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Theory of the Poloidal Spin-up Precursor to Transport Barrier Formation[1] G.M. STAEBLER, General Atomics — The phenomenon of a sudden change in the poloidal flow prior to the reduction in transport and the steepening of temperature and density profiles has been observed both at the edge (high-modes) and in the core (enhanced reversed shear (ERS-modes) of tokamaks. The poloidal spin-up precursor is narrowly localized in the (radial) direction across magnetic flux surfaces. Although the reduction of turbulent transport is consistent with the theory of $E \times B$ flow shear suppression, the localized poloidal spin-up precursor has not been explained by the theory until now. It will be shown that the observed flow pattern is well described by a new class of bifurcation to the momentum balance equations. The new physics follows from extending the standard neoclassical theory of poloidal flow damping to include the turbulent viscous stress. The new bifurcation results from balancing the non-linear turbulent viscous tress with the linear poloidal flow damping due to the neoclassical parallel viscous stress. The new bifurcation results in a mono-polar $E \times B$ flow structure (with a large poloidal component) which is narrowly localized in the radial direction. The peak in the flow is shown to reduce and finally disappear as the diamagnetic velocity shear increases.

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- [ ] Prefer Oral Session
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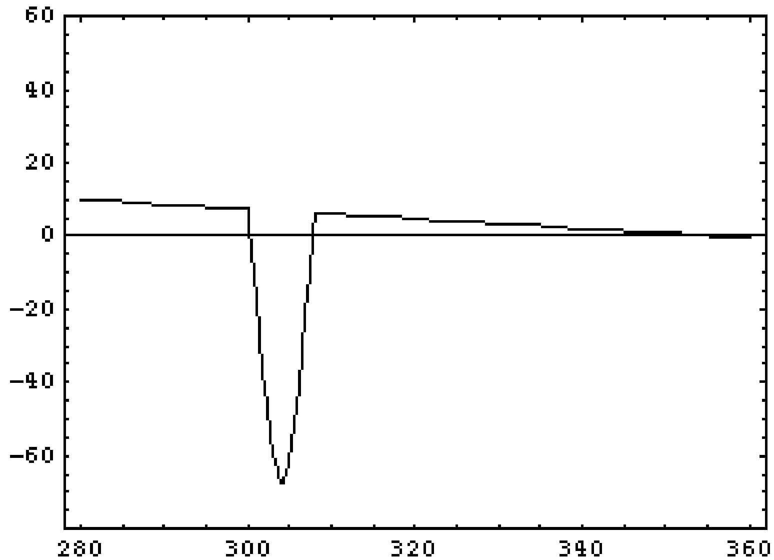
THEORY OF THE POLOIDAL SPIN-UP PRECURSOR TO TRANSPORT BARRIER FORMATION

- The poloidal velocity of carbon ions has been observed to spin-up in a narrow layer as a precursor to both H-mode and Type I ERS [1] transport barrier formation. The carbon is a tracer for changes in the ExB velocity.
  [1] R.E. Bell, F. M. Levinton, S. H. Batha, E. J. Synakowski and M. C. Zarnstorff, Phys. Rev. Lett. 81 (1998) 1429.]

Theoretical model for U_{ExB}



R (cm)

PROPERTIES OF THE POLOIDAL SPIN-UP PRECURSOR

- The precursor is a monopolar excursion of the ExB velocity.
- The temperature and density profiles are not observed to change over the time it takes the spin-up to occur. Thus, the spin-up is in the perpendicular velocity contribution to the ExB velocity not the diamagnetic velocity.
- The contribution to the ExB velocity from the perpendicular velocity diminishes as the diamagnetic velocity builds during the transport barrier formation.
- Transport barriers without poloidal spin-up precursors are also observed (Type II ERS, DIII-D NCS, and slow H-modes)
- It will be shown that all of these properties are reproduced by a new class of bifurcation to the momentum balance equations, which will be called the jet bifurcation due to its localized mono-polar structure across magnetic flux surfaces.

