MULTI-DEVICE DIMENSIONLESS SCALING
OF NEOCLASSICAL TEARING MODE BETA LIMIT

by
R.J. La Haye

in collaboration with
R.J. Buttery,* S. Guenter,† G.T.A. Huysmans,‡ and H.R. Wilson*

*EURATOM/UKAEA Fusion Assoc. Culham Science Center
†Max Planck Institut fur Plasmaphysik
‡JET Joint Undertaking, currently at CEA

Presented at
the American Physical Society
Division of Plasma Physics Meeting
Seattle, Washington

November 15–19, 1999
Multi-Device Dimensionless Scaling of Neoclassical Tearing Mode Beta Limit

R.J. LA HAYE, General Atomics, R.J. BUTTERY, H.R. WILSON, Euratom/UKAEA Fusion Association Culham, S. GUENTER, MPI f. Plasmaphysik, G.T.A HUYSMANS, Jet Joint Undertaking (now at CEA, Cadarache) — To extrapolate the neoclassical tearing mode (NTM) beta limit to reactor grade tokamaks, a multi-device database has been compiled from Asdex-Upgrade, DIII-D, and JET. The key issue in predicting the NTM beta limit is the relative scaling of the “seed” island $w_s$ to the threshold island $w_{th}$. For sawtooth induced $m/n = 3/2$ NTM, the relative threshold island width is taken from the polarization/inertial model as $w_{th}/r \propto \rho_i^* g^{1/2}(\epsilon, \nu)$ where $g$ is a function of collisionality $\nu = \nu_i/\epsilon \omega_i^*$ that increases from 1 at low $\nu$ to $\epsilon^{-3/2} \gg 1$ at high $\nu$. The relative seed island scaling, allowing for the dynamics of geometrically coupled perturbations as a function of magnetic Reynolds number $S$, is taken as $w_s/r \propto \beta_g^2 S^{-\alpha} \propto \rho_i^{3\alpha} \nu^\alpha$ for $\gamma = \alpha/2$. Thus the scaling of $w_s/w_{th} \propto \rho_i^{3\alpha-1} \nu^\alpha$ with $\rho_i$ depends critically on whether $\alpha \ll 1/3$. Best fits of experimental data will be presented.

1Work supported in part by U.S. DOE Contract DE-AC03-99ER54463 and the U.K. Dept. of Trade and Industry and Euratom.


q = 1 sawtooth induced m/n = 3/2 NTM; beta decreases by up to 30%
HELICALLY PERTURBED BOOTSTRAP CURRENT CAN EXCITE NEOCLASSICAL TEARING MODE

\[ \tau_R \frac{dw}{dt} = \Delta' + \varepsilon^{1/2} \left( \frac{L_q}{L_p} \right) \beta_\theta \left( \frac{w}{w^2 + w_d} - \frac{w_{\text{thresh}}^2}{w^3} \right) \]

★ Modified Rutherford Eq. with \( w_{\text{thresh}} \approx \varepsilon^{1/2} \rho_\theta \text{i} \), the ion banana width

\[ \beta_\theta, \text{ crit } \approx \left[ -\Delta' / (\varepsilon^{1/2} L_q/L_p) \right] \left( \frac{w_{\text{thresh}}}{r} \right) \left[ \frac{w_{\text{seed}}/w_{\text{thresh}}}{1 - (w_{\text{thresh}}/w_{\text{seed}})^2} \right], \quad w_{\text{thresh}}^2 \gg w_d^2 \]

★ if \( w_{\text{seed}}/w_{\text{thresh}} = \text{constant} > 1, \quad \beta_\theta, \text{ crit } \propto w_{\text{thresh}}/r \propto \rho_\theta^* \)

★ if \( w_{\text{seed}}/w_{\text{thresh}} < 1, \quad \beta_\theta, \text{ crit } \rightarrow \infty, \quad \text{i.e. plasma remains metastable} \)

\[ \sqrt{3} w_{\text{seed}}/w_{\text{thresh}}, \quad w_{\text{sat}}/w_{\text{thresh}} \]
DIMENSIONLESS SCALING MODEL

- $w_{\text{thresh}}$ from polarization/inertial model (Wilson, et al., 1996)
  
