CP1 2:00 MONDAY

Abstract Submitted
for the DPP99 Meeting of
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Sorting Category: (Theoretical)

Algorithms for Finding 2D MHD Equilibria with Given "Almost Ideal" MHD Constraints[1] T.H. JENSEN, General Atomics — Under the constraints of ideal MHD, the flux surface topology cannot change. This not true under the constaints of almost ideal MHD (AIMHD). Equilibrium algorithms observing AIMHD constraints can therefore be used for the study of nonlinear properties of tearing modes.[2] Only the simplest cases are considered for which (i) $\partial/\partial z = 0$; (ii) $\bar{\nabla} p = 0$; (iii) $(\bar{B} \times \hat{z})^2/(\bar{B} \cdot \hat{z})^2 \ll 1$. It is the aim of this work to find algorithms which can deal with cases for which the current density may be different functions of the flux function on the two sides of a singular surface so that a finite gradient of the current density at the singular surface may exist. For such cases, island formation may result in irreversible, quantifiable changes of the AIMHD constraints and a nonlinear instability of tearing modes may exist.[3] Several approaches to making suitable algorithms for this purpose will be discussed.

[1]Supported by U.S. DOE Grant DE-FG03-95ER54309.
[2]C. Ren, T.H. Jensen, and J.D. Callen, Phys. Plasmas **5**, 2574 (1998).
[3]T.H. Jensen and W.B. Thompson, Phys. Fluids **30**, 3052 (1987).

☐ Prefer Oral Session
☒ Prefer Poster Session

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OBJECT OF WORK:

FIND MEANS FOR STUDYING NONLINEAR PROPERTIES OF TEARING MODES. (ULTIMATELY TO BE ABLE TO HANDLE CASES WITH A FINITE CURRENT DENSITY GRADIENT AT THE SINGULAR SURFACE)

FRAMEWORK:

SINGLE FLUID MHD ; $\bar{\nabla} p = 0$

"INITIAL EQUILIBRIUM": $\partial/\partial z = \partial/\partial y = 0$

"FINAL (FORCED) EQUILIBRIUM": $\frac{\partial}{\partial z} = 0$; PERIODIC IN y

$$(\bar{B} = B_z \hat{z} + \bar{\nabla}\psi \times \hat{z})$$

TOKAMAK ORDERING: $(\bar{\nabla}\psi / B_z)^2 \ll 1$

RESULTS:

i) MEANS FOUND

ii) STABILITY LIMIT AGREES WITH STANDARD LINEAR THEORY[1)]

iii) SATURATED ISLAND WIDTHS ARE SENSITIVE TO DETAILS

REF. 1 FORTH, KILLEEN + ROSENBLUTH, PHYS. FLUIDS 6 459 (1963)

BASIC CONCEPTS:

i) AN MHD EQUILIBRIUM MAY BE DETERMINED BY SPECIFYING $B_z(\psi)$ AND BOUNDARY CONDITIONS THROUGH

$$\nabla^2\psi + \frac{1}{2}\frac{d}{d\psi}B_z^2(\psi) = 0$$

IF FOR A REAL PLASMA EXTERNAL CURRENTS ARE ADDED (OR BOUNDARY CONDITIONS CHANGED) $B_z(\psi)$ WILL CHANGE.

ii) AN MHD EQUILIBRIUM MAY INSTEAD BE DETERMINED BY SPECIFYING AN IDEAL MHD CONSERVED FUNCTION (e.g. $q(\psi)$). IF FOR A REAL PLASMA EXTERNAL CURRENTS ARE APPED (OR BOUNDARY CONDITIONS CHANGED), THIS CONSERVED (INVARIANT) FUNCTION WILL NOT CHANGE.

iii) THE RESPONSE OF A PLASMA EQULIBRIUM (GIVEN BY CONSERVED FUNCTION) TO EXTERNAL CURRENTS HAS INFORMATION ON STABILITY AND DIRECTION OF DYNAMIC DEVELOPMENT.

NO INFORMATION ON TIMESCALE OF DYNAMICS.