PERPENDICULAR VISCOUS STRESS DUE TO DRIFT WAVES

- Quasilinear calculations in a sheared slab magnetic field geometry [2] have shown that the viscous stress tensor due to drift waves (ITG,TEM) has the general form [2][R. R. Dominguez and G. M. Staebler, Phys. Fluids B5 (1993) 3876]

$$\Pi_{xy} = \eta_{yy}\,\gamma_{ExB} + \eta_{yz}\,\gamma_z\,, \quad \text{where} \quad \gamma_z = -du_z/dx$$
$$\Pi_{xz} = \eta_{zz}\,\gamma_z + \eta_{zy}\,\gamma_{ExB} \qquad\qquad \gamma_{ExB} = -du_{ExB}/dx$$

- The gradient lengths of the velocity (and electric field) must be short in order for the turbulent viscous stress to compete with the neoclassical viscous stress (poloidal flow damping term). Thus, only gradients of the flows will be retained and a thin slab approximation will be used for the perpendicular momentum balance equation:

$$\frac{d\Pi_{xy}}{dx} + mn\nu(u_{ExB} - u_{ExB}^{nc}) = 0 \qquad (1)$$

- The source free steady state toroidal momentum balance equation reads

$$d\Pi_{\mathbf{x}\varphi}/dx = 0,$$

- Integration with zero stress at the boundary obtains the non-trivial solution

$$\gamma_{\mathbf{z}} = C_{\mathbf{z}}\, \gamma_{\mathbf{ExB}}\,, \text{ where} \quad C_{\mathbf{z}} = (B_\theta \eta_{\mathbf{yy}} - B_\phi \eta_{\mathbf{zy}})/(B_\phi \eta_{\mathbf{zz}} - B_\theta\, \eta_{\mathbf{yz}})$$

- This is used to eliminate the parallel velocity making the perpendicular viscous stress only a function of the ExB velocity shear.

- The neoclassical flow is:

$$u^{nc}_{E\times B} = \left(u^{nc}_{\theta} \frac{B}{B_\phi} - \frac{1}{en}\frac{dp}{dx} \right) \Bigg/ \left(1 + \frac{B_\theta}{B_\phi} C_z \right) \quad \text{where } u^{nc}_{\theta} \propto dT/dx\,.$$

$du^{nc}_{E\times B}\big/dx = -\gamma^{nc},\ d\gamma^{nc}\big/dx = 0$, is assumed so that $u_{E\times B} = u^{nc}_{E\times B}$ is an exact solution to Eq.1

MODEL FOR THE VISCOUS STRESS DUE TO DRIFT WAVES

- The viscous stress due to drift waves has the general property that for low ExB velocity shear $|\gamma_{\mathbf{ExB}}| < \gamma_{\mathbf{L}}$ the local viscosity is large due to ion temperature gradient (ITG) modes.
- For large ExB shear $|\gamma_{\mathbf{ExB}}| > \gamma_{\mathbf{H}}$ the local viscosity is smaller since the ITG modes are quenched by the ExB shear.
- Electron temperature gradient modes could provide the viscosity since they are not stabilized by the level of ExB shear needed for the ITG modes.
- The following piecewise linear model realizes these properties.

$$\Pi_{\mathrm{xy}} = mn\gamma_{\mathrm{ExB}} \begin{cases} \mu_{\mathrm{L}} & \text{for } |\gamma_{\mathrm{ExB}}| < \gamma_{\mathrm{L}} \\ \dfrac{\beta}{|\gamma_{\mathrm{ExB}}|} - \alpha & \text{for } \gamma_{\mathrm{L}} \leq |\gamma_{\mathrm{ExB}}| \leq \gamma_{\mathrm{H}} \\ \mu_{\mathrm{H}} & \text{for } |\gamma_{\mathrm{ExB}}| > \gamma_{\mathrm{H}} \end{cases} \qquad (2)$$

where $\beta = \gamma_{\mathrm{L}}\gamma_{\mathrm{H}}(\mu_{\mathrm{L}}-\mu_{\mathrm{H}})/(\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}})$, $\alpha = (\mu_{\mathrm{L}}\gamma_{\mathrm{L}}-\mu_{\mathrm{H}}\gamma_{\mathrm{H}})/(\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}})$

Multiplying Eq.1 by ($\gamma_{E\times B}$-γ^{nc}), a first integral may be obtained using this model

