Minimal Plasma Response Models for Design of Tokamak Equilibrium Controllers with High Dynamic Accuracy

D.A. Humphreys, J.A. Leuer, M.L. Walker
General Atomics

41st Annual Meeting of Division of Plasma Physics
American Physical Society
Seattle, WA
November 15-19, 1999

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Abstract Submitted
for the DPP99 Meeting of
The American Physical Society

Sorting Category: 5.1.1.2 (Experimental)

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D.A. HUMPHREYS, M.L. WALKER, J.A. LEUER, General Atomics

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[1]Supported by U.S. DOE Contract DE-AC03-99ER54463.

☐ Prefer Oral Session
☒ Prefer Poster Session

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Special instructions: DIII-D Poster Session 2, immediately following JA Leuer

Date printed: July 16, 1999

Electronic form version 1.4

Abstract

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We describe a model of linearized plasma shape and position response which is based on low poloidal mode number (m≤2, approximately vertical and major radial) displacements of the plasma current distribution. The model introduces minimal plasma degrees of freedom while providing sufficient accuracy for high performance controller design. The effects of significant variation in plasma poloidal beta, internal inductance, and separatrix configuration are taken into account. Models which can predict plasma shape and position variation with reasonable accuracy are particularly important for design of dynamic controllers in devices with significant variation in auxiliary heating input power and plasma shape - conditions common in the DIII-D tokamak. Model predictions are validated using experimental response data from DIII-D. Application of the plasma response model to design of multivariable dynamic plasma controllers recently implemented on DIII-D is described.

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OVERVIEW

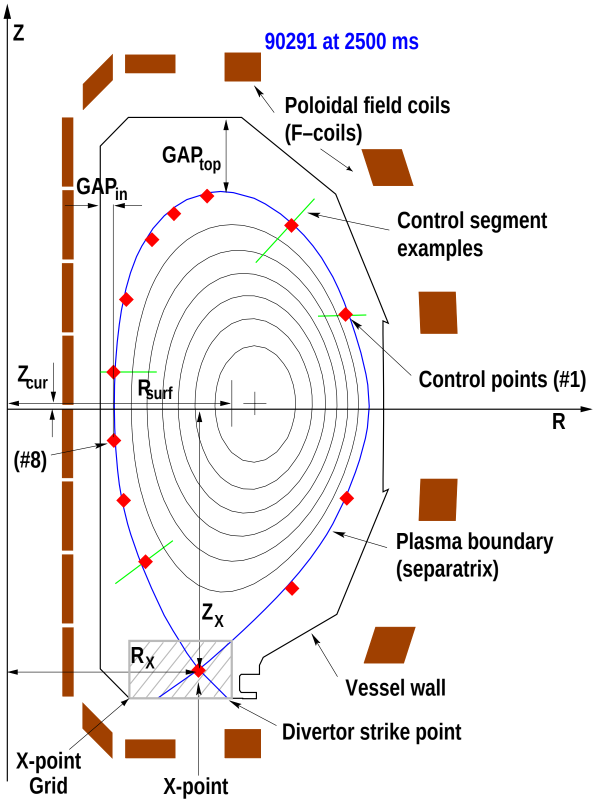
- Accurate models of dynamic plasma response are needed for design of multivariable model-based controllers.

- Validated, analytically-based plasma models:
  → Reveal important/unimportant physics effects;
  → Improved physics understanding enables model extension to:
    ⇒ New operating regimes in existing devices;
    ⇒ Next-generation tokamak experiments.

- Minimal complexity, minimal order dynamic plasma response models:
  → Have been developed and extensively validated against DIII-D experiments with varying β_p and l_i;
  → Enable rapid calculation and controller design.

- Validated minimal plasma response models have been used to design successful dynamic shape controllers for DIII-D. [See poster JP1.33, this session]

Isoflux Shape Control System on DIII-D Regulates Flux at Specified Control Points on Boundary



- Real time EFIT (Equilibrium FITing code) calculates flux and field very accurately near plasma boundary.

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Realtime Equilibrium Reconstruction Provides High Accuracy Required by Isoflux Control Scheme

- Realtime version of EFIT algorithm provides solution to Grad-Shafranov tokamak equilibrium equation consistent with magnetic measurements:

  ⇒ Best fit to magnetic measurements from 39 flux loops, 31 B_p probes, Rogowski loop measurement of I_p, 16 Motional Stark Effect measurements of plasma current density, 18 shaping coil currents.

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  ⇒ Boundary determined within ~ few mm;
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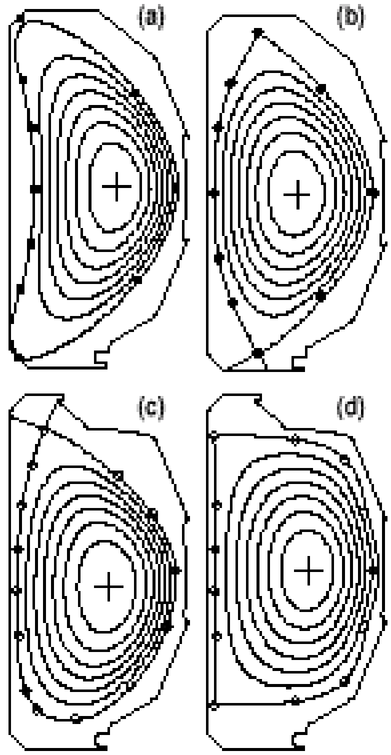
Isoflux Control Scheme Used With Realtime EFIT Provides High Shaping Flexibility, High Shape Control Accuracy



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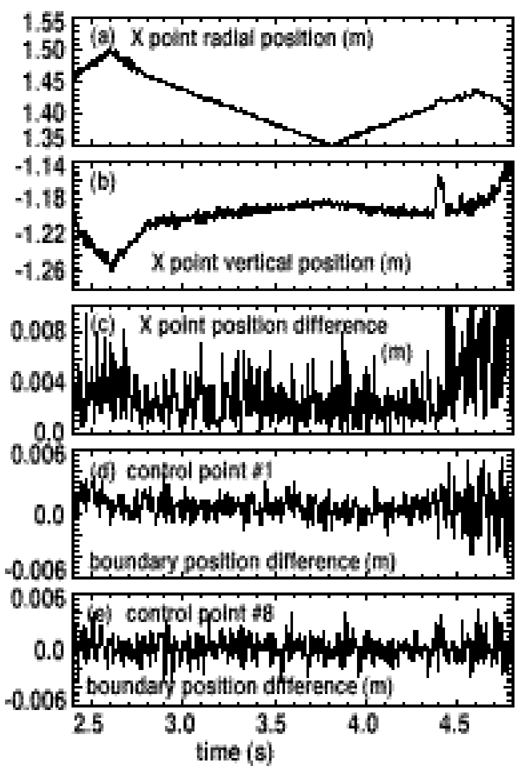


Motivation: New DIII-D Control System Will Use Model-Based Approach to Controller Design

- Design philosophy:
  - → Develop and validate linearized and nonlinear models of plasma/conductor/power supply responses;
  - → Design controllers based on minimal linear models;
  - → Test and optimize controller performance with linear and nonlinear model simulations;

- Advantages:
  - → Controllers allow significant variation in plasma equilibrium without switching to new controller;
  - → New equilibrium/shape development without consuming valuable experimental time on tokamak;
  - → Improved control performance: accuracy/precision, dynamic response, disturbance/noise rejection.
  - → Ability to balance tokamak constraints with desired control.

- Disadvantages:
  - → Requires reasonably accurate system response models;
  - → Possible increase in difficulty of tuning control algorithms "by hand" between shots (but with theoretical reduction in need for such tuning);

Plasma Response Model Based on Rigid Plasma Displacements, Simple Force Balance Relations

- Goal: Use "minimally complex" models which provide sufficiently accurate representation of experiment.

- Simple radial force balance:

$$B_{zm}^{appl} = B_z^{Shafranov} = -\frac{\mu_0 I_p}{4\pi R_m}\Gamma(R_m, \beta_p, \ell_i)$$

$$\equiv -\frac{\mu_0 I_p}{4\pi R_m}\left(\ln\left|\frac{8R_m}{a\sqrt{\kappa}}\right| + \frac{2\kappa}{\kappa^2+1}\beta_p + \frac{\ell_i}{2} - 1.5\right)$$

- Simple vertical force balance:

$$0 = -2\pi R_m I_{p0}\frac{\partial B_R}{\partial z}\delta z + I_{p0}\frac{\partial M_{pc}}{\partial z}\delta I_c + \frac{\partial F_z}{\partial \beta_p}\delta\beta_p + \frac{\partial F_z}{\partial \ell_i}\delta\ell_i$$

- Plasma response objects from perturbed force balance:

R_m Current-Resp
$$\frac{\partial R_m}{\partial I_c} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1}\frac{\partial B_{zm}}{\partial I_c} + \frac{\partial R_m}{\partial z}\frac{\partial z}{\partial I_c}$$

R_m I_p-Response
$$\frac{\partial R_m}{\partial I_p} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1}\frac{\mu_0\Gamma_0}{4\pi R_0}$$

$$\frac{\partial R_m}{\partial \beta_p} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1}\frac{\mu_0 I_{p0}}{4\pi R_0}\left(\frac{2\kappa}{\kappa^2+1}\right) + \frac{\partial R_m}{\partial z}\frac{\partial z}{\partial \beta_p}$$

$$\frac{\partial R_m}{\partial \ell_i} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1}\frac{\mu_0 I_{p0}}{8\pi R_0}$$

Z_c Curr-Resp $\dfrac{\partial z}{\partial I_c} \equiv \dfrac{(\partial M_{pc} / \partial z)}{-\mathbf{2}\pi B_{zJ\mathbf{0}} n_J}$, ***$Z_c$ β_p-Resp*** $\dfrac{\partial z}{\partial \beta_p} \approx \dfrac{\int R B_{R\mathbf{0}} J_\varphi dA_p}{\beta_{p\mathbf{0}} B_{zJ\mathbf{0}} n_J}$

Rigid Radial and Vertical Plasma Displacement Assumption Closes Plasma Response Equations

- Plasma current distribution assumed to shift rigidly in both radial and vertical directions:

$$\delta J_\varphi = \left.\frac{\partial J_\varphi}{\partial R_m}\right|_{Rigid} \delta R_m + \left.\frac{\partial J_\varphi}{\partial z}\right|_{Rigid} \delta z$$

- Variations in flux at conductors, isoflux segment control points derived from simple Green functions and rigid shift:

$$\delta \psi_c = \frac{\partial \psi_c}{\partial J_\varphi}\left(\left.\frac{\partial J_\varphi}{\partial R_m}\right|_{Rigid} \delta R_m + \left.\frac{\partial J_\varphi}{\partial z}\right|_{Rigid} \delta z \right)$$

$$\delta \psi_{iso} = \frac{\partial \psi_{iso}}{\partial J_\varphi}\left(\left.\frac{\partial J_\varphi}{\partial R_m}\right|_{Rigid} \delta R_m + \left.\frac{\partial J_\varphi}{\partial z}\right|_{Rigid} \delta z \right)$$

- Variations in B-field (Z,R) components at X-point grid locations similarly derived from simple Green functions and rigid shift:

$$\delta B_{z,R}^{grid} = \frac{\partial B_{z,R}^{grid}}{\partial J_\varphi}\left(\left.\frac{\partial J_\varphi}{\partial R_m}\right|_{Rigid} \delta R_m + \left.\frac{\partial J_\varphi}{\partial z}\right|_{Rigid} \delta z \right)$$

Certain Effects Included in the Minimal Model are Key to Producing Accurate Plasma Response

- Explicit effects of κ significantly greater than 1:

$$B_z^{Shafranov} \equiv -\frac{\mu_0 I_p}{4\pi R_m}\left(\ln\left|\frac{8R_m}{a\sqrt{\kappa}}\right| + \frac{2\kappa}{\kappa^2+1}\beta_p + \frac{l_i}{2} - 1.5\right)$$

- Radial response to variation in β_p and l_i :

$$\frac{\partial R_m}{\partial \beta_p} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1} \frac{\mu_0 I_{p0}}{4\pi R_0}\left(\frac{2\kappa}{\kappa^2+1}\right) + \frac{\partial R_m}{\partial z}\frac{\partial z}{\partial \beta_p}$$

$$\frac{\partial R_m}{\partial l_i} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1} \frac{\mu_0 I_{p0}}{8\pi R_0}$$

- Response of ***vertical*** position to variation in β_p:

$$\frac{\partial z}{\partial \beta_p} \approx \frac{\int R B_{R0} J_\varphi dA_p}{\beta_{p0} B_{zJ0} n_J}$$

- Cross-coupling of radial and vertical responses:

$$\frac{\partial R_m}{\partial I_c} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1} \frac{\partial B_{zm}}{\partial I_c} + \frac{\partial R_m}{\partial z}\frac{\partial z}{\partial I_c}$$

- Plasma (resistive) current response:

$$L_p \frac{dI_p}{dt} + R_p I_p + \frac{\partial \psi_p}{\partial R_m}\frac{dR_m}{dt} + \frac{\partial \psi_p}{\partial z}\frac{dz}{dt} + \frac{\partial \psi_p}{\partial I_c}\frac{dI_c}{dt} + I_{p0}\frac{\partial L_p}{\partial l_i}\frac{dl_i}{dt} = V_L$$

Plasma Perturbation Experiments Were Performed on DIII-D to Validate Plasma Response Model

- Wide range of equilibria perturbed in dedicated or piggyback experiments:
  - ⇒ Lower/upper single null, double null;
  - ⇒ Ohmic→High β_p; Low→High l_i.

