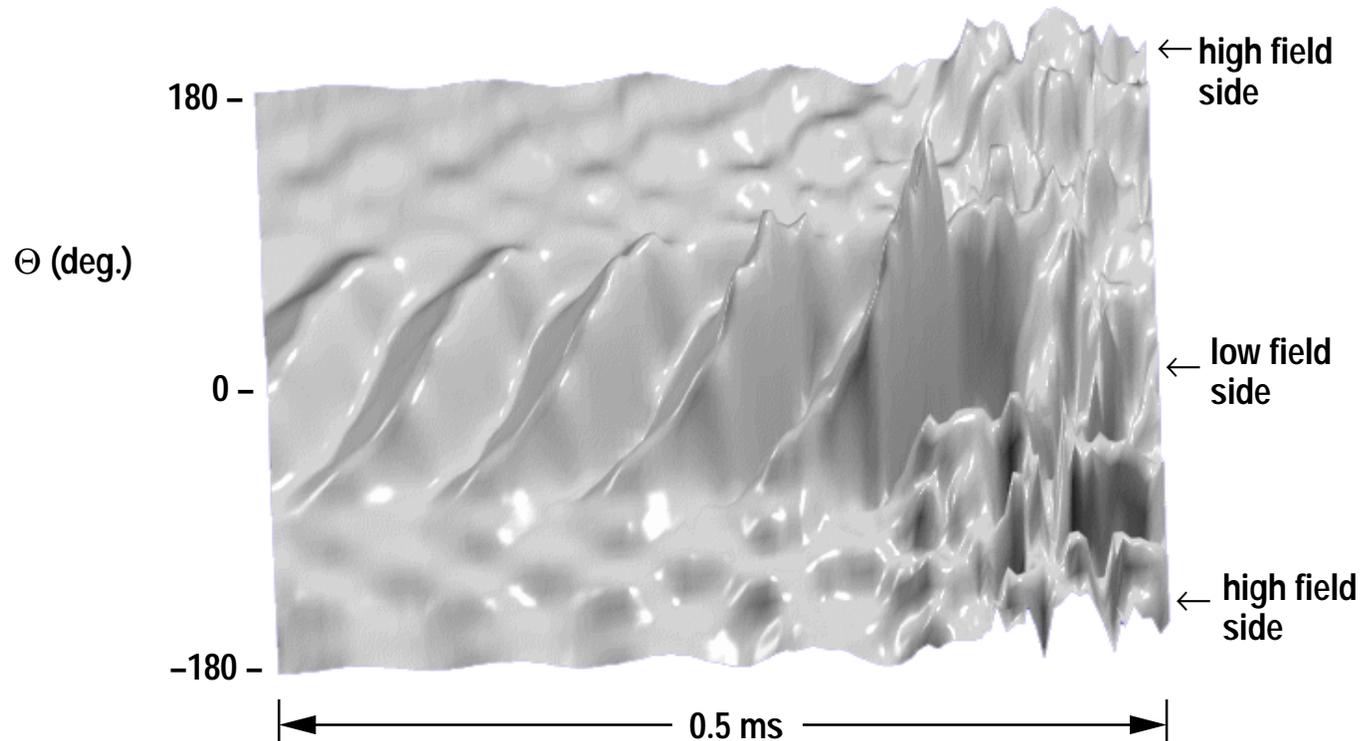


M.S. Chu, Phys. Rev. Lett. 77, 2710 (1996)

- DIII-D L-mode NCS discharges develop internal transport barrier
 - Leads to strong central pressure peaking
 - ⇒ invariably leads to disruption at $\beta_N \leq 2$ near calculated ideal and resistive β limits

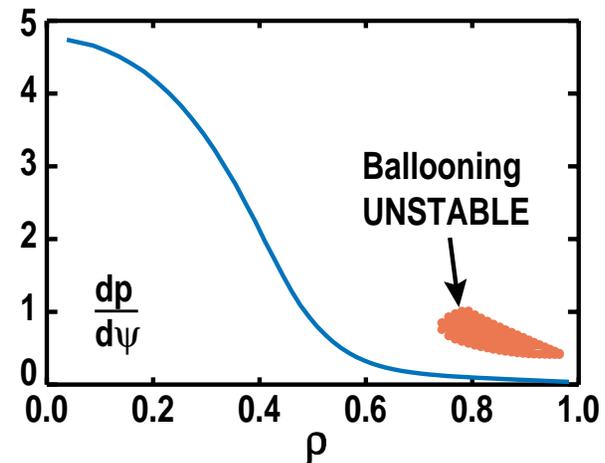
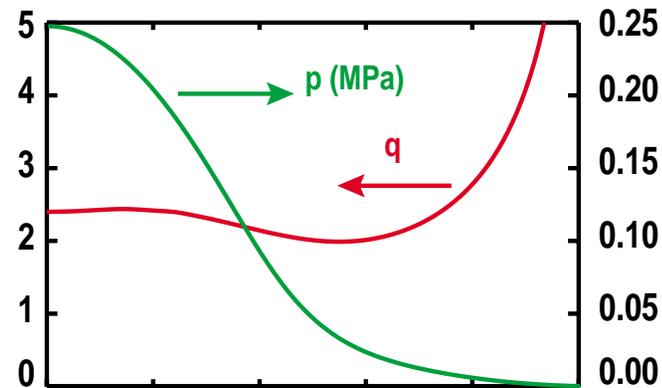
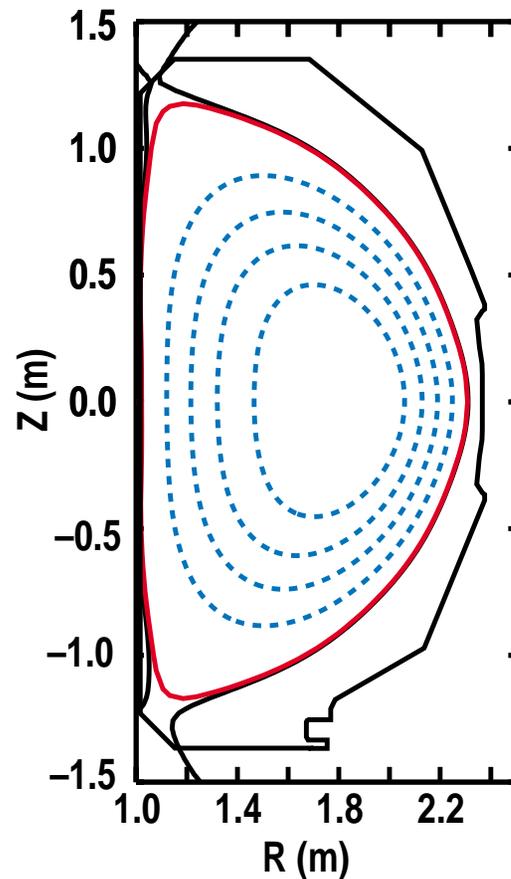
FAST-GROWING $n=1$ INSTABILITY CAUSES DISRUPTION

- Rapid growth: $\gamma^{-1} \sim 0.15$ ms
- Large amplitude: $\tilde{B}_\theta/B_\theta = 10\%$ at outer wall
- Strongly ballooning toward low field side



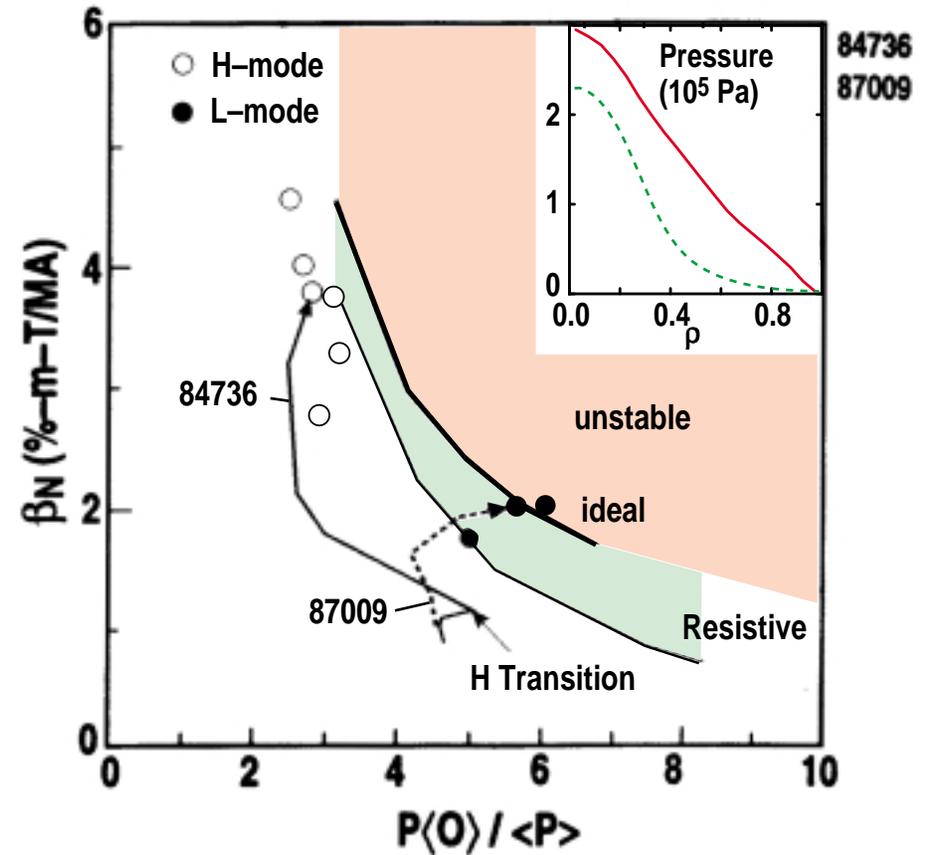
STABILITY CALCULATIONS SHOW SENSITIVITY TO PRESSURE STEEPNESS IN MINIMUM SHEAR REGION

- Standard fit indicates discharge 87009 is 25% below ideal β limit
- Fit with pressure steepened by 15% at $r \sim 0.3$ finds marginal stability



EXPERIMENTAL β LIMITS CONSISTENT WITH CALCULATED DEPENDENCE ON $p_o/\langle p \rangle$

- DIII-D high $p_o/\langle p \rangle \sim 6.0$ (L-mode):
 $\beta_N \lesssim 2.5$
 - Limited by fast $n = 1$ disruption
- Discharge #87009 disrupted as it approached ideal stability boundary at high $p_o/\langle p \rangle$
- DIII-D low $p_o/\langle p \rangle \sim 1.5$ (H-mode):
 reach $\beta_N \lesssim 4$
 - No disruption limited by ELM-like activity from finite edge pressure gradients



