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The Non-Bounce-Averaged Calculation of Electron Cyclotron Current Drive Efficiency for Recent DIII-D Experiments[1] Y.R. LIN-LIU, V.S. CHAN, General Atomics, O. SAUTER, CRPP/EPFL, R.W. HARVEY, CompX — The standard approach of modeling electron cyclotron current drive (ECCD) in tokamaks has been based on the bounce-averaged Fokker-Planck theory. It assumes that the effective collision frequency is much smaller than the bounce frequency for trapped electrons at all energies. This assumption is invalid at low energies and gives pessimistic estimate of ECCD efficiency. A non-bounce-averaged Fokker-Planck code (CQLP) is used to calculate electron response for the DIII-D ECCD experimental parameters.[2] The ECCD efficiency at finite collisionality is evaluated and compared with a scaling law deduced from a boundary-layer analysis in the small inverse apect ratio limit, which predicts a modest improvement in current drive efficiency. The numerical calculation is extended to consider an enhancement of ECCD efficiency in the presence of dc electric field.

[1]Work supported by U.S. DOE Contract DE-AC03-99ER54463 and in part by the Swiss National Science Foundation.
[2]T.C. Luce *et al.*, General Atomics Report GA-A23018 (1999).

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- [ ] Prefer Oral Session
- [X] Prefer Poster Session

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INTRODUCTION

- Electron Cyclotron Current Drive (ECCD) is a leading candidate for current profile control in Advanced Tokamak (AT) operation

- Localized off-axis ECCD, in both directions of the plasma current, was clearly demonstrated in recent proof-of-principle experiments on DIII–D

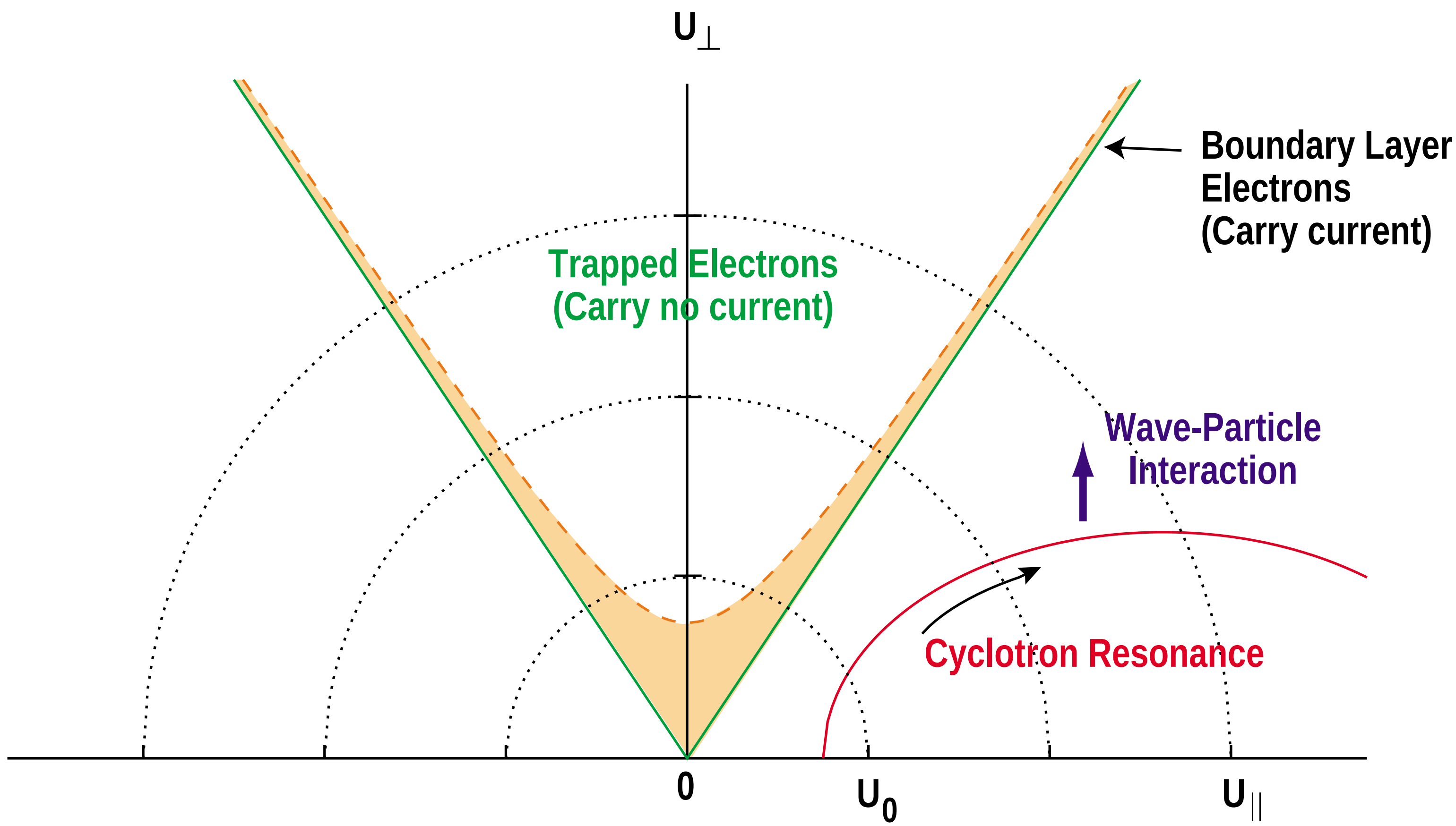
- The measured ECCD efficiency agrees with the coupled ray-tracing and bounce averaged Fokker-Planck calculations for the cases near magnetic axis; but it meets and exceeds the predicted value at larger radius.

