Effects of RF on Tokamak Plasma Near The Axis$^1$
S.C. CHIU, Sunrise R&M, Inc., V.S. CHAN, Y.R. LIN-LIU, Y.A. OMELCHENKO, General Atomics — Radiofrequency (RF) waves provide an attractive source of auxiliary power necessary for a tokamak plasma to reach and sustain reactor conditions. In addition to providing heating and current drive, RF may also affect confinement of the plasma. Thus, understanding the effect of RF on confinement is critical for optimizing RF applications in a reactor plasma. Specifically, RF was found experimentally to affect the toroidal rotations of tokamak plasmas. A conjecture for this effect was that RF induces enhanced radial fluxes due to large orbit effects [C.S. Chang, Bull. Amer. Phys. Soc. 43, 173 (1998)], which in turn induces a radial return current of the bulk plasma. The resulting $J \times B$ torque drives plasma rotation. In the core region of the plasma, many particles have orbits large compared with the minor radius. It is thus interesting to see how the RF affects particle orbits in that region. In this paper, we investigate the analytic behaviour of particle orbits due to RF near the core region. The diffusion of the orbits due to heating is investigated in the quasilinear approximation. Its effect on radial particle currents will be estimated.

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Radiofrequency (RF) waves provide an attractive source of auxiliary power necessary for a tokamak plasma to reach and sustain reactor conditions. In addition to providing heating and current drive, RF may also affect confinement of the plasma. Thus, understanding the effect of RF on confinement is critical for optimizing RF applications in a reactor plasma. Specifically, RF was found experimentally to affect the toroidal rotations of tokamak plasmas. A conjecture for this effect was that RF induces enhanced radial fluxes due to large orbit effects [C.S. Chang, Bull. Amer. Phys. Soc. 43, 173 (1998)], which in turn induces a radial return current of the bulk plasma. The resulting $J \times B$ torque drives plasma rotation. In the core region of the plasma, many particles have orbits large compared with the minor radius. It is thus interesting to see how the RF affects particle orbits in that region. In this paper, we investigate the analytic behaviour of particle orbits due to RF near the core region. The diffusion of the orbits due to heating is investigated in the quasilinear approximation. Its effect on radial particle currents will be estimated.
1  Analysis of orbits near the axis

2  Characteristics of ion cyclotron resonance heating

3  Asymmetrical mechanisms of ICRF

4  Summary and Discussions
MOTIVATION

1. Theory and experiment have indicated plasma rotation is beneficial to stability and confinement.

2. C-MOD have found strong rf driven rotation in the co-current direction even with symmetrical antenna phasing when resonance is located near the axis.

3. Small banana width theory indicates that the rf rotation drive is very small in the symmetrical phasing case. This is because it assumes symmetry in co-and-counter phase space resulting in symmetrical power absorption and radial drift excursion in the co-and-counter directions.

4. There is very little understanding of finite orbit rf effects.
MAIN CONCLUSIONS

1. The splitting of the co-current and counter-current orbits is enhanced in the core region.

2. The trapped-passing boundary in the core region is different from a thin-banana width consideration at moderate energies. For co-current particles, the trapped passing boundary disappears at a critical energy. Counter-current particles are always trapped at sufficiently high energies.

3. Orbit splitting in the core region causes preferential absorption by the co-passing particles and results in inward radial current in the core region, but an outward current in the edge region of the wave zone.

4. At high energies there is no trapped-passing boundary and both the co-and-counter particles travel outward on average as they are heated and a co-current rotation for the heated species (counter for bulk) should be induced at sufficiently high powers.
A. Orbits are well described by constants of motion:

\[ E = \frac{u^2}{2} + \mu B, \quad \mu = \frac{u^2}{2B}, \quad \zeta = \psi - \frac{lu}{\Omega}, \]

where

\[ (B, \Omega) = \frac{(B_0\Omega_0)}{1 + \varepsilon \cos \theta}, \quad I = \pm I_a = RB_T \]

Basic length scale: (\(\delta_1\) = elongation)

\[ R_1 = \frac{\delta l R_0}{2a_s} \]
Parameters of orbits:

(i) Flux surface parameter ($\varepsilon = \text{inverse aspect ratio}$):

$$\varepsilon^2 = \frac{\psi}{I_aR_1}$$

(ii) Invariant energy parameters:

$$\eta_E = \frac{2E}{\Omega_0^2 R_1^2}, \quad \eta_\perp = \frac{2\mu B_0}{\Omega_0^2 R_1^2}, \quad \eta_p = \eta_E - \eta_\perp.$$

We can also define pitch angle $\alpha_p = \eta_p / \eta_E$.

