INTERACTION OF AN EXTERNAL ROTATING MAGNETIC FIELD WITH THE PLASMA TEARING MODE SURROUNDED BY A RESISTIVE WALL

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Abstract Submitted
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Interaction of an External Rotating Magnetic Field with the Plasma Tearing Mode Surrounded by a Resistive Wall[1] S.C. GUO, Consorzio RFX, Padova, Italy, M.S. CHU, General Atomics — The effect of an externally rotating magnetic field on the plasma tearing mode surrounded by a resistive wall is studied. A pair of tearing mode evolution equations describing the magnetic energy and angular momentum balance across the magnetic island are used. The model is valid for both the RFP and the tokamak. The pair of equations is solved numerically to determine the equilibrium amplitude and phase of the tearing mode with respect to that of the external magnetic field and the phase stability of the combined system. When the external magnetic field amplitude is large, the tearing mode frequency is locked to that of the external field above minimum amplitude. Dependence of the critical unlocking amplitude and the phase stability on parameters relevant to present day experiments are obtained. In the opposite limit, when the amplitude is small, the external field is not sufficient to lock the tearing mode below a critical amplitude. Dependence of this critical locking amplitude on plasma characteristics and external wall distance is also obtained. Possible utilization of the external rotating field to stabilize the tearing mode is also discussed.[2]

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OUTLINE

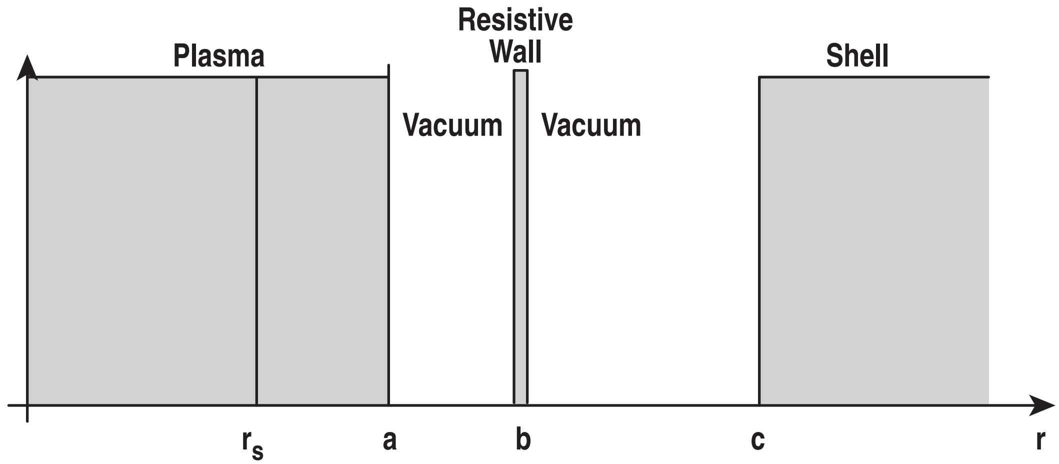
- Introduction
- Model Equations
- Steady state solutions and phase stability analysis
  - Mode locked to the external rotating applied field; unlocking threshold
  - Mode unlocked; locking threshold
- Intrinsically unstable modes and stable modes
- Summary and discussion

INTRODUCTION

- Key (MHD) issues in the operation of fusion devices:
  * Avoidance of locked mode induced by resistive wall and/or error field.
  * Active stabilization of low m resistive MHD modes.
- One of the possible approaches is to externally apply an rotating helical magnetic field, as some experiments have tested (DIII-D[1], JET[2], RFX[3]...).
- The problem involves non-linear interaction of plasma modes with external magnetic perturbation and/or resistive wall. Various theoretical studies have been done on this field.[4]
- In this work, we derive and solve the tearing mode evolution equations (equilibrium solution), which involve the effects of both resistive wall and rotating external magnetic field. The model equation can be applied to both tokamaks and RFPs.

MODEL EQUATIONS

- Cylindrically symmetric plasma ($r = a$) surrounded by a resistive wall $r_W = b$, and a perfect conducting shell $r_{sh} = c$



- The equation governing the magnetic perturbation due to a tearing mode (in outer region)

$$\frac{d}{dr}\left(f(r)\frac{d\psi}{dr}\right) - g(r)\psi = 0 \quad \text{where } \psi = rb_r$$

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MODEL EQUATIONS

- **Boundary Conditions**
  - ⁕ Thin shell approximation for the resistive wall

$$\frac{\delta_b}{b} \ll \omega\tau_b < \frac{b}{\delta_b}$$

  - ⁕ Externally applied rotating magnetic field ψ_{ex} at $r_{ex} = c$; with frequency ω_{ex}

- **Assumptions**
  - ⁕ The plasma is assumed to rotate only in the toroidal direction
  - ⁕ $\frac{1}{\omega_p^2}\frac{d\omega_p}{dt} \ll 1$
  - ⁕ Validity of Rutherford equation: $w/\delta_{VR} \geq 1$, Constant-ψ
    (Fitzpatrick, 1999; Riconda et al., 1999)