- The bounce averaged calculations assume that bounce frequency is much larger than collision frequency for trapped electrons at all energies, hence, underestimate current drive efficiency

- Preliminary finite-collisionality calculations using a velocity-space interpolation formula give modest increase in ECCD efficiency

- Non-bounce averaged calculations of ECCD efficiency for the Lorentz gas model (pitch-angle scattering only) are performed in the present work to gain better understanding of collisionality effects

SCHEMATIC OF FINITE COLLISIONALITY ON ECCD



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RADIAL AND POLOIDAL SCANS HAVE BEEN OBTAINED TO TEST THE EFFECTS OF TRAPPED PARTICLES

P_{ECH} = 0.95–1.14 MW

$\bar{n}$ = 1.66–1.85 10^{13} cm^{-3}

q_{95} = 5.95–6.33

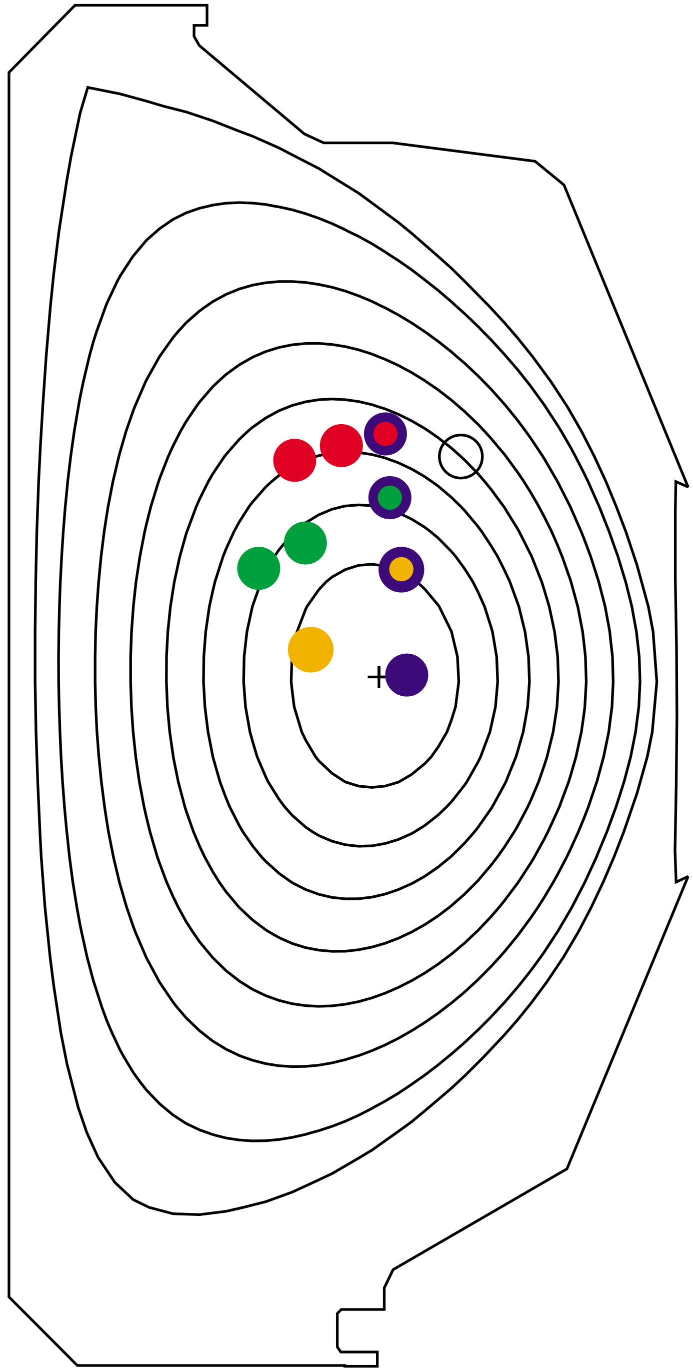
Radial Scan

Poloidal Scan ρ = 0.2

Poloidal Scan ρ = 0.34

Poloidal Scan ρ = 0.47

ECCD experiments were also conducted in the counter-current campaign



DIII–D NATIONAL FUSION FACILITY SAN DIEGO

GENERAL ATOMICS

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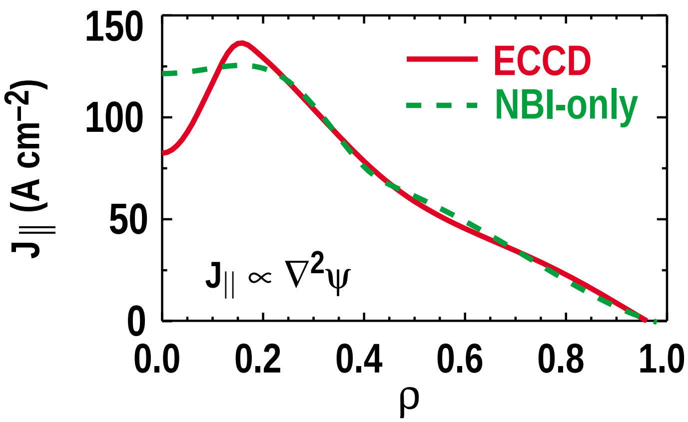
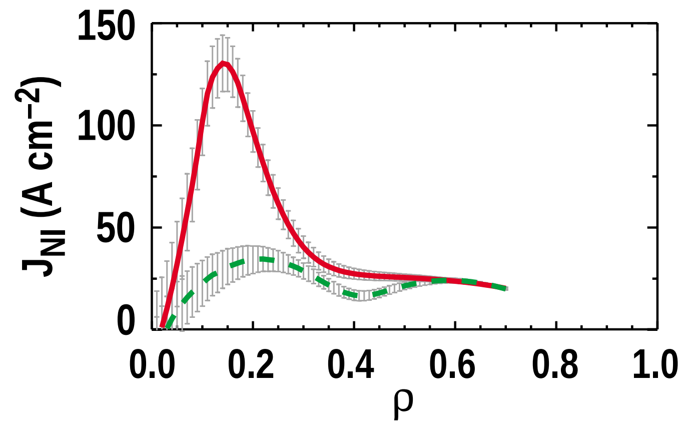
LOCALIZED CURRENT DRIVE IS CLEARLY MEASURED