- Many plasma position/shape parameters varied:
  - ⇒ Major radius, vertical position (rigid shifts);
  - ⇒ Inner/outer gaps (independently):
  - ⇒ Top gap (for lower single null);
  - ⇒ X-point radial/vertical position;
  - ⇒ Elongation (plasma height/width separately);

- Several different waveforms programmed to vary plasma position/shape parameters dynamically:
  - ⇒ Triangle, step, sinusoidal waveforms;
  - ⇒ Periods ranging from 20→400 msec.

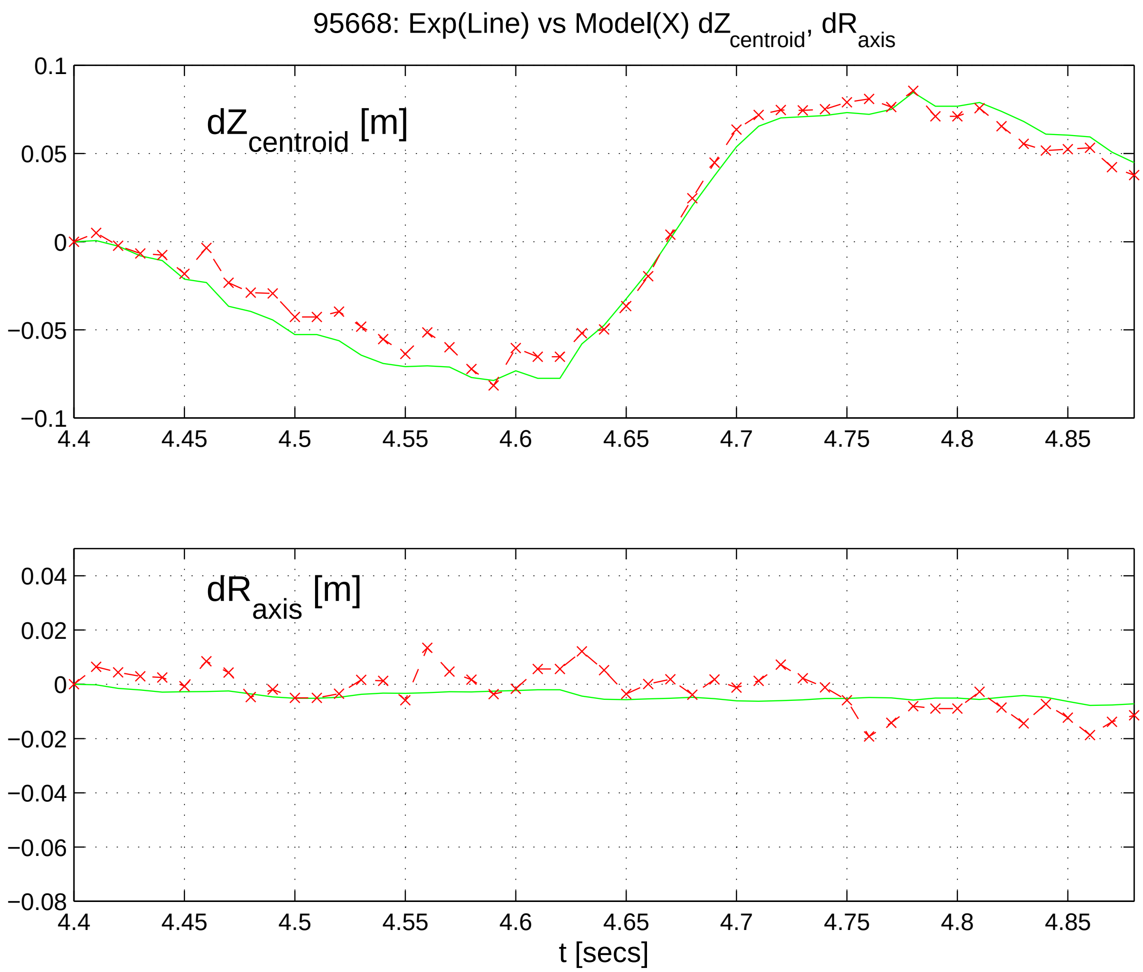
- Actual coil current, β_p history from experiment used to calculate model-predicted plasma/diagnostic response:

$$\begin{bmatrix} \delta R_m \\ \delta B_{probe} \\ \delta\psi_{isoflux} \\ \vdots \end{bmatrix} = [C] \begin{bmatrix} \delta\vec{I}_c \\ \delta\beta_p \\ \delta\ell_i \\ \vdots \end{bmatrix}$$

Simple Model Reproduces Experimental Ohmic Plasma Response with High Accuracy:

Δz, ΔR_m

- Experiment (equilib. reconstruction)=line, Model=X;
- $dZ_{centroid}$, dR_{axis} = centroid-vertical, axis-radial positions;
- Vertical position intentionally perturbed with pre-programmed waveform to obtain model test data:



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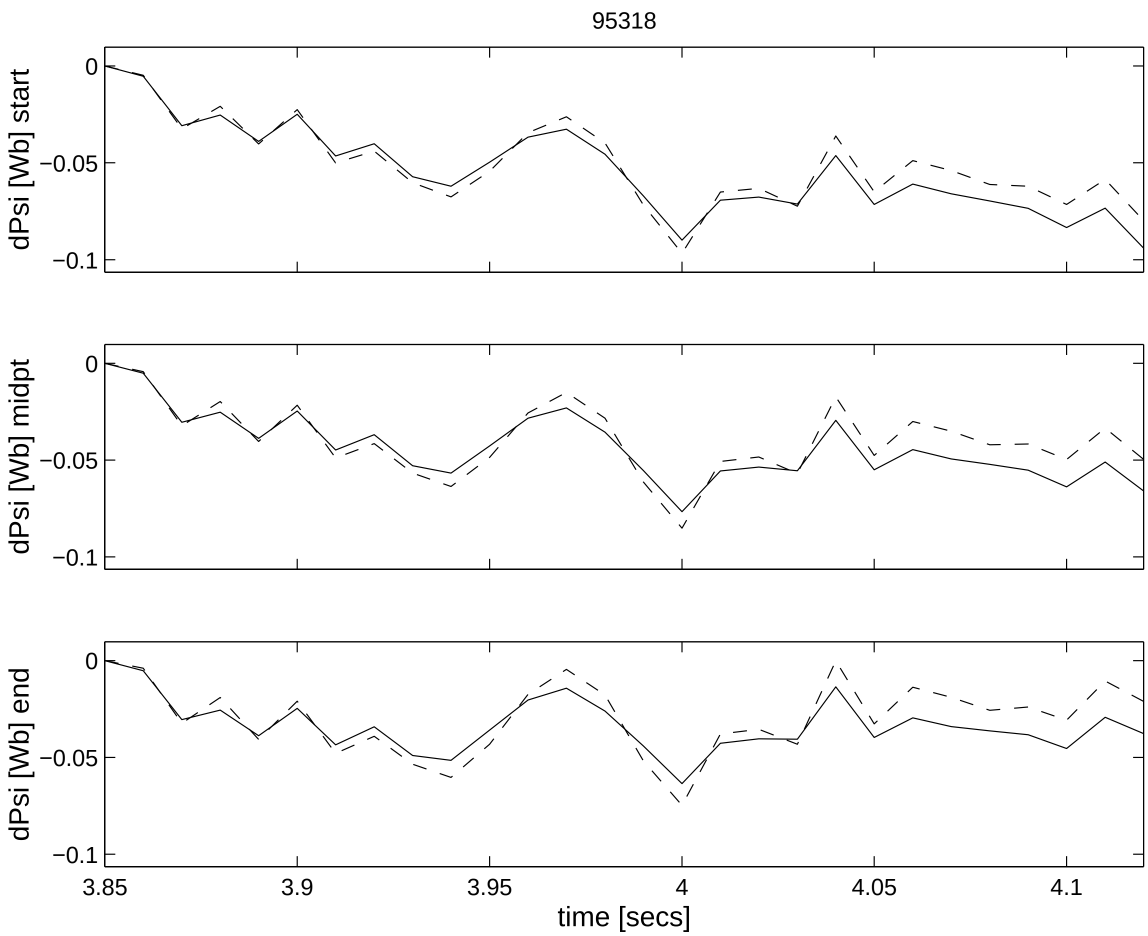
Simple Model Reproduces Experimental Ohmic Plasma Response with High Accuracy:

$\Delta\psi$(inboard-midplane isoflux segment)

- Model accurately reproduces $\Delta\psi$ inside/outside plasma, near plasma boundary, for ramping major radius;
- Experiment=solid, Model=dashed;



Simple Model Reproduces Experimental Ohmic Plasma Response with High Accuracy:

X-Point Δz_X, ΔR_X

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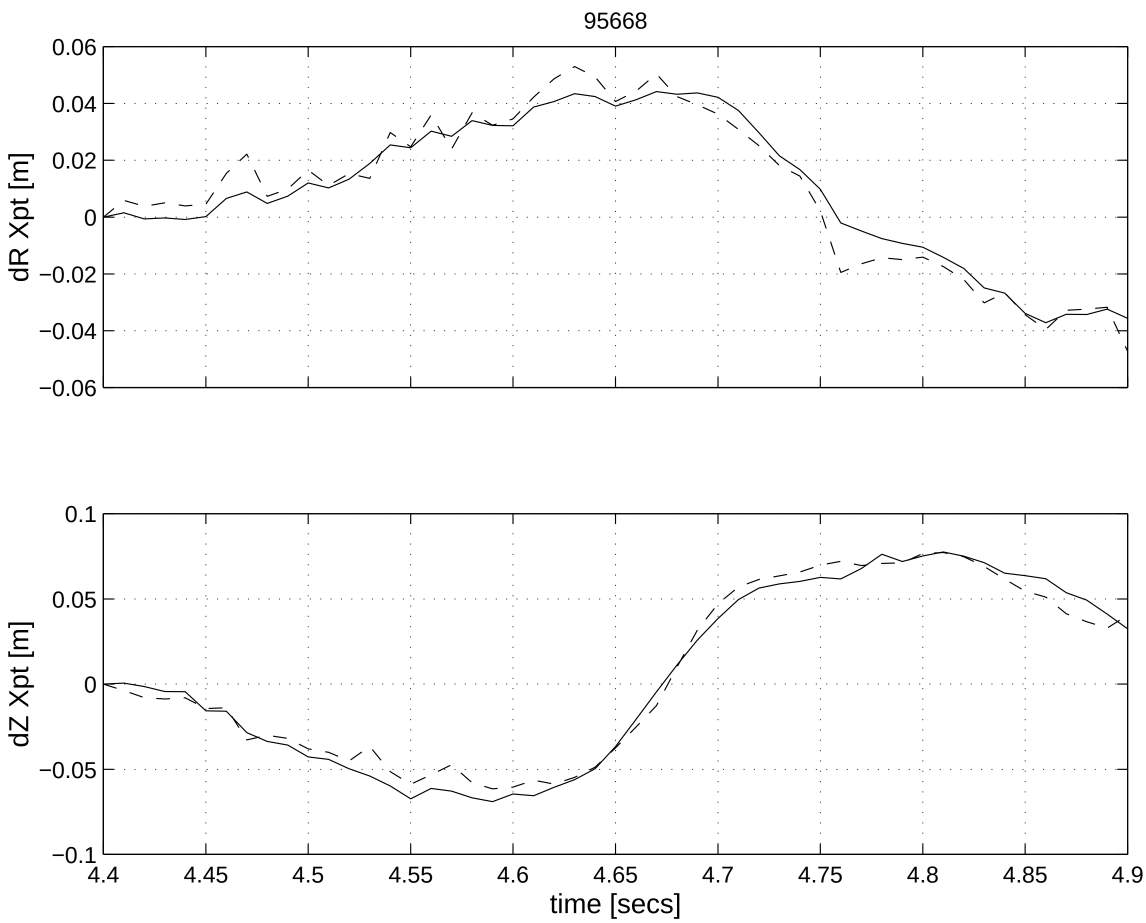