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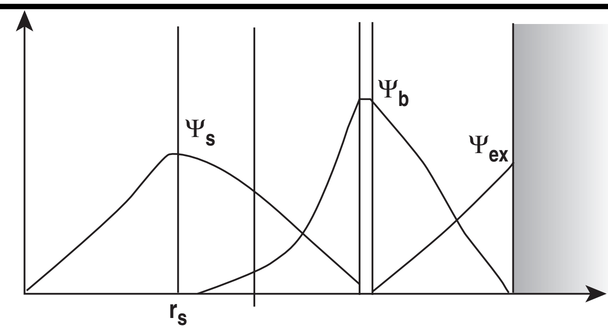
TEARING MODE EVOLUTION EQUATIONS



- Without loss of generality, $\psi_{m,n}$ can be separated into three parts:

$$\psi_{m,n} = \psi_s + \psi_b + \psi_{ex}$$
$$= \Psi_s \hat{\psi}_s(r) + \Psi_b \hat{\psi}_b(r) + \Psi_{ex} \hat{\psi}_{ex}(r)$$

$$T_{EM}^{m,n} = \frac{2\pi^2 R_0}{\mu_0} \frac{n}{m^2 + n^2 \varepsilon_s^2} I_m \left\{ \psi_{m,n}^* \left(r \frac{d\psi_{m,n}}{dr} \right) \Bigg|_{r_s^-}^{r_s^+} \right\}$$

$$\Delta = \Delta_s' r_s = r \left[\frac{d\psi_{m,n}}{dr} \psi_{m,n}^* + \frac{d\psi_{m,n}^*}{dr} \psi_{m,n} \right]_{r_s^-}^{r_s^+} \Bigg/ 2\Psi_s \Psi_s^*$$

- Evolution equations

$$4 I \tau_R \frac{d\sqrt{|\Psi|}}{dt} = \Delta_{plasma} + \Delta_{wall} + \Delta_{ex}$$

$$4\sqrt{|\Psi|} \frac{d\Delta\omega}{dt} = T_{plasma} + T_{wall} + T_{ex} \qquad \Delta\omega = \omega_0 - \omega_p$$

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TEARING MODE EVOLUTION EQUATIONS

$$\Delta_{plasma} = \Delta_b'(0)\left(1 - \frac{\sqrt{|\Psi_0|}}{\sqrt{\Psi_0}}\right)$$

$$\Delta_{wall} = -\frac{E_{sb}E_{bs}E_{bb}}{\omega_p^2\tau_b^2 + E_{bb}^2}$$

$$\Delta_{ex} = \frac{E_{sb}E_{bex}}{\sqrt{\omega_{ex}^2\tau_b^2 + E_{bb}^2}}\,\frac{\Psi_{ex}}{\Psi_s}\cos(\Delta\varphi - \theta)$$

$$T_{plasma} = \frac{G\nu}{\tau\nu}\left(\omega_0 - \omega_p\right)$$

$$T_{wall} = Q\,\frac{E_{sb}E_{bs}\omega_p\tau_b}{E_{bb}^2 + \omega_p^2\tau_b^2}$$

$$T_{ex} = -Q\,\frac{E_{sb}E_{bex}}{\sqrt{E_{bb}^2 + \omega_{ex}^2\tau_b^2}}\,|\Psi_s||\Psi_{ex}|\sin(\Delta\varphi - \theta)$$

$$E_{ij} = \left[r\frac{d\hat{\psi}_j}{dr}\right]_i \qquad i, j = s,\ b,\ ex$$

$$\Delta\phi = \int_0^t\left(\omega_{ex} - \omega_p\right)dt' + \phi_{0ex} - \phi_{0p}$$

$$\theta = \tan^{-1}\frac{\omega_{ex}\tau_b}{|E_{bb}|}$$

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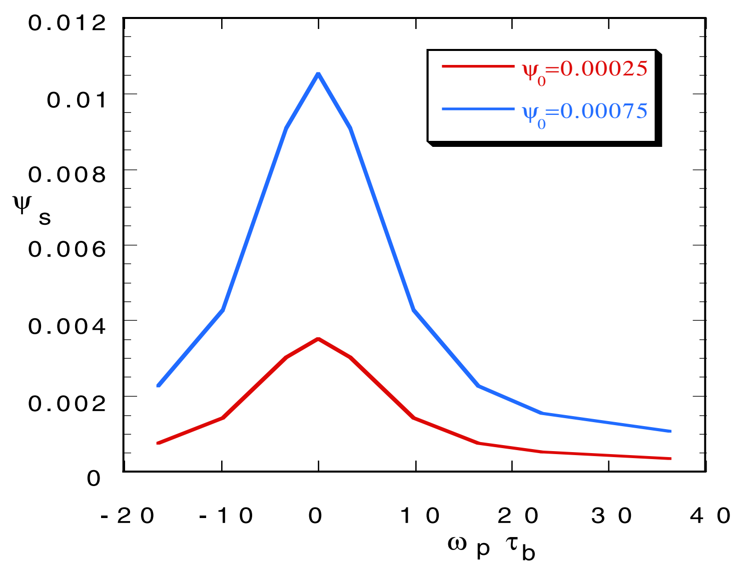
TEARING MODE EVOLUTION EQUATIONS

(a) Resistive wall stabilization

$$\Psi_s = \Psi_0 \left[1 - \frac{E_{sb}E_{bs}E_{bb}}{\omega_p^2\tau_b^2 + E_{bb}^2} \frac{1}{\Delta_b'(0)} \right]^2$$

$$\tau_b \to \infty \quad \Psi_s = \Psi_0 \, ; \quad \tau_b \to 0 \quad \Psi_{s0} = \Psi_0 \left[1 - \frac{E_{sb}E_{bs}}{E_{bb}\Delta_b'(0)} \right]^2 = \Psi_0 \left[\frac{\Delta_c'(0)}{\Delta_b'(0)} \right]^2$$