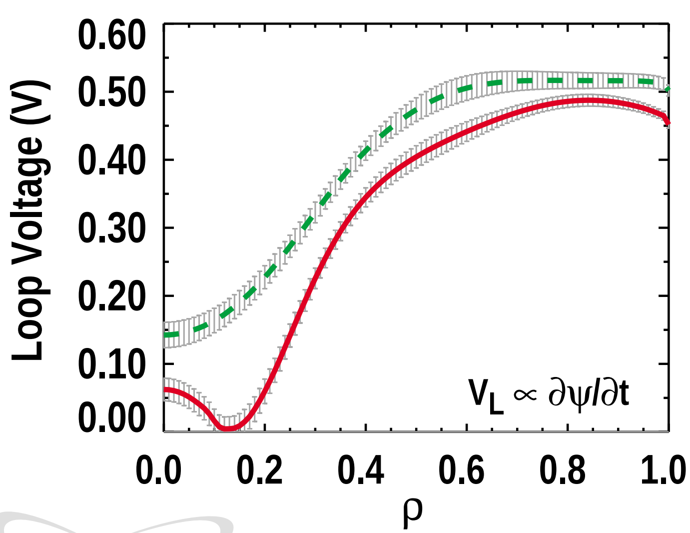


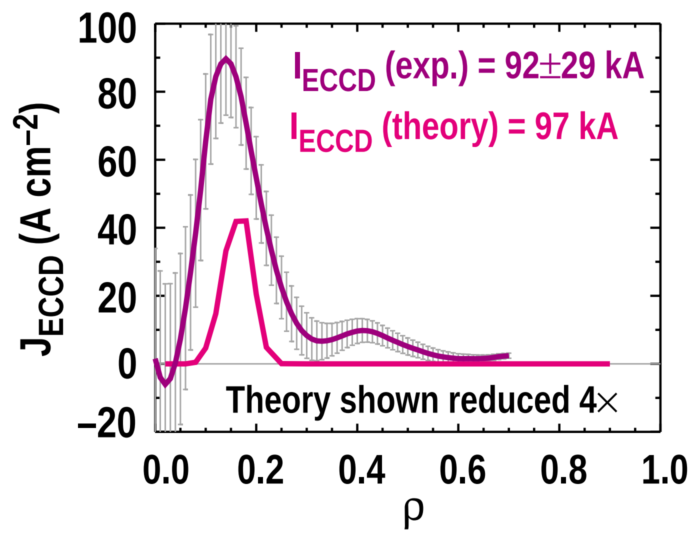
$J_{||}$ and loop voltage obtained from magnetic reconstructions with high resolution motional Stark effect spectroscopy (MSE)



Assumption of neoclassical resistivity gives $J_{NI} \equiv J_{||} - \sigma E_{||}$

Comparison of ECCD case with NBI-only fiducial separates ECCD from bootstrap and NBCD





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GENERAL ATOMICS

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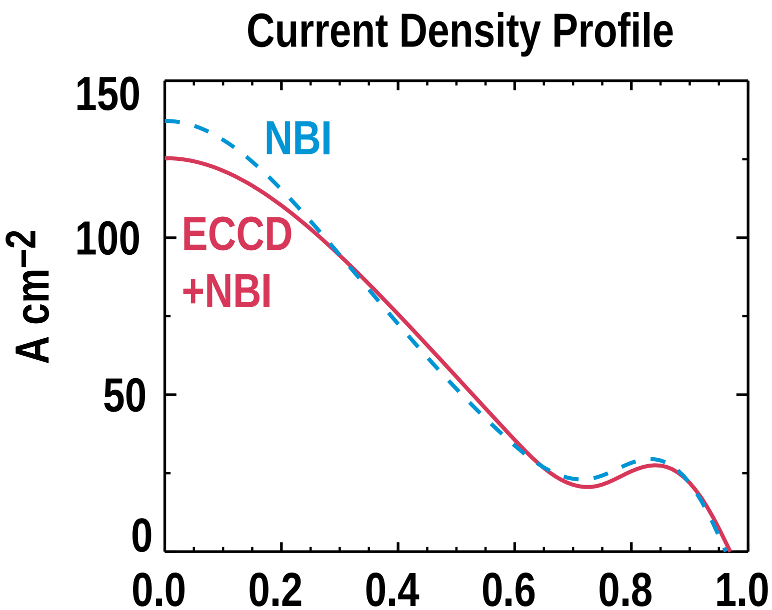
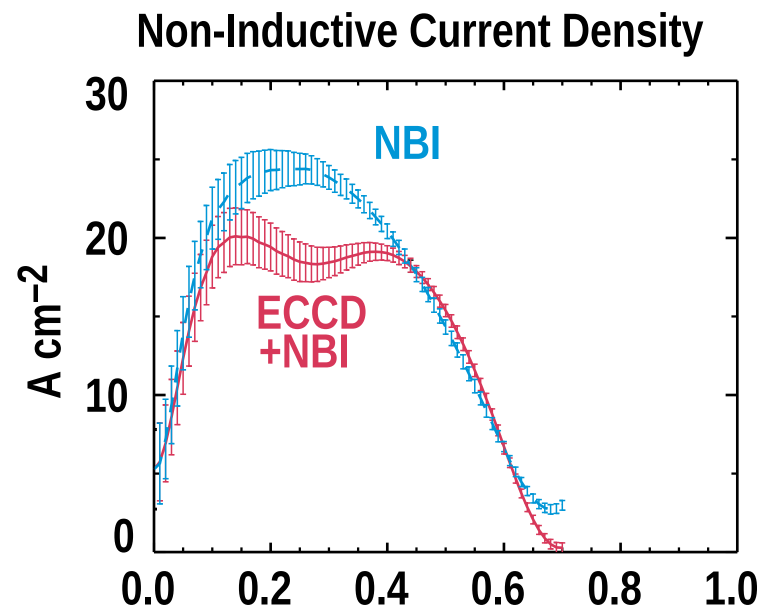
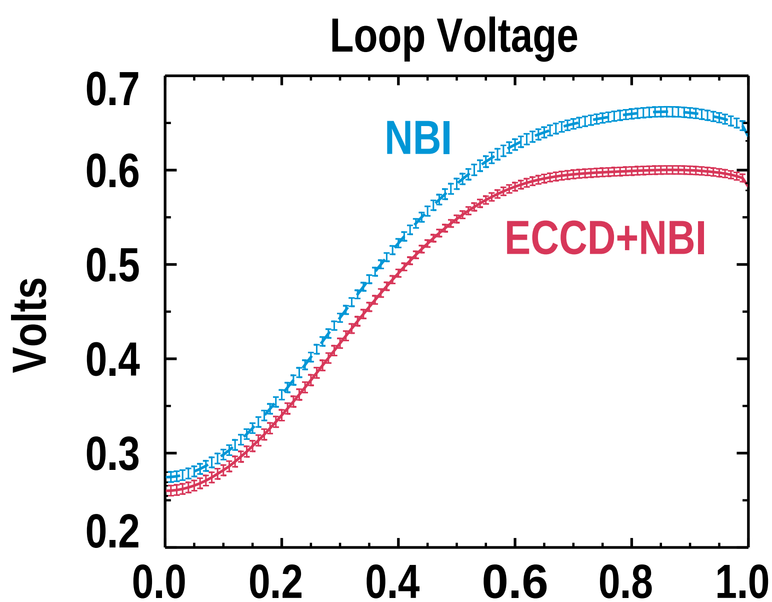
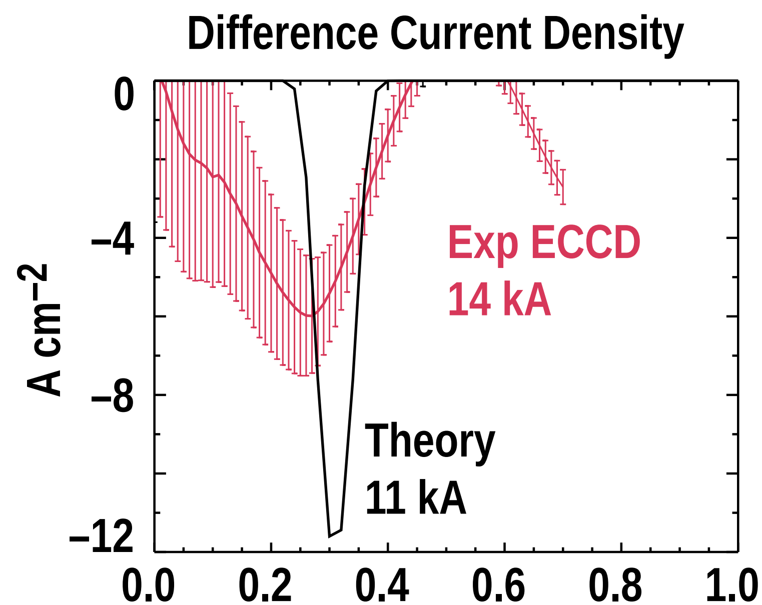
OFF-AXIS COUNTER ECCD WAS MEASURED