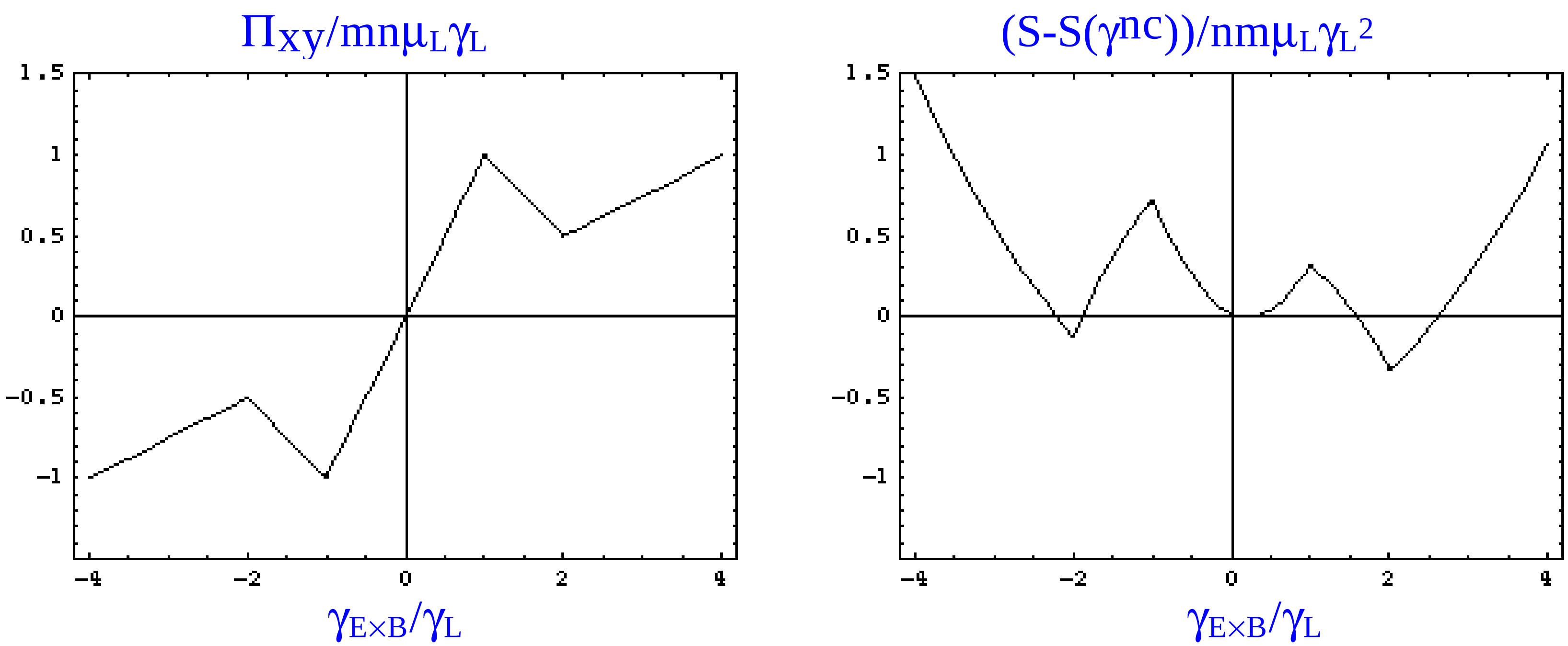
$$E = S - \frac{1}{2}\mathrm{mn}\nu(u_{ExB} - u_{ExB}^{nc})^2,$$

where

$$S = (\gamma_{ExB} - \gamma^{nc})\Pi_{xy} - F,$$

and

$$F = \frac{1}{2}\mathrm{mn}\begin{cases} \mu_L \gamma_{ExB}^2 & \text{for } |\gamma_{ExB}| < \gamma_L \\ -\alpha\gamma_{ExB}^2 + (\alpha + \mu_L)\gamma_L^2 & \text{for } \gamma_L \le |\gamma_{ExB}| \le \gamma_H \\ \mu_H \gamma_{ExB}^2 - (\alpha + \mu_H)\gamma_H^2 + (\alpha + \mu_L)\gamma_L^2 & \text{for } |\gamma_{ExB}| > \gamma_H \end{cases}.$$



Parameter: $\gamma_H/\gamma_L = 2.0$, $\mu_L/\mu_H = 4.0$, $\gamma^{nc}/\gamma_L = 0.2$

DUAL GENERALIZED PHASE TRANSITION

- The jet bifurcation is a solution to a new type of generalized phase transition model, which is dual to the usual Ginzburg-Landau model.