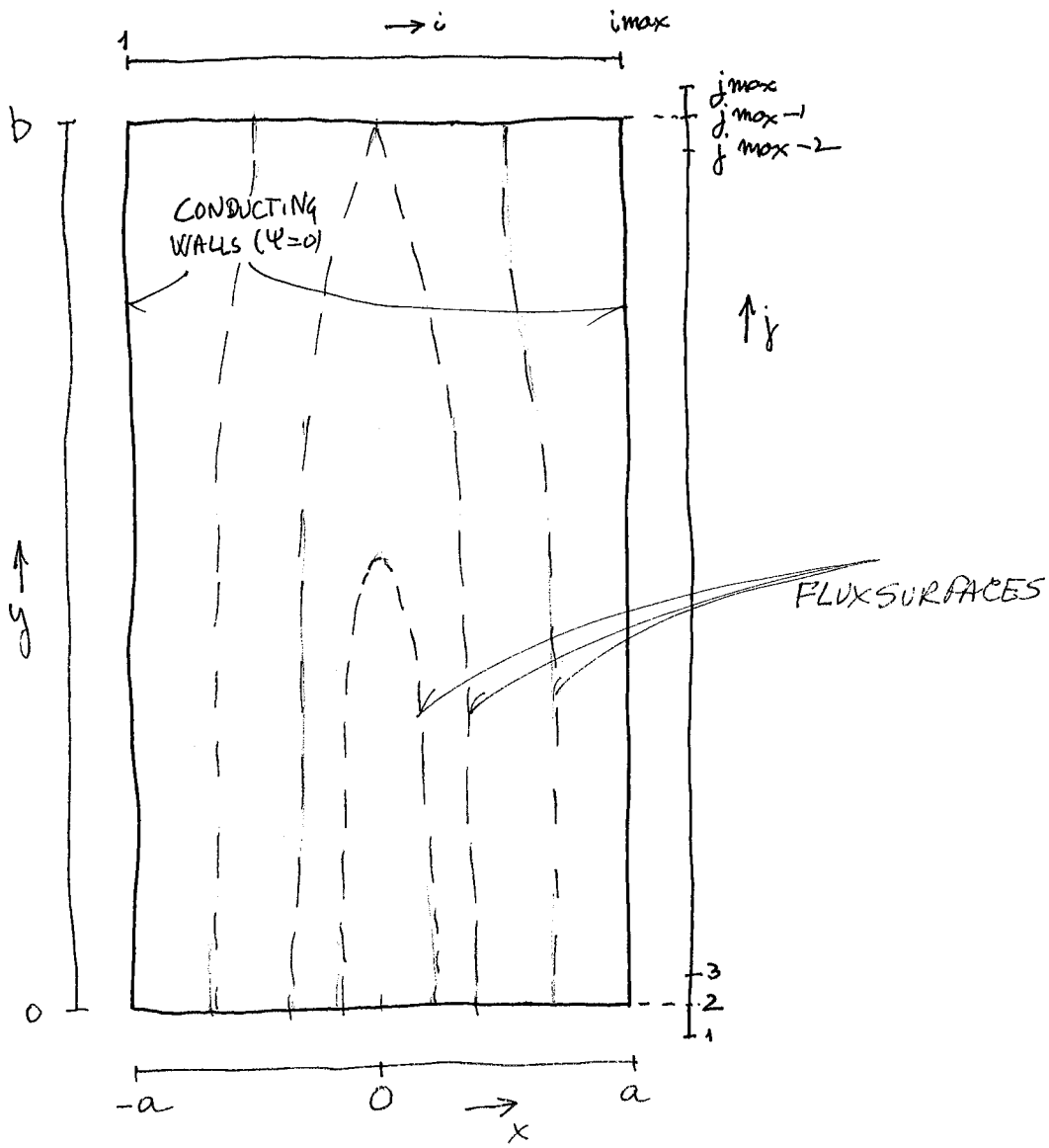
iv) IDEAL MHD CONSTRAINTS CANNOT BE USED FOR THE STUDY OF TEARING MODES SINCE CHANGE OF FLUX FUNCTION TOPOLY IS IMPOSSIBLE UNDER SUCH CONSTRAINTS.

b) "ALMOST IDEAL MHD"[2] EMPLOYS A FINITE NUMBER OF CONSERVED QUANTITIES (INSTEAD OF A CONSERVED FUNCTION (IDEAL MHD)).

ALMOST IDEAL MHD ALLOWS TOPOLOGICAL CHANGES OF FLUXFUNCTIONS AND CAN THEREFORE BE USED FOR STUDYING TEARING MODES.

SPECIFICS:

GEOMETRY:



PERIOD IN y: 2b

REF. 2: JENSEN, LEONARD, LAHAYE + CHU, PHYS. FLUIDS B 3 1650 (1991)

CONSERVED QUANTITIES:

$$KO_\nu \equiv \int G_\nu(\underset{INT}{\psi}(x,y))\,dx\,dy \quad ; \quad \nu = 1, 2, \ldots$$

ψ_{INT} IS INITIAL EQUILIBRIUM; $G_\nu(\psi)$ IS A SET OF FUNCTIONS TO BE CHOSEN.