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GENERAL ATOMICS

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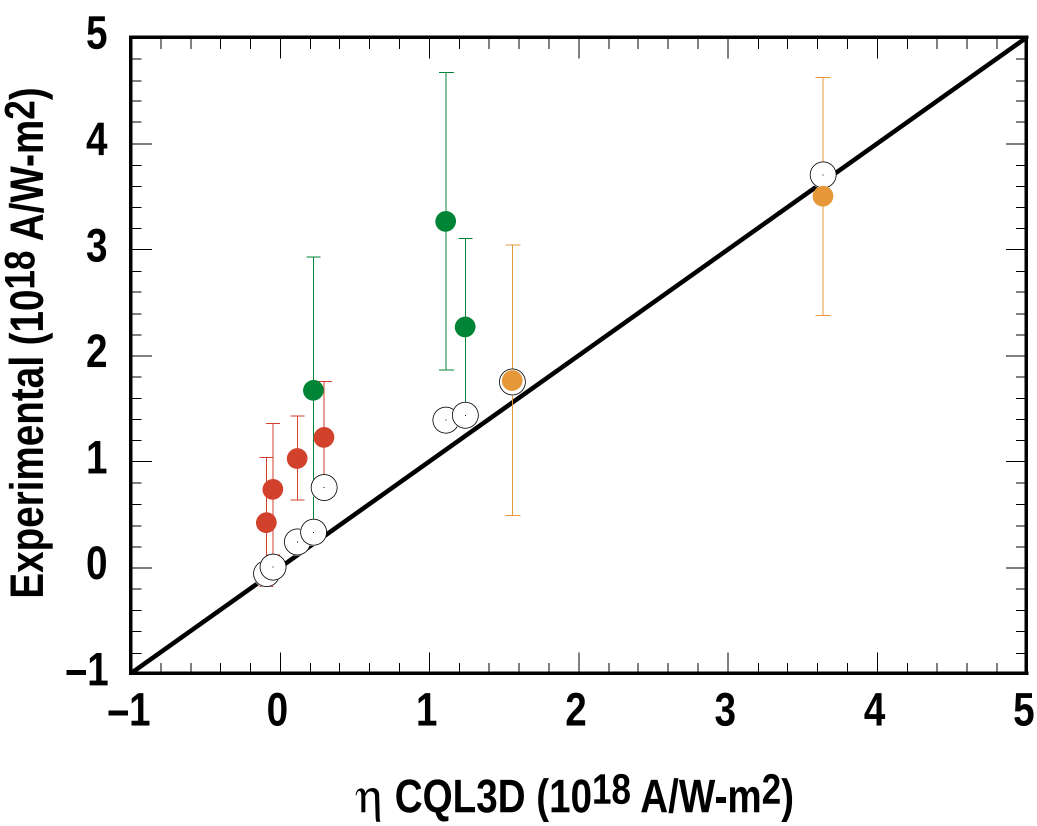
BOUNCE AVERAGED FOKKER-PLANCK CALCULATIONS UNDERESTIMATE OF ELECTRON CYCLOTRON CURRENT DRIVE EFFICIENCY

- The bounce averaged calculations ae based on the zero-collision theory, i.e., $\tau_b << \tau_e$ for all energies; the assumption is clearly not valid for low energy electrons

- Collisionality effectively reduces trapped electron fraction and increases current drive efficiency



Open circles correspond to the calculated ECCD efficiency in the presence of the measured Ohmic field $E_{||}$