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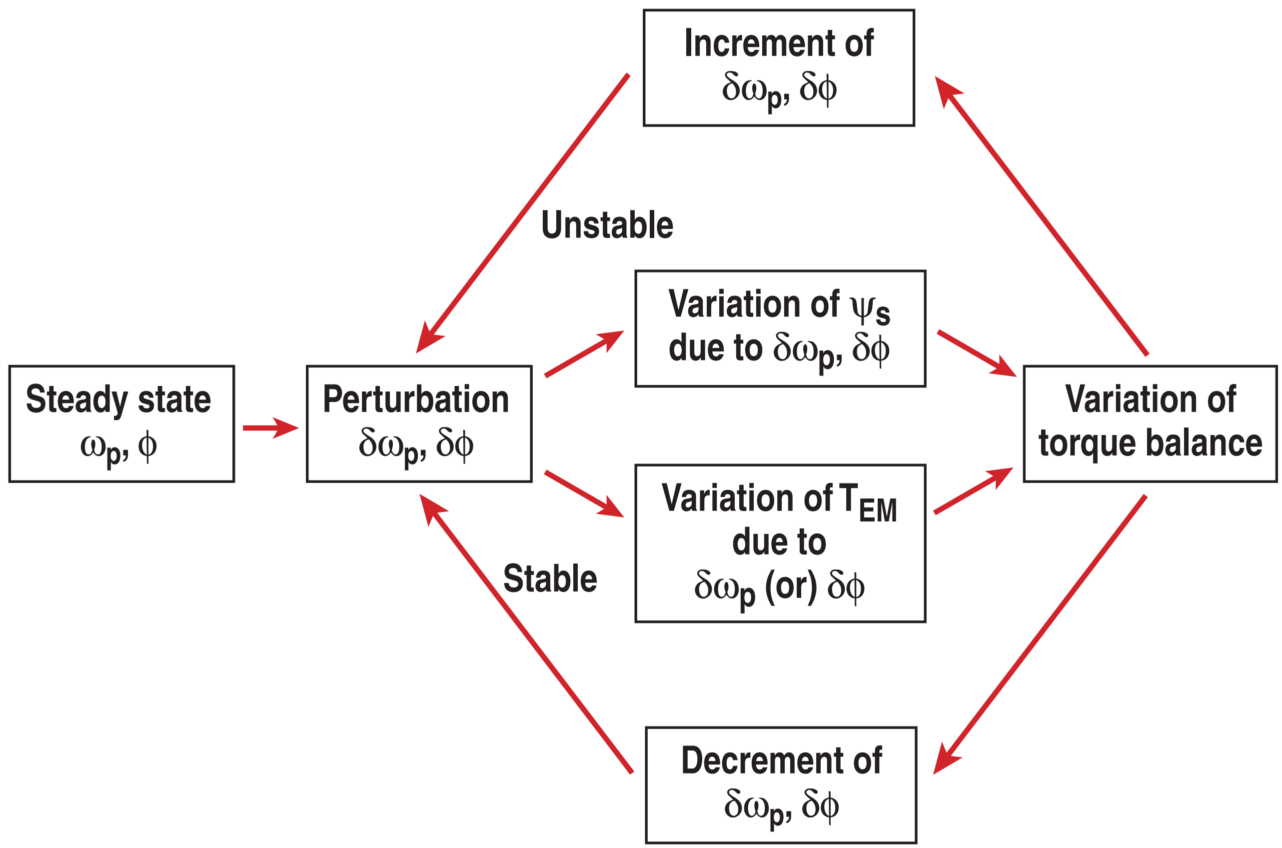
PHASE STABILITY ANALYSIS



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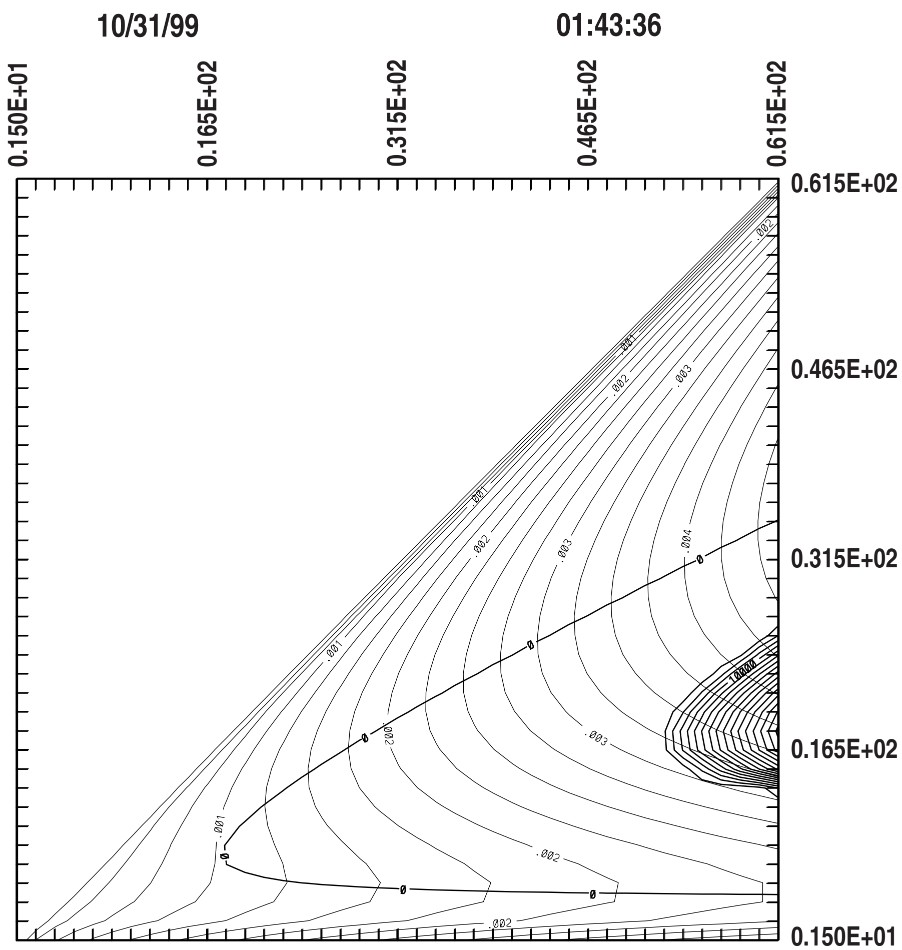
STEADY STATE SOLUTION I: FOR $\Psi_{ex} = 0$