- The non-linearity is in the field gradient (kinetic energy) rather than the field (potential energy)

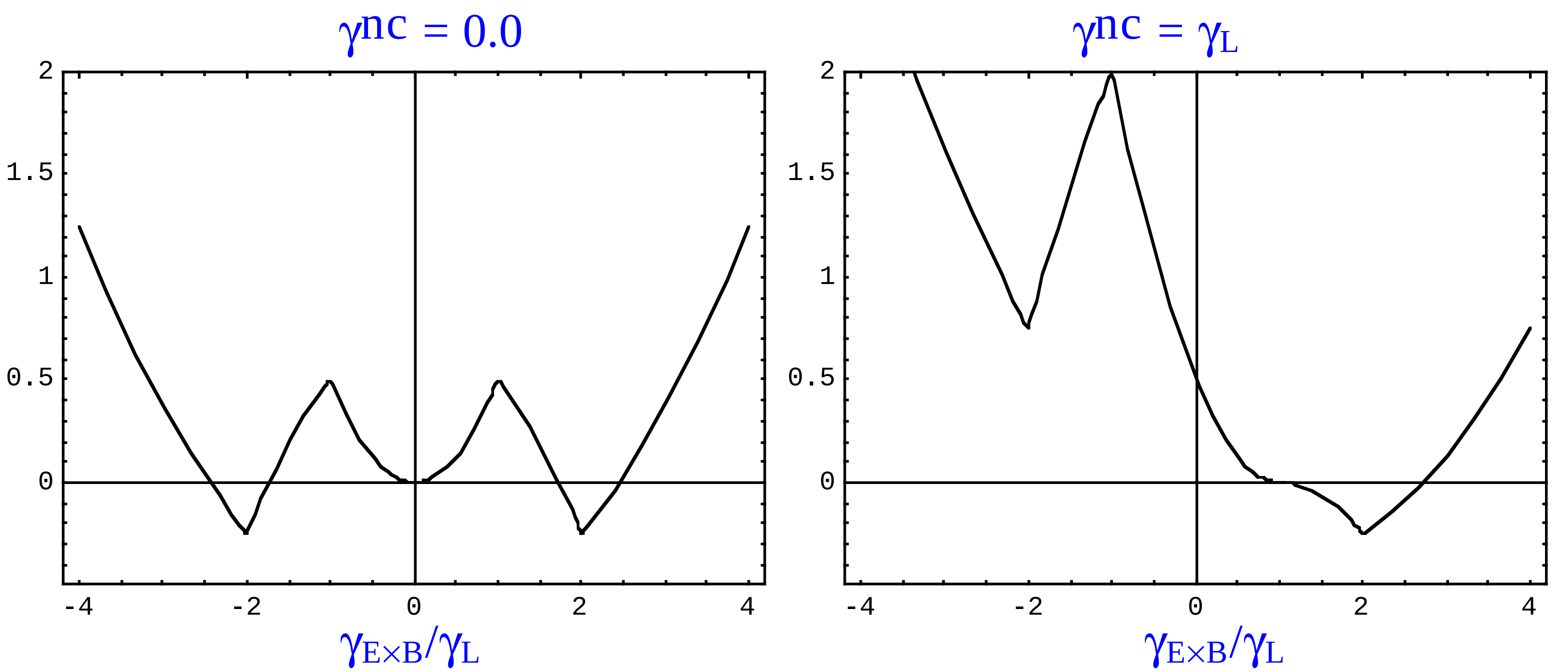
- The jet solution is analagous to a topological solition. It connects the two ground states at mγ_H

$$\frac{1}{2}mn\nu(u_{ExB} - u_{ExB}^{nc})^2 = \begin{cases} S - S(-\gamma_H) \text{ for } -\gamma_H \le \gamma_{ExB} \le -\gamma_C \\ S - S(\gamma_H) \text{ for } \gamma_L \le \gamma_{ExB} \le \gamma_H \end{cases}$$