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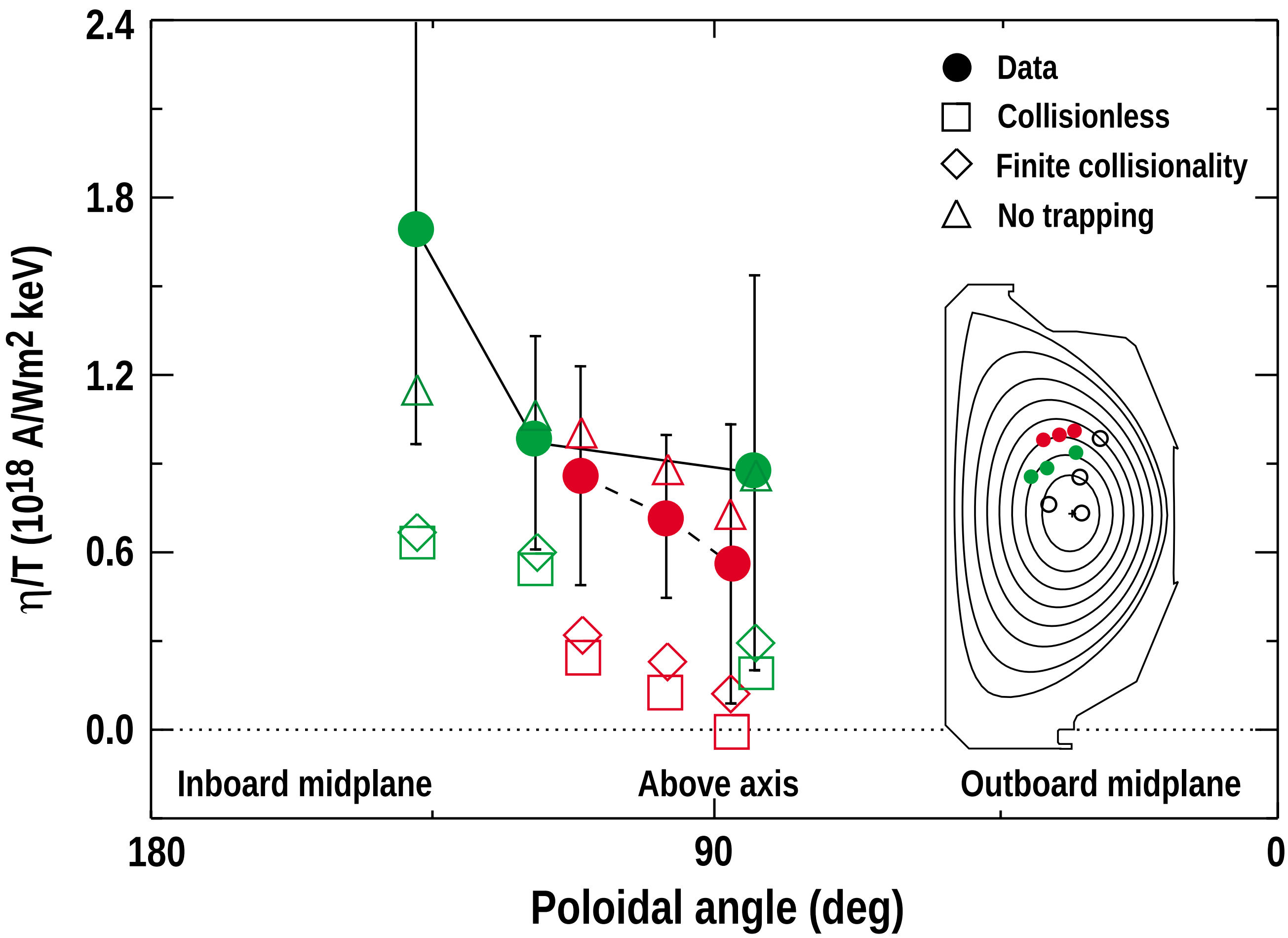
FINITE COLLISIONALITY CALCULATIONS USING A LINEAR ADJOINT FUNCTION SHOW AN INCREASE IN EFFICIENCY



- Quasi-linear and $E_{||}$ effects are not included
- The connection formula is $\chi = \chi_c + F\,(\chi_B - \chi_c)$ where χ_c is the no trapping limit and χ_B is the collisionless limit

$$F \equiv \left[1 + \alpha \sqrt{\nu_{*,e}} \left(\frac{u_e}{u}\right)^2\right]^{-1}$$

The results shown are for $\alpha = 2$ which reproduces finite collisionality results for the conductivity and bootstrap current

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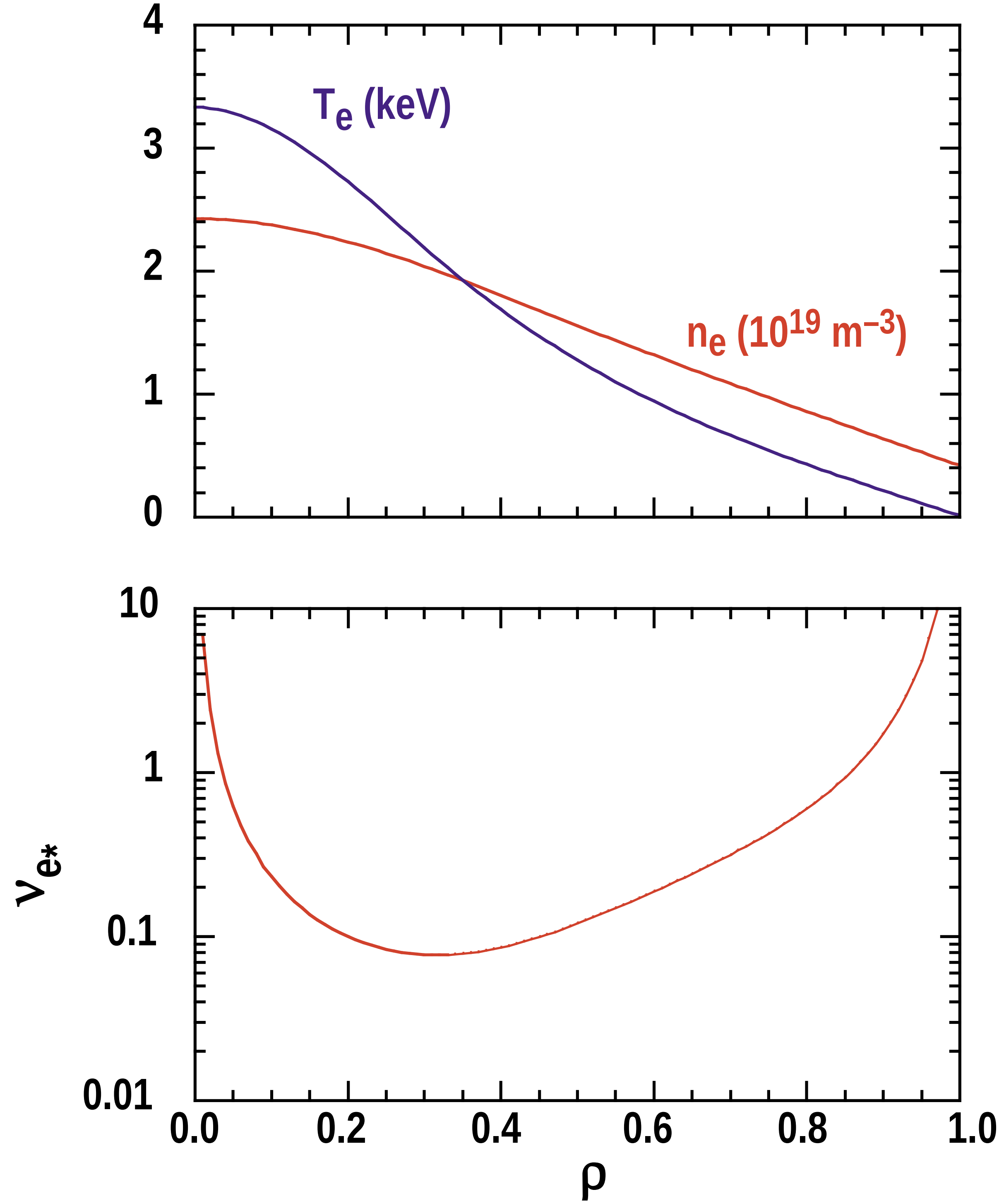
KINETIC PROFILES OF EXPERIMENTAL L–MODE DISCHARGE



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THEORETICAL MODEL FOR ECCD

- **Quasilinear Fokker-Planck equation:**

$$v_{\parallel}\hat{b}\nabla f - C_e f = S_{rf}(f) + eE_{\parallel}\frac{\partial f}{\partial p_{\parallel}}$$

where f is the electron distribution function, $v_{\parallel}$ and $p_{\parallel}$ are respectively the parallel velocity and momometum.

