SUSTAINMENT OF PLASMA ROTATION BY ICRF

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- **Representative Experimental Results**
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- **ICRF Model and Torque Density**
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- **Diamagnetic Scaling of Rotation Velocity**
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BACKGROUND AND MOTIVATION

- **Alcator C-Mod and JET observe development of co-current plasma rotation in ICRF-heated discharges.**

- **ICRF heating introduces zero (or negligible) angular momentum to the plasma.**

  - **Experiments have a symmetric $k_{||}$ - spectrum and contribute no net angular momentum.**

  - **Even if the $k_{||}$ spectrum lauched is one sided, the angular momentum input is negligible** ($k_{||} = n/R$; $n \approx 12$ for C-Mod)

$$\left(\Delta T\right)_{RF} = \text{RF Torque} = M \, \Delta v_{||} \, R = n \;\; \Delta E / \omega$$

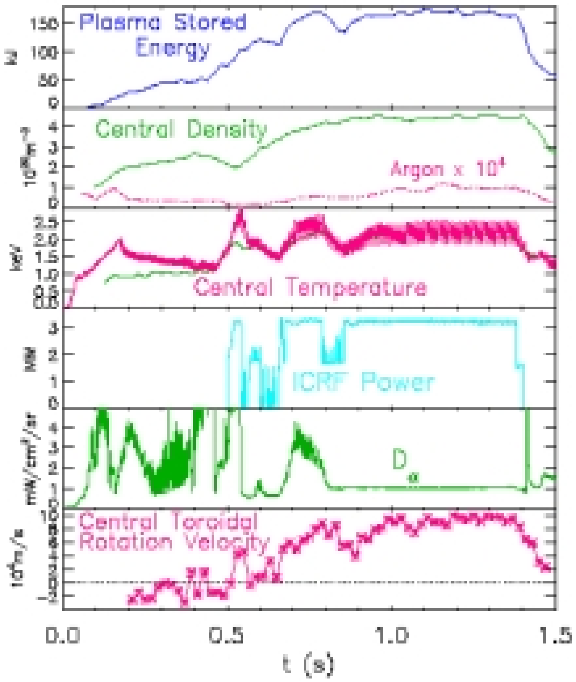
$$\left(\Delta T\right)_{NBI} = \text{typical NBI torque} = \Delta E \; R \; v_{beam}^{-1}$$

$$\frac{\Delta T_{RF}}{\Delta T_{NBI}} = \frac{n \, v_{beam}}{\omega \, R} \approx \frac{1}{15} \quad << 1$$

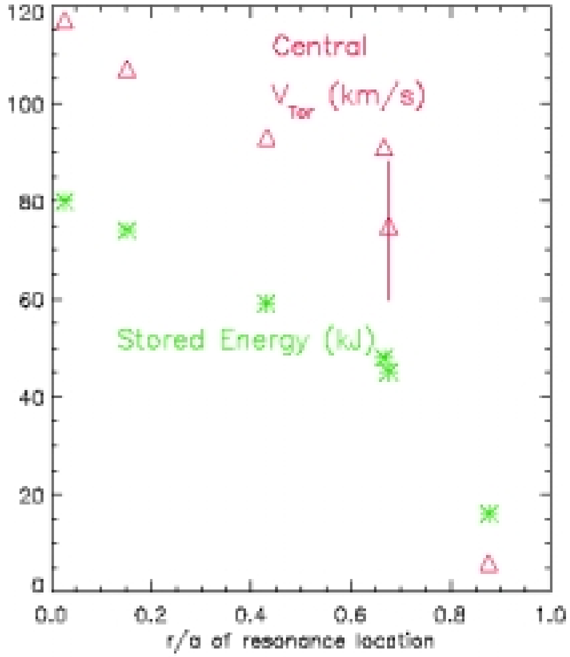
- **What is the mechanism for developing toroidal rotation and how does it scale?**

REPRESENTATIVE EXPERIMENTS

1. Paper by J. E. Rice, et al. [*Nuclear Fusion* 39 (1999) 1175] reports rotation observations and scaling.



5.7 T, 1.0 MA D(H) Discharge



Resonance location scan with B varying and q_{95} = 4.7

MODEL OVERVIEW - 1

1. **Even though ICRF heating introduces no net torque, there remains the possibility of creating positive and negative torque density regions.**