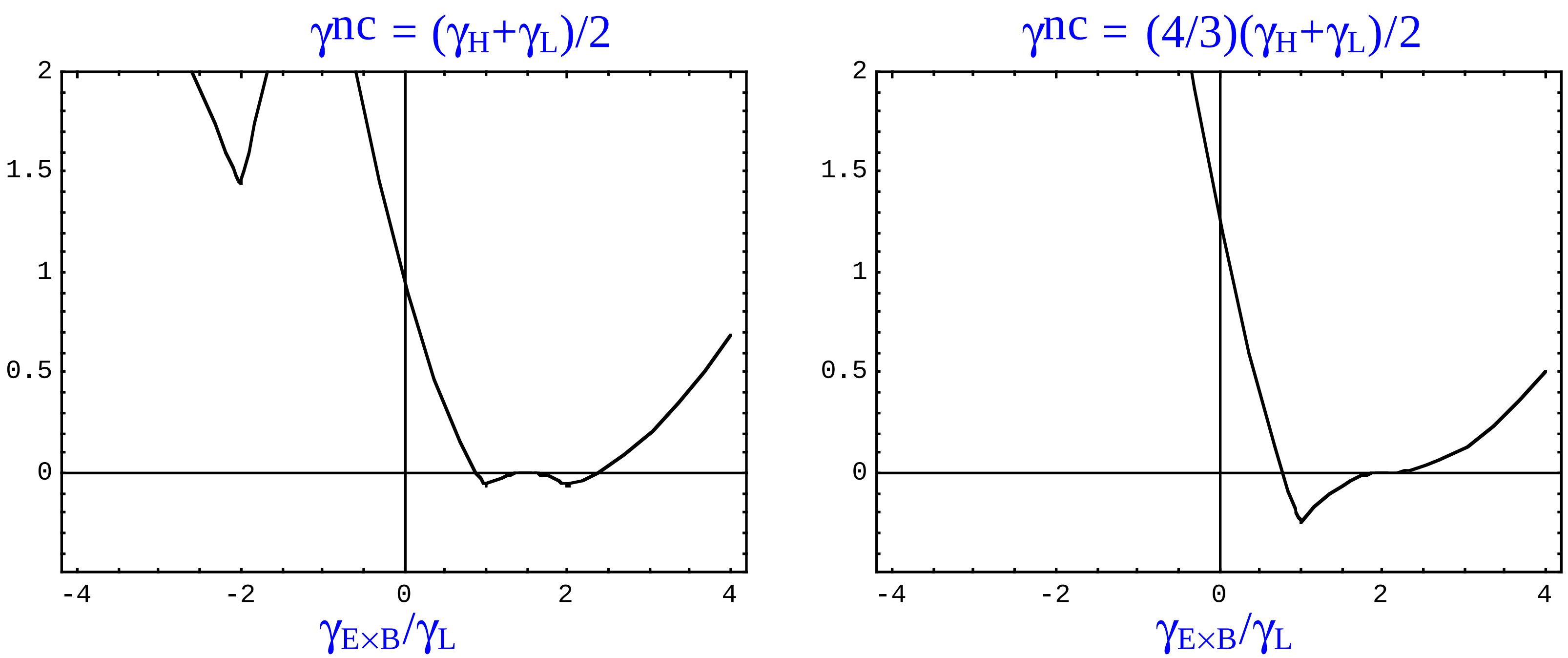
- The jet solution has the topological invariant

$$\Delta\Pi_{xy} = \int_{x_0}^{x_2} dx \frac{d\Pi_{xy}}{dx} = \Pi_{xy}(\gamma_L) - \Pi_{xy}(\gamma_H) + \Pi_{xy}(-\gamma_H) - \Pi_{xy}(-\gamma_C)$$

THE STRESS ENERGY EVOLVES WITH γ^{nc}



- The stress energy is symetric for $\gamma^{nc} = 0.0$
- For $\gamma^{nc} > \gamma_L$ the point γ^{nc} becomes unstable (no seed perturbation is required for a jet)



- For $\gamma^{nc} > (\gamma_H+\gamma_L)/2$ no jet is possible since $S(\gamma_H) < S(\gamma_L)$.
- A jet is energetically favorable if $S(m\gamma_H) < S(\gamma^{nc})$.
  - A jet may be energetically unfavorable if only one minimum is below $S(\gamma^{nc})$.