OBSERVED MODE GROWTH IS CONSISTENT WITH IDEAL $n = 1$ KINK DRIVEN THROUGH INSTABILITY THRESHOLD

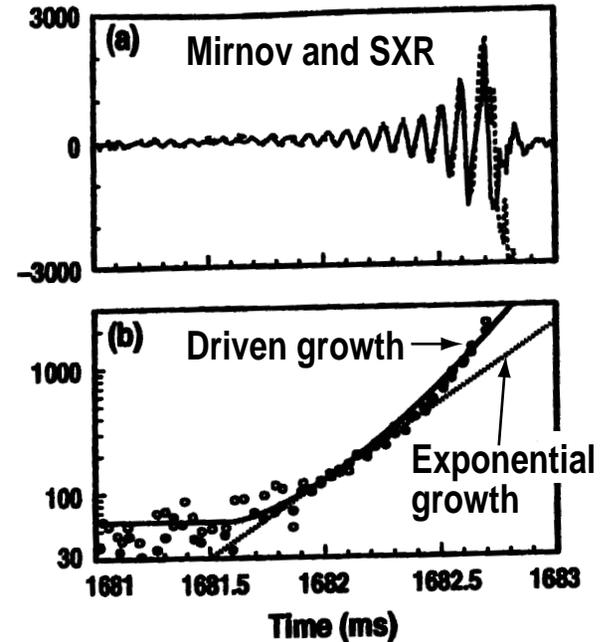
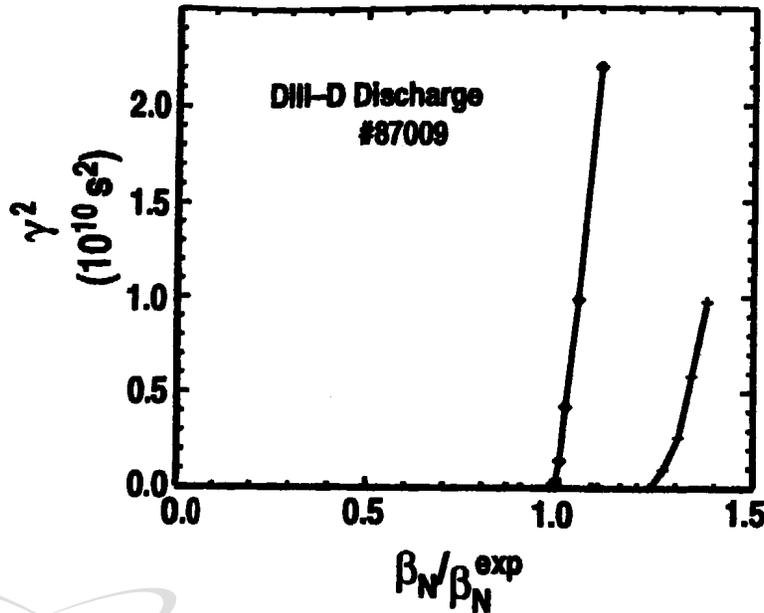
- Driven instability model (Callen et al, 1999)

$$\gamma \sim \exp [(t/\tau)^{3/2}] \quad \tau \sim (2/3)^{2/3} \hat{\gamma}_{\text{MHD}}^{-2/3} \gamma_h^{-1/3}$$

- $\hat{\gamma}_{\text{MHD}}$ predicted from stability calculations $\sim 3 \times 10^5 \text{ s}^{-1}$

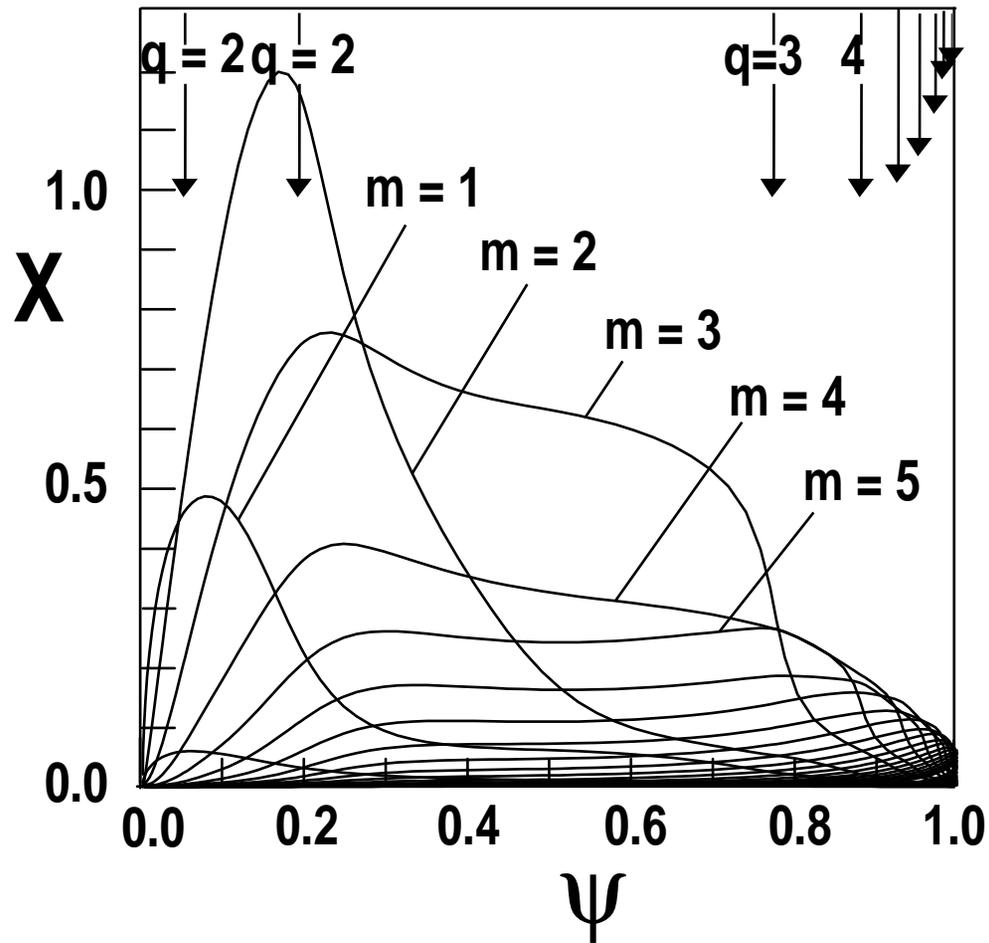
- Growth fits model with

$$\gamma_{\text{MHD}} \sim 1.10 \times 10^5 \text{ s}^{-1}$$



COMPUTED INSTABILITY WITH STEEPENED PRESSURE IS A GLOBAL $n = 1$ KINK PEAKED IN THE CORE

- Steepened fit is consistent with diagnostics
- Predicted ideal kink: p^l ($\rho = 0.3$) steepened by 15%
 β_N increased by 10% above marginal



CASE STUDY #6

L-MODE NCS

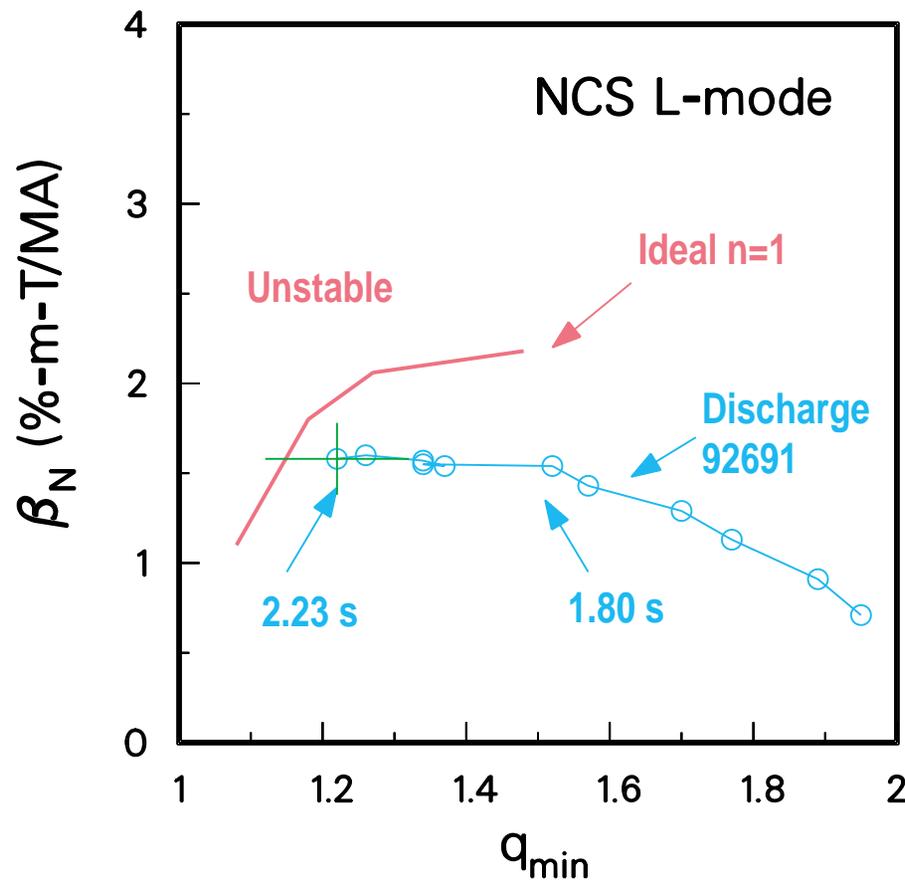
DIII-D DISCHARGE

92691

Case Study #6
L-mode
#92691

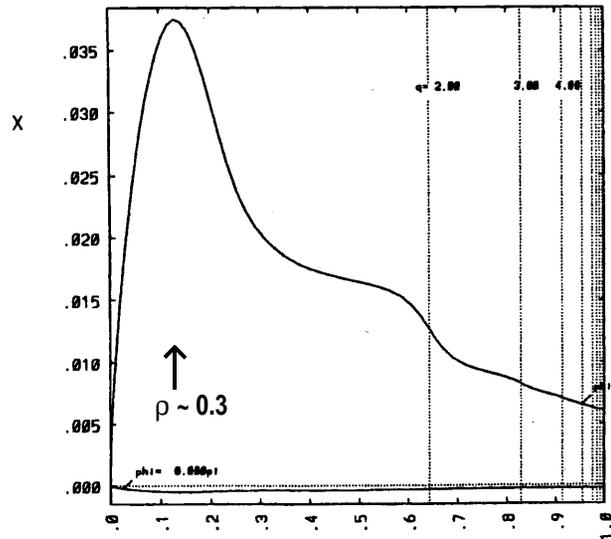
DISCHARGE #92691 WITH LOW q_{\min} AND HIGH $p_0/\langle p \rangle$ DISRUPTED WHEN THE IDEAL STABILITY BOUNDARY WAS CROSSED

- Ideal simulations using GATO based on equilibrium at 2.2 s with $P_0/\langle P \rangle = 6.6$

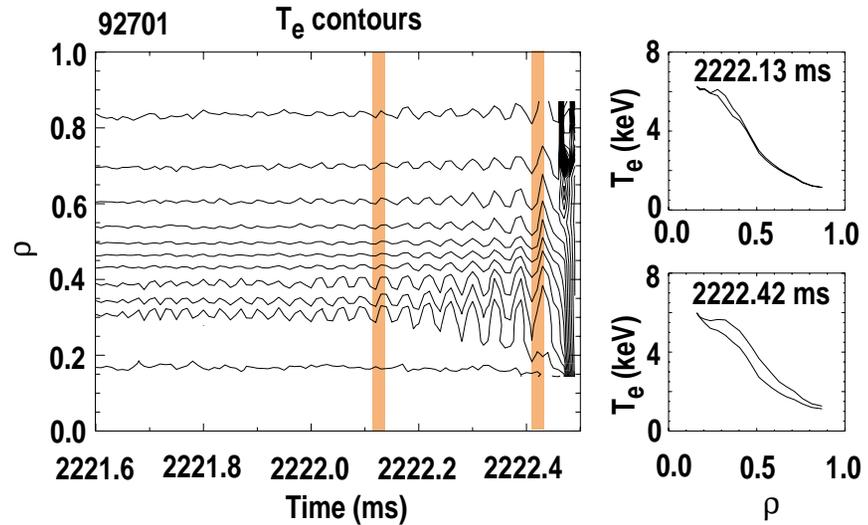


INTERNAL MODE STRUCTURE IS CONSISTENT WITH IDEAL INSTABILITY

- The observed instability has an ideal MHD character
 - No evident island structure
- Radial structure is confirmed by SXR and BES measurements