$\Delta'_{plasma} + \Delta'_{wall} = 0$ $\quad$ $T_{plasma} + T_{wall} = 0$



- For $\omega_0 < \omega_c$, continuous spectrum
- $\omega_\psi < \omega_c$, bifurcated spectrum

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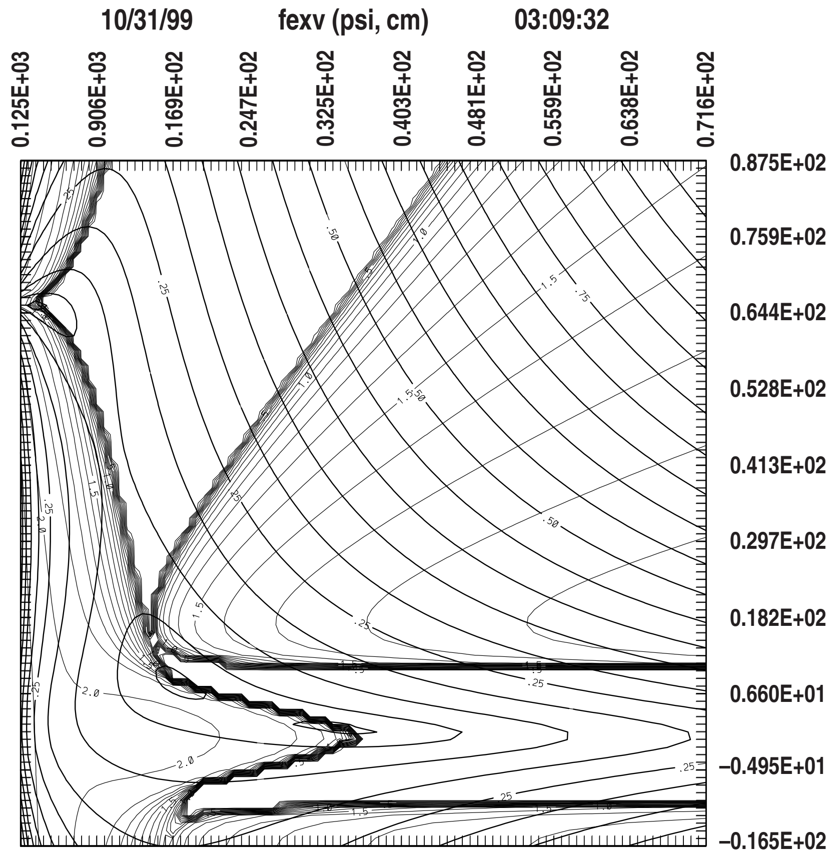
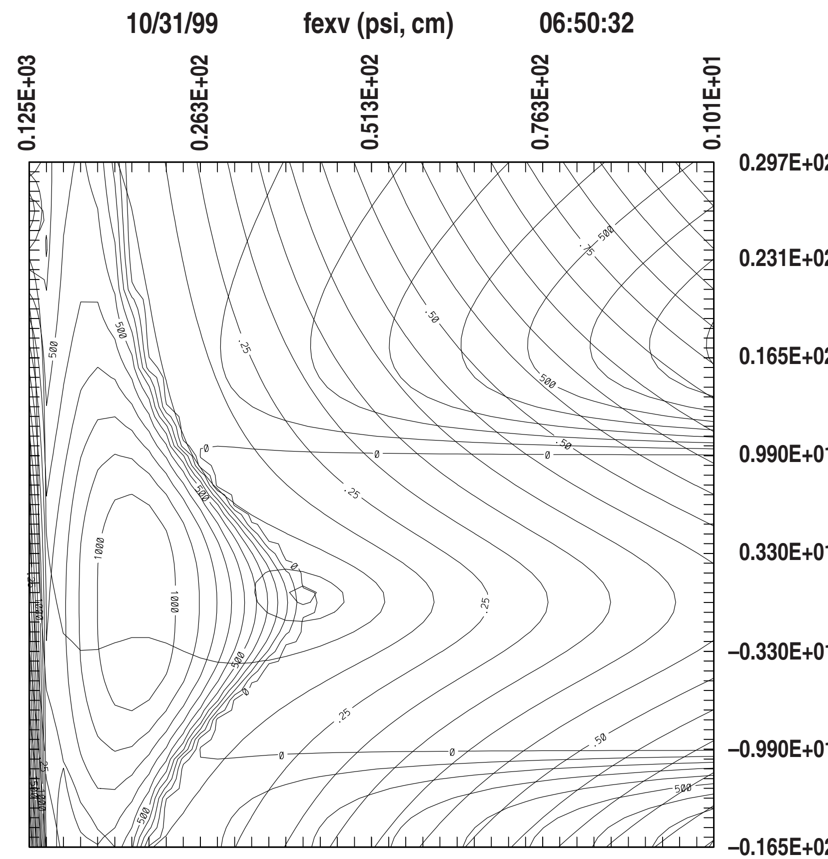
STEADY STATE SOLUTION II: LOCKED UNSTABLE MODE