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THE FINAL (FORCED) EQUILIBRIUM MUST SATISFY

$$\nabla^2\psi + j(\psi) = 0$$

AND BOUNDARY CONDITIONS. A FUNCTION $j(\psi)$ MUST BE FOUND SO THAT:

$$K_\nu \equiv \int G_\nu(\psi(x,y))\,dx\,dy = KO_\nu$$

AN EXPANSION AROUND THE FINAL SOLUTION SHOWS[2] THAT IF

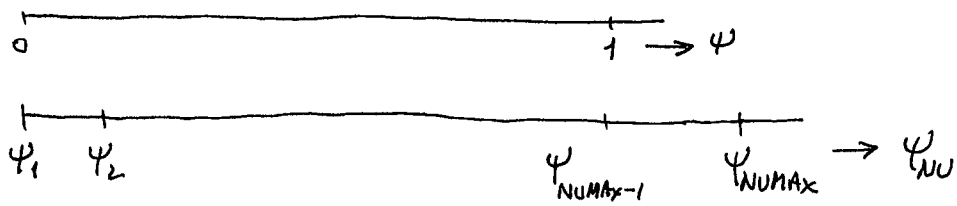
$$j(\psi) = \sum_\nu \alpha_\nu \frac{dG_\nu(\psi)}{d\psi}$$

THEN THE MATRIX, $M_{\nu\mu}$, RELATING THE $\delta\alpha_\nu$'S AND THE δK_ν's $(= K_\nu - KO_\nu)$ IS POSITIVE DEFINIT (ONE CAVEAT, SEE REF. 2)

THEN: FOR A GUESSED SET α_ν ONE CAN FIND $\psi(x,y)$ (USING JACOBI ALGORITHM); SUBSEQUENTLY ONE CAN FIND THE K_ν'S (AND THE δK_ν'S); THEN ONE CAN FIND A <u>BETTER</u> SET OF α_ν'S <u>WITHOUT INVERTING $M_{\nu\mu}$.</u>

NUMERICS SPECIFICS:

A SET OF FIXED ψ-VALUES ARE CHOSEN:



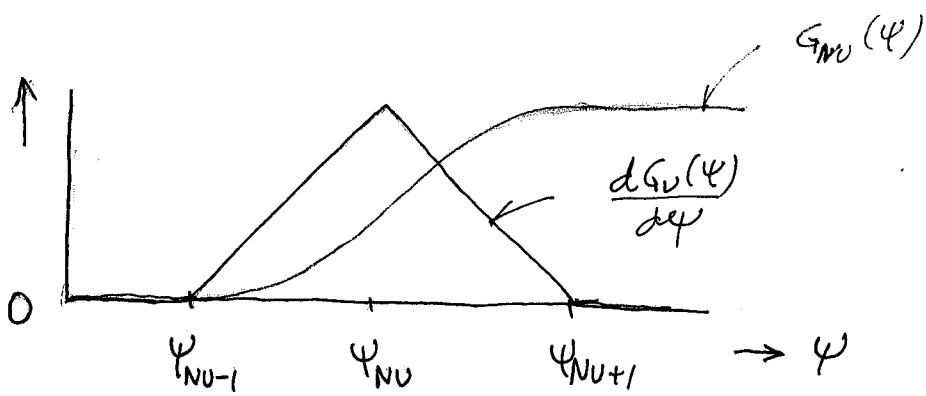
$\psi_{NU+1} > \psi_{NU}$; $\psi_1 = 0.0$; $\psi_{NUMAX-1} \simeq 1.0$

ALMOST SAME NUMBER OF GRIDPOINTS WITH

$$\psi_{NU} < \psi(i,j) < \psi_{NU+1} \qquad (NU < NUMAX-2)$$

THE SET $G_\nu(\psi)$ IS CHOSEN SO THAT $(G_\nu = G_{NU})$

$$\frac{dG_{NU}}{d\psi} = \begin{cases} \dfrac{\psi - \psi_{NU-1}}{\psi_{NU} - \psi_{NU-1}} & \text{FOR } \psi_{NU-1} < \psi < \psi_{NU} \\ 1 - \dfrac{\psi - \psi_{NU}}{\psi_{NU+1} - \psi_{NU}} & \text{FOR } \psi_{NU} < \psi < \psi_{NU+1} \\ 0 & \text{OTHERWISE} \end{cases}$$



INITIAL (UNFORCED) EQUILIBRIUM: $\psi = \dfrac{\cos\kappa x - \cos\kappa a}{1 - \cos\kappa a}$; $k^2 \equiv \kappa^2 - (\pi/b)^2$

$\Delta' a = -2ka \dfrac{\cos ka}{\sin ka}$. INITIAL EQUILIBRIA CONSIDERED DEPEND ON $\Delta' a$ AND ASPECT RATIO, b/a ONLY.

