# THEORY AND SIMULATION OF ROTATIONAL SHEAR STABILIZATION OF TURBULENCE

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**GENERAL ATOMICS**

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# WHAT THIS TALK IS ABOUT

- **This talk is about the quench rule for rotational shear stabilization of turbulence in tokamaks: when the ExB shear rate** $\gamma_E=(r/q)\partial(qV_{ExB}/r)/\partial r$ **exceeds the maximum local ballooning mode growth rate** $\gamma_{max}$ **turbulent diffusion is quenched.**

- **This rule is the basis for transport models** $\chi=(1-\gamma_E/\gamma_{max})\chi_0$ **describing core transport barriers** (Waltz, Staebler, Dorland, Hammett, Kotschenreuther & Konings 1997)

- **This talk is not about the experimental fidelity of the rule. For this see**
  - **Burrell (APS review talk 1996 );**
  - **Synakowski et al & Greenfield et al (APS invited papers 1996 )**
  - **Lao et al (APS invited paper 1995)**

- **This talk is about the numerical and theoretical evidence and limitations for the rule.**

# BACKGROUND TO ANALYTIC THEORIES

**ExB shear rate : $\gamma_E=(r/q)\partial(qV_{ExB}/r)/\partial r$** **complex ballooning mode growth rate: $\gamma_0$**

| | **nonlinear 2 point renormalization** | **linear eigenmode stability** |
|---|---|---|
| $\rho_* \rightarrow 0$<br><br>**$V_{ExB} >> V_*$**<br>**EXB shear only**<br><br>**x-variation only**<br>***Im* $(d\gamma_0/dx) = k_y \gamma_E$** | Biglari, Diamond, &Terry 1990<br>Shaing, Crume,& Houlberg 1990<br>Zhang & Mahajan 1992<br>• turbulence suppression<br>• $\gamma_{E}$_norm $\approx\Delta\omega(\Delta k_x/\Delta k_y)$<br>$\Delta\omega\equiv D\Delta k_x^2$<br>• no distinction slab / torus | Connor,Taylor,&Wilson 1993<br><br>• toroidal process at smallest $\gamma_E$ reduces eigenmode rate to small ballooning mode angle average rate<br><br>• transport follows eigenmode rate resulting in slab-like levels |
| $\rho_*$ **finite**<br>**$V_{ExB} = O(V_*)$**<br>**x-variation**<br>**$(d\gamma_0/dx) = \gamma_0'$**<br>profile shear<br>**$x^2$-variation**<br>**$(d^2\gamma_0\, dx^2) = \gamma_0''$**<br>profile curvature | | Rominelli & Zonca 1993<br>Dewar 1996<br><br>• profile curvature *Re*$(\gamma_0'')$ works against profile shear stabilization *Im*$(\gamma_0')$ |

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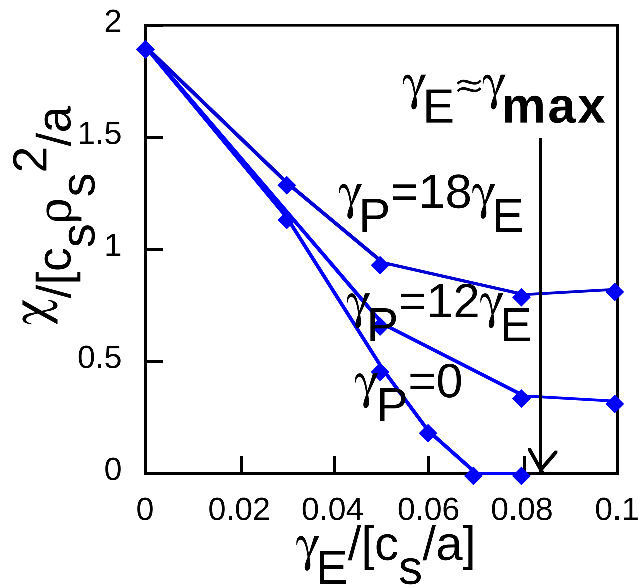
# PREVIOUS NONLINEAR SIMULATIONS