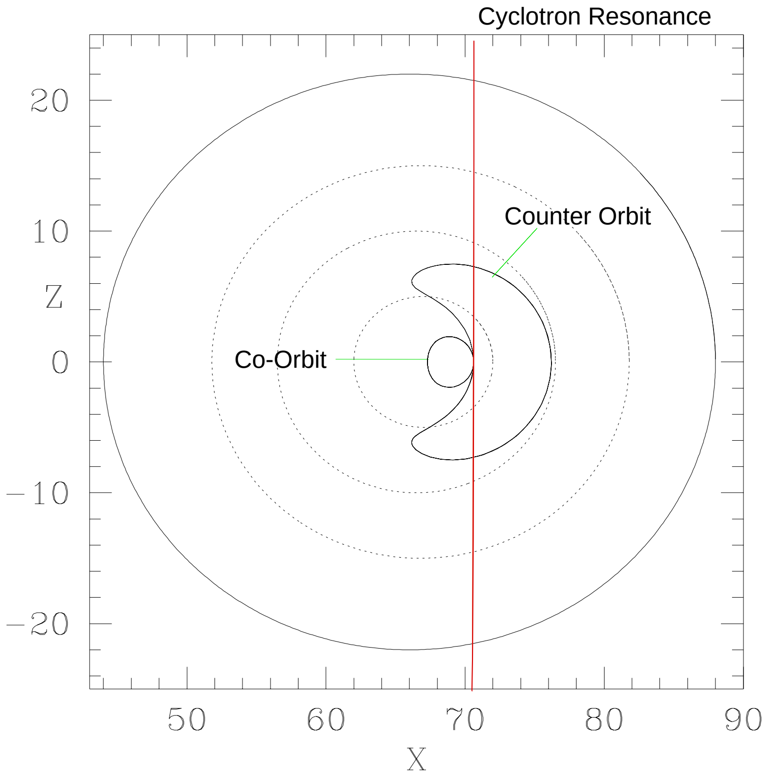
2. **Describe plasma response to torque density by an angular momentum diffusion equation.**

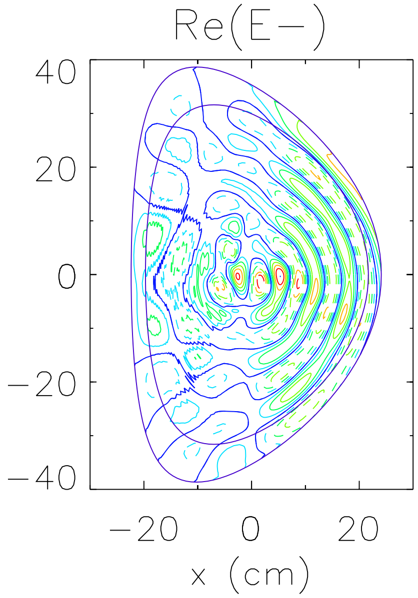
   - **Separated torque density regions lead to finite central rotation**

3. **Model ICRF heating by the introduction of energetic particles on the equatorial plane and the removal of an equal number of cold particles.**

   - **Particles are introduced at a particular flux surface — the resonance location— with equal numbers of co- and counter- velocities so there is no angular momentum input.**

REPRESENTATIVE INITIAL ORBITS





(Plot by P. Bonoli)

- **Fast-wave refraction leads to midplane heating.**

MODEL OVERVIEW - 2

4. Follow particles by ORBIT with ion-ion pitch angle and drag collisions

- **Record particle's flux-surface when Energy → 0.**

- **Particle's displacement from originating flux-surface drives a radial neutralizing current and a $j_r B_\theta$ torque density in the background plasma**

  - **Continuous creation of energetic particles drives steady j_r current**

- **ORBIT also computes the torque density imparted to the background by energetic-ion collisions**

- **Total volume-integrated applied torque vanishes to $2 \cdot 10^{-4}$ accuracy.**