THREE STEPS ARE TAKEN AT EACH ITERATION:

1. JACOBI STEP:

$$\psi^{N+1} = \psi^N + C_1 \left[D^2 \psi^N + \sum_\nu \alpha_\nu^N \frac{dG_\nu(\psi^N)}{d\psi} \right]$$

2. IMPROVE α_ν's:

$$\alpha_\nu^{N+1} = \alpha_\nu^N + C_2 \left[K_\nu^N - K_{0\nu} \right]$$

3. IMPOSE EXTERNAL CURRENTS (BOUNDARY CONDITION)

$$\psi^{N+1}_{AFTER}(x,b) = \psi^{N+1}_{BEFORE}(x,b) \frac{MAX\{\psi^{N+1}_{BEFORE}(x,0)\}}{MAX\{\psi^{N+1}_{BEFORE}(x,b)\}} \psi_{sep}$$

THE EXTERNAL (FORCING) CURRENT (RELATIVE TO TOTAL PLASMA CURRENT) IS APPROXIMATELY (SIGN IS CORRECT)

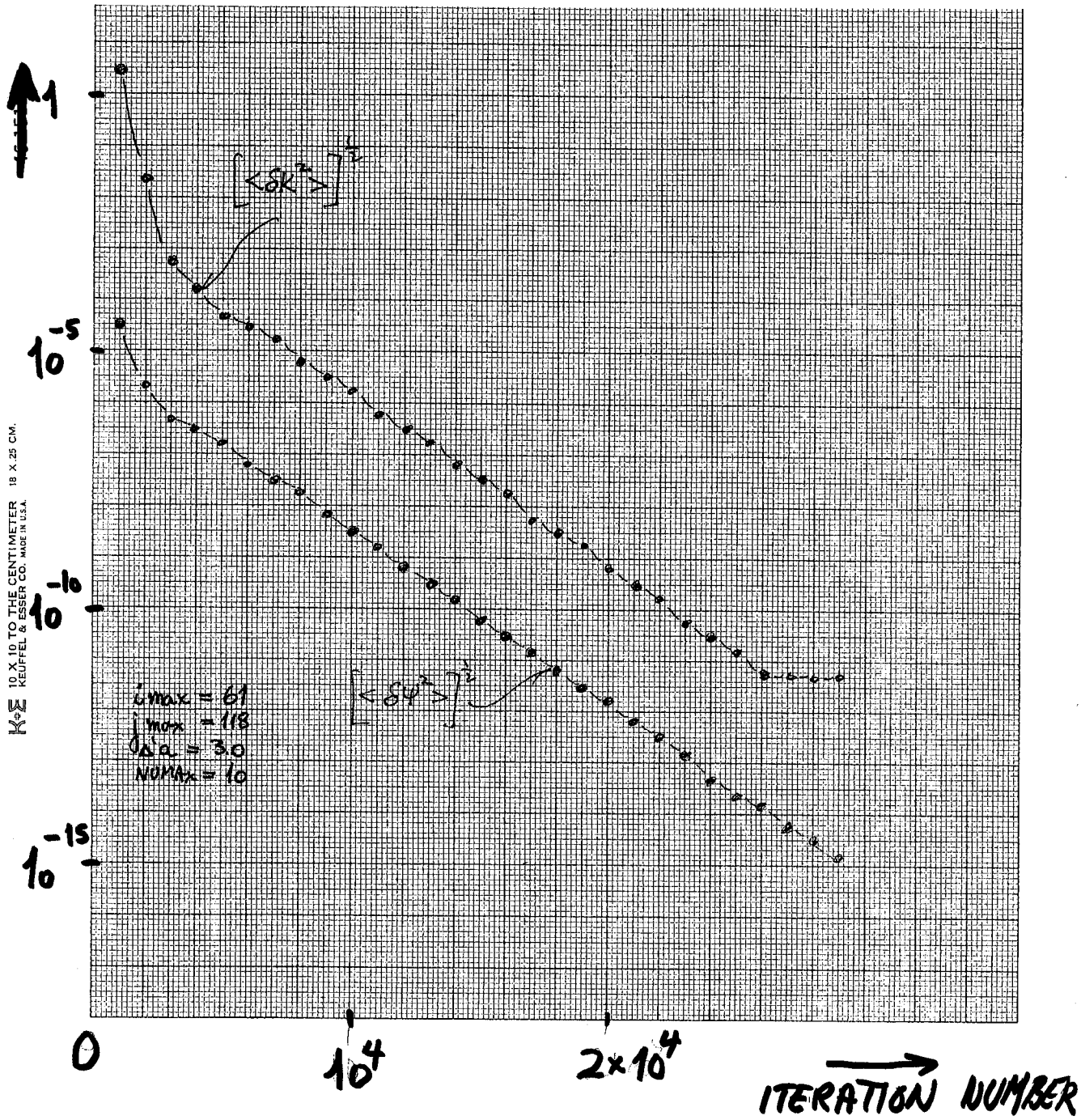
$$J_{ex} = \left[\frac{MAX\{\psi_{BEFORE}(x,0)\}}{MAX\{\psi_{BEFORE}(x,b)\}} \psi_{sep} - 1 \right] a$$