ExB shear rate : $\gamma_E = (r/q)\partial(qV_{ExB}/r)/\partial r$ complex ballooning mode growth rate: $\gamma_0$

| | gyrofluid ITG ballooning mode or "flux tube" code | |
|---|---|---|
| $\rho_* \to 0$<br><br>$V_{ExB} >> V_*$<br>EXB shear only<br><br>x-variation only<br><br>$Im\,(d\gamma_0/dx) = k_y\,\gamma_E$ | Waltz, Kerbel, & Milovich 1994<br>• turbulence quenched<br>• $\gamma_{E\_crit} \approx \gamma_{max} = max\ \{Re\,(\gamma_0)\}$<br><br>• $\chi = (1 - \gamma_E/\gamma_{max})\chi_0$ basis of models for core transport barriers | • Get quench for fixed $\gamma_P$ but for pure toroidal rotation, parallel shear rate $\gamma_P = (Rq/r)\gamma_E$ can increase $\gamma_{max}$ ($\gamma_P$) faster than $\gamma_E$ avoiding quench.<br><br>• Superficial resemblance to Biglari, Diamond,&Terry rule since $\Delta\omega$ ($\Delta k_x/\Delta k_y$) tracks $\gamma_{max}$ for isotropic turbulence<br><br>• Appears not to follow eigenmode stability by Connor, Taylor,& Wilson rule since eigenmodes are stable at very small $\gamma_E$ here. |

# OUTLINE TO NEW WORK

- Review of the ExB shear in ballooning mode representations
  - Coupling in ballooning angle $\theta_0$ versus convection in ballooning angle $\theta_0$
  - How a new mode centered Floquet ballooning representation avoids numerical "box mode" instabilities.
- Gyrofluid ITG ballooning mode or $\rho_* \to 0$ "flux tube" numerical illustrations showing how the $\gamma_{E\_crit} \approx \gamma_{max}$ quench rule results from a nonlinear convective amplification process unique to toroidal geometry.
- Likely modifications and limitations on the $\gamma_{E\_crit} \approx \gamma_{max}$ quench rule for finite $\rho_*$ and general profile shear and profile curvature: lessons from the ballooning-Schrodinger eigenmode equation.

# • BALLOONING MODE AND REPRESENTATIONS

$dF/dt = \partial F/\partial t + \tilde{v}_{\mathbf{ExB}} \bullet \nabla_\perp F$ ExB nonlinear coupling

• x'-space radial direction $V_{\mathbf{ExB}} = \gamma_{\mathbf{E}}\, x'$ $\gamma_{\mathbf{E}} = (r/q)\partial(qV_{ExB}/r)/\partial r$

$dF/dt = -(\gamma_E\, x')\, i\, k_y F \quad + \quad L\, F$ $L$ linear operations

• transform $k_x$'-space or ballooning mode -space $F(\theta,\theta_0)$

$x'f = -i\ \partial F/\partial k_x'$

$k_x' \equiv k_y \hat{s}\theta_0$ $k_y \equiv nq / r$ $\hat{s} \equiv (r/q)(d\,q/dr)$

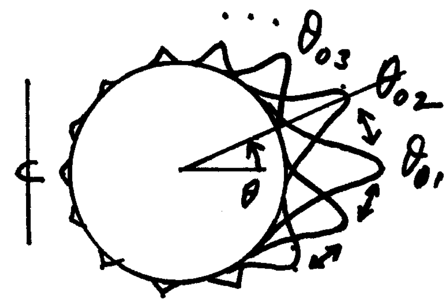
$dF(\theta,\theta_0)/dt = -(\gamma_{\mathbf{E}}/\hat{s})\, \partial F(\theta,\theta_0)/\partial\theta_0 + L[\theta - \theta_0, \cos(\theta), \partial/\partial\theta]\, F(\theta,\theta_0)$

_each $F(\theta,\theta_0)$ centered at $\theta \approx \theta_0$

_For finite $\gamma_{\mathbf{E}}$, $\theta_0$ is no longer a "good quantum number"

_ExB shear $\gamma_{\mathbf{E}}$ linearly "couples" $\theta_0$ 's

_$\theta_0$ discrete: $\partial F/\partial\theta_0 \approx \Delta F/\Delta\theta_0$