GATO Simulation



Strait, APS, 1997

CASE STUDY #7

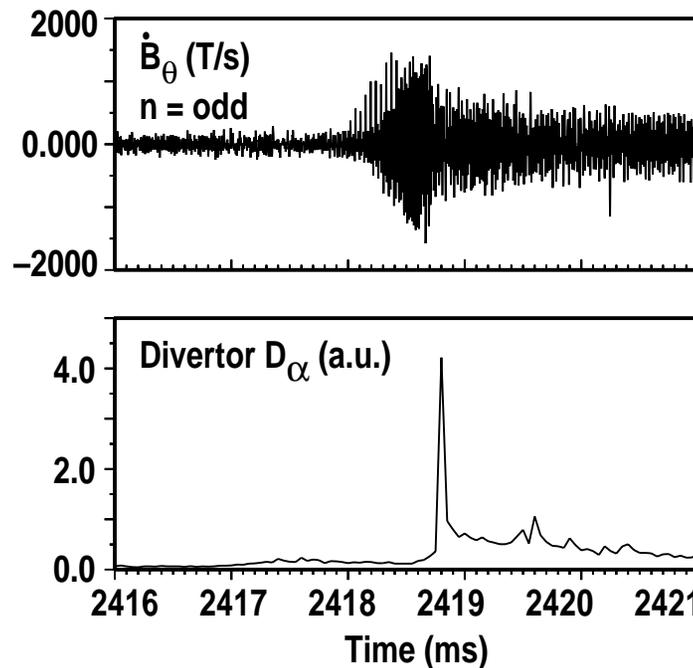
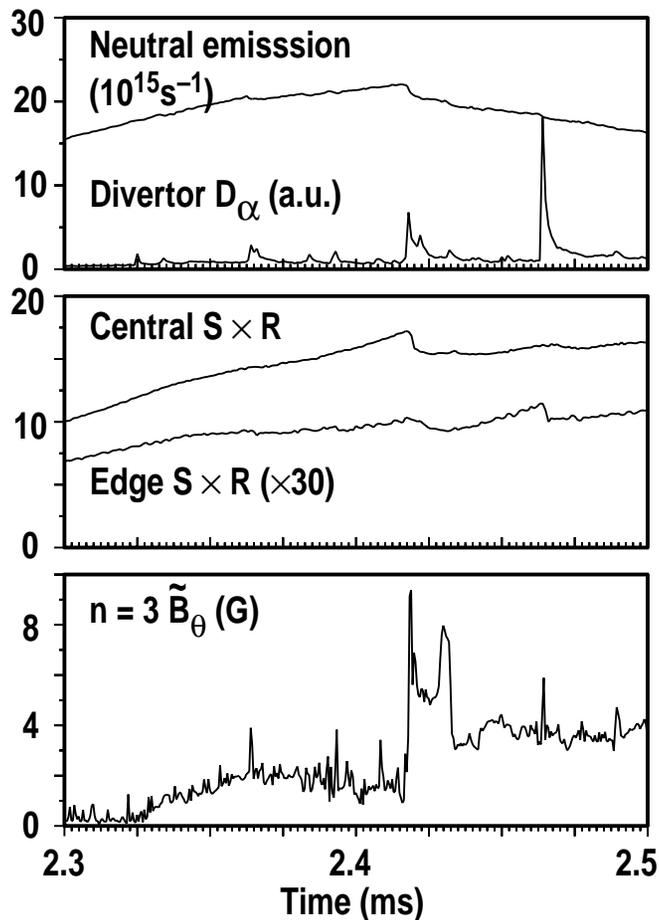
H-MODE NCS

DIII-D DISCHARGE

87937

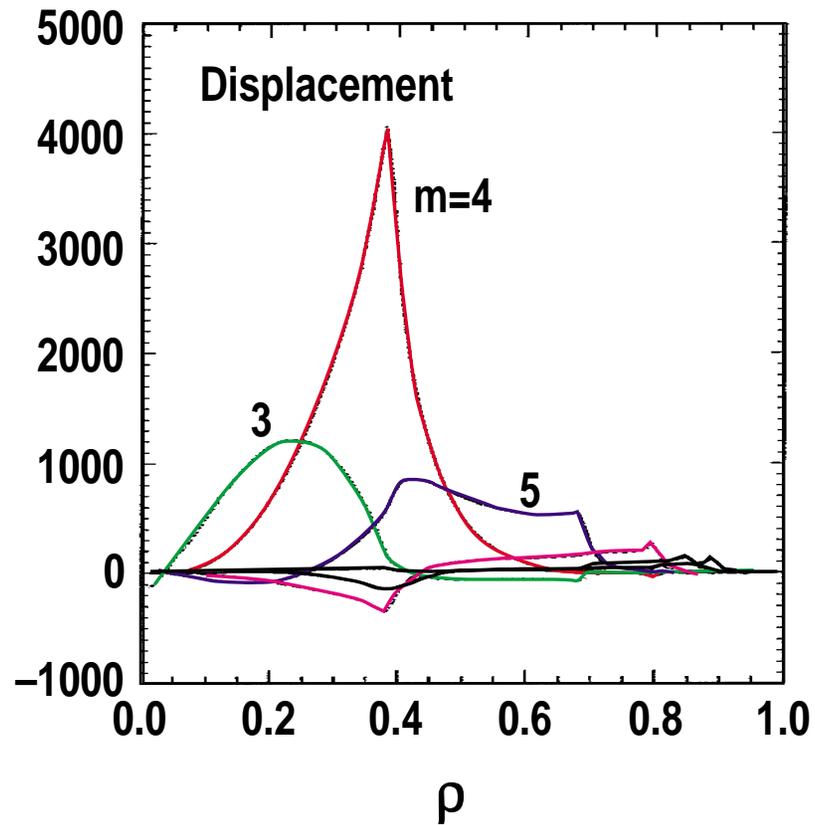
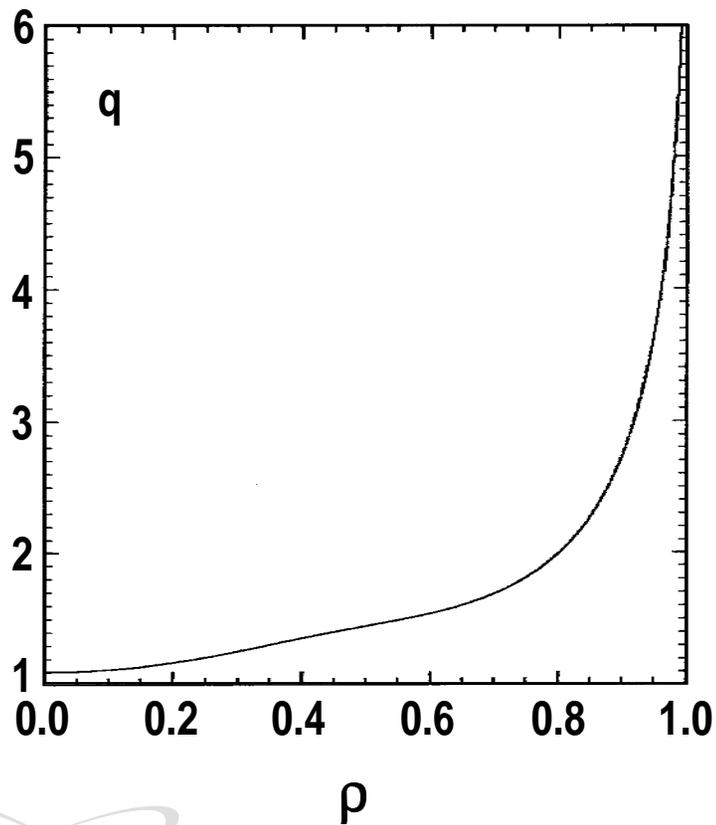
INTERNAL $n = 3$ MODE LIMITS THE CENTRAL PRESSURE IN SOME H-MODE NCS DISCHARGES

- Rapid growth = $\gamma^{-1} \sim 0.2$ ms
- Initial energy loss occurs in the plasma core



IDEAL $n = 3$ INFERNAL MODE IS PREDICTED MOST UNSTABLE

- $n=3$ becomes unstable when β is raised to 1.05 times the experimental value. ($n=1$ and 2 remain stable)
- Dominant $m/n = 4/3$ agrees with experimental mode identification
- Centrally peaked mode structure is consistent with observed energy loss