5. Compute non-vanishing central rotation from torque density

6. Investigate scaling of central rotation and sensivity to initial conditions

- **Particle energy and pitch, resonance location and q.**

ANGULAR MOMENTUM DIFFUSION EQUATION-1

1. General Form of angular rotation rate Ω response to torque density τ

$$\frac{\partial}{\partial t}\left(M\, nR^2\Omega\right) = \nabla \cdot \left\{ n\, M\, R^2\, \chi_M\, \nabla\, \Omega \right\} + \tau$$

2. Steady-state axisymmetric version

$$\frac{1}{V'}\frac{\partial}{\partial \psi}\left\{ V' \left\langle n\, M\, R^2 \chi_M \left(\nabla\, \psi\right)^2 \right\rangle \frac{\partial \Omega}{\partial \psi} \right\} = -\left\langle \tau \right\rangle$$

$$V' = \oint \frac{d\ell\, 2\pi R}{\nabla \psi} = \frac{\partial V}{\partial \psi}$$

and <> denotes magnetic surface average

3. $\langle \tau \rangle$ is torque density on bulk plasma and has two sources:

- **$j_r B_\theta$ torque arising from radial curents which neutralize energetic particle displacements**

- **Collisional Angular momentum transfer from energetic particles.**

ANGULAR MOMENTUM DIFFUSION EQUATION-2

4. First integral of angular momentum equation

$$V' \left\langle n\, M\, R^2 \chi_M \left(\nabla\psi\right)^2 \right\rangle \frac{\partial\Omega}{\partial\psi} = -\int_0^{\psi} \langle\tau\rangle V'\, d\psi \;\; = T(\psi)$$

- $T(\psi)$ **= torque exerted inside ψ-surface**
- **No net torque condition: $T(\psi_{max}) = 0$**

5. Apply no-slip boundary condition at surface

- **Field lines outside separatrix line-tied to vessel; toroidal rotation not permitted**

6. Torque proportional to rate of creation of energetic particles $\dot{N}$ and angular momentum transferred per particle.

$$\langle\tau\rangle \propto \dot{N}$$

ANGULAR MOMENTUM DIFFUSION EQUATION-3

7. Angular rotation rate (use toroidal flux Φ as independent variable)

$$\Omega(\Phi) = \int_{\Phi}^{\Phi_{\mathrm{max}}} \frac{d\Phi}{q\,V'} \frac{T(\Phi)}{\left\langle n\,M\,R^2\,\chi_M\,(\nabla\psi)^2 \right\rangle}$$

8. Conclude:

For regions of separated postive and negative torque density, $T(\Phi)$ is non-zero and toroidal rotation can develop, even though the total torque $T(\Phi_{\mathrm{max}})=0$

ION-CYCLOTRON HEATING MODEL

1. **The ion-cyclotron heating process changes a particle's prependicular energy, while leaving $v_{||}$ and the canonical angular momentum unchanged.**

   - **No net angular momentum is introduced**

2. **Our ICRF model replaces cold particles by energetic particles constrained to have an equal number of co- and counter velocity particles.**

   - **Particles are created on the same flux surface on the midplane**

   - **Mimics ICRF heating for particles whose orbit is tangent to the resonant surface at the midplane where the fast wave intensity is high.**

   - **Pitch at creation is fixed to be low: $v_{||}/v = (0.25 - 0.40)$**

3. **Energetic particles will spatially diffuse via banana diffusion and will collisionally transfer angular momemtum to the bulk plasma.**

   - **ORBIT code follows these processes via a Monte Carlo approach.**

ORBIT CODE

1. ORBIT code has been developed to follow energetic particle orbits in toroidal confinement geometries of arbitrary cross section.