$\omega_p = \omega_{ex}$

(a) Bifurcated solution



(b) Continuous solution



$[m = 1, n = 0, \psi_0 = 0.25\times10^{-3}, \Delta_b(0) = 0.3]$

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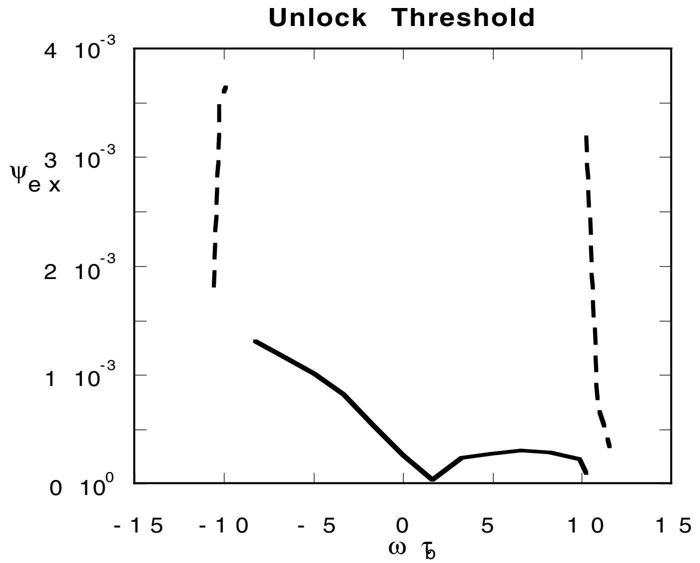
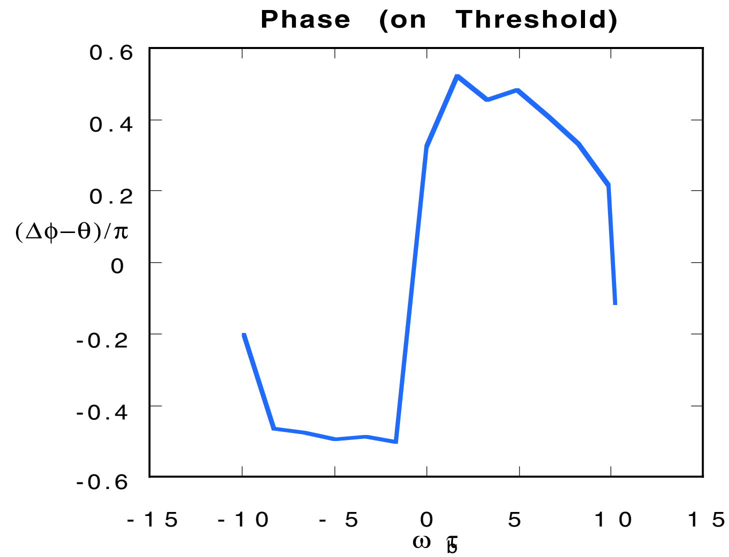
STEADY STATE SOLUTION II: LOCKED UNSTABLE MODE

- **Stability Boundary [for case (a) lower frequency solution]**





- $\Delta\varphi_0 - \theta \leq \dfrac{\pi}{2}$, $\omega_p \sim \omega_p^L \sim 0$ $\Delta\varphi_0 - \theta \lesssim \dfrac{\pi}{4}$, $\omega_p \sim \omega_p^H \sim \omega_0$