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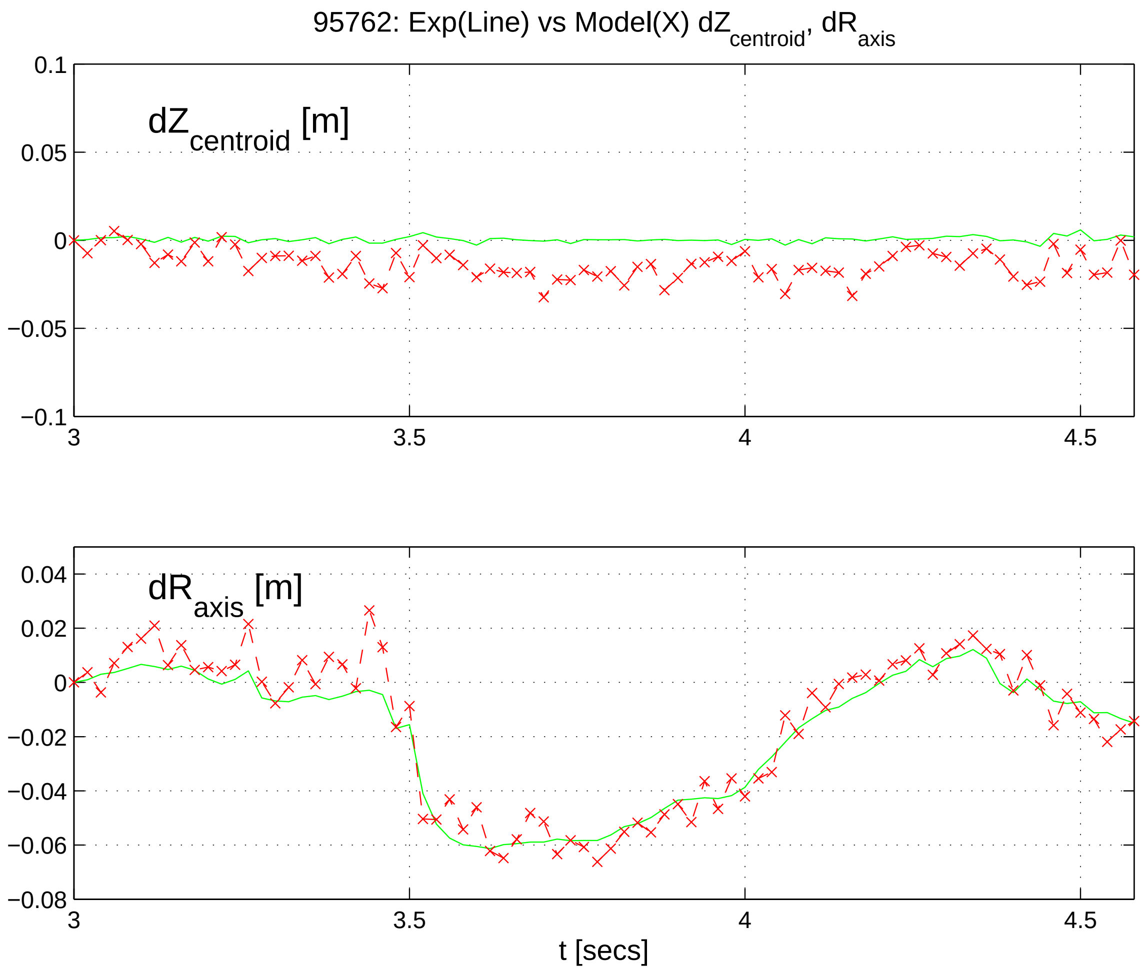
Simple Model Reproduces Experimental High-β_p Plasma Response with High Accuracy: Δz, ΔR_m

- Experiment (equilib. reconstruction)=line, Model=X;
- $dZ_{centroid}$, dR_{axis} = centroid-vertical, axis-radial positions;
- β_p dropped from ~1.1 to ~0.5 at t~3.5 s due to Ar gas puff
  $\Rightarrow$ plasma moved radially inboard:



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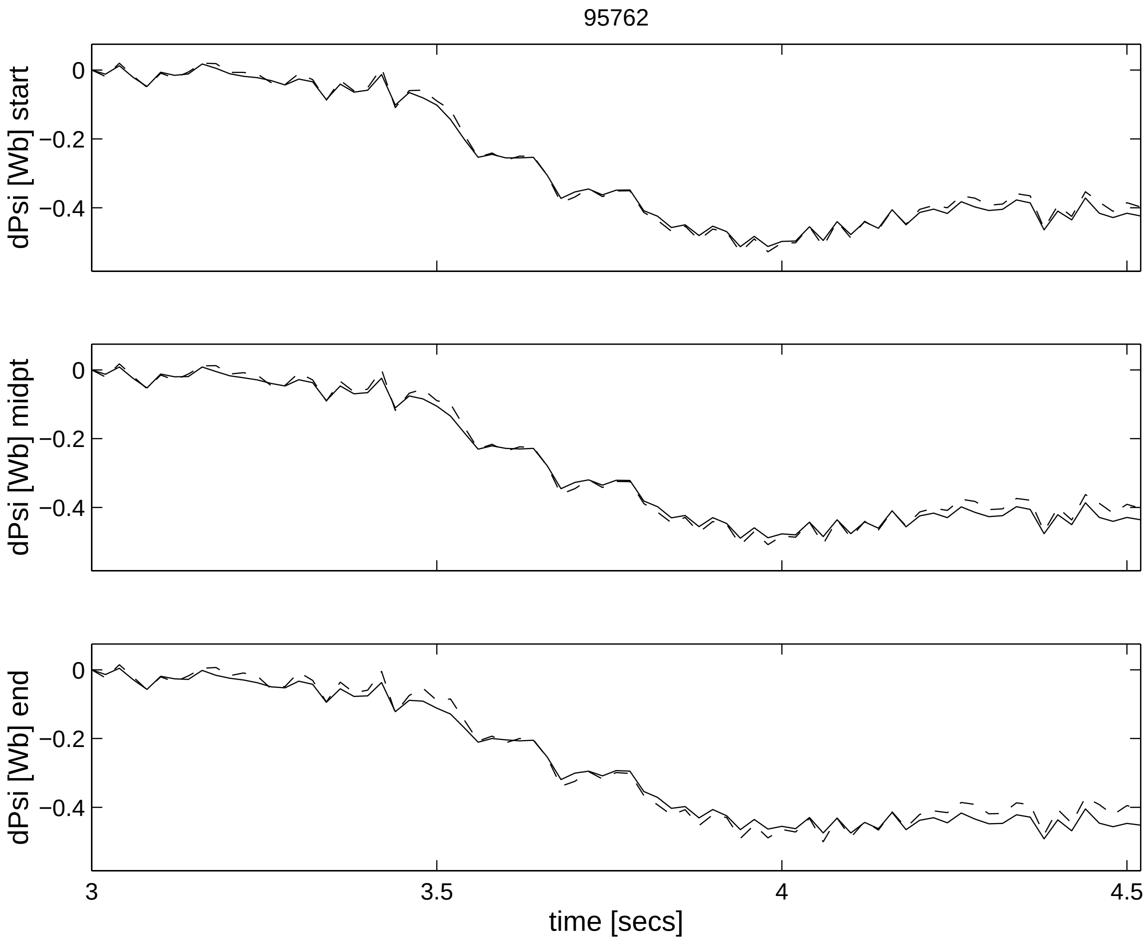

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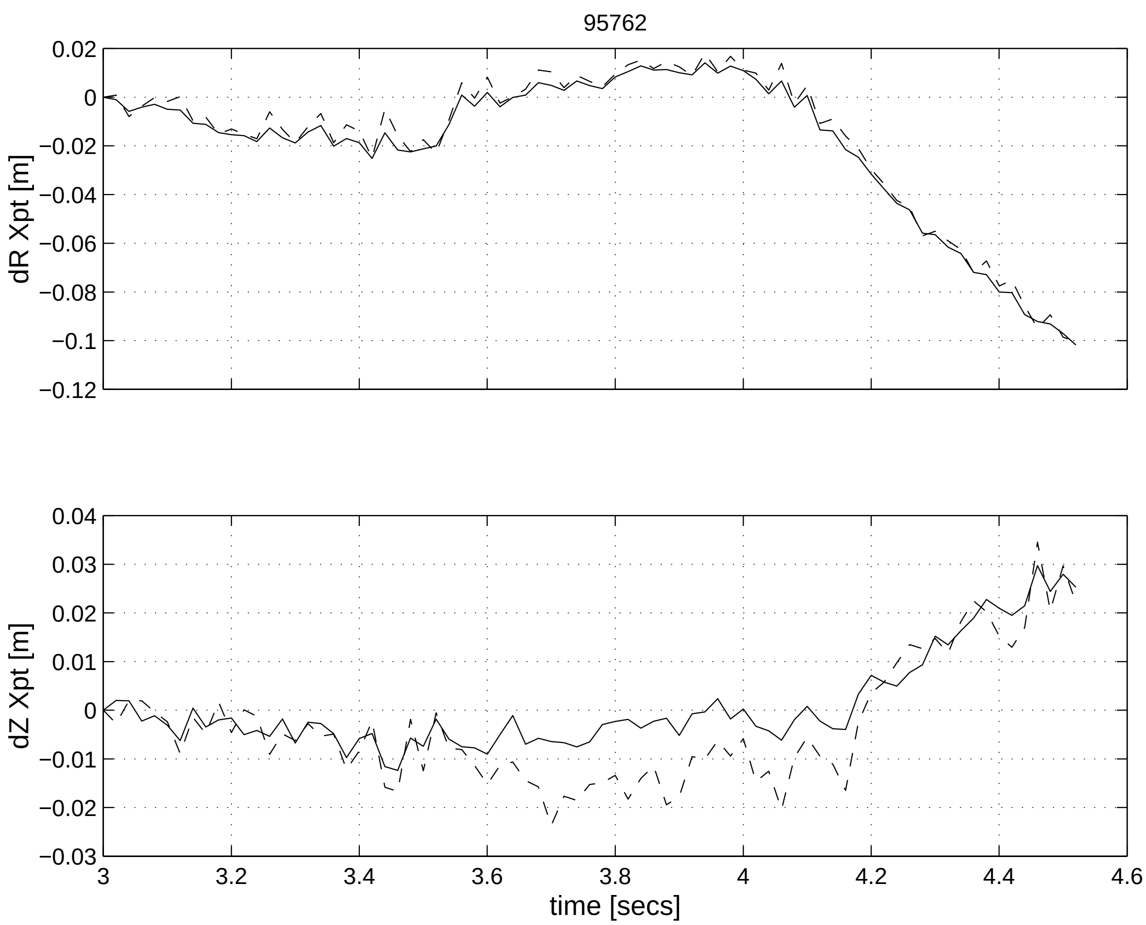

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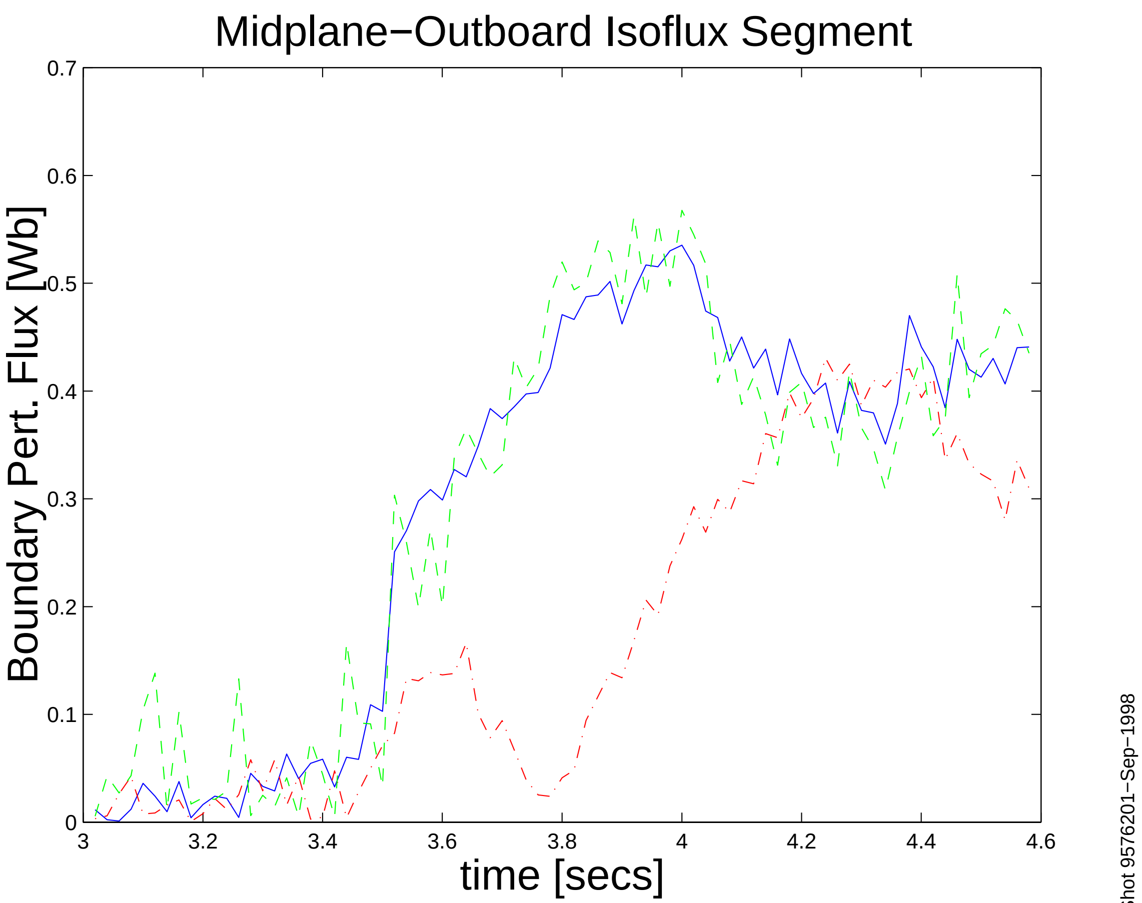
A “Response Model” Based on Vacuum Response (No Plasma Contribution) Can Be VERY Inaccurate

- Example: Radial shift of high-β_p single null plasma;
- Experiment (solid blue), plasma model (dashed green) differ significantly from vacuum model (dash-dot red) over interval 3.5<t<4.2 s:



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Accurate Model of Vertical Plasma Response is Important for Designing Good Dynamic Controller

- Present DIII-D plasma control has dynamically "separated" vertical position and stability control:
  $\rightarrow$ *Stability* control uses velocity ($\propto dZ/dt$) feedback provided by fast (~60 μsec) separate cpu;
  $\rightarrow$ *Position* control provided as part of shape control with longer cycle time (~1 msec) cpu;

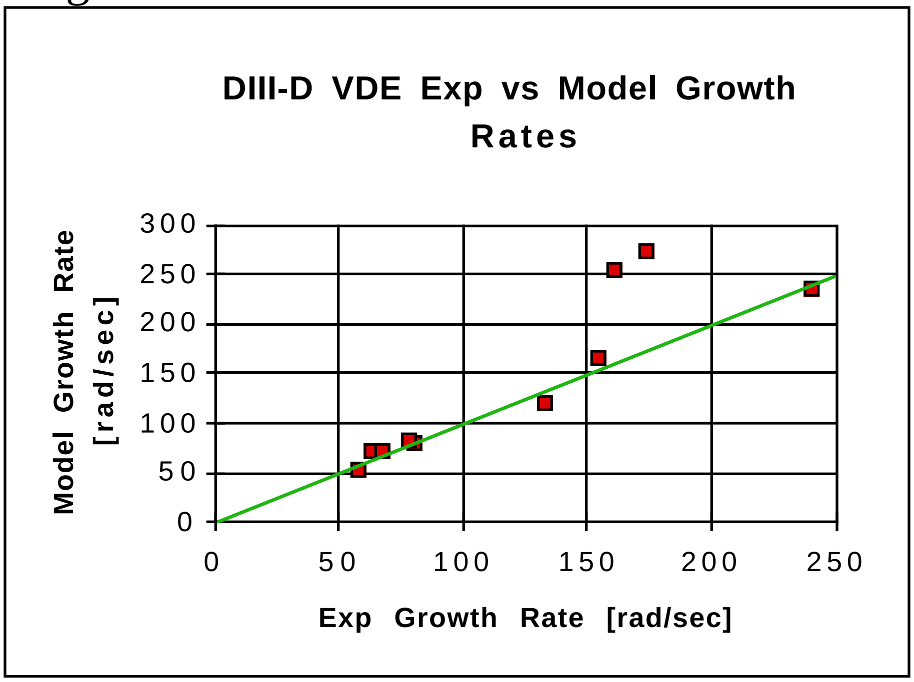
- MIMO controllers on DIII-D must be designed to provide good shape control and minimize interaction with (fast) stability control loop;

- Good dynamic control therefore requires accurate model of vertical plasma response:
  $\rightarrow$ Open loop vertical growth rate;
  $\rightarrow$ Effect of stabilization (by fast velocity feedback) on vertical plasma displacement mode.

- Effect of vertical motion is also important in order to produce correct *radial* plasma response;

Rigid Current-Conserving Model Vertical Growth Rate Matches DIII-D VDE Experiments

- Growth rates from current-conserving (δI_p=0) rigid vertical model compared with experimentally measured VDE growth rates:

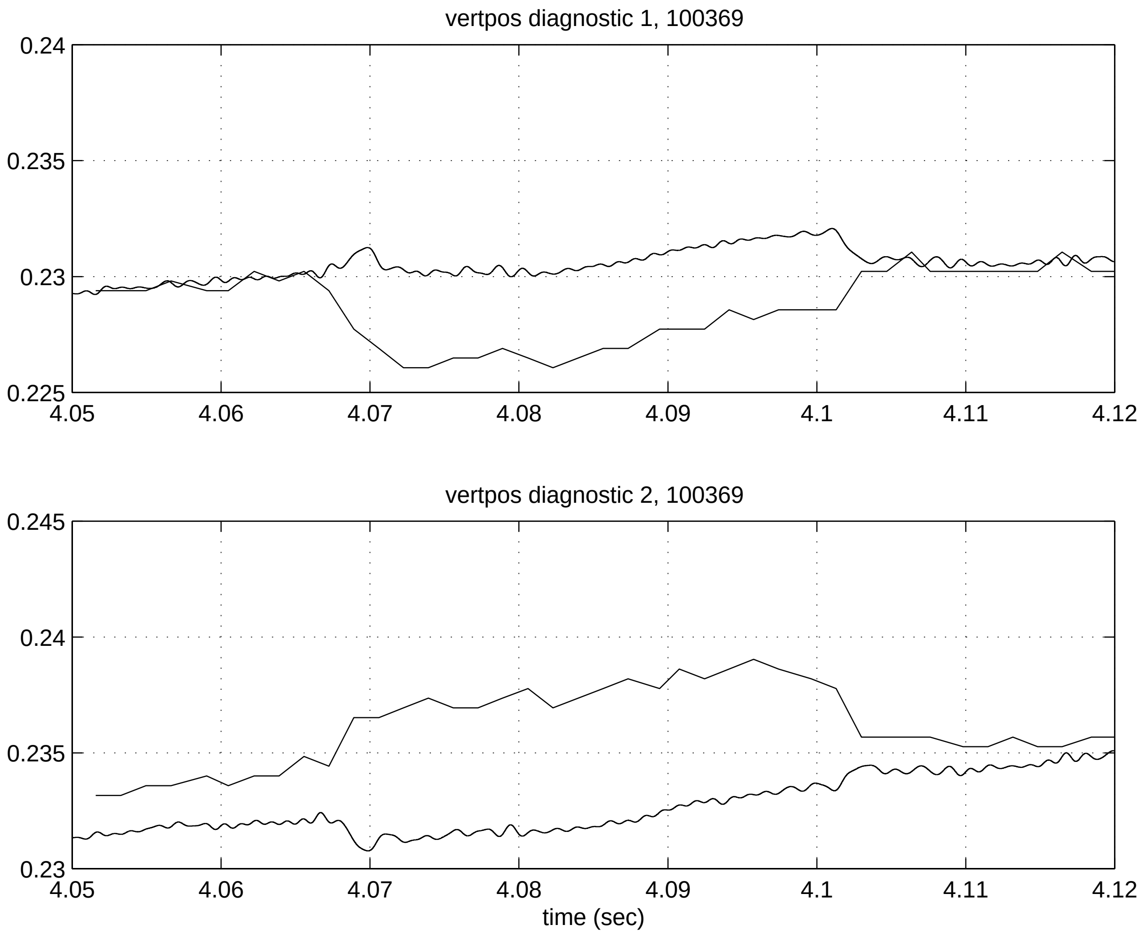


- Most VDE's yield vertical growth rate within ~10% of predicted value;

- Small subset of discharges produce VDE's with vertical growth rates differing by as much as 50% from predicted values:
  $\longrightarrow$ Source of difference not yet well understood;
  $\longrightarrow$ ~ 50% error in growth rate is tolerable for dynamic controller design in such cases.

Incorrect Model of Vertical Instability Response Produces Incorrect B-Field Measurements

- Open loop simulation with vertical instability *removed* (to roughly model effect of fast vertical velocity control loop) shows incorrect response of outboard poloidal field probes:



- Vertical model affects both vertical and*radial* response: cross-coupling terms are significant in up-down asymmetric plasmas (this case = Upper Single Null).

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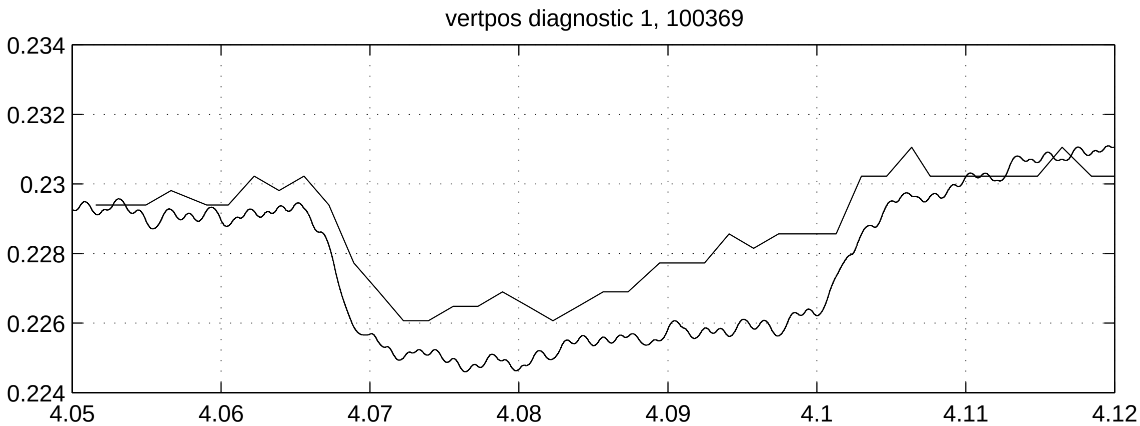
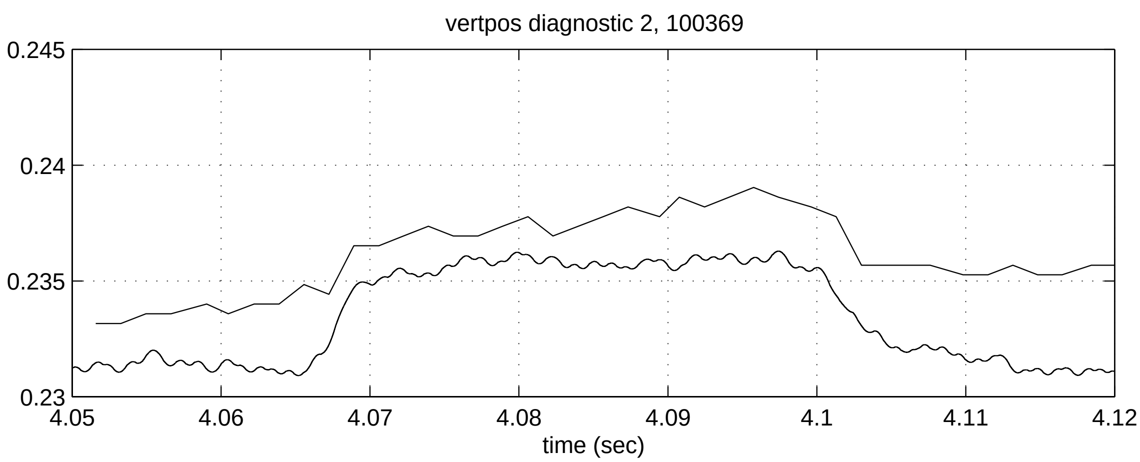
Inclusion of Accurate Vertical Instability Mode in Plasma Model Produces Correct Probe Measurements

- Closed loop simulation with vertical instability *included* (and effect of fast velocity control loop correctly included in model) shows correct response of outboard poloidal field probes:





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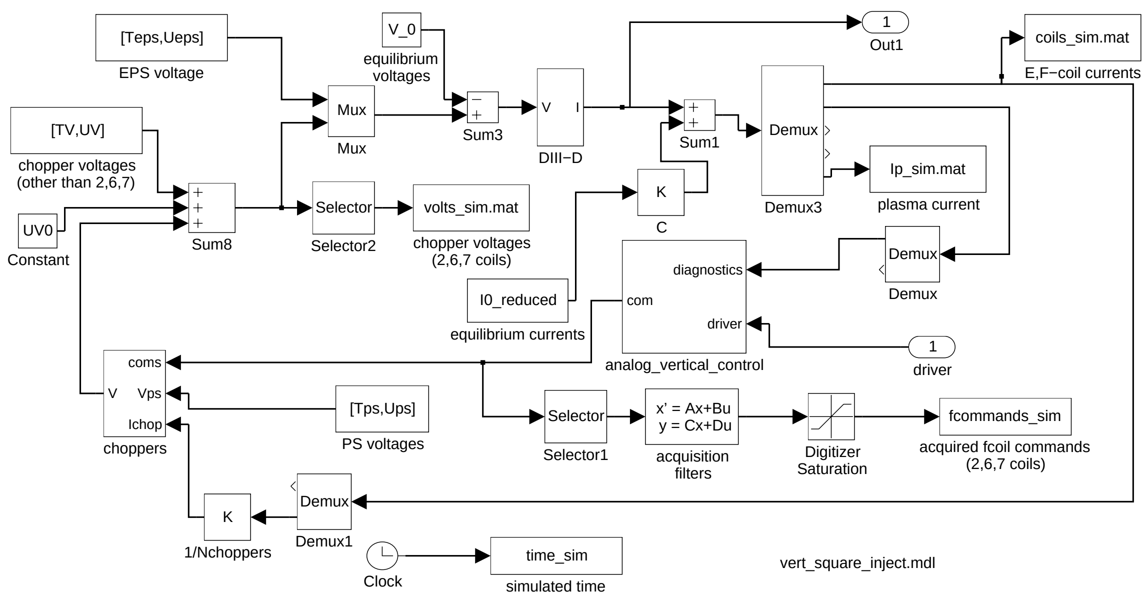
Accurate Representation and Simulation of Vertical Response in Tokamaks is Complex Problem

- Simulation/validation of intrinsically unstable open-loop <u>**plasma model**</u> is difficult: little more than growth rate can be validated;



- Simulation/validation of closed-loop response in DIII-D requires accurate modeling of highly nonlinear <u>**chopper power supplies**</u> and <u>digital controller</u> action (as well as unstable plasma).