C_e: Coulomb collision operator

S_{rf}: quasilinear rf diffusion operator

$E_{\parallel}$: Ohmic electric field

- **In tokamak geometry, the driven current has a simple poloidal angle dependence:**

$$j_{\parallel} \equiv -e\int d\Gamma\, f_1 v_{\parallel}$$

$$= \frac{\langle j_{\parallel}B\rangle}{\langle B^2\rangle}B$$

i.e., $j_{\parallel}/B$ is a flux-surface quantity

STANDARD THEORETICAL TREATMENTS

- **Collisionless model** $\left(\tau_b \approx \frac{qR}{v} << \tau_e\right)$:

$$\hat{b} \cdot \nabla f = 0 \; ; \qquad f = f(\varepsilon, \mu)$$

$$-\left\langle \frac{B}{v_\parallel} C_e \right\rangle f = \left\langle \frac{B}{v_\parallel} S_{rf} \right\rangle (f) + \left\langle e E_\parallel B \right\rangle \frac{\partial f}{\partial \varepsilon}$$

where ε is the particle energy and μ is the magnetic moment. For a given flux surface, to solve the bounce-averaged equation is a 2-D problem

- **Linear Regime:**

$$f = f_M + f_1^{ohm} + f_1^{rf} + \ldots$$

$$j_\parallel = j_\parallel^{ohm} + j_\parallel^{rf} + \ldots$$

Here f_M is the Maxwellian distribution, f_1^{ohm} is the equivalent Spitzer function in toroidal geometry, and

$$v_\parallel \hat{b} \nabla f_1^{rf} - C_e^{\ell} f_1^{rf} = S_{rf}(f_M)$$

We expect the linear approximation to be justified for

$$p_{rf} \tau_e << n_e T_e$$

where p_{rf} is absorbed rf power density

ADJOINT TECHNIQUES CAN BE USED TO EXAMINE COLLISIONALITY EFFECTS ON ECCD IN LINEAR REGIME

- **Introducing the adjoint equation:**

$$-v_\parallel \hat{\mathbf{b}}\nabla\chi - \mathbf{C}_\mathbf{e}^{\ell+}\chi = \frac{v_\parallel \mathbf{B}}{\langle \mathbf{B}^2\rangle}$$

Here, $\mathbf{C}_\mathbf{e}^{\ell+}$ is the adjoint collision operator defined by

$$\left\langle \int d\Gamma f\, \mathbf{C}_\mathbf{e}^{\ell}\, g \right\rangle = \left\langle \int d\Gamma g\, \mathbf{C}_\mathbf{e}^{\ell+}\, f \right\rangle$$

Therefore,

$$\begin{aligned}
\frac{j_\parallel}{B} &= -e\left\langle \int d\Gamma\, f_1 \frac{v_\parallel B}{\langle B^2\rangle} \right\rangle \\
&= -e\left\langle \int d\Gamma\, f_1 \left(-v_\parallel \hat{b}\nabla\chi - C_e^{\ell+}\chi\right)\right\rangle \\
&= -e\left\langle \int d\Gamma\, \chi\left(v_\parallel \hat{b}\nabla f_1 - C_e^{\ell} f_1\right)\right\rangle \\
&= -e\left\langle \int d\Gamma\, \chi S_{rf}\left(f_M\right)\right\rangle
\end{aligned}$$

- **Absorbed power density:**

$$Q = \left\langle \int d\Gamma\, \varepsilon S_{rf}\left(f_M\right)\right\rangle$$

ADJOINT FORMULATION:
AN EFFICIENT METHOD FOR MODELING ECCD IN LINEAR REGIME

- **Current drive efficiency:**

write $\chi = \nu_{e0}^{-1}\left(\dfrac{v_e}{B_{max}}\right)\tilde{\chi}$,

with $v_e \equiv \left(\dfrac{2T_e}{m}\right)^{1/2}$ and $\nu_{e0} = \dfrac{e^4 n_e \ell n\Lambda}{4\pi\varepsilon_0^2 m^2 v_e^3}$

$$\frac{\eta}{T_e} \equiv \frac{n_e\langle j_{\parallel}\rangle}{2\pi Q}$$

$$= \frac{4\varepsilon_0^2}{e^3 \ell n\Lambda}\left\langle\frac{B}{B_{max}}\right\rangle \frac{\left\langle\int d\Gamma\tilde{\chi} S_{rf}(f_M)\right\rangle}{\left\langle\int d\Gamma\left(\varepsilon / m v_e^2\right) S_{rf}(f_M)\right\rangle}$$

- **Note that there is no dependence on S_{rf} in χ; once evaluated, it can be used to calculate η for any given S_{rf}**

- **Moreover, χ is the Spitzer function in toroidal geometry. It can be used to evaluate σ_{neo} and the bootstrap coefficients**

SIMPLIFIED COLLISION MODEL

- To solve the adjoint equation with the full coulomb collision operator for arbitrary collisiionality is a 3-D numerical problem

- Small inverse aspect ratio limit ($\delta \equiv \frac{a}{R} \rightarrow 0$)

  — pitch-angle scattering dominant

  — analytic solutions possible in the banana regime

  — using Hinton-Rosenbluth boundary layer analysis the leading order collisionality corrections to ECCD [+]

  $$\Delta j \cong \sqrt{\delta \nu_{*e}}\, j_c$$

  where j_c is the ECCD in the no-trapping limit

- The collisionality correction to ECCD efficiency estimated by the velocity-space connection formula appears to be consistent with the above scaling law

[+] V.S. Chan (unpublished, 1981 APS)