(iii) Angular momentum parameters:

$$\eta_\zeta = \frac{P_\zeta}{I_aR_1}$$
Then orbit is well described by

\[(\varepsilon^2 - \eta_\zeta)^2 = \eta_p + \eta_E \varepsilon \cos \theta\]

Alternatively, this can be written as

\[\eta_\zeta = \varepsilon^2 - \sigma \sqrt{\eta_p + \eta_E \varepsilon \cos \theta}\]

- \(\eta_E, \eta_p, \eta_\zeta\) and \(\sigma\) completely determines the orbits and thus forms the phase space of orbits

- \(\sigma = +1\) is for a co-current moving particle, and \(\sigma = -1\) is for a counter-current moving particle
It can be shown that for a given set of $\eta$'s, a passing co-orbit always encloses a passing counter-orbit.

The orbits of co-and-counter orbits are split, in contrast to thin banana theory, in which they are degenerate in the lowest approximation. This split results in asymmetry in power deposition and subsequent radial excursions.
B Analysis of orbits:

On the equatorial plane, the orbit intercepts are given by

\[
\left(\varepsilon^2 - \eta \zeta\right)^2 = \eta_p + \eta_E \varepsilon
\]

- \( \varepsilon > 0 \) (\(<0\)) represent intercepts at \( \theta = 0 \) (\(= \pi \)). Orbit topology is easily deduced graphically.

An alternative label for \( \eta \zeta \) is intercept at \( \theta = 0 \), i.e., \( \varepsilon_0 \)

\[
\eta \zeta = \varepsilon_0^2 - \sigma \sqrt{\eta_p + \eta_E \varepsilon_0},
\]

then

\[
\left[\left(\varepsilon^2 - \varepsilon_0^2\right)(\varepsilon + \varepsilon_0) + 2\sigma \sqrt{\eta_p + \eta_E \varepsilon_0}(\varepsilon + \varepsilon_0)\right] - \eta_E = 0
\]
(a) $\sigma = +1$ (co-current)

Critical energy $\eta_{EC} = \frac{\delta \varepsilon^3}{27}$. There exists two transitions of orbit topologies for $\eta_E < \eta_{EC}$ as a particle pitch angle is increased above the transition values $\alpha_{pc2}$ and $\alpha_{pc1}$.

$$\alpha_{pc1} = \varepsilon_0 \left( \frac{4 \varepsilon^3 0 \alpha^2_{Tci}}{9 \eta_E} - 1 \right), \quad i = 1, 2$$

$$\alpha_{Tci} = \frac{1}{4} + \frac{1}{2} \sqrt{2 \alpha_E + 1 \cos \left[ \theta_r - (i - 1) \frac{2\pi}{3} \right]}$$

where

$$\theta_r = \tan^{-1} \left( \frac{\sqrt{\alpha_E (4 - \alpha_E)}^3}{2 - 10 \alpha_E - \alpha_E^2} \right).$$
For $\alpha_p < \alpha_{pc2}$, particles are trapped. For $\alpha_{pc1} > \alpha_p > \alpha_{pc2}$, a co-passing orbit encloses a counter-passing orbit. For $\alpha_p > \alpha_{pc1}$ only one co-passing orbit exits

There is a line in the $\alpha_p$-$\eta_E$ plane:

$$\alpha_{p3} = \left( \frac{\sqrt{\eta_E}}{2 \varepsilon_0} - \frac{\varepsilon_2}{2 \sqrt{\eta_E}} \right)^2.$$

For values of $\eta_E$ greater than those on this line, co-passing particles are confined on the low-field side of the magnetic axis

(b) $\sigma = +1$ (counter-current)

There is only one transition of orbit topologies marking the trapped-passing boundary of co-and-counter particles at $\alpha_p = \alpha_{pc4}$ which is given by

$$\alpha_{pc4} = \varepsilon_0 \left( \frac{4 \varepsilon_0^3 \alpha_2 Tc4}{9 \eta_E} - 1 \right)$$
where

\[
\alpha_{Tc4} = \frac{1}{4} \left\{ -1 + \left[ \frac{\alpha_E^2 + 10\alpha_E - 2}{2} + \frac{1}{2} \sqrt{\alpha_E(\alpha_E - 4)^3} \right]^{1/3} + \left[ \frac{\alpha_E^2 + 10\alpha_E - 2}{2} \right] - \frac{1}{2} \sqrt{\alpha_E(\alpha_E - 4)^3} \right\}^{1/3} \text{ for } \alpha_E > 4
\]

\[
= \frac{1}{4} + \frac{1}{2} \sqrt{2\alpha_E + 1} \cos(\theta_{T4}) \text{ for } \alpha_E > 4
\]

\[
\theta_{T4} = \frac{1}{3} \tan^{-1} \left( \frac{\sqrt{\alpha_E(4 - \alpha_E)^3}}{\alpha_E^2 + 10\alpha_E - 2} \right)
\]

- There is an \( \eta_{E4} \) at which \( \alpha_{p\alpha 4} = 1 \). For \( \eta_E \geq \eta_{E4} \), all counter-particles starting from \((\theta=0, \varepsilon=E_0)\) are trapped
## ORDER OF MAGNITUDE