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CONCLUSIONS

- High dynamic accuracy of performance in DIII-D isoflux boundary/X-point control requires accurate plasma response models for controller design.

- "Minimal" analytically-based plasma models:
  - $\rightarrow$ Are sufficient to accurately describe plasma response for control;
  - $\rightarrow$ Have been developed and **extensively validated** against DIII-D experiments.
  - $\rightarrow$ Have been used to design successful dynamic shape controllers for DIII-D. [See poster JP1.33, this session]

- Physics terms important to plasma response have been identified:
  - $\rightarrow$ Effect of plasma elongation;
  - $\rightarrow$ Effects of variation in β_p and l_i;
  - $\rightarrow$ Cross-coupling of vertical and radial displacement;
  - $\rightarrow$ Validated physics understanding allows models to be extended to other operating regimes and devices more reliably.

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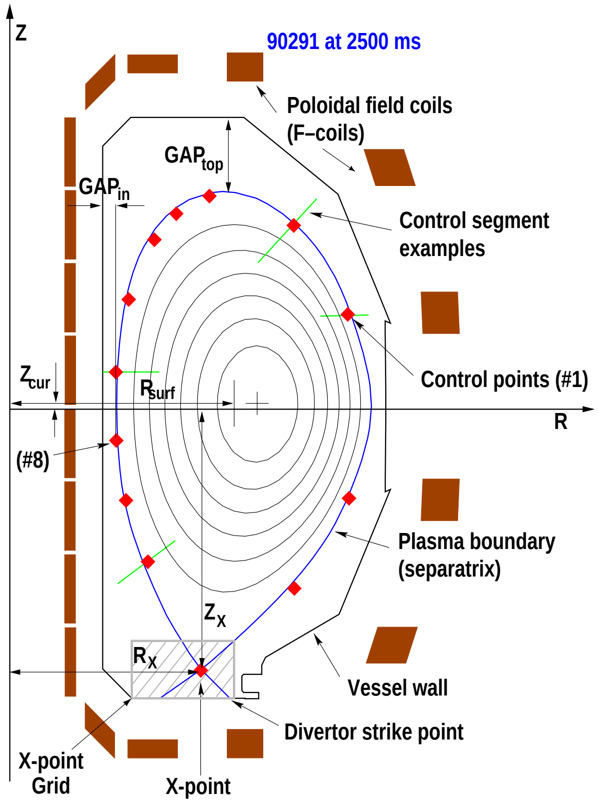

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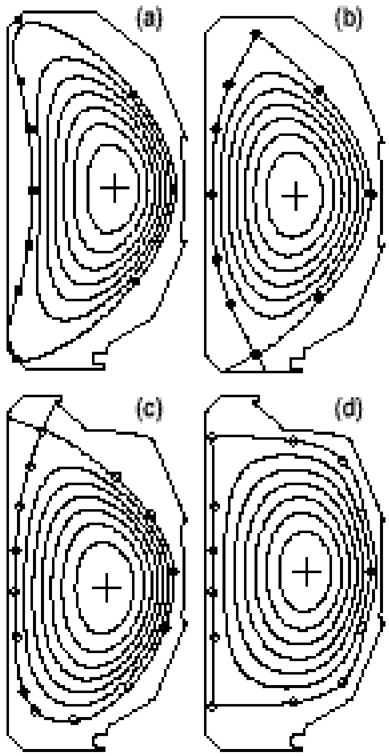
Isoflux Control Scheme Used With Realtime EFIT Provides High Shaping Flexibility, High Shape Control Accuracy



⇐ Discharges produced include:
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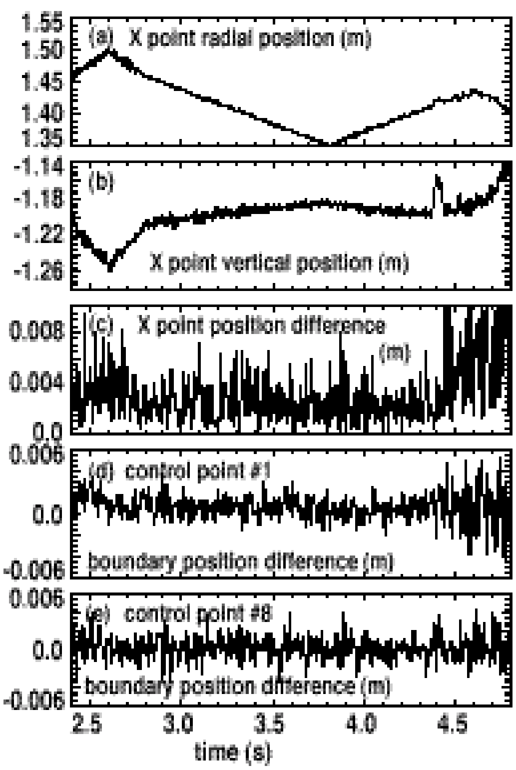
- c) Upper SN;
- d) High squareness DN;



Motivation: New DIII-D Control System Will Use Model-Based Approach to Controller Design

- Design philosophy:
  - → Develop and validate linearized and nonlinear models of plasma/conductor/power supply responses;
  - → Design controllers based on minimal linear models;
  - → Test and optimize controller performance with linear and nonlinear model simulations;

- Advantages:
  - → Controllers allow significant variation in plasma equilibrium without switching to new controller;
  - → New equilibrium/shape development without consuming valuable experimental time on tokamak;
  - → Improved control performance: accuracy/precision, dynamic response, disturbance/noise rejection.
  - → Ability to balance tokamak constraints with desired control.

- Disadvantages:
  - → Requires reasonably accurate system response models;
  - → Possible increase in difficulty of tuning control algorithms "by hand" between shots (but with theoretical reduction in need for such tuning);

Plasma Response Model Based on Rigid Plasma Displacements, Simple Force Balance Relations

- Goal: Use "minimally complex" models which provide sufficiently accurate representation of experiment.

- Simple radial force balance:

$$B_{zm}^{appl} = B_z^{Shafranov} = -\frac{\mu_0 I_p}{4\pi R_m}\Gamma(R_m, \beta_p, \ell_i)$$

$$\equiv -\frac{\mu_0 I_p}{4\pi R_m}\left(\ln\left|\frac{8R_m}{a\sqrt{\kappa}}\right| + \frac{2\kappa}{\kappa^2+1}\beta_p + \frac{\ell_i}{2} - 1.5\right)$$

- Simple vertical force balance:

$$0 = -2\pi R_m I_{p0}\frac{\partial B_R}{\partial z}\delta z + I_{p0}\frac{\partial M_{pc}}{\partial z}\delta I_c + \frac{\partial F_z}{\partial \beta_p}\delta\beta_p + \frac{\partial F_z}{\partial \ell_i}\delta \ell_i$$

- Plasma response objects from perturbed force balance:

R_m *Current-Resp* $$\frac{\partial R_m}{\partial I_c} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1}\frac{\partial B_{zm}}{\partial I_c} + \frac{\partial R_m}{\partial z}\frac{\partial z}{\partial I_c}$$

R_m I_p*-Response* $$\frac{\partial R_m}{\partial I_p} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1}\frac{\mu_0 \Gamma_0}{4\pi R_0}$$

$$\frac{\partial R_m}{\partial \beta_p} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1}\frac{\mu_0 I_{p0}}{4\pi R_0}\left(\frac{2\kappa}{\kappa^2+1}\right) + \frac{\partial R_m}{\partial z}\frac{\partial z}{\partial \beta_p}$$

$$\frac{\partial R_m}{\partial \ell_i} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1}\frac{\mu_0 I_{p0}}{8\pi R_0}$$

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Z_c Curr-Resp $\dfrac{\partial z}{\partial I_c} \equiv \dfrac{(\partial M_{pc} / \partial z)}{-2\pi B_{zJ0} n_J}$, **$Z_c$ β_p-Resp** $\dfrac{\partial z}{\partial \beta_p} \approx \dfrac{\int R B_{R0} J_\varphi dA_p}{\beta_{p0} B_{zJ0} n_J}$

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GENERAL ATOMICS

Rigid Radial and Vertical Plasma Displacement Assumption Closes Plasma Response Equations

- Plasma current distribution assumed to shift rigidly in both radial and vertical directions:

$$\delta J_\varphi = \left.\frac{\partial J_\varphi}{\partial R_m}\right|_{Rigid} \delta R_m + \left.\frac{\partial J_\varphi}{\partial z}\right|_{Rigid} \delta z$$

- Variations in flux at conductors, isoflux segment control points derived from simple Green functions and rigid shift:

$$\delta \psi_c = \frac{\partial \psi_c}{\partial J_\varphi}\left(\left.\frac{\partial J_\varphi}{\partial R_m}\right|_{Rigid} \delta R_m + \left.\frac{\partial J_\varphi}{\partial z}\right|_{Rigid} \delta z \right)$$

$$\delta \psi_{iso} = \frac{\partial \psi_{iso}}{\partial J_\varphi}\left(\left.\frac{\partial J_\varphi}{\partial R_m}\right|_{Rigid} \delta R_m + \left.\frac{\partial J_\varphi}{\partial z}\right|_{Rigid} \delta z \right)$$

- Variations in B-field (Z,R) components at X-point grid locations similarly derived from simple Green functions and rigid shift:

$$\delta B_{z,R}^{grid} = \frac{\partial B_{z,R}^{grid}}{\partial J_\varphi}\left(\left.\frac{\partial J_\varphi}{\partial R_m}\right|_{Rigid} \delta R_m + \left.\frac{\partial J_\varphi}{\partial z}\right|_{Rigid} \delta z \right)$$

Certain Effects Included in the Minimal Model are Key to Producing Accurate Plasma Response

- Explicit effects of κ significantly greater than 1:

$$B_z^{Shafranov} \equiv -\frac{\mu_0 I_p}{4\pi R_m}\left(\ln\left|\frac{8R_m}{a\sqrt{\kappa}}\right| + \frac{2\kappa}{\kappa^2+1}\beta_p + \frac{l_i}{2} - 1.5\right)$$

- Radial response to variation in β_p and l_i :

$$\frac{\partial R_m}{\partial \beta_p} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1} \frac{\mu_0 I_{p0}}{4\pi R_0}\left(\frac{2\kappa}{\kappa^2+1}\right) + \frac{\partial R_m}{\partial z}\frac{\partial z}{\partial \beta_p}$$

$$\frac{\partial R_m}{\partial l_i} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1} \frac{\mu_0 I_{p0}}{8\pi R_0}$$

- Response of ***vertical*** position to variation in β_p:

$$\frac{\partial z}{\partial \beta_p} \approx \frac{\int R B_{R0} J_\varphi dA_p}{\beta_{p0} B_{zJ0} n_J}$$

- Cross-coupling of radial and vertical responses:

$$\frac{\partial R_m}{\partial I_c} \equiv \left[\frac{B_{zm0}}{R_0}n + \frac{\mu_0 I_{p0}}{4\pi R_0^2}(\Gamma_0 - 1)\right]^{-1} \frac{\partial B_{zm}}{\partial I_c} + \frac{\partial R_m}{\partial z}\frac{\partial z}{\partial I_c}$$

- Plasma (resistive) current response:

$$L_p\frac{dI_p}{dt} + R_p I_p + \frac{\partial \psi_p}{\partial R_m}\frac{dR_m}{dt} + \frac{\partial \psi_p}{\partial z}\frac{dz}{dt} + \frac{\partial \psi_p}{\partial I_c}\frac{dI_c}{dt} + I_{p0}\frac{\partial L_p}{\partial l_i}\frac{dl_i}{dt} = V_L$$

Plasma Perturbation Experiments Were Performed on DIII-D to Validate Plasma Response Model

- Wide range of equilibria perturbed in dedicated or piggyback experiments:
  ⇒ Lower/upper single null, double null;
  ⇒ Ohmic→High β_p; Low→High l_i.

- Many plasma position/shape parameters varied:
  ⇒ Major radius, vertical position (rigid shifts);
  ⇒ Inner/outer gaps (independently):
  ⇒ Top gap (for lower single null);
  ⇒ X-point radial/vertical position;
  ⇒ Elongation (plasma height/width separately);

- Several different waveforms programmed to vary plasma position/shape parameters dynamically:
  ⇒ Triangle, step, sinusoidal waveforms;
  ⇒ Periods ranging from 20→400 msec.