$C_1 = 0.2 D^2$ (D IS GRID DISTANCE); C_2 IS ADJUSTED FOR GOOD CONVERGENCE. GOOD (EXPONENTIAL) CONVERGENCE WAS FOUND IN PARAMETER SPACE EXPLORED.

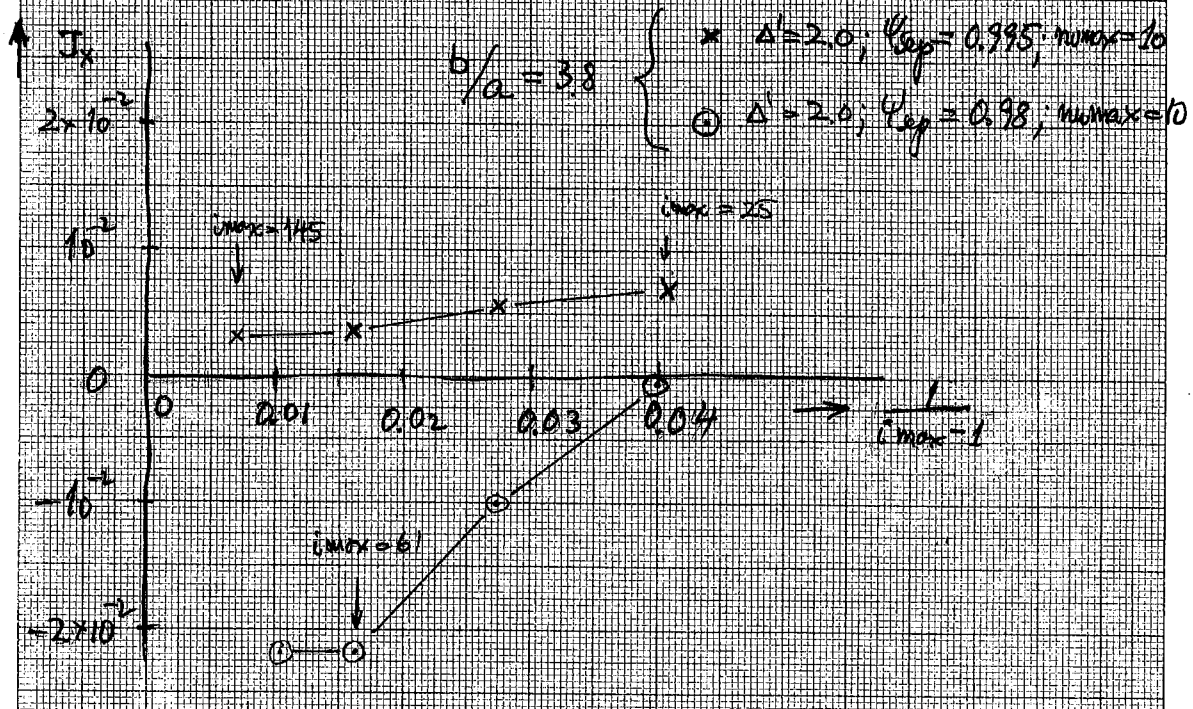
<u>OBJECT OF CALCULATION IS TO FIND:</u>

$$J_x(\Delta' a, b/a, \psi_{sep}, NUMAX)$$

EXAMPLE OF CONVERGENCE OF ALGORITHM:



SENSITIVITY TO GRID SIZE



CHOICE : $i_{max} = 61$; $j_{max} = 118$

IF, FOR A REAL PLASMA, THE FORCING CURRENT WERE SUDDENLY REMOVED, A SINGULAR CURRENT WITH THE SAME SIGN AS THE FORCING CURRENT WILL APPEAR AT THE SEPARATRIX. WITH ANY RESISTIVITY PRESENT THE Ψ-VALUE AT THE SEPARATRIX WILL INCREASE OR DECREASE DEPENDING ON SIGN OF THE SINGULAR CURRENT.

SIGNS ARE CHOSEN SO THAT A NEGATIVE J_x MAKES Ψ AT SEPARATRIX INCREASE $\rightarrow$ ISLAND WILL SHRINK.