2. Hamiltonian formalism developed

- **Rigorous Hamiltonian form found:**
  - **R. B. White and M.S. Chance, Phys. Fluids 27, 2455 (1984)**
  - **R. B. White, Phys. Fluids B2, 845 (1990)**

$$dP_\zeta/dt = -\partial H/\partial\zeta \qquad d\zeta/dt = \partial H/\partial P_\zeta$$
$$dP_\theta/dt = -\partial H/\partial\theta \qquad d\theta/dt = \partial H/\partial P_\theta$$

3. Monte- Carlo collisions after A. Boozer et al. Phys. Fluids 24, 851 (1981)

4. Collision model: Energetic proton ion-ion collisions with cold deuterons.

$$\frac{d\langle\theta^2\rangle}{dt} = \nu_o\left(\frac{E_o}{E}\right)^{3/2} \qquad \frac{1}{E}\frac{dE}{dt} = -\nu_o\left(\frac{E_o}{E}\right)^{3/2}\frac{M_{proton}}{M_{deuteron}} \qquad \nu_o = \frac{2^{3/2}\pi n e^4 \ell n\Lambda}{(M_p)^{1/2}E_o^{3/2}}$$

ORBIT CODE MODIFICATIONS

- **Plasma divided into $5 \cdot 10^4$ bins in toroidal flux (magnetic surface label)**

- **For each time step, momentum transfer from particles to plasma through pitch angle scattering and drag recorded in each bin.**

- **Final particle momentum and density recorded in each bin**

- **Integrals over toroidal flux (bins) needed for angular rotation performed**

- **Angular momentum check accurate to 1 part in 5000.**

NONDIMENSIONAL CENTRAL ROTATION RATE-1

1. Let Φ_o denote the toroidal flux value where energetic particles of energy E are introduced at a rate $\dot{N}$.

2. Let $v = \left(2E/M\right)^{1/2}\left(\Omega_a R_a\right)^{-1}$ **denote a nondimensional particle speed.**

3. ORBIT computes F(Φ) = fraction of particles ending up inside flux surface Φ and the integral T_1 of the $j_r \times B_\theta$ torque.

$$T_1(\Phi) = \frac{1}{v}\int_0^{\Phi} \frac{d\Phi'}{q}\, G(\Phi') \qquad G(\Phi) = \begin{cases} F(\Phi) & \Phi \le \Phi_o \\ F(\Phi) - 1 & \Phi \ge \Phi_o \end{cases}$$

4. ORBIT also calculates v $T_2(\Phi)$ = mechanical angular momentum deposited inside Φ.

5. Standard circular tokamak formulas, an assumed constant momentum diffusivity $\chi_M = a^2/6\tau_M$, and $\dot{N} E\tau_E = P\tau_E = W$ are employed to calculate the central rotation frequency

NONDIMENSIONAL CENTRAL ROTATION RATE-2

6. On-axis rotation rate is expressed in terms of the nondimensional rotation rate I*

$$\frac{\Omega(0)}{\dot{N}} = v^2 I^* \qquad I^* = \frac{1}{v}\int_0^{\Phi_{max}} \frac{d\Phi}{\Phi} T \qquad T = T_1 + T_2$$

$$v = \left(2E / M\right)^{1/2} \left(R_a \omega_{c,a}\right)^{-1}$$

7. Analytic considerations motivate the introduction of v so that I* is insenstive to physics parameters

ROTATION IN PHYSICAL UNITS

1. Select baseline initial particle values used in computing I* to be representative of Alcator C-Mod.

• E=48 keV, pitch = 0.25, rho = 0.165, low-field midplane, and N=2000. • Result:

$I^* = 22.5$

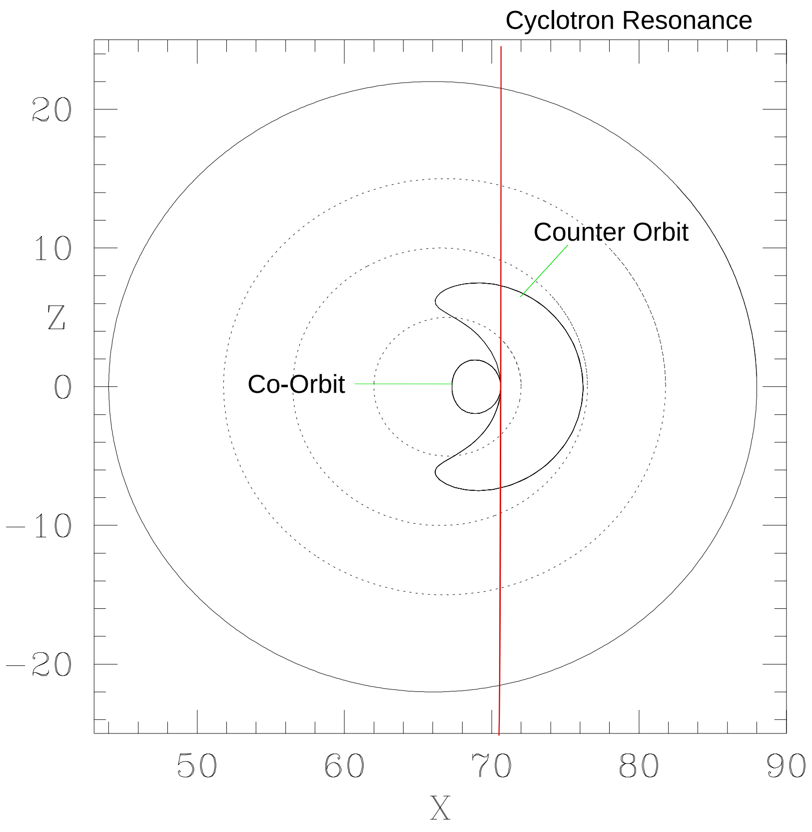
2. In physical variables the rotation rate is