DETAILS OF THE JET SOLUTION

- Domain: $x<x_0$ and $x>x_2$, $|\gamma_{ExB}| < \gamma_L$

  in this domain Eq. 1 becomes: ($f = u_{E\times B} - u^{nc}_{E\times B}$)

$$\alpha \frac{d^2 f}{dx^2} + \nu f = 0\,.$$

  The complete solution in the domain x={x_0,x_1} with $f(x_0) = 0$ and $\gamma_{E\times B}(x_0) = \gamma_H$ is given by ($\lambda_m = \sqrt{\alpha/\nu}$)

$$u_{E\times B} = u^{nc}_{E\times B} - \lambda_m\left(\gamma_H - \gamma^{nc}\right)\sin\left[\left(x - x_0\right)/\lambda_m\right].$$

  The width (x_1–x_0) is determined by the boundary condition $\gamma_{E\times B} = \gamma_L$ at x_1.

$$\cos\left[\left(x_1 - x_0\right)/\lambda_m\right] = \frac{\gamma_L - \gamma^{nc}}{\gamma_H - \gamma^{nc}}\,.$$

The width scales like $(\mu_L/\nu)^{0.5}$ so smaller neoclassical damping make the jet wider.

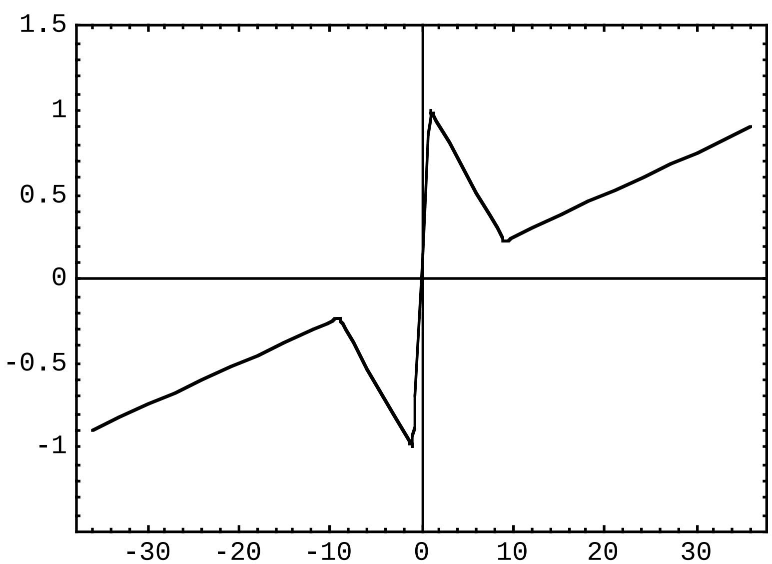
- The solution in the domain $x=\{x_1,x_2\}$ where $\gamma_{E\times B} = -\gamma_H$ at x_2, and $f(x_2) = 0$ is

$$u_{ExB} = u_{ExB}^{nc} + \lambda_m(\gamma_H + \gamma^{nc})\sin[(x - x_2)/\lambda_m]$$