CASE STUDY #8

H-MODE NCS

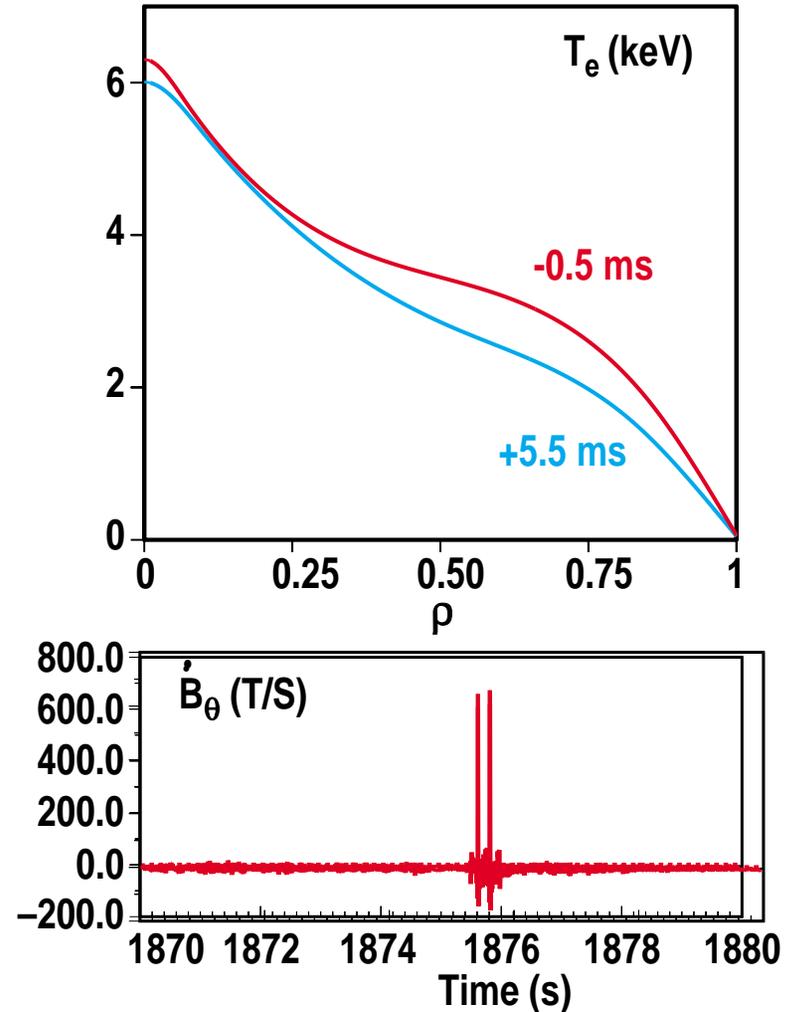
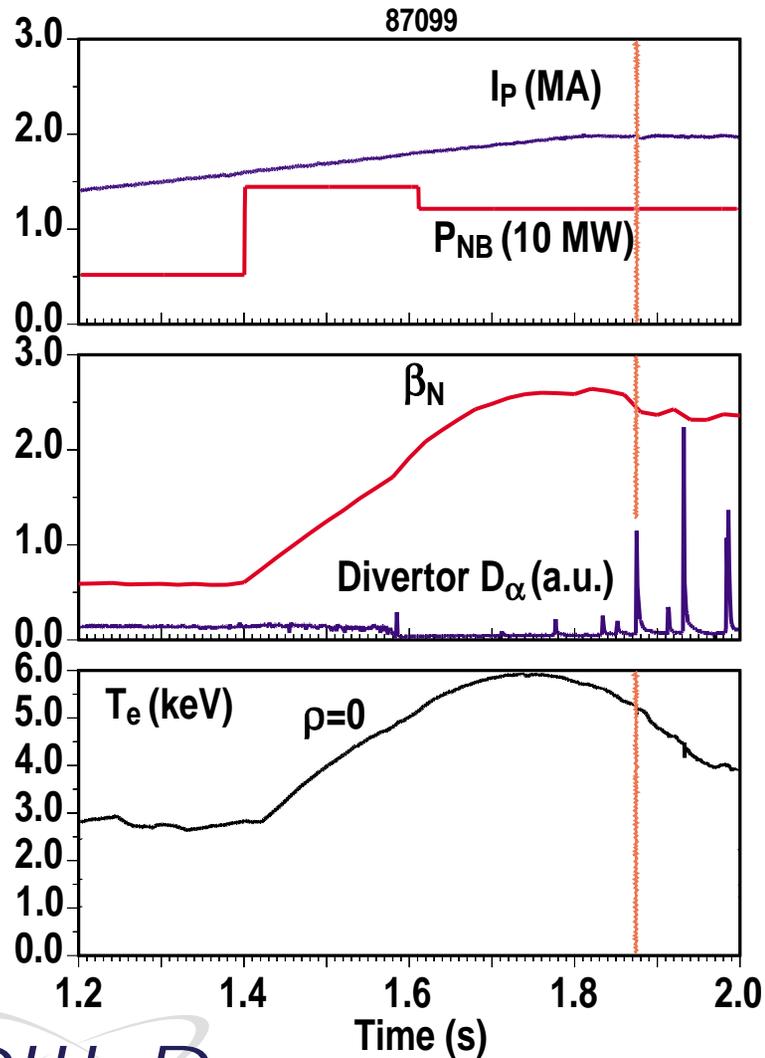
DIII-D DISCHARGE

87099



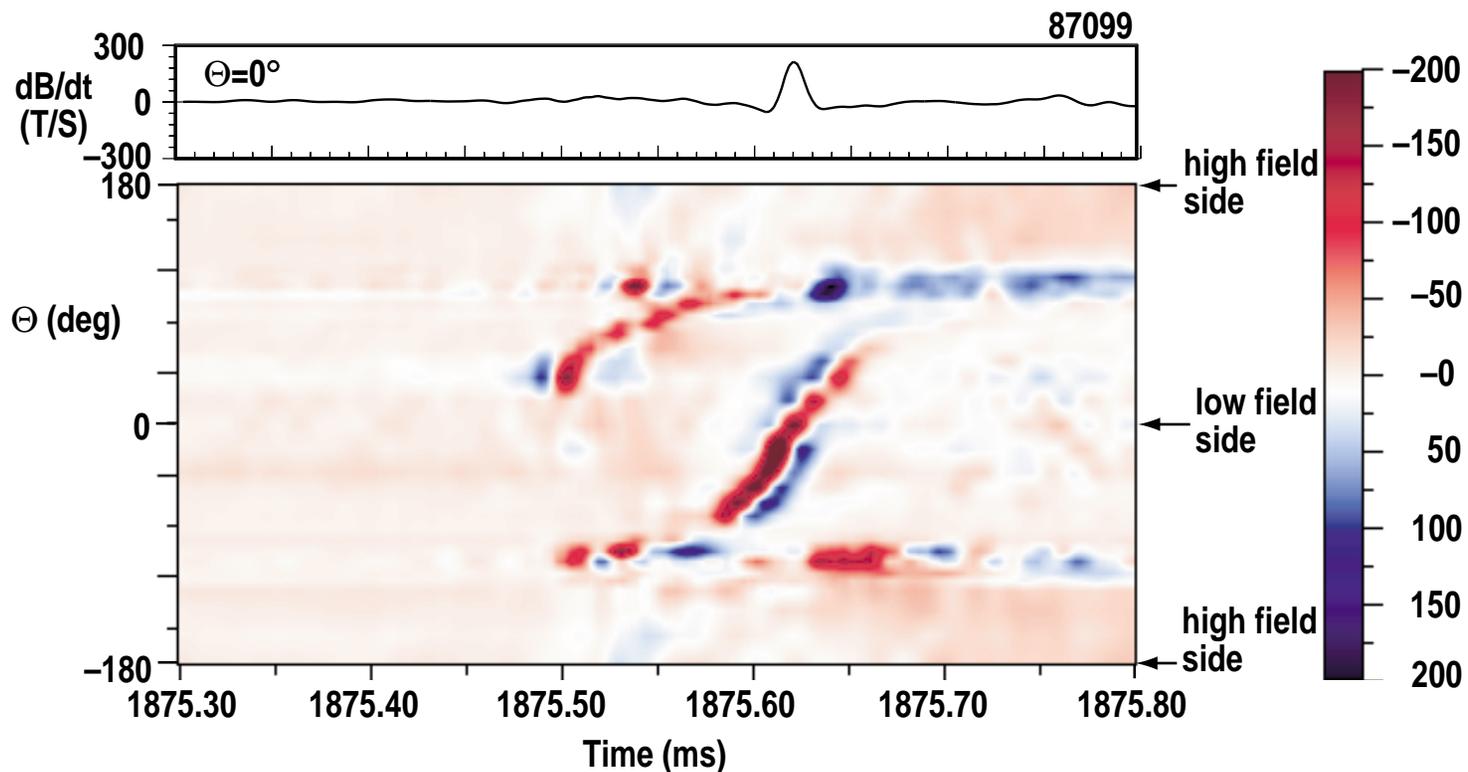
MODERATE n MODES LIMIT EDGE PRESSURE GRADIENT IN HIGHEST PERFORMANCE H-MODE NCS DISCHARGES

- Early heating creates broad current density profile ($I_i \sim 0.75$)
- Initial energy loss occurs at the edge



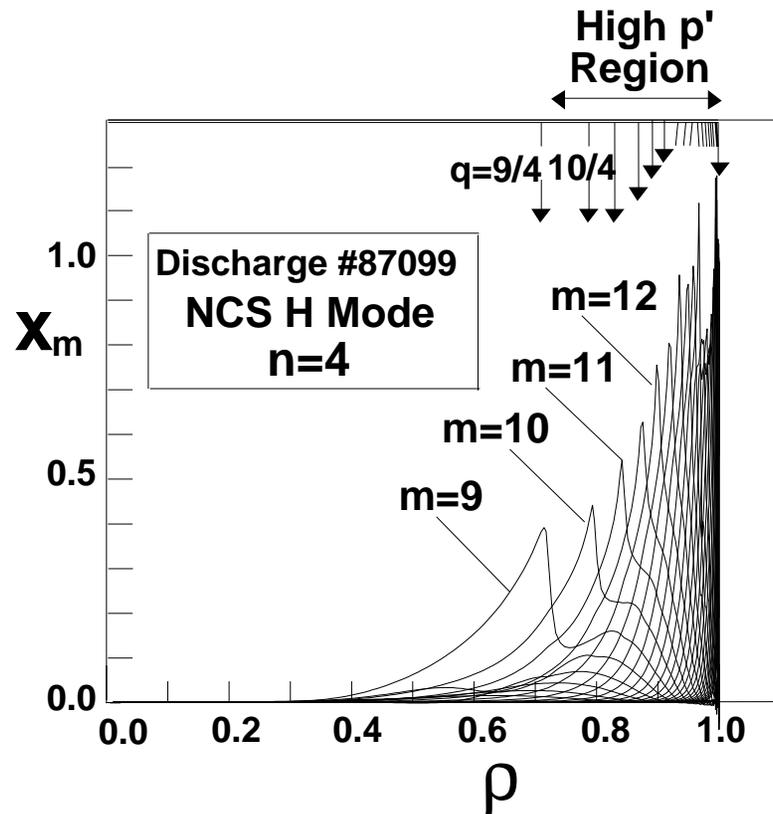
EDGE-DRIVEN MODE HAS A LOCALIZED/BALLOONING CHARACTER

- Instability consists of a single pulse
 - Wavelength corresponds to $n \sim 5$
- Amplitude peaks at the low field side (out board midplane)
- Rapid growth: $\gamma^{-1} \lesssim 0.15$ ms
- Rotation in the electron diamagnetic direction is consistent with localization near the plasma edge

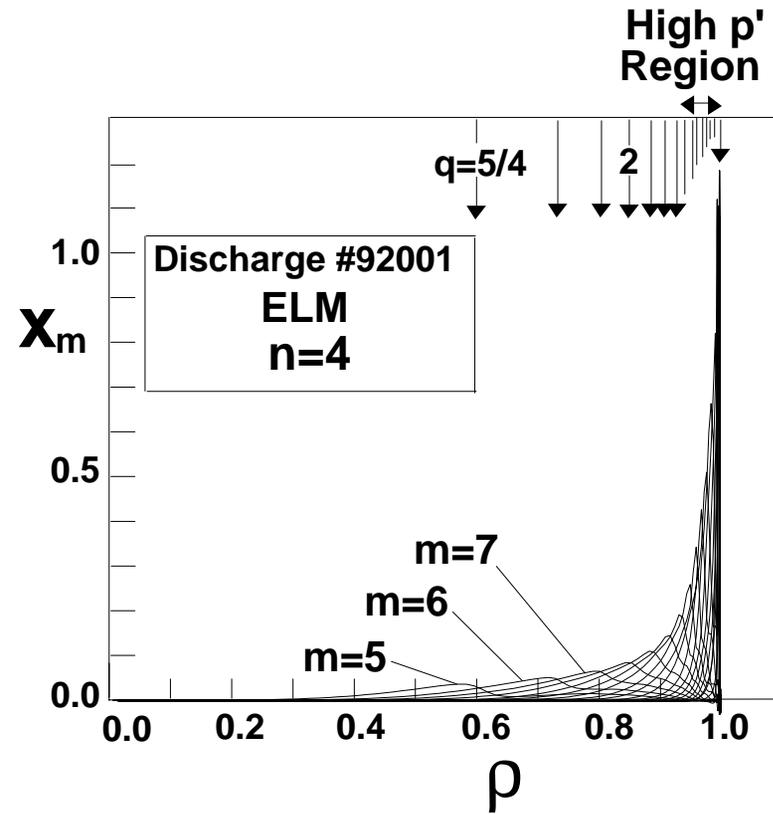


NCS H MODE TERMINATION INSTABILITY LESS LOCALIZED THAN STANDARD H MODE ELM

- **NCS H Mode Termination:**
Discharge #87099 n=4
($\beta_N \times 110\%$ DIII-D Wall)