$$\Omega(0) = \left\{ \frac{6\,W}{e\,B_a\,R_a^{\;3}\,a^2\,\bar{n}\,(2\pi)^2} \left(\frac{\tau_M}{\tau_E}\right) \right\} I^*$$

For the shot on sheet 4, this gives $v_{tor} = \Omega(0)\,R_a = 7\cdot10^4$ m/s, in good accord with the reported value. Results insensitive to E, pitch, and N.

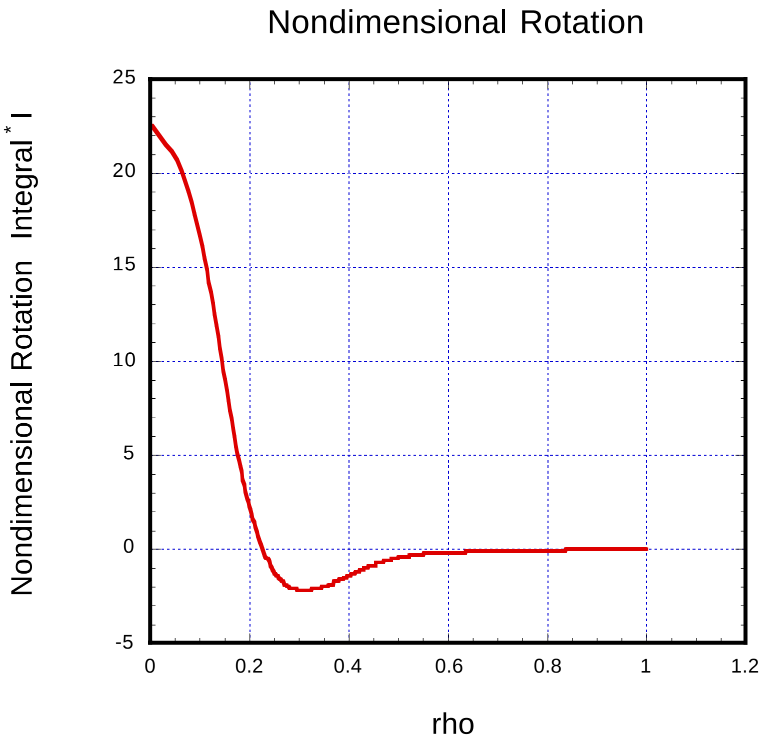
INITIAL ORBITS

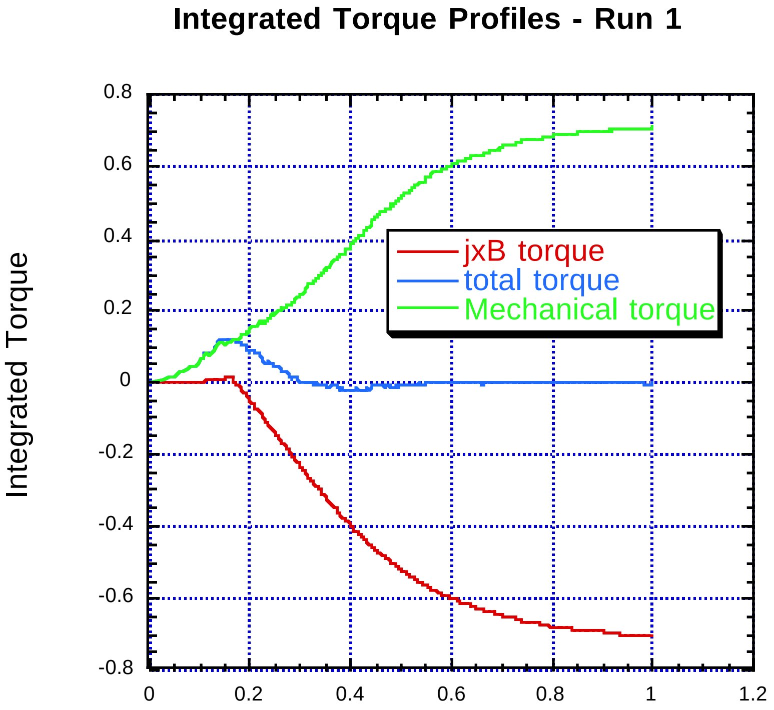
1. Initial orbits are characteristic of orbits near the magnetic axis