- Actual coil current, β_p history from experiment used to calculate model-predicted plasma/diagnostic response:

$$\begin{bmatrix} \delta R_m \\ \delta B_{probe} \\ \delta \psi_{isoflux} \\ \vdots \end{bmatrix} = [C] \begin{bmatrix} \delta \vec{I}_c \\ \delta \beta_p \\ \delta l_i \\ \vdots \end{bmatrix}$$

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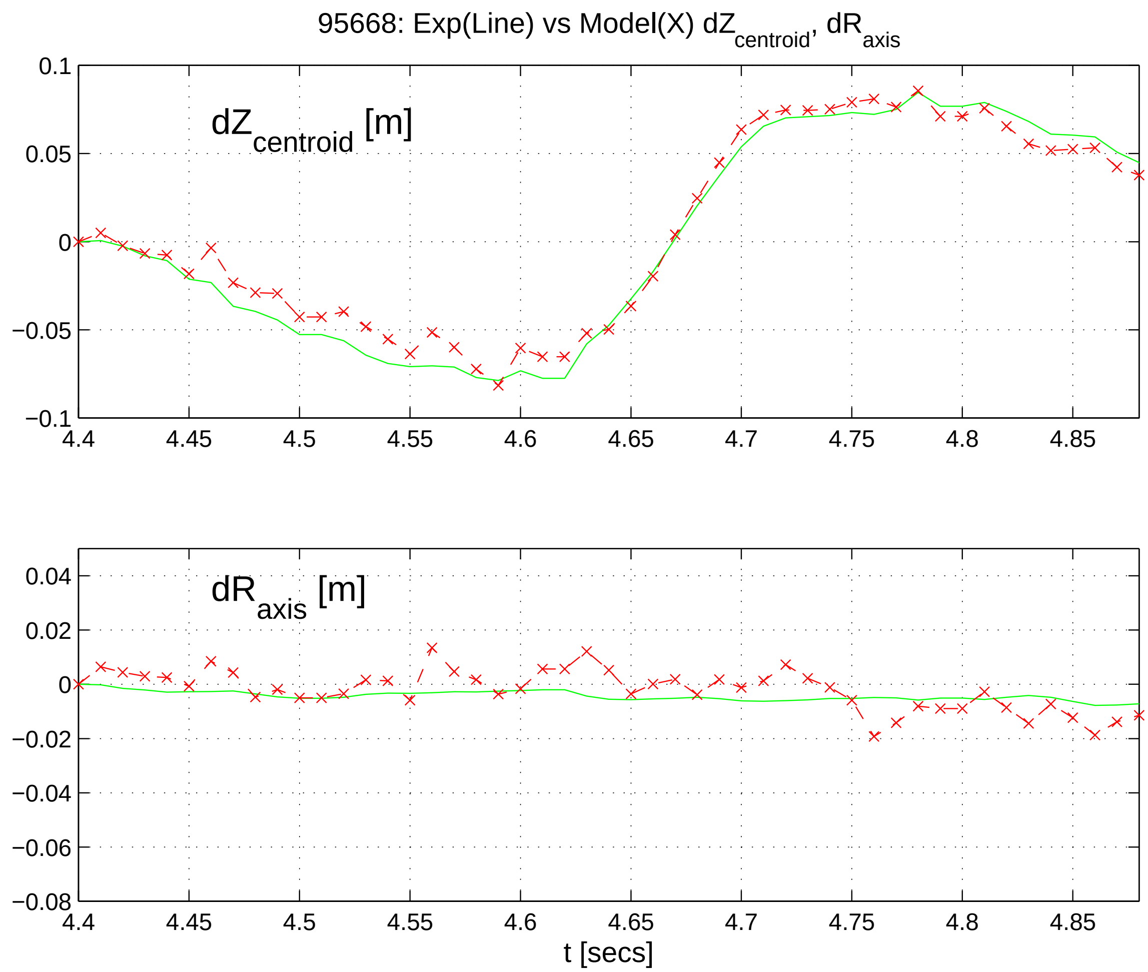
GENERAL ATOMICS

Simple Model Reproduces Experimental Ohmic Plasma Response with High Accuracy: Δz, ΔR_m

- Experiment (equilib. reconstruction)=line, Model=X;
- $dZ_{centroid}$, dR_{axis} = centroid-vertical, axis-radial positions;
- Vertical position intentionally perturbed with pre-programmed waveform to obtain model test data:



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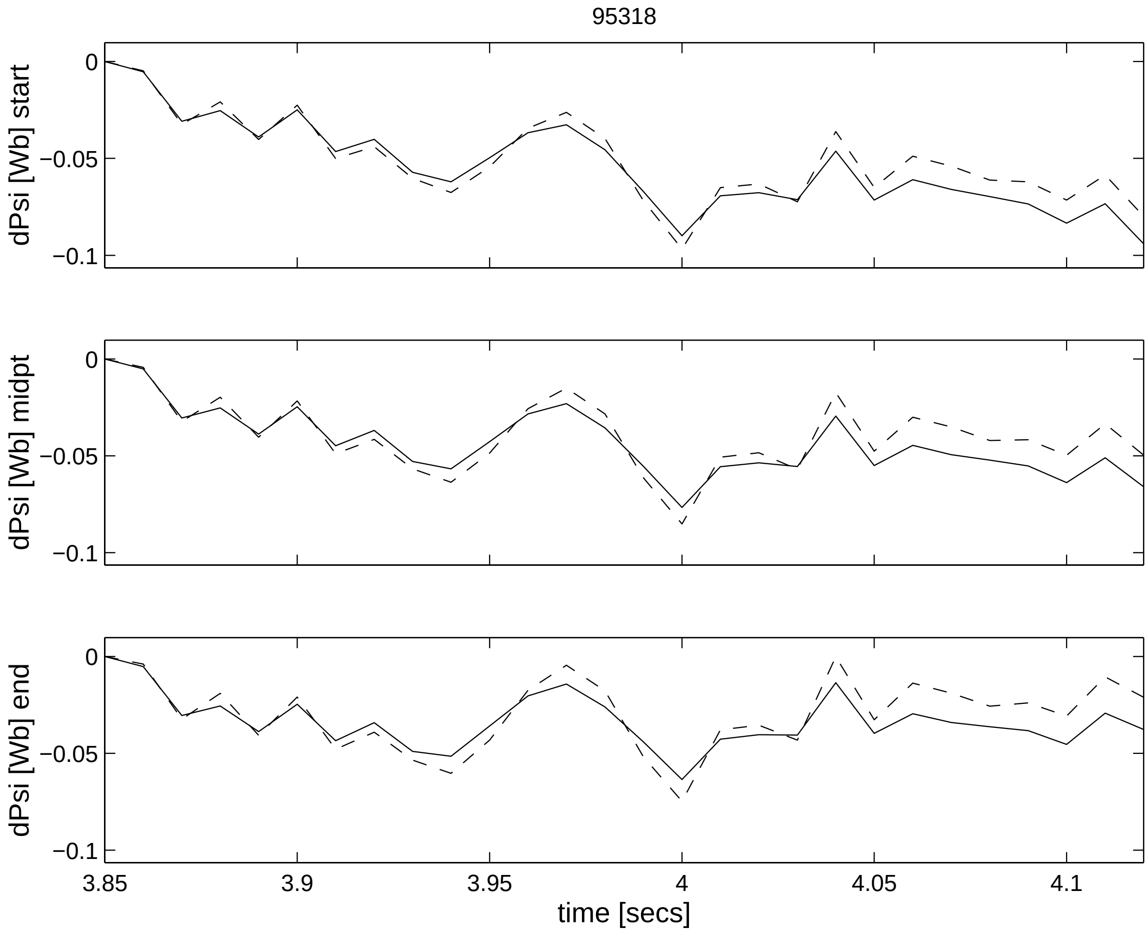
Simple Model Reproduces Experimental Ohmic Plasma Response with High Accuracy:
Δψ(inboard-midplane isoflux segment)

- Model accurately reproduces Δψ inside/outside plasma, near plasma boundary, for ramping major radius;
- Experiment=solid, Model=dashed;



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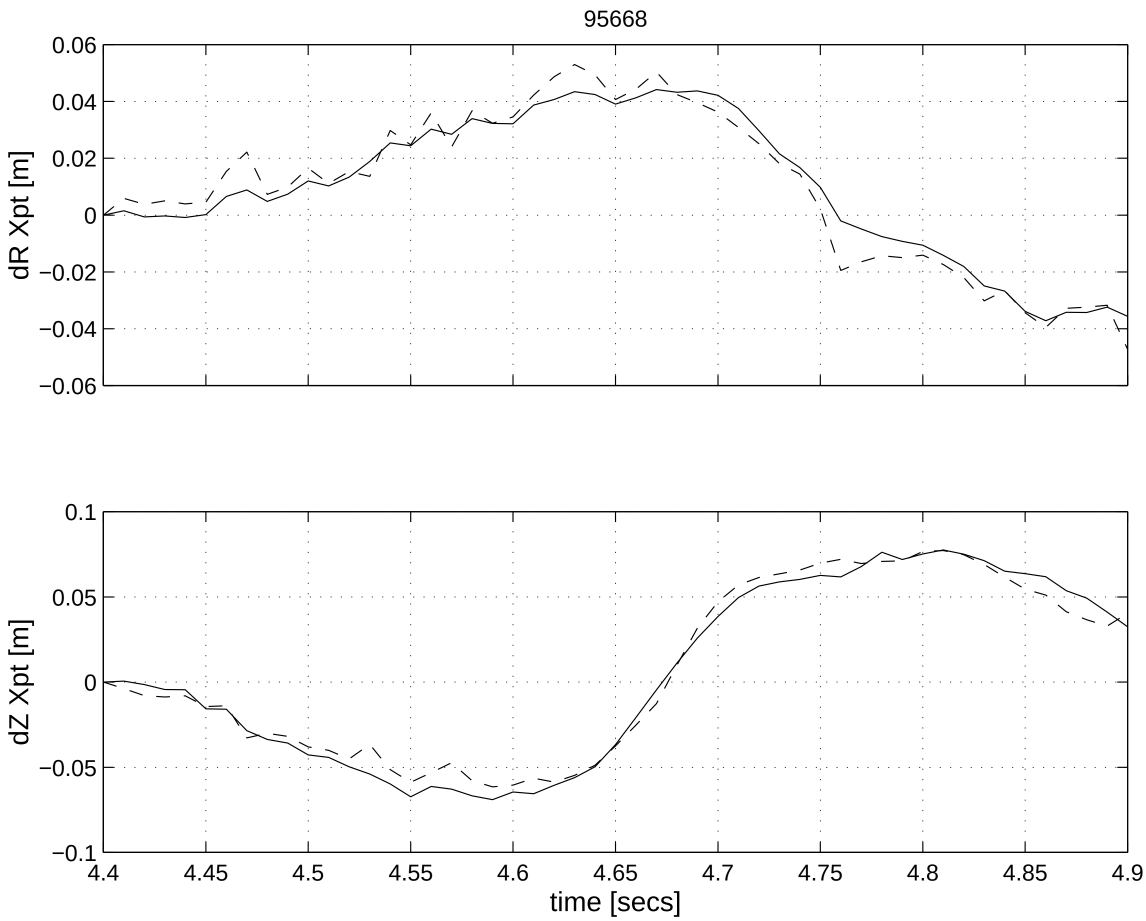
GENERAL ATOMICS

Simple Model Reproduces Experimental Ohmic Plasma Response with High Accuracy:

X-Point Δz_X, ΔR_X

- Experiment (equilib. reconstruction)=solid, Model=X;
- dZ_X, dR_X = X-point position;
- Vertical position intentionally perturbed with pre-programmed waveform to obtain model test data:



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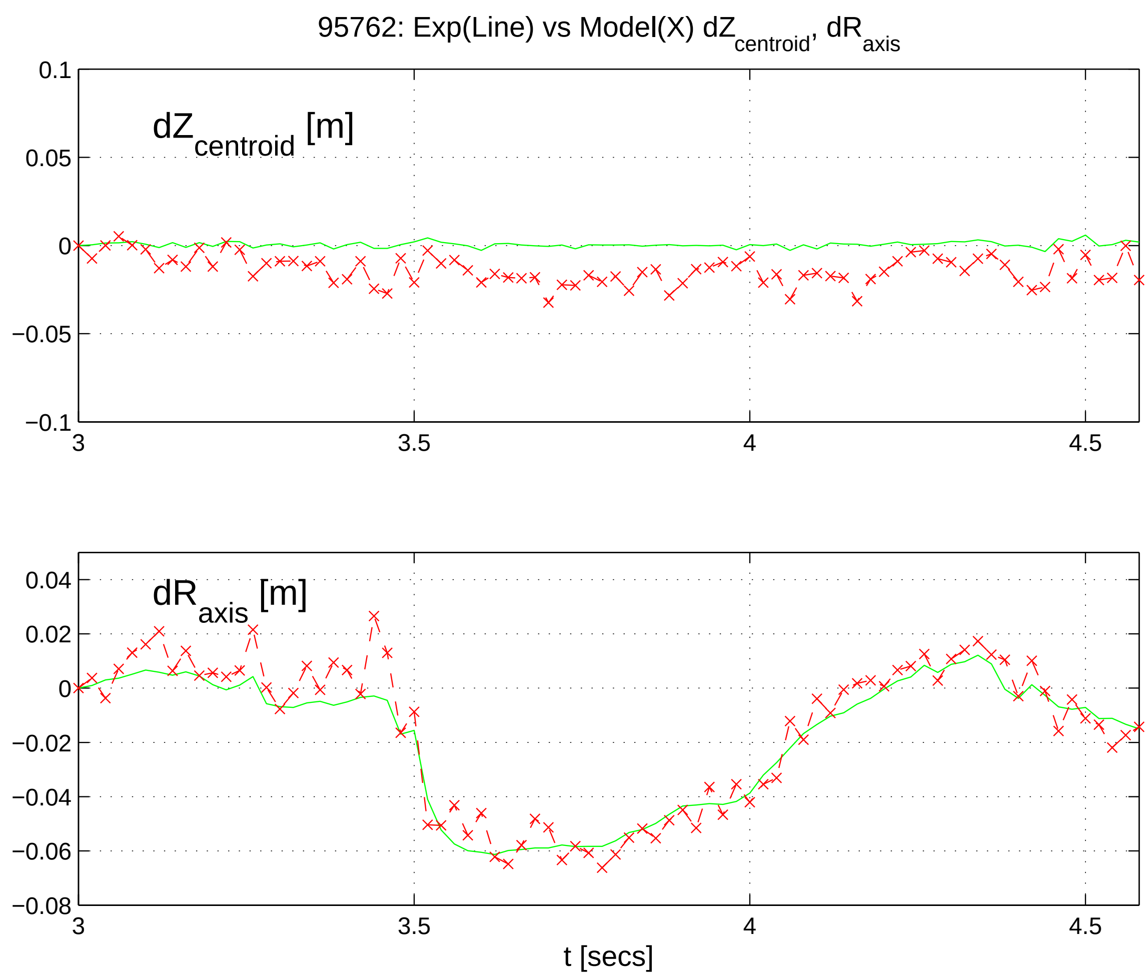
GENERAL ATOMICS