- **ELM:**
Discharge #92001 n=4



CASE STUDY #9

WALL STABILIZATION

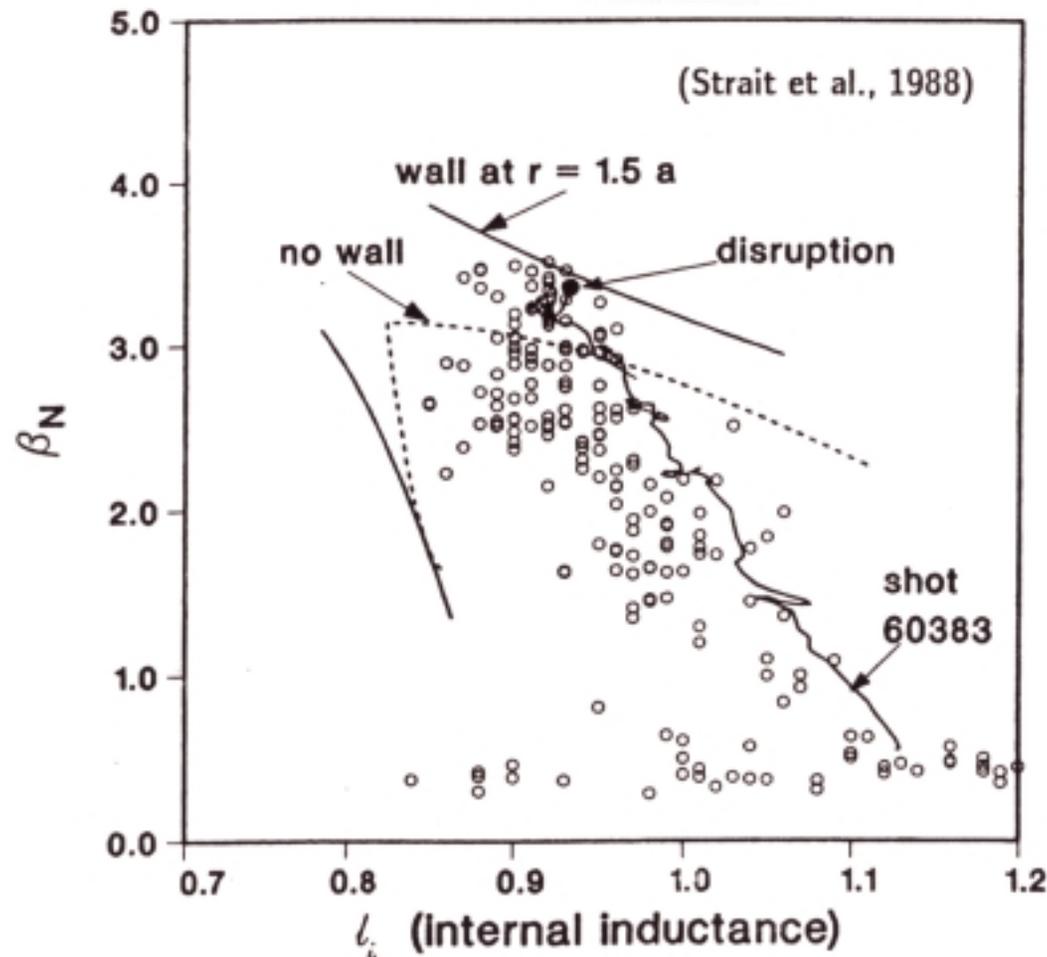
DIII-D DISCHARGE

870111



PREVIOUS DIII-D EXPERIMENTS ARE CONSISTENT WITH WALL STABILIZATION

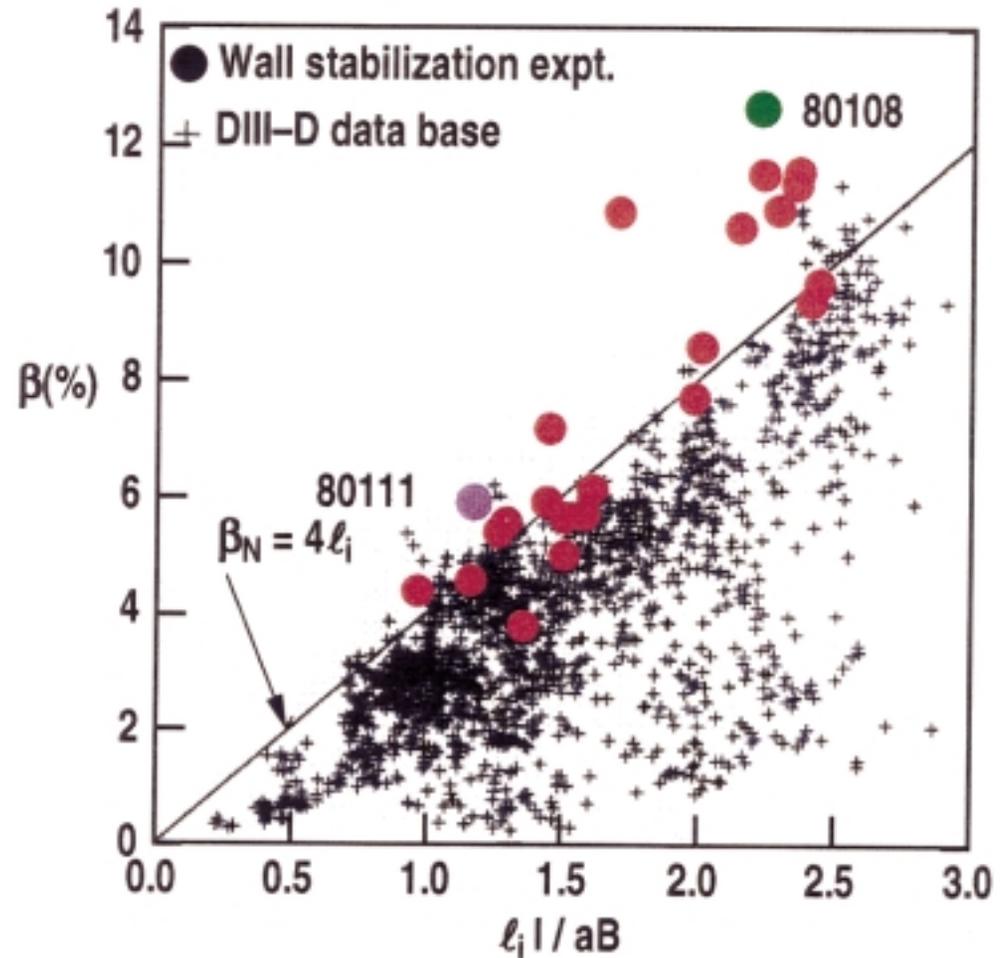
- Experimental β_N value never exceeds stability limit computed with an ideal wall but often exceeds limit computed with no wall (E.J. Strait, et al., EPS 1988)



- Stabilization from well known profile effects could not be ruled out

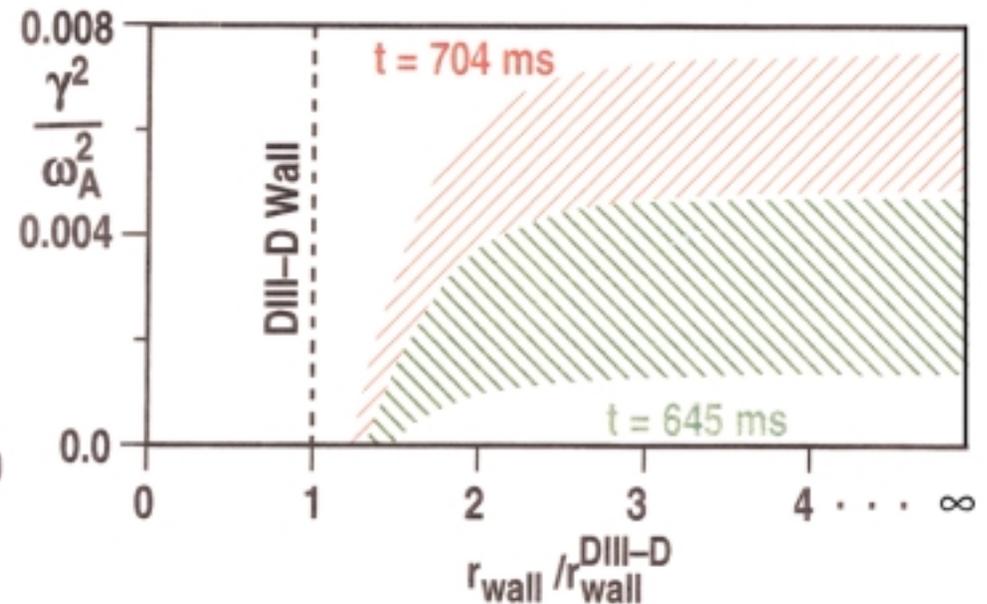
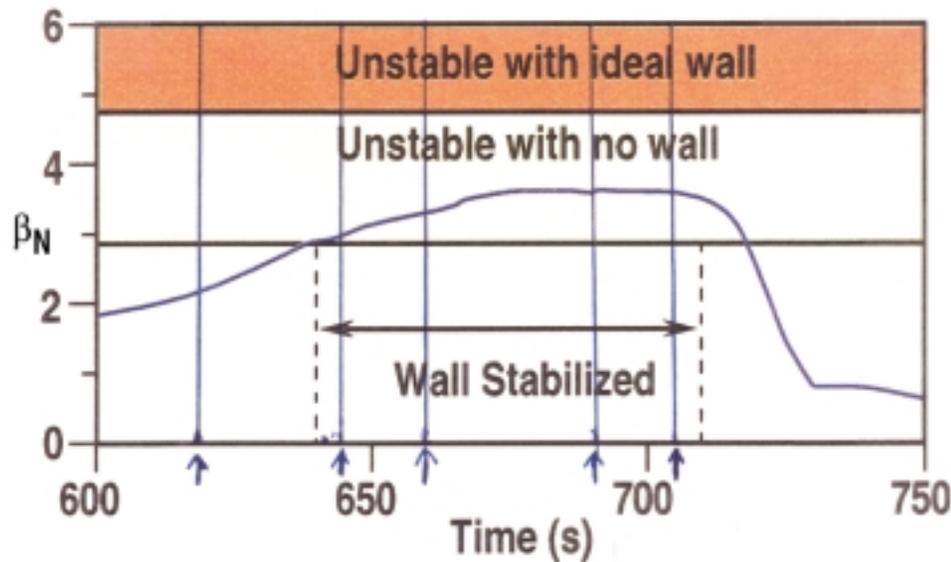
DIII-D WALL STABILIZATION EXPERIMENTS IN 1994 EXCEEDED THE EARLIER $4I_i$ SCALING

- Previous $\beta_N \sim 4I_i$ scaling for DIII-D is an operational β limit for plasmas with moderate to high I_i
- Wall stabilization experiments used current ramp and strong shaping to increase coupling between plasma and wall