ROTATION AND TORQUE PROFILES - RUN 1

- **Rotation Profile is peaked.**



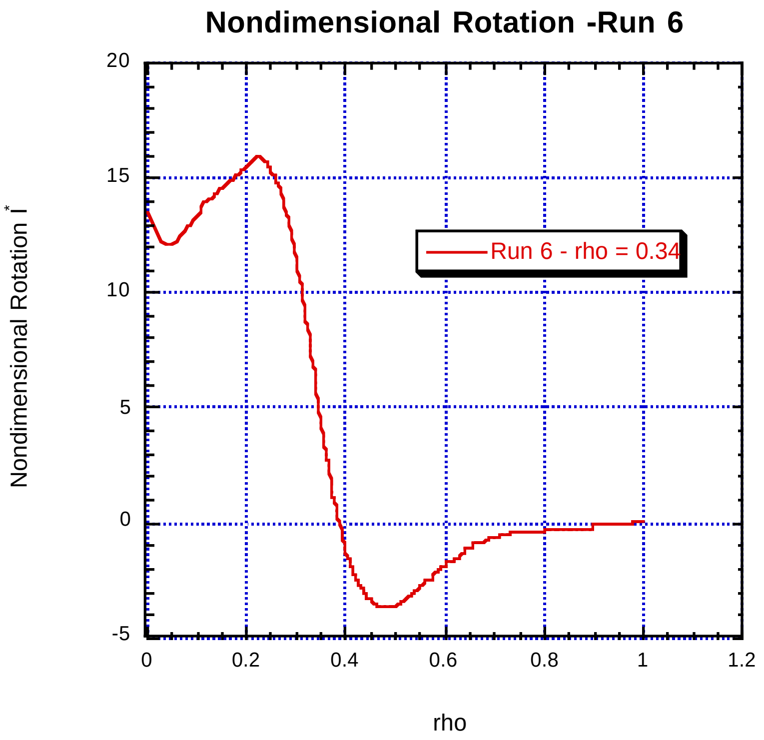


SENSIVITY STUDIES

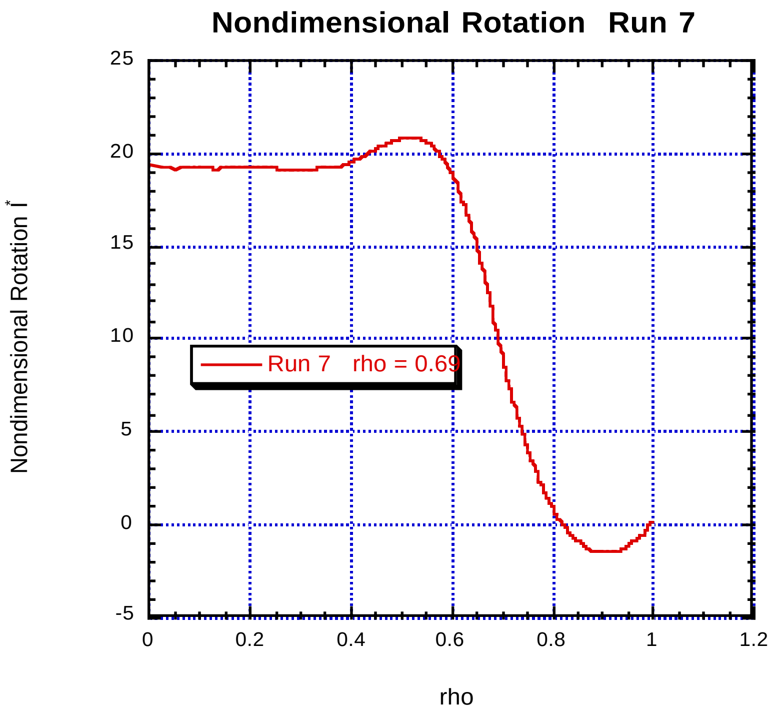
- How much does the central rotation change as E, rho, pitch, q-profile, and initial surface (ICRF resonance surface) vary ? Results expressed as I*.

Run	Objective (N=500)	I*	rho	E (keV)	pitch	q_{max}	resonance
1	Baseline (N=2000)	22.5	0.165	48	0.25	4.0	LFS
1.1	Baseline (N=200)	24.9	0.165	48	0.25	4.0	LFS
2	Pitch variation	28.3	0.165	48	0.35	4.0	LFS
3	Energy dependence	24.6	0.165	24	0.34	4.0	LFS
4	HFS vs LFS (run 1)	-18.6	0.165	48	0.25	4.0	HFS
5	q_{max}	17.5	0.165	48	0.25	8.0	LFS
6	initial rho	13.5	0.34	48	0.5	4.0	LFS
7	initial rho	19.3	0.69	48	0.64	4.0	LFS
8	Banana vs Circulating (run 6)	11.4	0.34	48	0.32	4.0	LFS
9	HFS vs LFS (run 7)	-22	0.69	48	0.64	4.0	HFS
10	On axis	7.3	0.0	48	0.35	4.0	On-axis
11	On axis - pitch	-1.9	0.0	48	0.25	4.0	On-axis

ROTATION CURVES vs INITIAL RHO



Nondimensional Rotation -Run 6



Nondimensional Rotation Run 7

- **As resonance layer is moved outward, rotation profile broadens and is lower in magnitude than baseline case.**

SUMMARY - 1

1. **Separated regions of positive and negative torque density can generate central rotation**

   - **General property of a diffusion equation**

2. **ICRF generates two types of torque density on bulk plasma which are comparable in magnitude and integrate to zero net torque**