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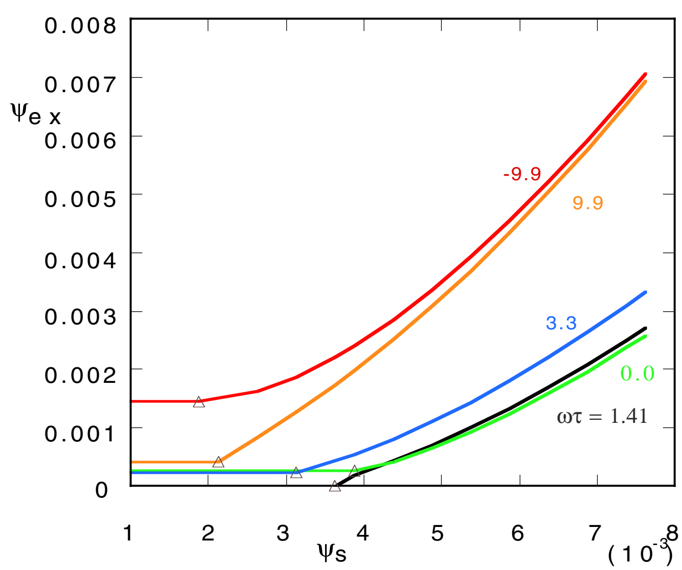
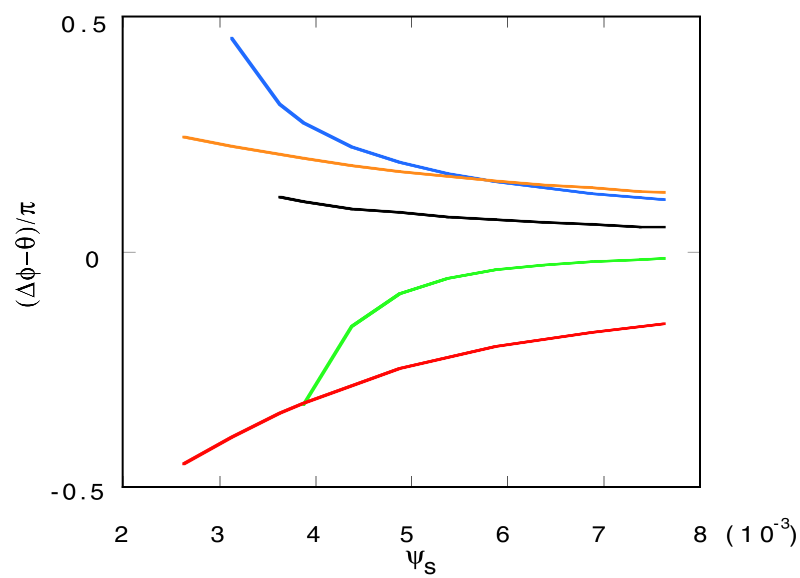
STEADY STATE SOLUTION II: LOCKED UNSTABLE MODE

- Stability Boundary [for case (a) lower frequency solution]





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STEADY STATE SOLUTION III: UNLOCKED UNSTABLE MODE

$$\left(\omega_p \neq \omega_{ex}\right)$$

(a) Analysis

$$\Psi_s = \Psi_{s0} + \delta\psi_s$$

$$\Psi_{s0} = \Psi_0 \left[1 + \frac{\Delta_{wall}}{\Delta_b'(0)} \right]^2$$

$$\delta\psi_s = \frac{1}{2I\tau_R} P \frac{|\Psi_{ex}|}{|\Psi_{s0}|^{1/2}} \frac{1}{\sqrt{\alpha^2 + (\omega_{ex} - \omega_p)^2}} \cos(\Delta\phi - \theta - \beta)$$

- $\delta\psi_s$ oscillates in time, so the torque T_{ex} also oscillates in time

- Assumption: Plasma is sufficiently viscous that it responds only to the steady components

$$\langle T_{ex} \rangle = \frac{QP^2}{4I\tau_R} \frac{|\Psi_{ex}|^2}{|\Psi_{s0}|^{1/2}} \frac{(\omega_{ex} - \omega_p)}{\alpha^2 + (\omega_{ex} - \omega_p)^2}$$

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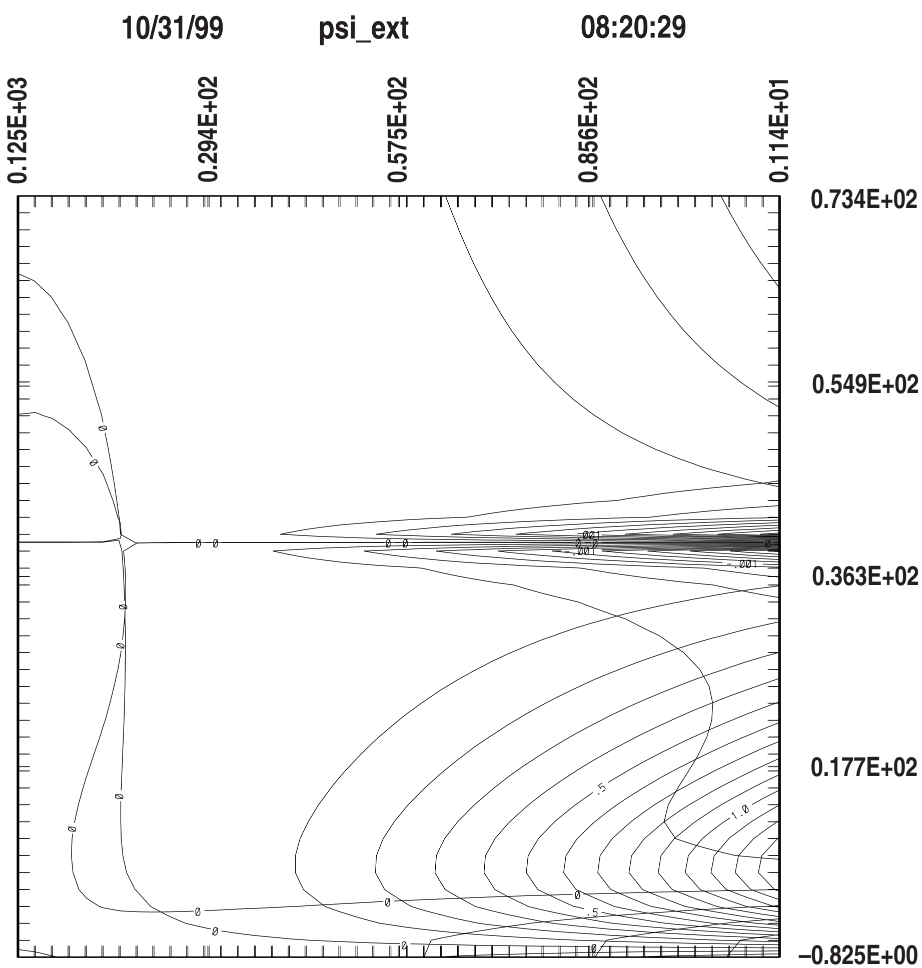
STEADY STATE SOLUTION III: UNLOCKED UNSTABLE MODE