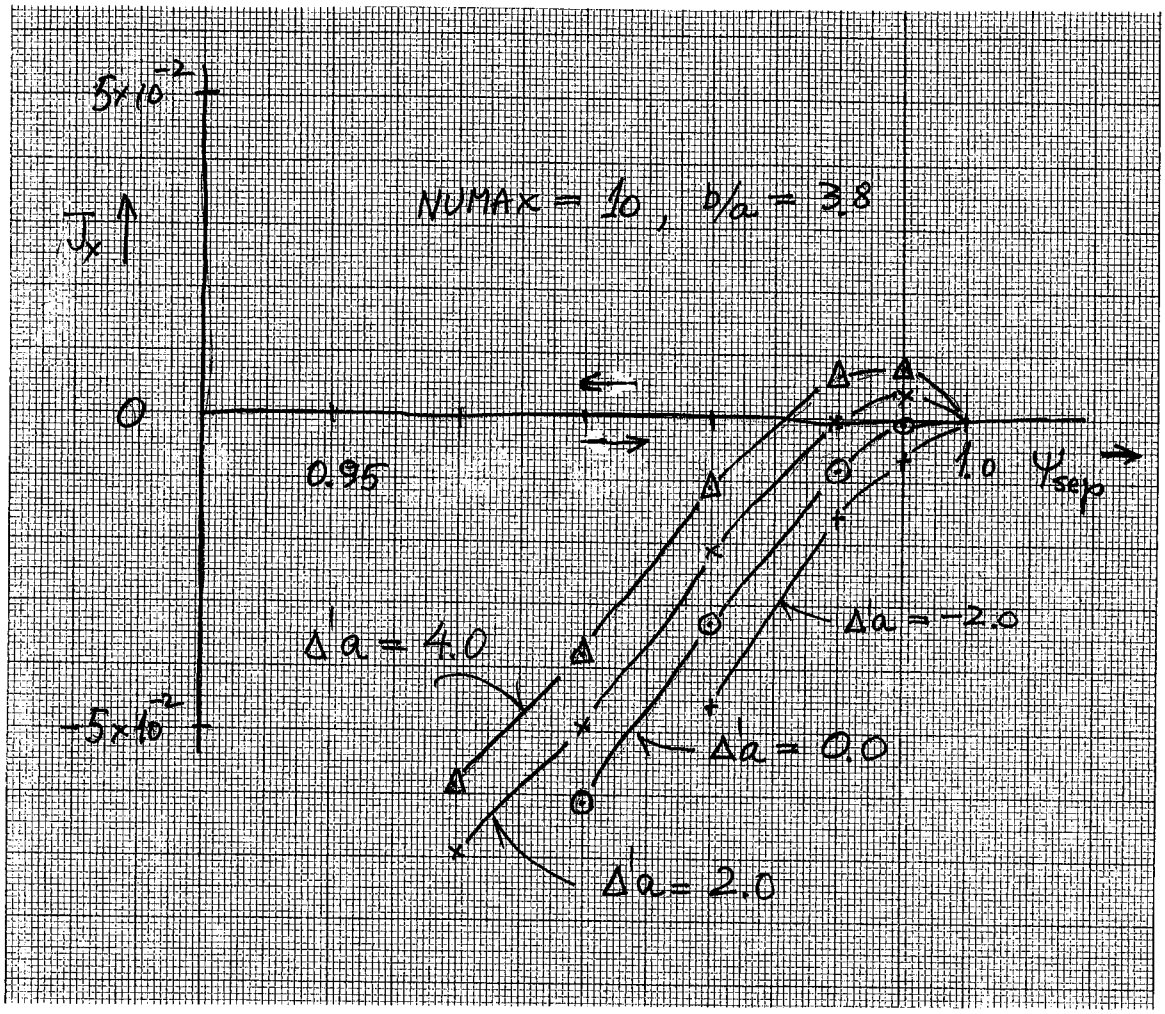
CONVERSELY, A POSITIVE J_x MEANS A GROWING ISLAND.

FOR $J_x = 0$ THE EQUILIBRIUM IS STABLE IF $\frac{\partial J_x}{\partial \Psi_{sep}} > 0$, A SATURATED ISLAND EQUILIBRIUM. FOR $\frac{\partial J_x}{\partial \Psi_{sep}} < 0$ EQUILIBRIUM IS UNSTABLE.