LORENTZ GAS MODEL

- **Lorentz gas model (e-i pitch-angle scattering only):**

$$C_e f \approx \nu_{ei} L f$$

$$L = \frac{1}{2}\frac{\partial}{\partial \xi}\left(1 - \xi^2\right)\frac{\partial}{\partial \xi} \quad ; \qquad \xi \equiv \frac{u_\parallel}{u}$$

$$\nu_{ei}(v) = Z_{eff}\ \nu_{e0}\ \gamma\left(\frac{u_e}{u}\right)^3; \qquad \vec{p} \equiv m\vec{u} = m\gamma\vec{v}$$

- **Adjoint equation (2-D numerical problem):**

$$B = \frac{B_0}{h(\theta)}; \qquad \hat{b}\cdot\nabla = \frac{B_\theta}{rB}\frac{\partial}{\partial \theta}$$

$$\chi = \chi_c + \chi_t = \frac{vB_0}{\nu_{ei}\langle B^2\rangle}\left(\frac{\xi}{h} + G_t\right)$$

$$-\sigma\frac{\partial}{\partial \theta}G_t - \nu\frac{\partial}{\partial \lambda}\left(\lambda|\xi|\right)\frac{\partial}{\partial \lambda}G_t = \alpha$$

with $\lambda \equiv \left(1 - \xi^2\right)h;$ $\qquad \sigma = \mathrm{sgn}\left(u_\parallel\right)$

$$\nu \equiv \frac{2rB_0}{B_\theta v}\nu_{ei} \approx \frac{2qR}{v}\nu_{ei}; \qquad \alpha \equiv \frac{\partial}{\partial \theta}\frac{|\xi|}{h}$$

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LORENTZ GAS MODEL
(NUMERICAL SCHEME)

- **Following Hinton and Rosenbluth,+**

Define $G_t^{\pm} = \frac{1}{2}\left\{G_t(\sigma = -1) \pm G_t(\sigma = +1)\right\}$,

$$\frac{\partial}{\partial\theta} G_t^{-} - \nu \frac{\partial}{\partial\lambda}\left(\lambda|\xi|\right)\frac{\partial}{\partial\lambda} G_t^{+} = \alpha$$

$$\frac{\partial}{\partial\theta} G_t^{+} - \nu \frac{\partial}{\partial\lambda}\left(\lambda|\xi|\right)\frac{\partial}{\partial\lambda} G_t^{-} = 0$$

Introduce $\Phi = \int_0^{\lambda} d\lambda' G_t^{+}(\lambda', \theta)$, **which satisfies**

$$\frac{\partial}{\partial\theta}\left(\frac{\partial\Phi}{\partial\theta}\Big/\nu\lambda|\xi|\right) - \nu\frac{\partial^2}{\partial\lambda^2}\left(\lambda|\xi|\right)\frac{\partial^2\Phi}{\partial\lambda^2} = \frac{\partial\alpha}{\partial\lambda}$$

The equation is 2nd order in θ and 4-th order in λ

- **The problem of solving the above equation can be formulated in terms of a variational principle, and**

$$G_t^{+} = \frac{\partial\Phi}{\partial\lambda}; \qquad \frac{\partial G_t^{-}}{\partial\lambda} = \frac{1}{\nu\lambda|\xi|}\frac{\partial\Phi}{\partial\lambda}$$

+Hinton and Rosenbluth (1973)

NUMERICAL SCHEME (CONTINUED)

- **Numerical solution:**
  - finite difference in θ and $|\xi|$.
  - ADI iteration scheme based on Hinton-Rosenbluth variational principle
  - use a primitive multi-grid scheme
  - simplified magnetic geometry ($B = \dfrac{B_0}{1+\delta\cos\theta}$)

- **Analytic solution in the banana regime:**

write $$\chi = \frac{vB_0}{\nu_{ei}\langle B^2\rangle}\left(\frac{\xi}{h} + G_t\right) = \frac{vB_0}{\nu_{ei}\langle B^2\rangle} G$$

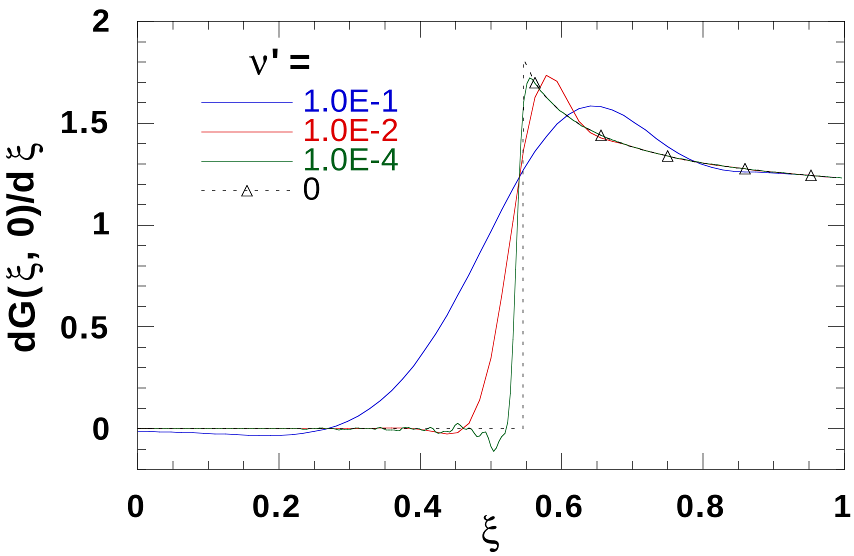
$$G = \frac{1}{2}\,\mathrm{sgn}(u_\parallel)\int_\lambda^{\lambda_c} \frac{d\lambda'}{\left\langle \sqrt{(1-\lambda'/h)}\right\rangle} \qquad (\lambda < \lambda_c)$$

$$G = 0 \qquad (\lambda > \lambda_c)$$

where $\lambda_c \equiv 1-\delta$

COLLISIONALITY CORRECTIONS LOCALIZED NEAR BOUNDARY LAYER

- **Numerical results for Lorentz gas model:**
  - $B = \dfrac{B_0}{1+\delta\cos\theta}$; $\delta = 0.175$
  - $\nu' \equiv \dfrac{\nu}{2\delta^{3/2}} = \dfrac{qR}{Z_{\text{eff}}\delta^{3/2}v}\nu_{\text{ei}}$
  - $\nu' = 0$ **corresponds to the analytic banana-regime solution**



- $dG/d\xi$ **as a function of** ξ **at ploidal angle** $\theta = 0$ **(outboard midplane) for various** ν'