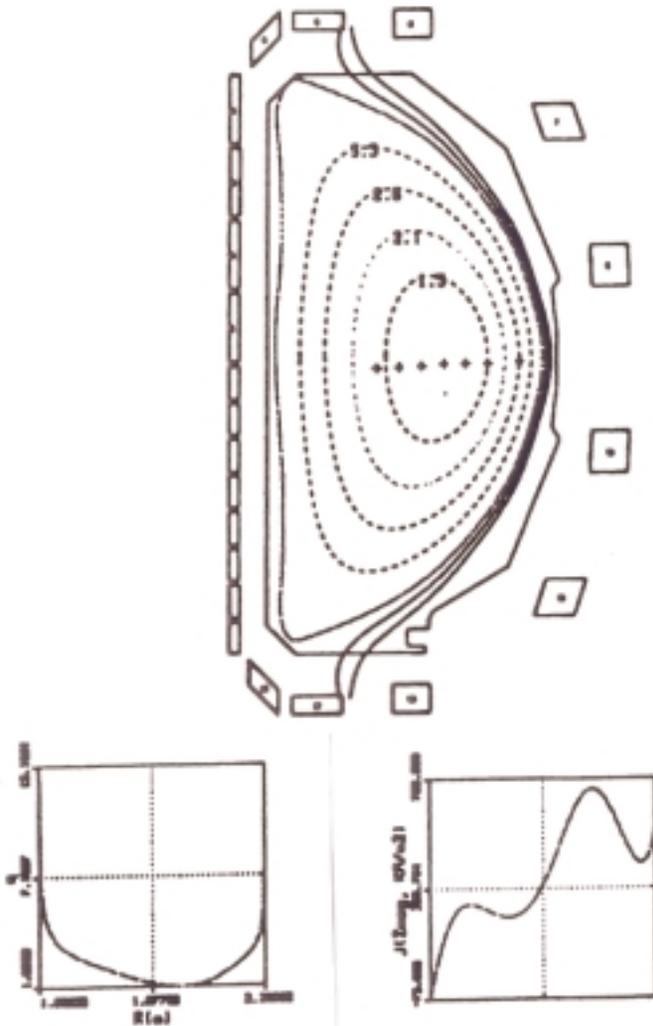
DISCHARGE 80111 IS WALL STABILIZED FOR 30 WALL PENETRATION TIMES

- Discharge 80111 is wall stabilized for at least 60 ms, from 640 to 704 msc
- No ideal or resistive mode observed during this time
- Equilibrium reconstructions and ideal stability calculations performed at 5 time slices



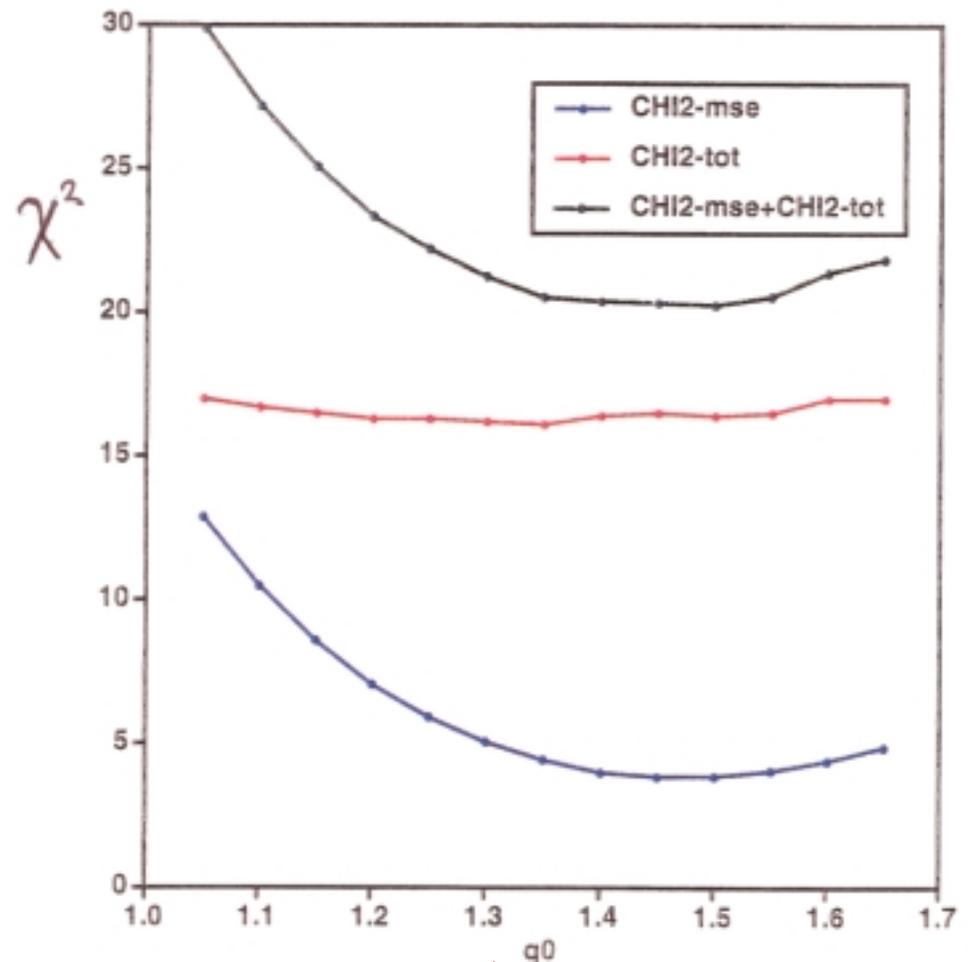
COMPLETE EQUILIBRIUM RECONSTRUCTION AND SENSITIVITY STUDIES DEMONSTRATE DISCHARGE 80111 IS ABOVE NO WALL LIMIT