RESULTS:

i) $\Delta' a = 0.0$ IS STABILITY LIMIT FOR INITIAL EQUILIBRIUM

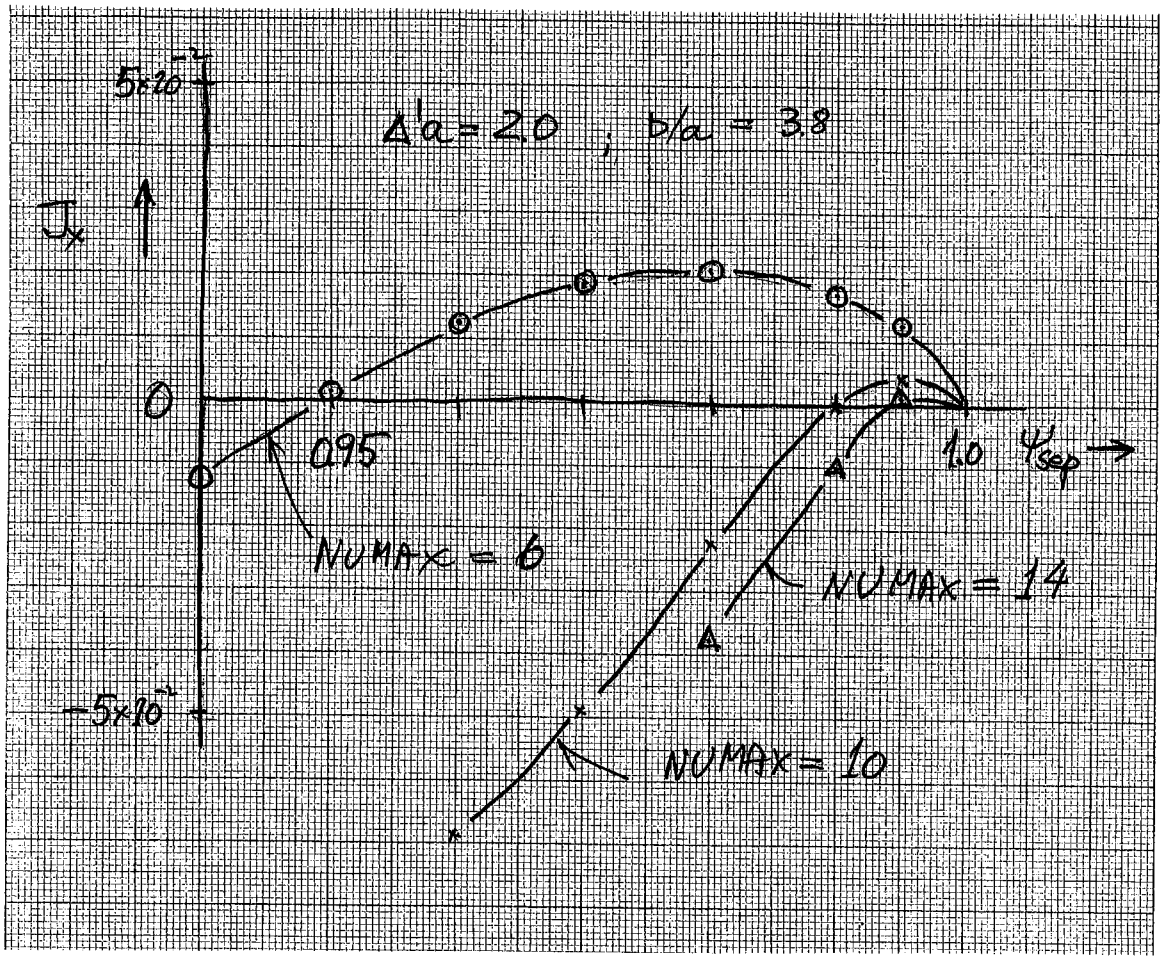
ii) FOR $\Delta' a > 0.0$ THE SATURATED ISLAND WIDTH INCREASES WITH $\Delta' a$



iii) J_x IS INSENSITIVE TO ASPECT RATIO (b/a)

ψ_{sep} \ b/a	2.5	3.8	5.8
0.995	3.24×10^{-3}	3.01×10^{-3}	3.18×10^{-3}
0.990	5.68×10^{-4}	-3.37×10^{-4}	-5.58×10^{-4}
0.980	-2.02×10^{-2}	-2.18×10^{-2}	-2.00×10^{-2}
0.970	-5.29×10^{-2}	-4.97×10^{-2}	-4.23×10^{-2}

iv) THE SATURATED ISLAND WIDTH IS SENSITIVE TO NUMAX, i.e. THE ASSUMPTIONS ON HOW FAR CIRCUMSTANCES ARE FROM THOSE OF IDEAL MHD. (SURPRISE ?[3,4,5])



REF. 3 WHITE, MONTICELLO, ROSENBLUTH + WADDELL, PHYS. OF FLUIDS, 20 800 (1977)
REF. 4 CARRERAS, WADDELL + HICKS, NUCL. FUS. 19 1423 (1979)
REF. 5 REN, JENSEN + CALLEN, PHYS. OF PLASMAS 5 2574 (1998)

ACKNOWLEDGEMENT: JOE FREEMAN MADE THIS WORK POSSIBLE BY HIS KIND AND INFINITELY PATIENT WAY OF TEACHING ME SOME FORTRAN.