Simple Model Reproduces Experimental High-β_p Plasma Response with High Accuracy: Δz, ΔR_m

- Experiment (equilib. reconstruction)=line, Model=X;
- $dZ_{centroid}$, dR_{axis} = centroid-vertical, axis-radial positions;
- β_p dropped from ~1.1 to ~0.5 at t~3.5 s due to Ar gas puff
  $\Rightarrow$ plasma moved radially inboard:



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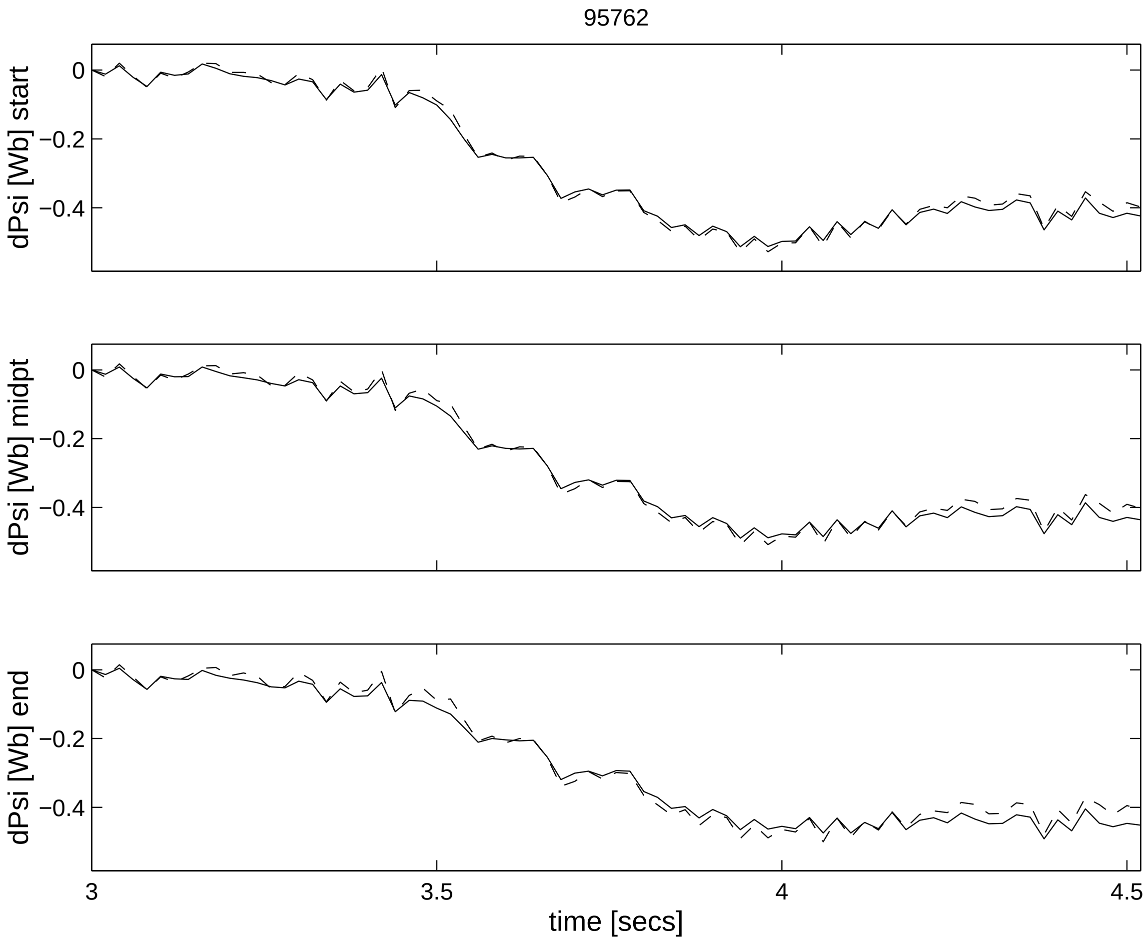
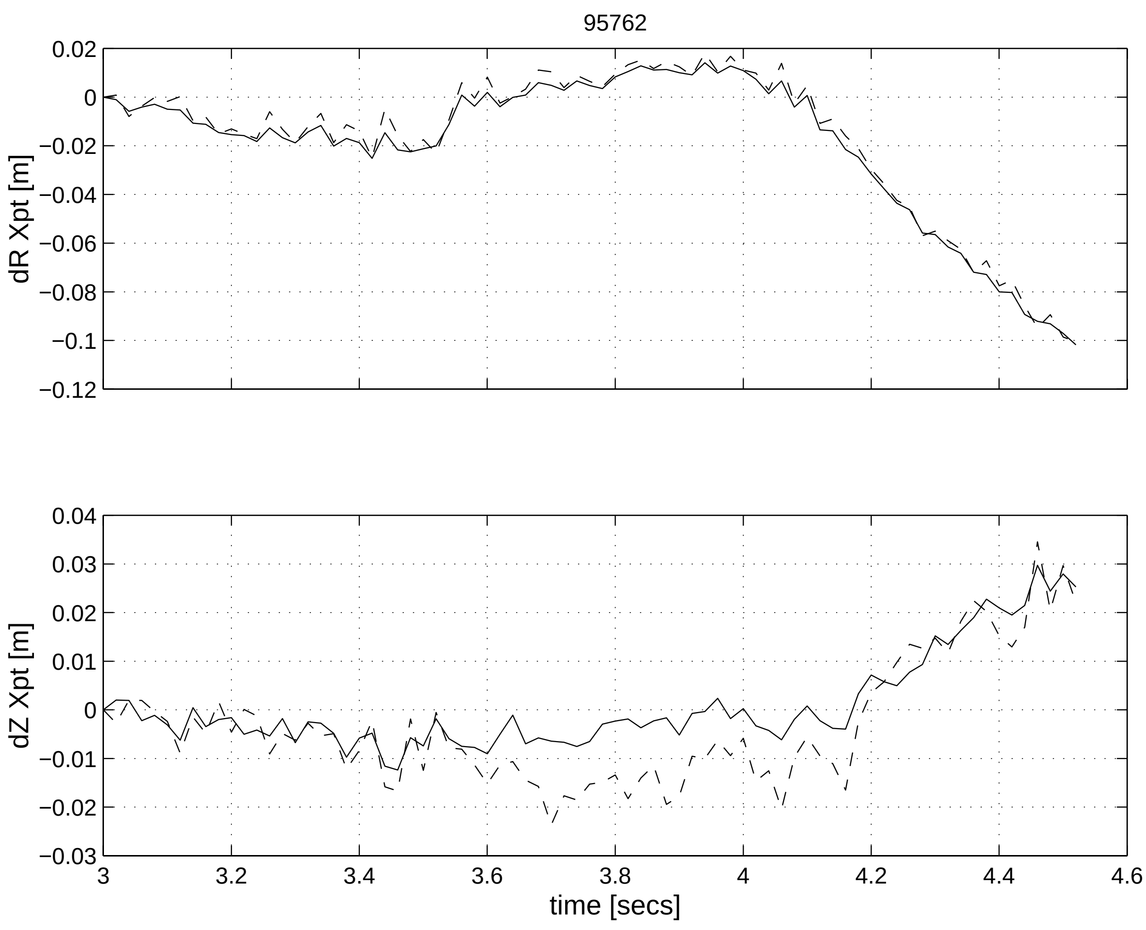
Simple Model Reproduces Experimental High-β_p Plasma Response with High Accuracy: $\Delta\psi$(inboard-midplane isoflux segment)

- Model accurately reproduces $\Delta\psi$ inside/outside plasma, near plasma boundary, for ramping major radius;
- Experiment=solid, Model=dashed:



Simple Model Reproduces Experimental High-β_p Plasma Response with High Accuracy: X-Point Δz_X, ΔR_X

- Experiment (equilib. reconstruction)=solid, Model=X;
- dZ_X, dR_X = X-point position;
- β_p dropped from ~1.1 to ~0.5 at t~3.5 s due to Ar gas puff
  $\Rightarrow$ plasma moved radially inboard:



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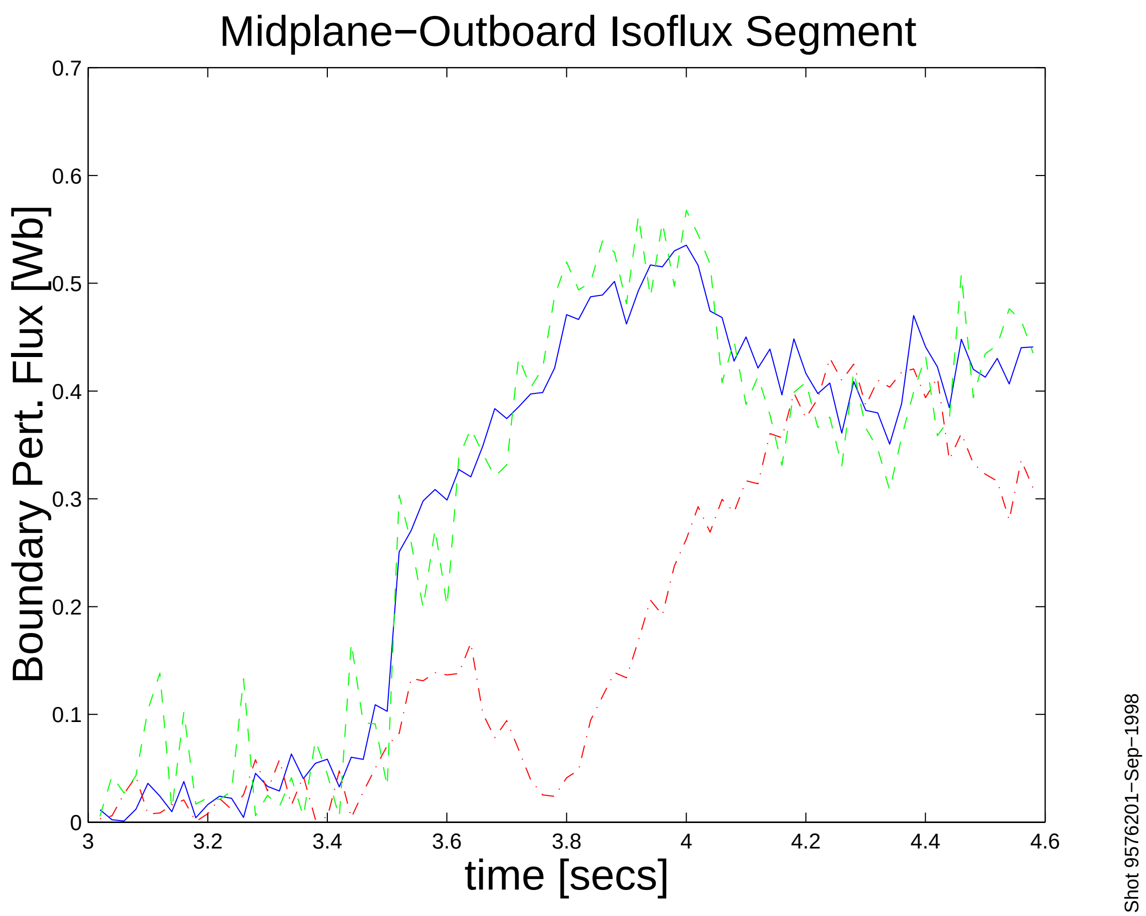
A "Response Model" Based on Vacuum Response (No Plasma Contribution) Can Be VERY Inaccurate

- Example: Radial shift of high-β_p single null plasma;
- Experiment (solid blue), plasma model (dashed green) differ significantly from vacuum model (dash-dot red) over interval 3.5<t<4.2 s:



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Accurate Model of Vertical Plasma Response is Important for Designing Good Dynamic Controller

- Present DIII-D plasma control has dynamically "separated" vertical position and stability control:
  $\rightarrow$ *Stability* control uses velocity ($\propto dZ/dt$) feedback provided by fast (~60 μsec) separate cpu;
  $\rightarrow$ *Position* control provided as part of shape control with longer cycle time (~1 msec) cpu;

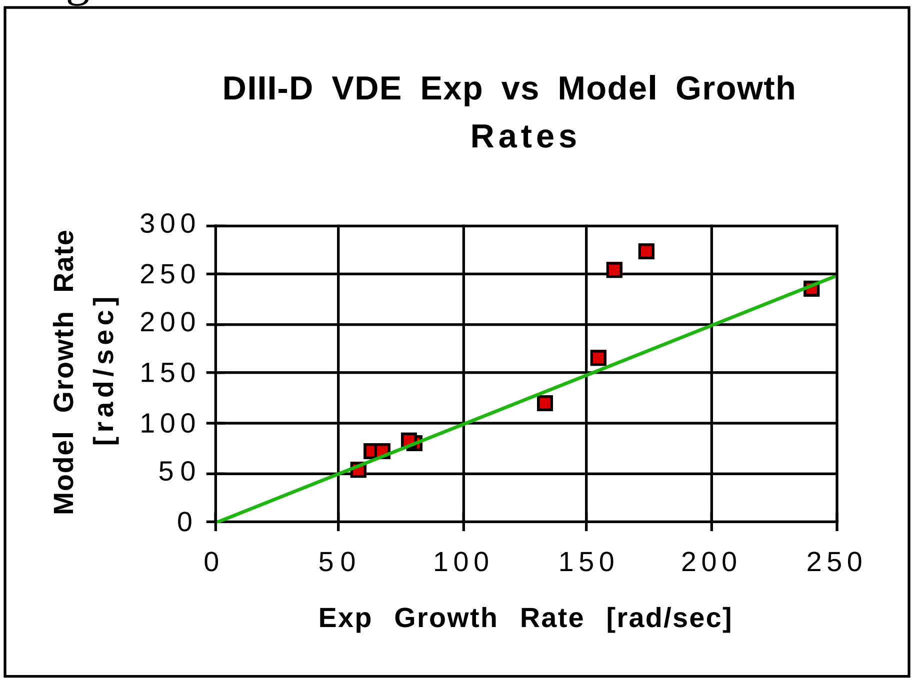
- MIMO controllers on DIII-D must be designed to provide good shape control and minimize interaction with (fast) stability control loop;

- Good dynamic control therefore requires accurate model of vertical plasma response:
  $\rightarrow$ Open loop vertical growth rate;
  $\rightarrow$ Effect of stabilization (by fast velocity feedback) on vertical plasma displacement mode.