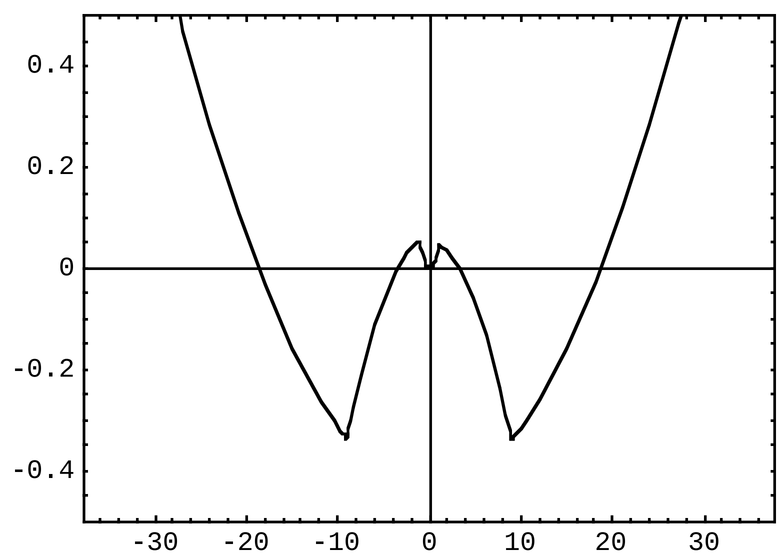
- Matching the two solutions at x_1 determines the width $(x_2 – x_1)$. This completes the jet. The position x_0 is arbitrary unless spatial variations of γ^{nc} are retained.

Viscous Stress and Stress energy for the best fit to the TFTR data.

$\Pi_{xy}/mn\mu_L\gamma_L$ $(S-S(\gamma^{nc}))/nm\mu_L\gamma_L^2$



$\gamma_{E\times B}/\gamma_L$



$\gamma_{E\times B}/\gamma_L$

TFTR ERS DATA IS WELL FIT BY THE JET

- The carbon poloidal velocity data gives the approximate ExB velocity in the region of the localized excursion.
  R. E. Bell, F. M. Levinton, S. H. Batha, E. J. Synakowski and M. C. Zarnstorff, Phys. Rev. Lett. **81**, 1429 (1998).

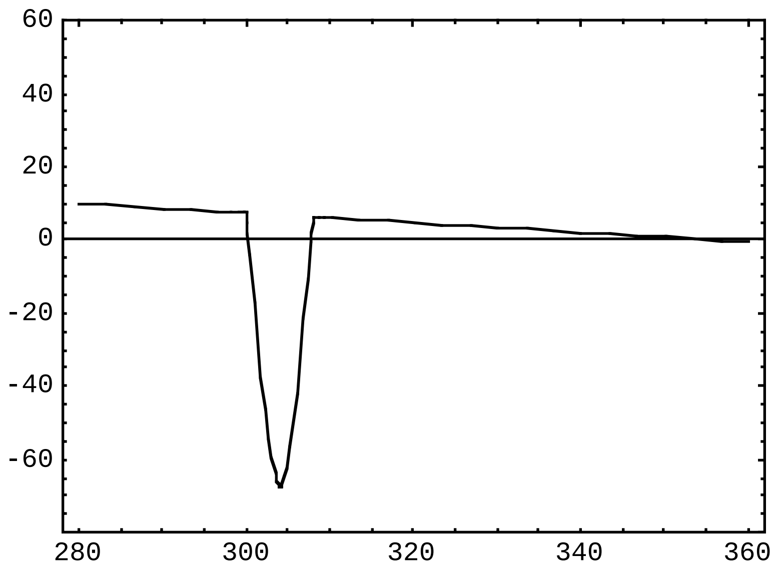
- TFTR data for a particular discharge:
  r = 304cm-R, R=260cm, $n_{\mathbf{e}} = 1.45\text{x}10^{13}/\text{cm}^3$, $T_{\mathbf{i}}$ = 5.7keV, q=2.6, $m_{\mathbf{i}} = 2\ M_{\mathbf{proton}}$

- Fit parameters for the jet solution:
  $\gamma_{\mathbf{L}} = 3\text{x}10^5/\text{s}$, $\gamma_{\mathbf{H}} = 9\,\gamma_{\mathbf{L}}$, $\mu_{\mathbf{L}} = 2.8\text{m}^2/\text{s}$, $\mu_{\mathbf{H}} = \mu_{\mathbf{L}}/40$, $X_{\mathbf{0}}$ = 300cm,
  $u^{\mathbf{nc}} = 10^6\text{cm/s} - \gamma^{\mathbf{nc}}(\text{X-280cm})$, $\gamma^{\mathbf{nc}} = 1.3\text{x}10^4/\text{s}$, ν= 346/s computed from