<table>
<thead>
<tr>
<th></th>
<th>DIII–D</th>
<th>C–MOD</th>
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<tr>
<td>$R_0$</td>
<td>1.65 M</td>
<td>0.67 M</td>
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<td>$a$</td>
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<td>0.22 M</td>
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<tr>
<td>$B$</td>
<td>2 T</td>
<td>8 T</td>
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<tr>
<td>$q_s$</td>
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<td>2.1</td>
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<td>$R_1$</td>
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<td>0.16 M</td>
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<td>$\varepsilon_0$</td>
<td>0.12 (20 cm)</td>
<td>0.15 (10 cm)</td>
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<td>$\eta_{\text{Ec}}$</td>
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<td>$1.0 \times 10^{-3}$</td>
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<td>$\rho_{\text{Ec}^2}$</td>
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<td>0.253 cm$^2$</td>
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<tr>
<td>$E_{\text{c (keV)}}$</td>
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<td>~78</td>
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2. CHARACTERISTICS OF ION CYCLOTRON RESONANCE HEATING

A. Direction of heating in phase space

From quasi-linear operator

\[ Q_{RF}(F_0) \approx \frac{1}{J_z} \left[ \left( \frac{\Omega_0}{\omega B_0} \frac{\partial}{\partial \mu} + \frac{\partial}{\partial E} \right) J_z D_{RF} \left( \frac{\Omega_0}{\omega B_0} \frac{\partial}{\partial \mu} + \frac{\partial}{\partial E} \right) F_0 \right] \]

assume \( \Omega_0 \approx \omega \), then resonant kicks are such that

\[ \Delta E = \Delta (\mu B_0) \]

\[ \Rightarrow \]

\[ \Delta (\eta_E - \eta_\perp) = \Delta \eta_p = 0 \]

So when resonance is at center, \( \eta_p \) is constant
When \( \omega - \Omega_0 = \Delta \omega \Omega_0 \),

\[
\Delta \eta_p = \Delta \omega \Delta \eta_E
\]

Also \( P_\zeta \) or \( \eta_\zeta \) is roughly converved by ICRF kicks.

**B  Resonance location:**

\[
\omega - \Omega - k_\phi \, u_\phi = 0, \quad k_\phi = \frac{n}{R}, \quad \omega = \Omega_0 (1 + \Delta \omega),
\]

then resonance location is given by

\[
\varepsilon_R \cos \theta_R = \frac{n^2}{\delta q_s^2} \eta_E - \Delta \omega + \frac{\eta_\sigma \sqrt{\eta_E}}{4 q_s} \sqrt{4 \alpha_p + \frac{\eta_\sigma^2 \eta_E}{4 q_s^2} - 4 \Delta \omega}
\]
3. ASYMMETRICAL MECHANISMS OF ICRF

A Direction of rotational drive

• Rotation is driven by radial current according to \[ \Delta \left( \frac{I_u}{\Omega} \right) = \Delta \psi \]

• Inward radial current of heated species causes a counter rotation drive (bulk reaction is in opposite direction)

• In addition to the continuous \( \Delta \psi \) of heated particles in the trapped and passing regions, there is a large sudden \( \Delta \psi \) as particles are energized across the trapped-passing boundary. This \( \Delta \psi \) is inward for co-current particles and outward for counter-current particles

• Orbit splitting in the core region causes preferential absorption by the co-passing particles and results in inward radial current in the core region, but an outward current in the edge region of the wave zone

• At high energies there is no trapped-passing boundary and both the co-and-counter particles travel outward on average as they are heated and a co-current rotation for the heated species (counter for bulk) should be induced at sufficiently high powers
B  Asymmetry due to orbit splitting

- Whenever a counter-passing particle is heated, there is a balancing absorption by its co-passing counterpart; however, a co-passing orbit can be heated without the counter-passing counterpart being heated. This asymmetry of absorption is enhanced in the core region because the splitting is larger as the core is reached. The induced radial current is thus inward in the core region.

- At the edge of the wave zone, the counter-passing particles can be heated without a balancing absorption by its co-passing counterpart; thus radial current is outward near the edge region of the rf wave zone.

C  Asymmetry at higher energies

- At high energies, there exist no trapped-passing boundary; however both the co-and-counter particles travel radially outward on average as heated. Thus, the induced radial current should be outward at high powers.
SUMMARY AND DISCUSSION

1. The splitting of the co-current and counter-current orbits is enhanced in the core region.

2. The trapped-passing boundary in the core region is different from a thin-banana width consideration at moderate energies. For co-current particles, the trapped passing boundary disappears at a critical energy. Counter current particles are always trapped sufficiently high energies.

3. In addition to the continuous $\Delta \psi$ of heated particles in the trapped and passing regions, there is a large sudden $\Delta \psi$ as particles are energized across the trapped-passing boundary. This $\Delta \psi$ is inward for co-current particles and outward for counter-current particles.

4. Orbit splitting in the core region causes preferential absorption by the co-passing particles and results in inward radial current in the core region, but an outward current in the edge region of the wave zone.

5. At high energies there is no trapped-passing boundary and both the co-and-counter particles travel outward on average as they are heated and a co-current rotation for the heated species (counter for bulk) should be induced at sufficiently high powers.