- Equilibrium Reconstruction



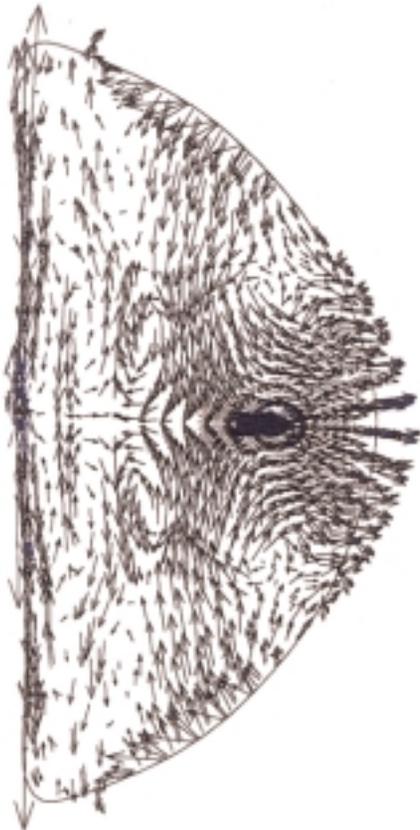
ADT-APS 99

- Sensitivity of equilibrium reconstruction to variations in q_0



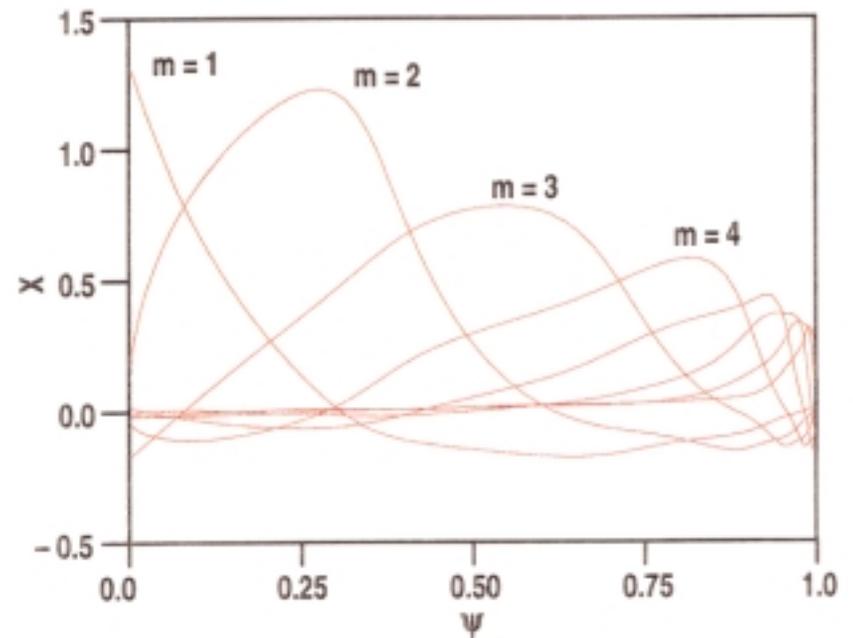
PREDICTED IDEAL INSTABILITY WITH NO WALL IS A GLOBAL $n = 1$ KINK MODE

- **Mode Displacement**



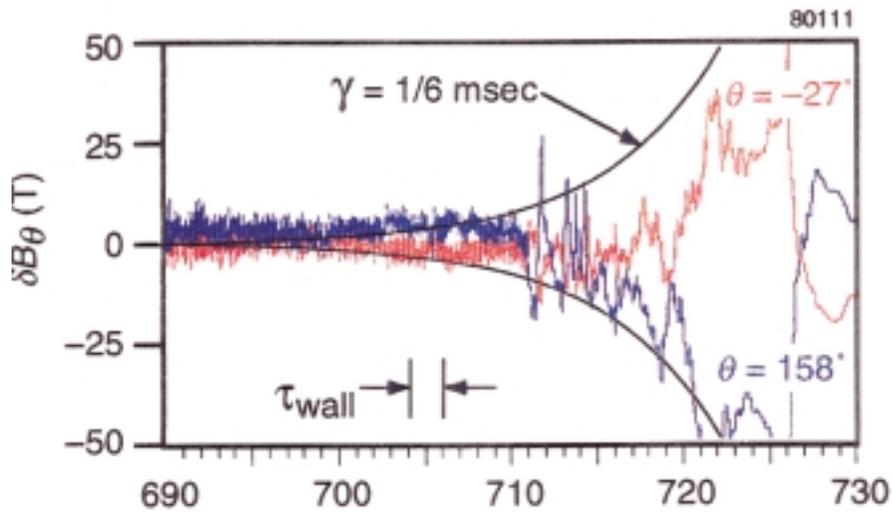
- **Fourier Decomposition**

$$X = \underline{\xi} \cdot \nabla \Psi / |\nabla \Psi|$$

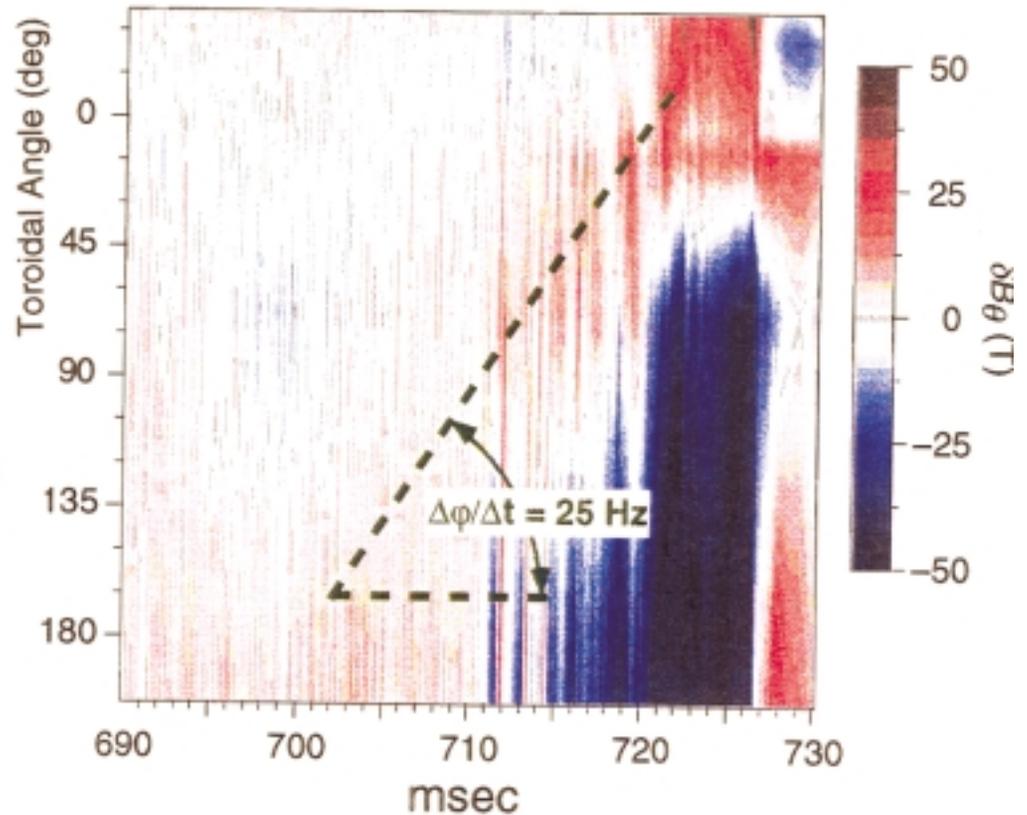


β COLLAPSE IS DUE TO A SLOWLY ROTATING AND SLOWLY GROWING 3/1 MODE

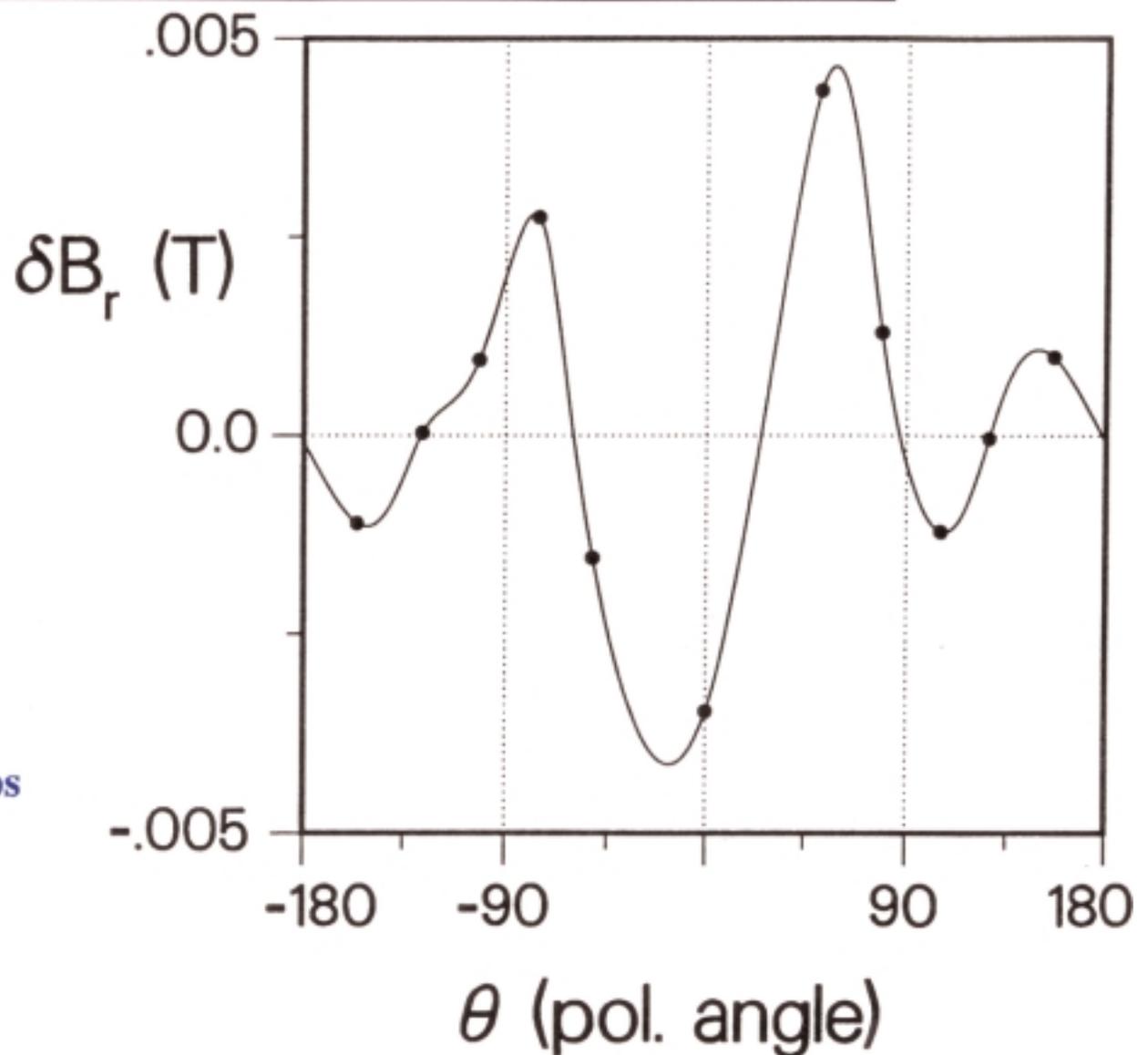
Saddle loop growth rate ~ 6 ms



Real frequency from SXR ~ 25 Hz



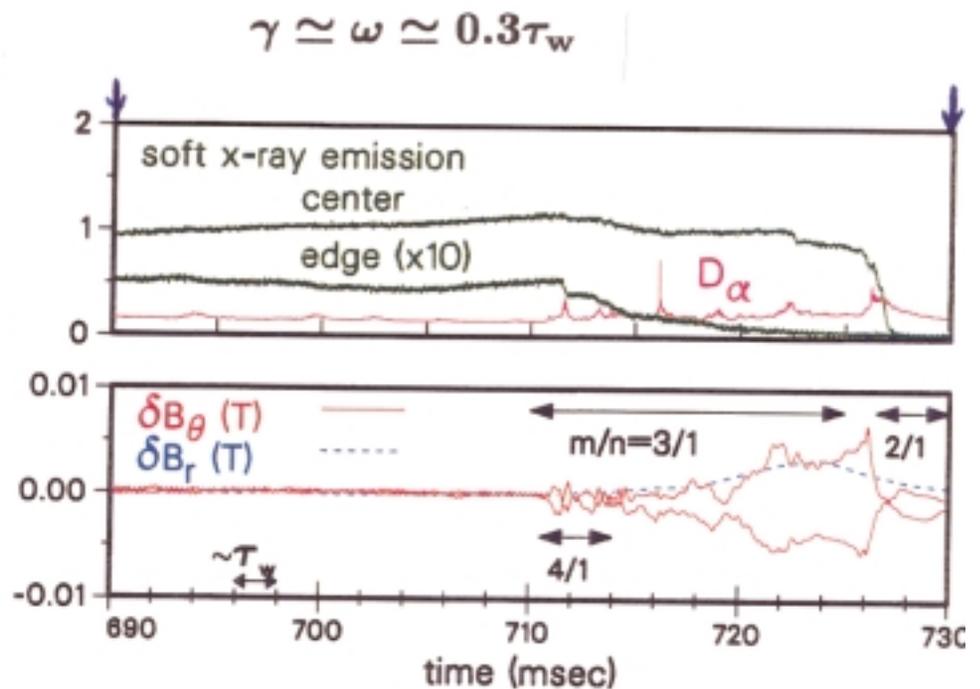
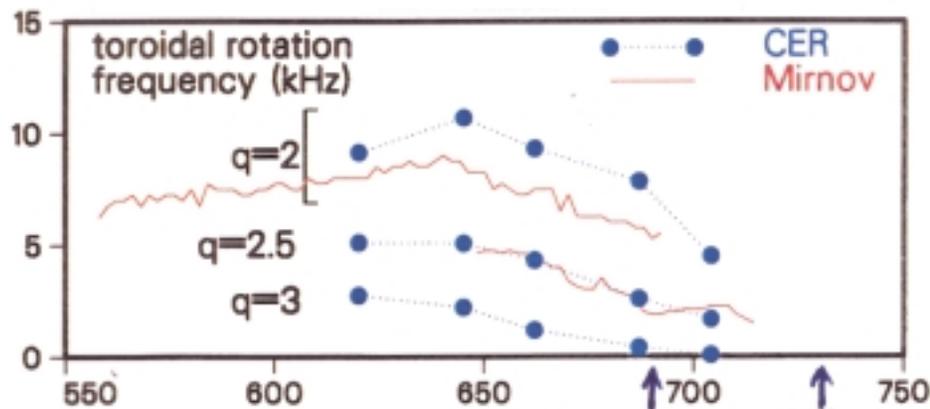
SADDLE LOOPS CONFIRM SLOWLY GROWING INSTABILITY IS AN $m/n = 3/1$ MODE



- Full toroidal saddle loop array $\Rightarrow n = 1$
- Difference of saddle loops 90° apart $\Rightarrow m = 3$

COLLAPSE AT HIGH β_N OCCURS WHEN PLASMA ROTATION NEAR $q = 3$ HAS DECAYED

- Theory predicts wall stabilization is lost when the plasma stops rotating
- Plasma rotation decays steadily during wall stabilized period
- SXR, saddle loops, and Mirnov array show mode grow in last phase