  $w_{\text{thresh}}/r \propto \rho_{i*} g^{1/2}(v, \varepsilon)$ with $g = (1+C_2 v)/(1+C_3 v)$ for $v \equiv v_i/\varepsilon \omega_{e*}$ and $C_2/C_3 \approx \varepsilon^{-3/2}
  
  - $w_d/r \propto (\chi_{\perp}/\chi_{||})^{1/4} \propto \rho_{i*}^{1/3}$ for $\chi_{\perp} \propto \chi_{\text{BOHM}}$ and $\chi_{||} \propto C_{\text{sw}}^{-1}$
  - ... Fitzpatrick et al., incomplete pressure flattening

- $w_{\text{seed}}$ from dynamical coupling model (Hegna et al., 1999)
  
  $\star \frac{w_{\text{seed}}}{r} \propto \left( \frac{\psi}{\psi_0} \right)^{1/2} \cdot f \left( \frac{r_1}{R}, \frac{r_{3/2}}{R}, \Lambda \right) \cdot S^{-\alpha} \propto \beta \gamma S^{-\alpha}, \alpha \text{ tbd}
  
  rel. sawtooth amp. geometric $m \pm 1$ dynamic shielding at $q = 3/2$ skin layer
  a function of $S$? coupling increases with mag. Reynold's no.
  
  - $S \propto \beta^{1/2}/\rho_{i*}^{3} v \Rightarrow w_{\text{seed}}/r \propto \rho_{i*}^{3\alpha} v^{\alpha}$ for $\gamma \equiv \alpha/2$

- $\frac{w_{\text{seed}}}{w_{\text{thresh}}} \propto \frac{3\alpha-1}{\rho_{i*}^{1/2}} \frac{\alpha}{g(v)} \approx \text{constant for } \alpha = 1/3$ and fixed $v$
  
  - $\alpha > 1/3$ would be favorable for a reactor-grade tokamak
EXAMINE DIMENSIONLESS SCALING IN AUG, DIII–D AND JET

- LSND, ELMing H–mode, q95 ≥ 3
- Sawtooth induced 3/2 NTM database
- Extrapolate to proposed ITER/FDR
A common separable power law of form \( \beta_{NC} \propto \rho_i^x (\nu_i/\epsilon \omega_{e*})^y \)

- Does not represent the scaling, thus \( \alpha \neq 1/3 \)

- \( \beta_N \propto \rho_i^* \) is support for polarization/inertial threshold model
The best fit of the database to the physics model has $\alpha \approx 4/9$.

$$\frac{\beta_{NC}}{C_0 \rho_{i*}} = \frac{C_1 \rho_{i*}^{3\alpha-1} \nu^\alpha}{1 + C_2^{2/3-6\alpha} \nu^{-2\alpha}} \frac{(1 + C_2^\nu)/(1 + C_3^\nu)}{C_1 \rho_{i*}^{2\alpha} - \nu^{2\alpha}}$$

- Seed island decreases faster with $\rho_{i*}$ than threshold for $\alpha \approx 4/9$.
- Stabilizes at very low $\rho_{i*}$, high $S$, i.e. $w_{seed}/w_{thresh} < 1$.
- ... nom op pt is stable for ITER/FDR.

The nom op pt is stable for ITER/FDR when $\rho_{i*}$ is low and $\nu = \nu_i/\epsilon \omega_{e*}$ is high.
CRITICAL BETA FOR NTM DEPENDS ON RELATIVE SCALING OF \( w_{\text{seed}} \) TO \( w_{\text{threshold}} \)

- \( w_{\text{seed}}/r \) decreases in dynamic shielding model at higher \( S \)
  - \( w_{\text{seed}}/r \propto S^{-4/9} \propto \rho_{i*}^{4/3} \) (at fixed \( \nu \))

- \( w_{\text{thresh}}/r \propto \rho_{i*} \) from polarization/inertial model

- \( w_{\text{seed}}/w_{\text{thresh}} \propto \rho_{i*}^{1/3} \) (at fixed \( \nu \))
  - Favorable for ITER-FDR, i.e. at small \( \rho_{i*} \)

★ Seed may be too small to excite NTM

... but depends on the difference of large extrapolations