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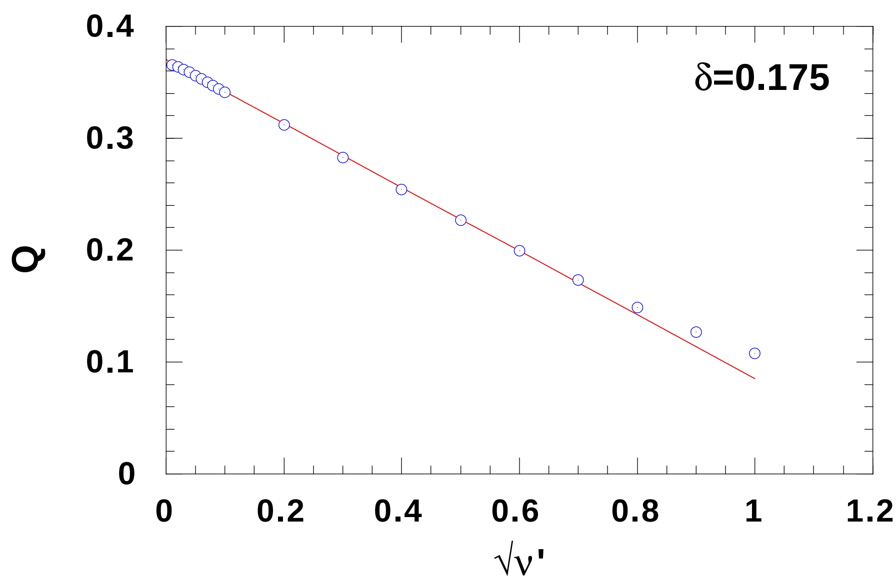
NUMERICAL RESULTS AGREE WITH THE BOUNDARY LAYER ANALYSIS IN THE LOW COLLISIONALITY LIMIT

- **Define** $Q \equiv h\int d\xi\, G_t(\xi, \theta, \nu')\xi$
  - Q is independent of poloidal angle θ
  - $Q = Q_0\left(1 - a\sqrt{\nu'} + \cdots\right)$ as $\nu' \to 0$
  - $a \approx 0.92$ in the small inverse aspect ratio limit ($\delta \to 0$)
  - $Q_0 = \dfrac{2}{3} f_t$ (f_t: the effective trapped particle fraction)



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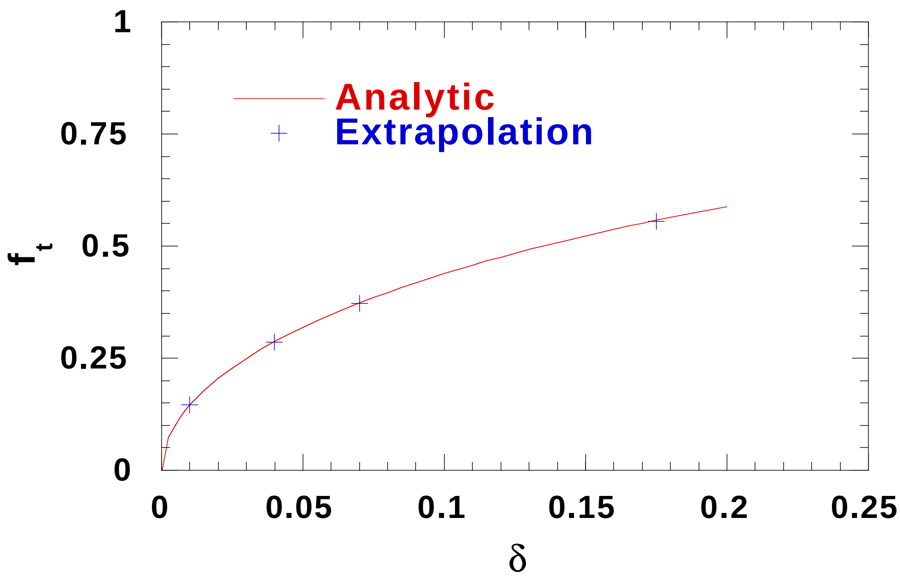
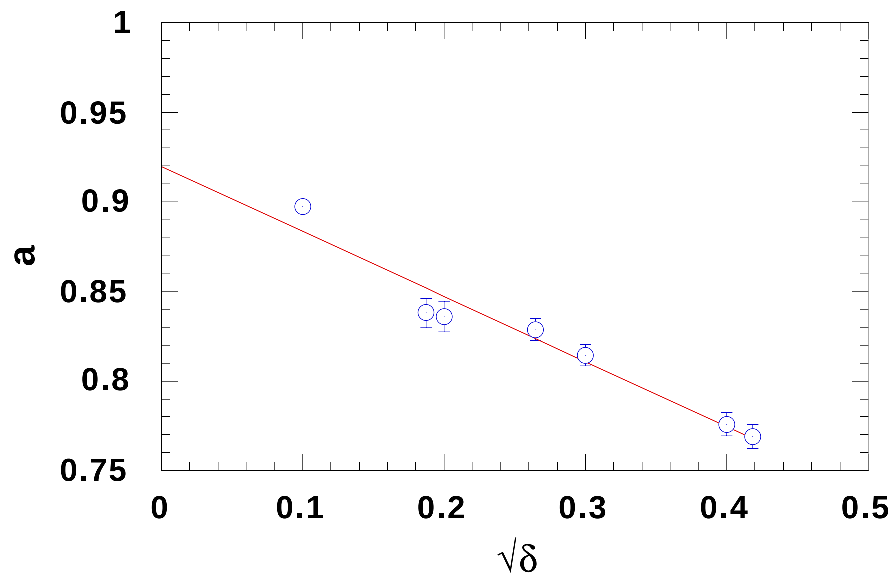
NUMERICAL RESULTS (CONTINUED)





- **The trapped particle fraction $f_t = \frac{3}{2}Q_0$ and the coefficient a as functions of inverse aspect ratio δ**

SUMMARY

- Comparing with the measured off-axis ECCD efficiency in recent DIII–D experiments, bounce-averaged calculations give lower estimates of the efficiency

- Collisionality reduces the trapped electron effects and will increase ECCD efficiency

- A velocity-space interpolation formula for estimating the collisionality effect gives only modest improvement in the theoretical situation

- The boundary layer analysis in the small inverse aspect ratio limit ($\delta \rightarrow 0$) indicates the collisionality correction is on the order of $\sqrt{\delta \nu_{*e}}$

- Numerical calculations of ECCD effieciency based on Lorentz gas model are in progress