- Effect of vertical motion is also important in order to produce correct *radial* plasma response;

Rigid Current-Conserving Model Vertical Growth Rate Matches DIII-D VDE Experiments

- Growth rates from current-conserving ($\delta I_p=0$) rigid vertical model compared with experimentally measured VDE growth rates:

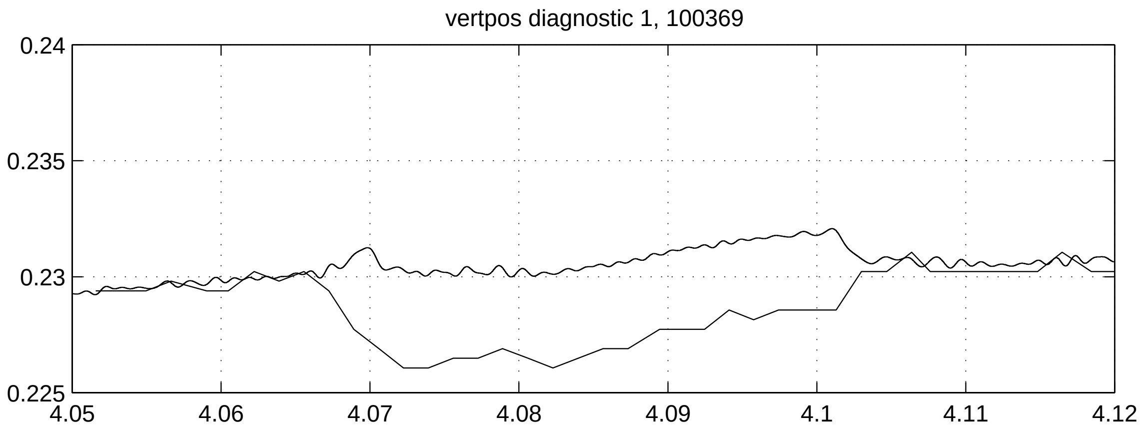


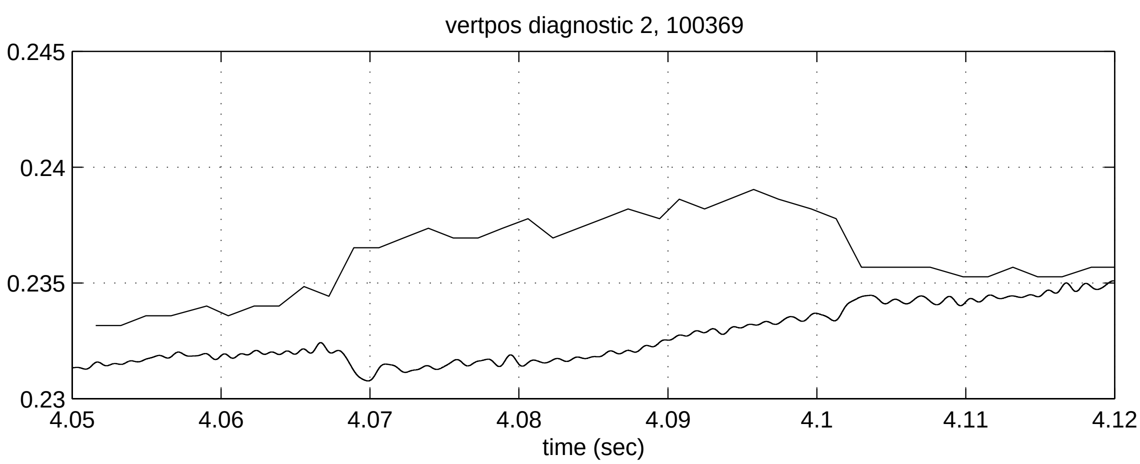
- Most VDE's yield vertical growth rate within ~10% of predicted value;

- Small subset of discharges produce VDE's with vertical growth rates differing by as much as 50% from predicted values:
  $\longrightarrow$ Source of difference not yet well understood;
  $\longrightarrow$ ~ 50% error in growth rate is tolerable for dynamic controller design in such cases.

Incorrect Model of Vertical Instability Response Produces Incorrect B-Field Measurements

- Open loop simulation with vertical instability *removed* (to roughly model effect of fast vertical velocity control loop) shows incorrect response of outboard poloidal field probes:





- Vertical model affects both vertical and*radial* response: cross-coupling terms are significant in up-down asymmetric plasmas (this case = Upper Single Null).

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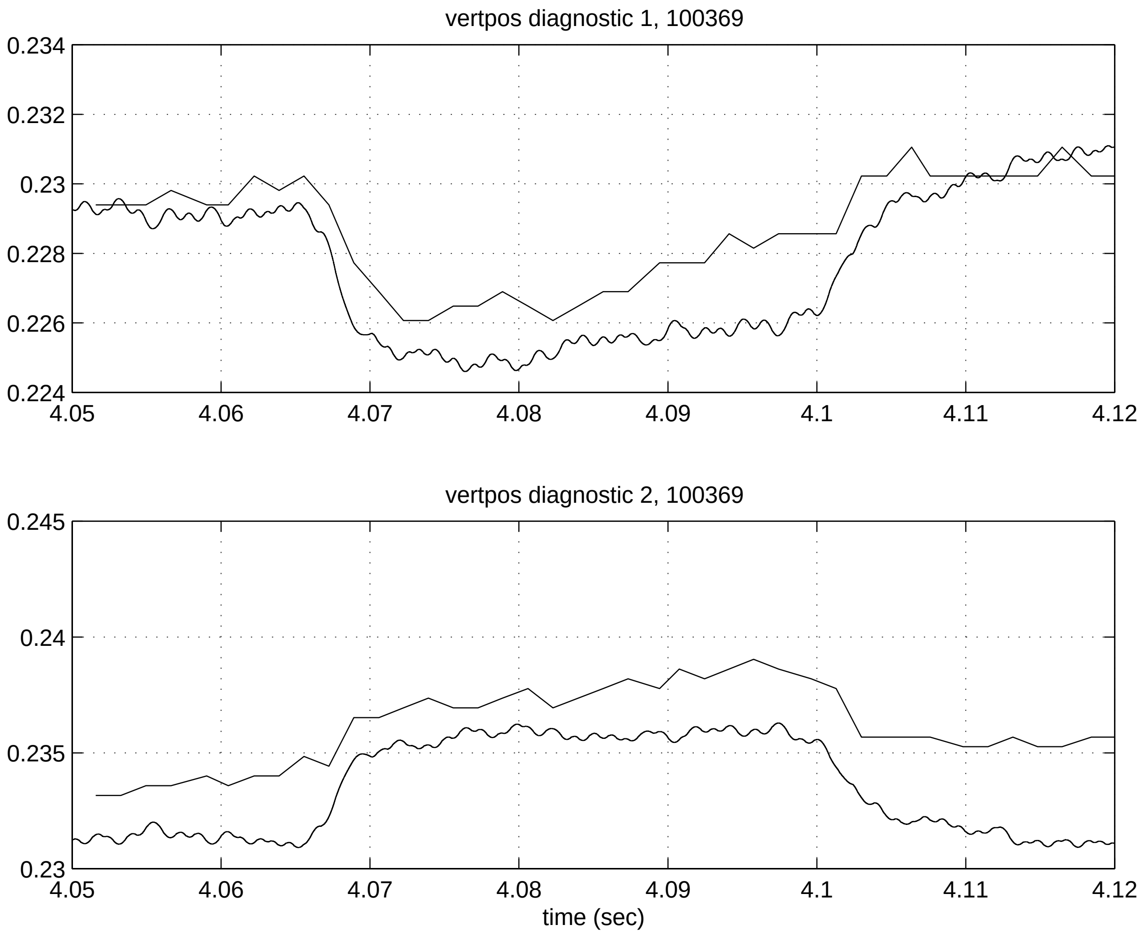
Inclusion of Accurate Vertical Instability Mode in Plasma Model Produces Correct Probe Measurements

- Closed loop simulation with vertical instability *included* (and effect of fast velocity control loop correctly included in model) shows correct response of outboard poloidal field probes:



- Vertical model affects both vertical and *radial* response: cross-coupling terms are significant in up-down asymmetric plasmas (this case = Upper Single Null).

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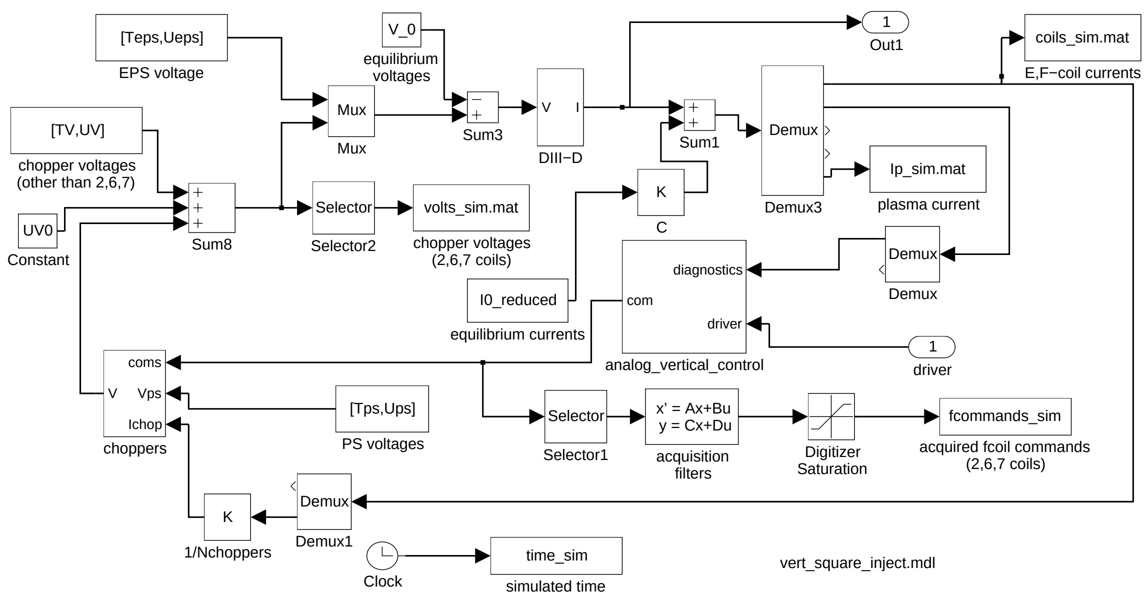
Accurate Representation and Simulation of Vertical Response in Tokamaks is Complex Problem

- Simulation/validation of intrinsically unstable open-loop **plasma model** is difficult: little more than growth rate can be validated;



- Simulation/validation of closed-loop response in DIII-D requires accurate modeling of highly nonlinear **chopper power supplies** and digital controller action (as well as unstable plasma).

CONCLUSIONS

- High dynamic accuracy of performance in DIII-D isoflux boundary/X-point control requires accurate plasma response models for controller design.

- "Minimal" analytically-based plasma models:
  - → Are sufficient to accurately describe plasma response for control;
  - → Have been developed and **extensively validated** against DIII-D experiments.
  - → Have been used to design successful dynamic shape controllers for DIII-D. [See poster JP1.33, this session]

- Physics terms important to plasma response have been identified:
  - → Effect of plasma elongation;
  - → Effects of variation in β_p and l_i;
  - → Cross-coupling of vertical and radial displacement;
  - → Validated physics understanding allows models to be extended to other operating regimes and devices more reliably.

PLASMA MODEL

VERTICAL MODE

MODEL VALIDATION EXAMPLES

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