   - $j_r \times B_\theta$ **and mechanical angular momentum transfer by collisions**

3. **ORBIT code follows individual particles and computes the torque densities**

   - **ICRF model (initial condition for ORBIT) replaces a cold particle by an energetic particle in the mid-plane.**

   - **Equal numbers of co- and counter energetic particles assure not net momentum injection. Angular momentum check to $2 \cdot 10^{-4}$ level.**

SUMMARY - 2

4. Central rotation arises

- **Co-current sense, magnitude, and scaling in accord with Alcator C-Mod**

  - $v_{exp}(0) = 10.0 \cdot 10^4$ m/s $\qquad$ $v_{model}(0) = 7 \cdot 10^4$ m/s

- **Insensitve to particle energy, pitch, q_{max} , N, and initial ρ.**

- **High-field-side initial ρ gives counter-current rotation.**

5. Summary formula

$$v_{tor}(0) = \left\{ \frac{6\,W}{e\,B_a\,R_a^2\,a^2\,n\,(2\pi)^2} \left(\frac{\tau_M}{\tau_E} \right) \right\} I^*$$

$I^* = 10\text{-}20$

CONCLUSIONS

- **A mechanism to create central rotation in tokamaks with ICRF heating has been indentified.**

- **Toroidal velocity scales diamagnetically**

  - Magnitude and sense in accord with C-Mod data.

- **Precise treatment of angular momentum needed and provided by ORBIT code**

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See Poster RP1.72 Thursday Afternoon

(Also, poster RP1.71 - Y. Omelchenkov)