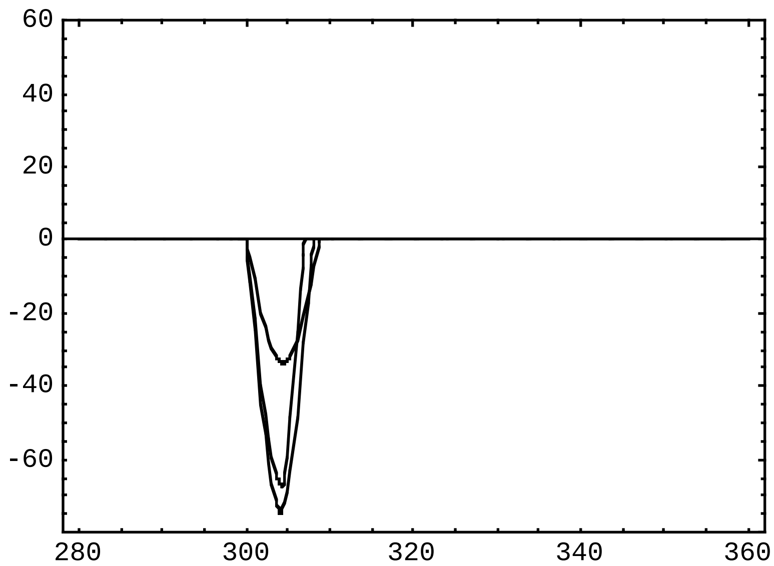
$$\nu \cong \frac{0.533\sqrt{2}\,f_{\mathrm{t}}}{\left(1+2\mathrm{q}^2\right)\left(1-f_{\mathrm{t}}\right)\ \tau_{\mathrm{i}}}\left(\frac{\mathrm{Rq}}{\mathrm{r}}\right)^2$$

- The linear growth rate ($\gamma_{\mathbf{L}}$) and effective diffusivity (from particle transport) ($\mu_{\mathbf{L}}$) are consistent with TFTR calculations prior to spin-up and ERS transition.

Best fit to TFTR data U_{ExB}



Decay of U_{EXB}-U^{nc} with increasing γ^{nc}



$\gamma^{\mathbf{nc}} = 1.3\text{x}10^4/\text{s}$, $\gamma^{\mathbf{nc}} = \gamma_{\mathbf{L}}$,

$\gamma^{\mathbf{nc}} = (\gamma_{\mathbf{L}}+\gamma_{\mathbf{H}})/2$ maximim

- The jet excursion shrinks and disappears as γ^{nc} increases.

JET PROPERTIES

- The jet bifurcation requires a seed shear perturbation if $\gamma^{nc} < \gamma_L$:
  - The smallest perturbation is in the direction of $\gamma^{\mathbf{nc}}$
  - Transport barriers can form without jets even with balanced NBI.

- There is another bifurcation path through the toroidal momentum balance equation.
  - The toroidal momentum balance equation does not have the neoclassical term ν so the timescale is slower (transport time) and it is not intrinsically localized.
  - DIII-D NCS and TFTR type II ERS transitions have no jet precursors.

- The strongly off-diagonal structure of the viscous stress tensor due to drift waves predicts that there should be a parallel velocity feature at the location of the ExB velocity excursion.
  - The parallel flow does not have to have the same monopolar structure of the ExB velocity since $C_{\mathbf{z}}$ is not constant. ($\gamma_{\mathbf{z}} = C_{\mathbf{z}}\ \gamma_{\mathbf{ExB}}$)

CONCLUSION

- The jet bifurcation does indeed reproduce the properties of the observed poloidal spin-up precursor.
- Transport barriers which form without jets can be accounted for.
- The fast growth of the jet (ν) and its disappearance once the temperature or density gradients increase, fits experimental observations.
- The monopolar character and strong radial localization of the jet fits the data.
- An analytic model solution for the jet can be fit to TFTR data with realistic parameters for the linear growth rates, and effective momentum diffusivities. The neoclassical poloidal damping rate is computed from theory so the width of the jet determines the momentum diffusivity.
- The jet theory predicts the existence of a parallel velocity excursion within the ExB jet due to the off-diagonal nature of the viscous stress.
- Triggering jets with localized perpendicular momentum sources (e.g. IBW) could dramatically lower the power required for a transport barrier.