(b) Results (for $\omega_p^H > \omega_{ex} > w_p^L$)



- The unlocked ω_p solution exists when

  $\omega_p < w_c^L$,
  or
  $\omega_p < w_c^H$

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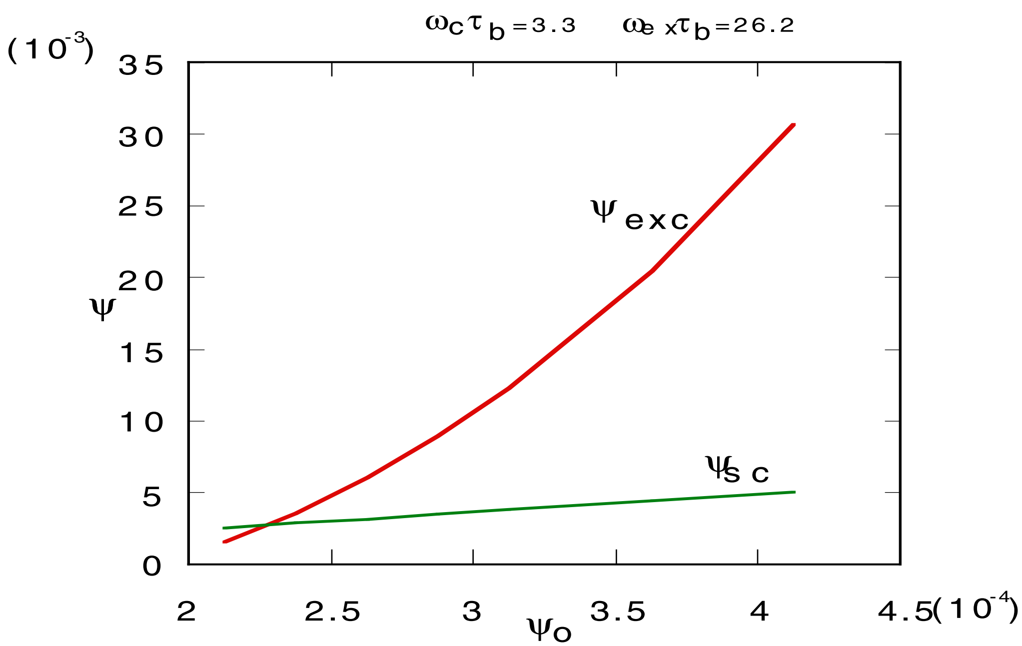
STEADY STATE SOLUTION III: UNLOCKED UNSTABLE MODE

(c) Locking threshold ψ_{exc} and corresponding ψ_{sc}



- Locking threshold ψ_{exc} is much larger than unlocking threshold

For $\psi_0 = 2.5\times10^{-4}$, $\psi_{exc}(\text{unlock}) = 2.4\times10^{-4}(\omega_p\tau_b = 3.3)$, $\psi_{exc}(\text{lock}) = 4.8\times10^{-3}$

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STEADY STATE SOLUTION IV: LOCKED STABLE MODE

- For intrinsically stable plasma, if the external field is rotating in the plasma frame, there will still be a torque acting on the resonant surface due to inertia and dissipative effects. This torque acts to bring the tearing mode to rotate together with ω_{ex}. Once the locking happens, a full reconnection occurs.