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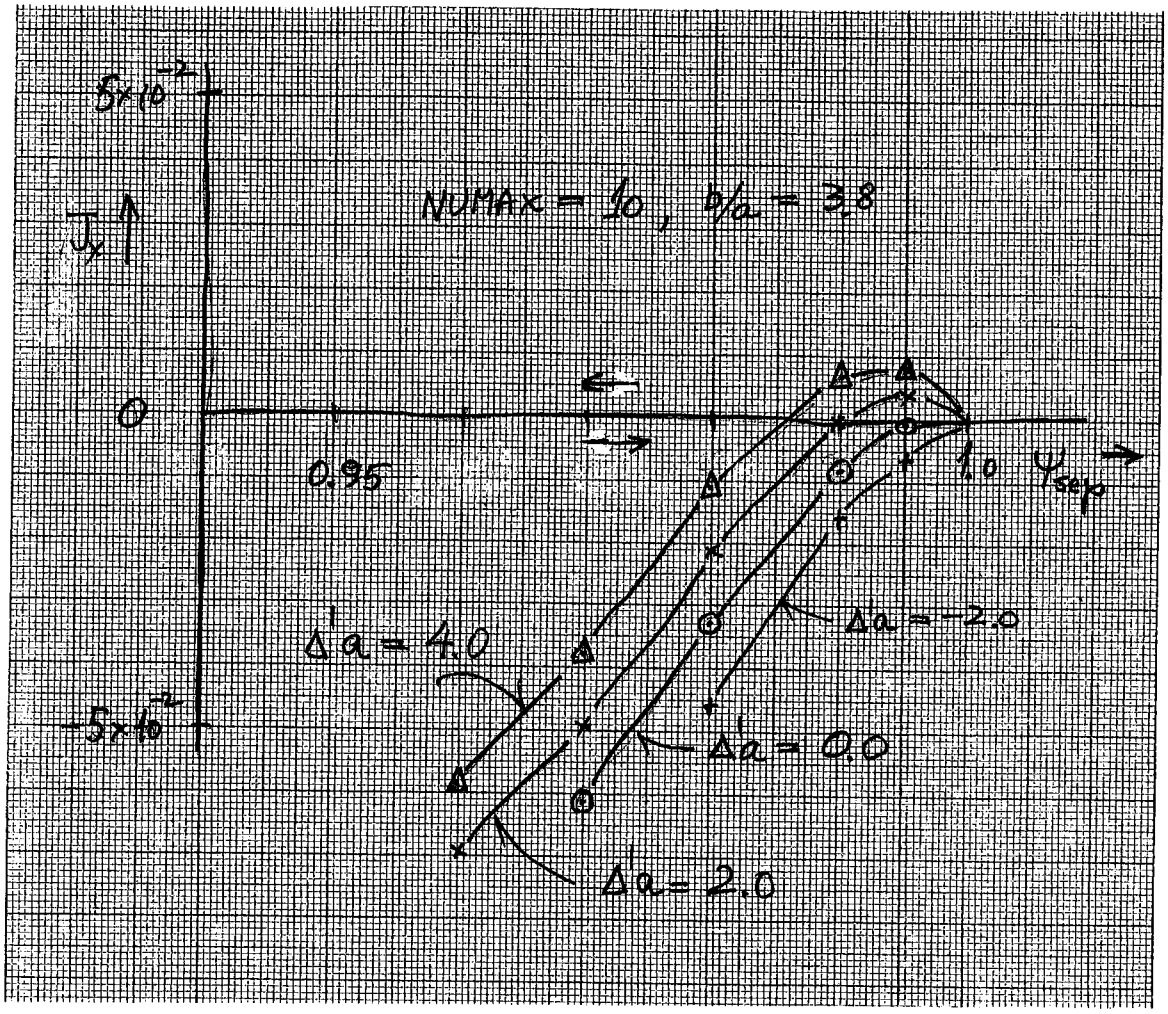

iii) J_x IS INSENSITIVE TO ASPECT RATIO (b/a); $\Delta' a \simeq 2.0$
NUMAX $\simeq 10$

ψ_{sep} \ b/a	2.5	3.8	5.8
0.995	3.24×10^{-3}	3.01×10^{-3}	3.18×10^{-3}
0.990	5.68×10^{-4}	-3.37×10^{-4}	-5.58×10^{-4}
0.980	-2.02×10^{-2}	-2.18×10^{-2}	-2.00×10^{-2}
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