CASE STUDY #10

WALL STABILIZATION

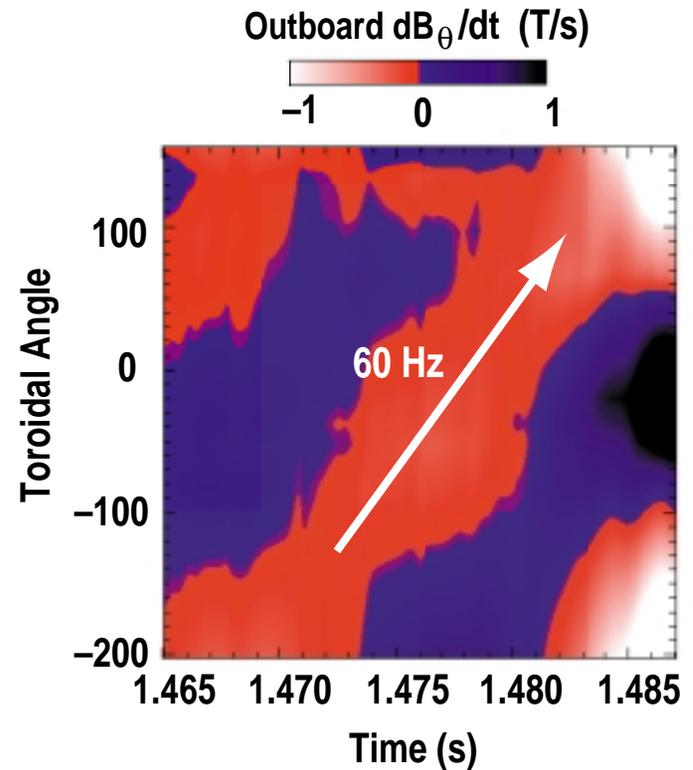
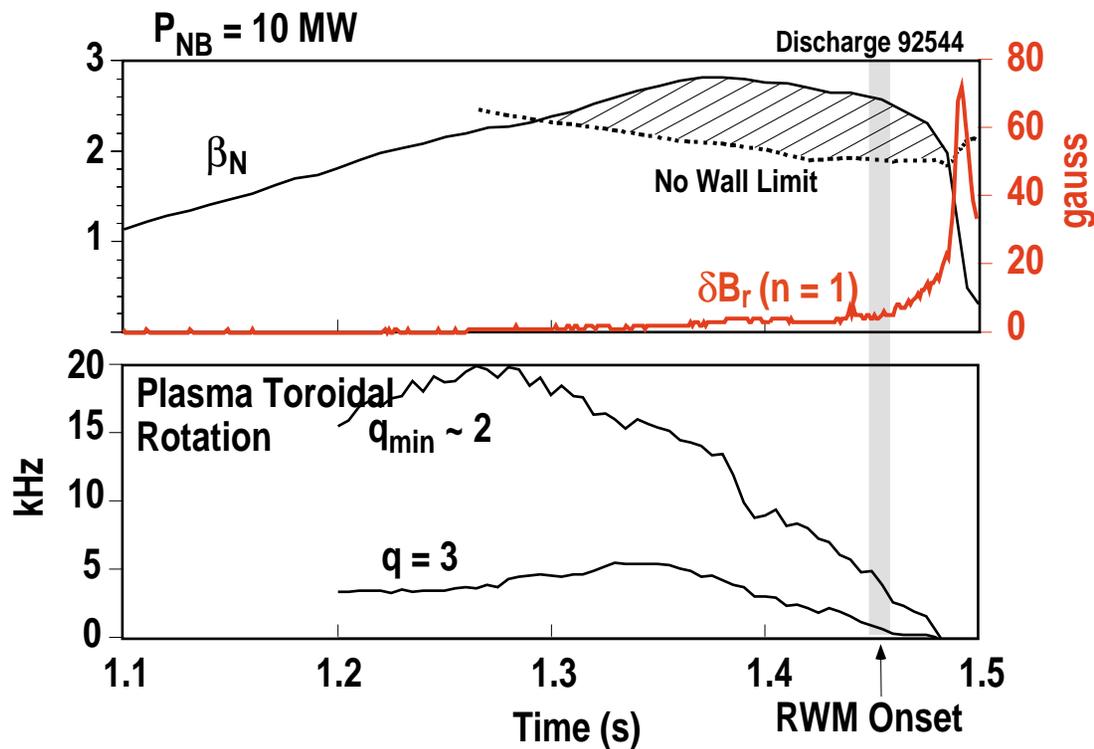
DIII-D DISCHARGE

92544



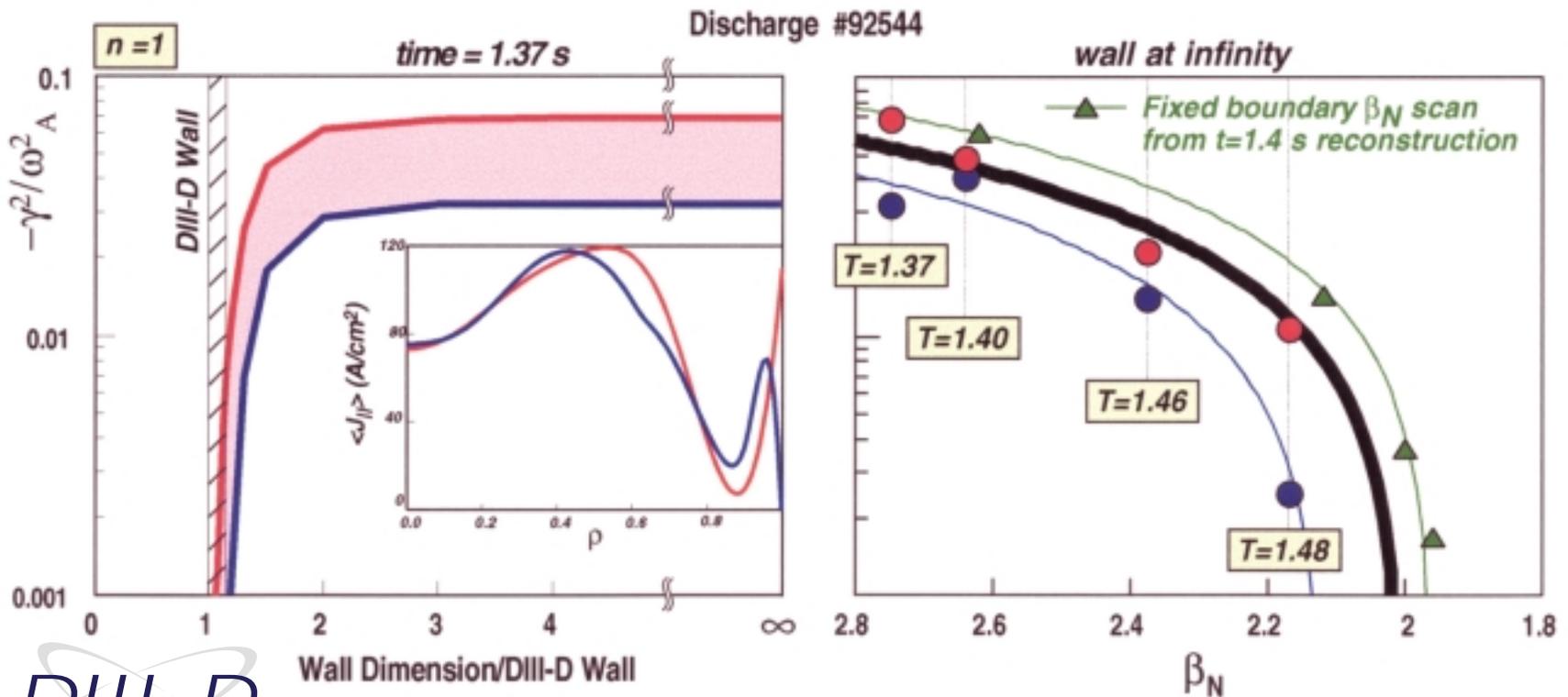
IDEAL KINK MODE STABILIZED BY ROTATION AND RESISTIVE WALL ABOVE NO-WALL β_N LIMIT FOR $> 30 \tau_{wall}$

- Wall stabilization sustained with β_N up to $1.4 \times \beta_N^{no-wall}$ computed by equilibrium reconstruction and full stability calculation
- Resistive wall mode grows when rotation drops below a critical value



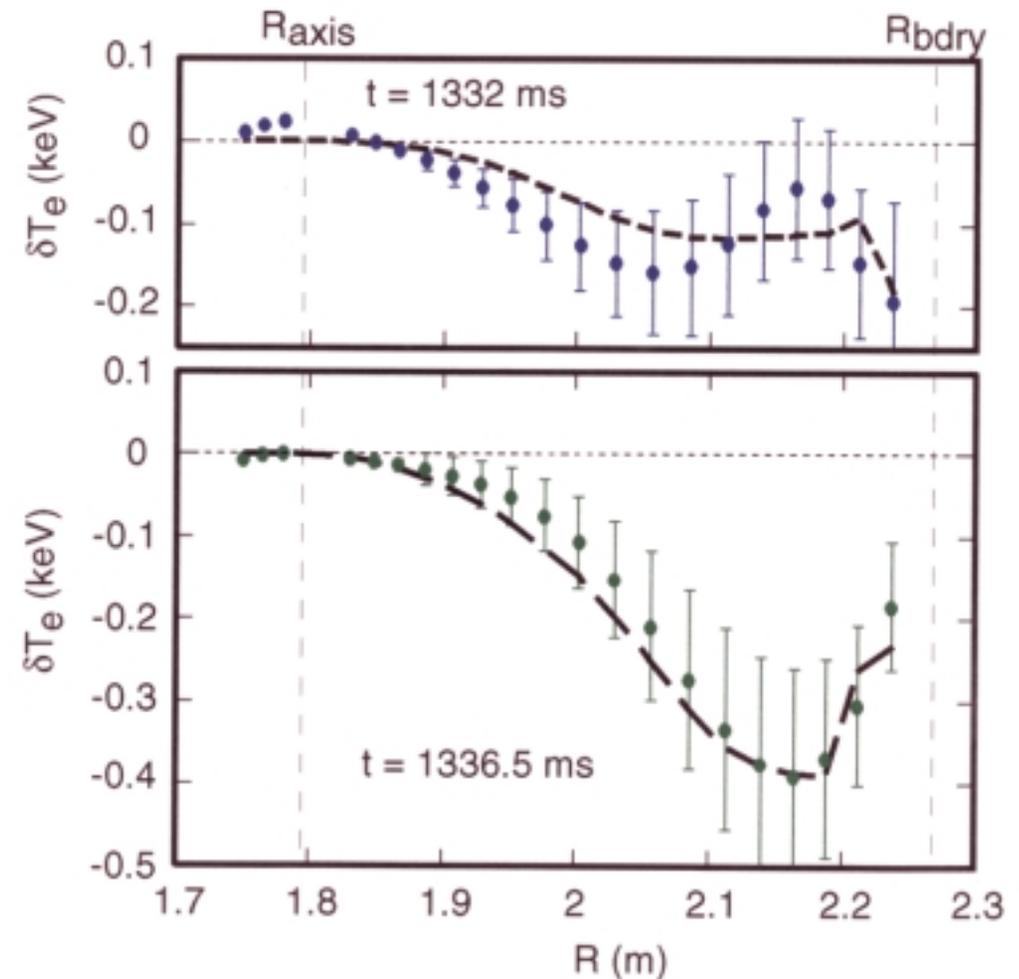
NO-WALL GATO STABILITY ANALYSIS FINDS GLOBAL $n = 1$ KINK MODE UNSTABLE DURING HIGH- β PHASE

- Instability little affected by edge current density within constraints of experimental data
- Mode near marginal stability at maximum experimental β_N with perfectly conducting wall at position of DIII-D wall
- Experiment clearly exceeds the $n = 1$ ideal stability limit calculated with wall at infinity



RESISTIVE WALL MODE RADIAL STRUCTURE PREDICTED FROM GATO CODE AGREES IN DETAIL WITH MEASURED ECE T_e FLUCTUATIONS

- Similar discharge #96519
- Predicted δT_e profile from $\delta T_e \propto \xi \nabla T_e$ with ξ from ideal stability calculation
- Amplitude and toroidal phase of calculated eigenfunction scaled from magnetic saddle loop data



CONCLUSIONS

- **Ideal MHD can predict many of the detailed features of the fastest growing instabilities in DIII-D**
- **Accurate reconstruction of the discharge equilibria is crucial to obtaining agreement in many cases**
 - **The more constrained the equilibrium, the better the agreement between predictions and observations**
- **Extensive equilibrium diagnostics and attention to details in the equilibrium reconstructions and stability calculations are the keys to accurate predictions**