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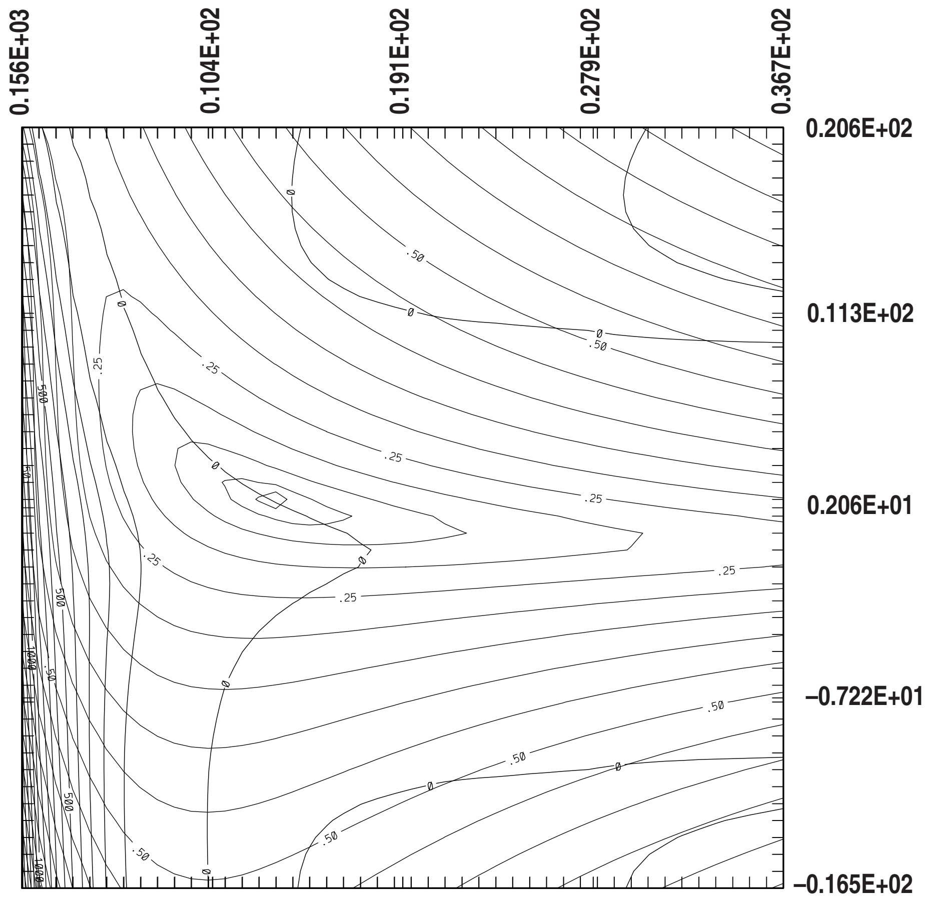

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STEADY STATE SOLUTION IV: LOCKED STABLE MODE

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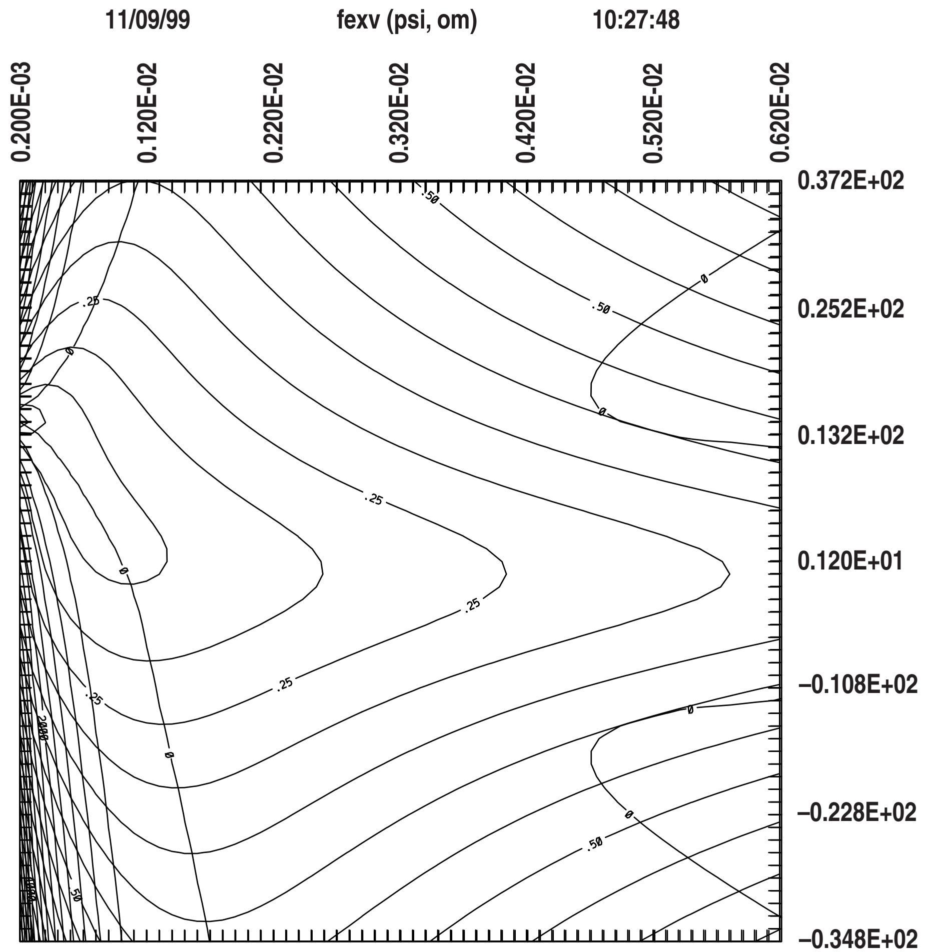

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SUMMARY AND DISCUSSION

- A pair of tearing mode evolution equations, which describe the magnetic energy and angular momentum balance across the magnetic island are derived

- Steady state solutions are obtained numerically:

  (a) The locking solution ($\omega_p = \omega_{ex}$) provides the required ψ_{ex} and $\Delta\phi_0$

  — When the external field is reduced beyond a critical amplitude (unlocked threshold), the locking is lost

  (b) The unlocking solution ($\omega_p \neq \omega_{ex}$) describes how the mode frequency and amplitude are influenced by the external field

  — When the external field is increased beyond a critical amplitude (locking threshold), the tearing mode will then become locked

- The resistive wall introduces an extra phase shift

- Discussion: Is it possible to force a transition from ω_p^L to ω_p^H by applying a rotating external magnetic field?

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