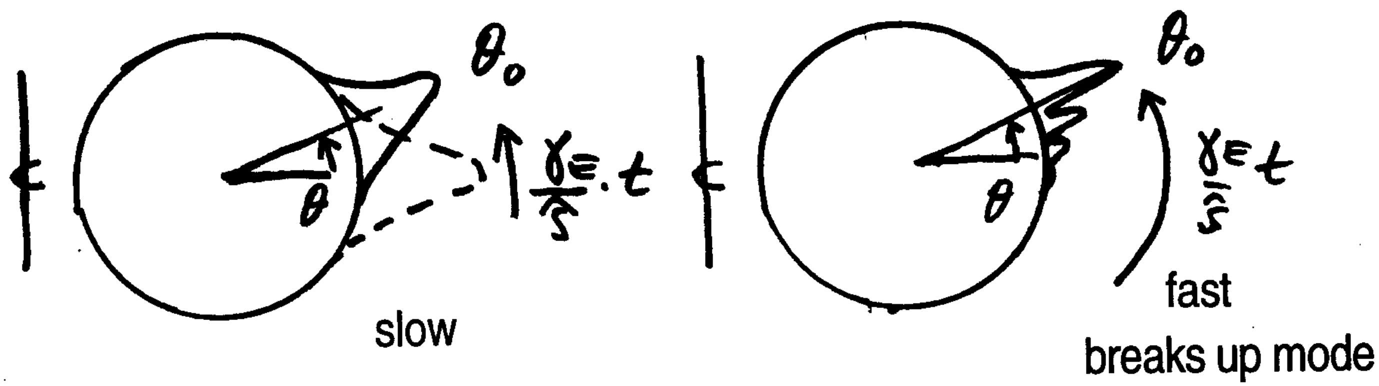


# • BALLOONING MODE AND REPRESENTATIONS continued

• Cooper transformation $\theta_0 \rightarrow \theta_0 + (\gamma_E/\hat{s})t$ "Floquet ballooning mode" $F'(\theta,\theta_0)$

$$dF'(\theta,\theta_0)/dt = L\,[\theta - (\theta_0 + (\gamma_E/\hat{s})t),\ \cos(\theta),\ \partial/\partial\theta\,]\ F'(\theta,\theta_0)$$

_ExB shear $\gamma_E$ linearly convects each $\theta_0$




_Easy to see Connor,Taylor, & Wilson rule for eigenmode or time average growth rate at small $\gamma_E/\hat{s}$ when modes not distorted or broken up:

$$\langle\gamma\rangle_0 = \oint \gamma_0(\theta_0)\,d\theta_0 / 2\pi << \gamma_0(0)$$

# • BALLOONING MODE AND REPRESENTATIONS continued

• Rotating frame transformation $\theta \to \theta + (\gamma_E/s\hat{})t$ "centered Floquet ballooning mode" $F''(\theta,\theta_0)$

$$dF''(\theta,\theta_0)/dt = (\gamma_E/s\hat{})\, \partial F''(\theta,\theta_0)/\partial\theta + L[\theta-\theta_0, \cos(\theta+(\gamma_E/s\hat{})t), \partial/\partial\theta]\, F''(\theta,\theta_0)$$

_Explicitly displays "poloidal breakup term " which stabilizes modes in the slab when cos( ) dependence is dropped or stabilizes toroidal modes at large $\gamma_E/s\hat{}$ even when $\langle\gamma\rangle_0 = \oint \gamma_0(\theta_0)d\theta_0/2\pi > 0$.

_Leaves nonlinear terms invariant, modes stay centered at $\theta \approx \theta_0$ and do not rotate out of the finite $\theta$ numerical box

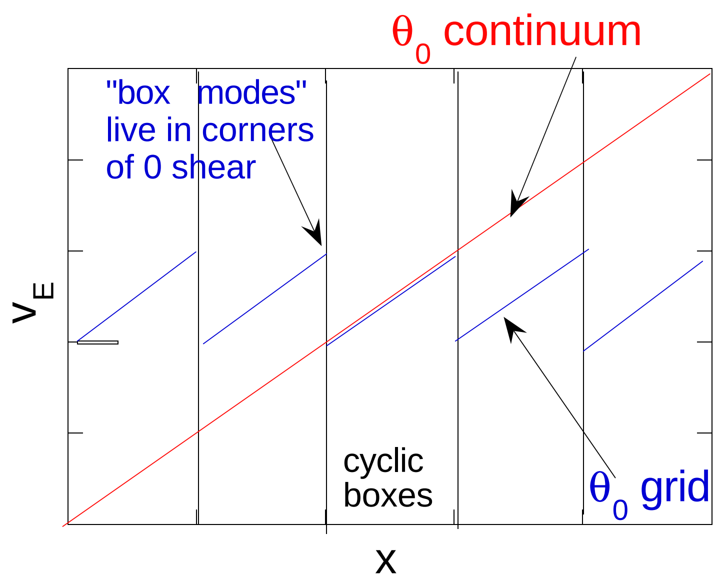
# • Why the centered Floquet ballooning mode representation is best for numerical simulations

_All represenations are equivalent if $\theta_0$ is a continuous variable !

_But to do ( $k_y$, $k_x$') nonlinear coupling  we must treat [$k_x'=k_y s\hat{}\theta_0$] $\theta_0$ as a discreet variable.

Discreteness in $\theta_0$ means we have a "cyclic box" in x-space

Two "visions" of "homogeneous" ExB shear.



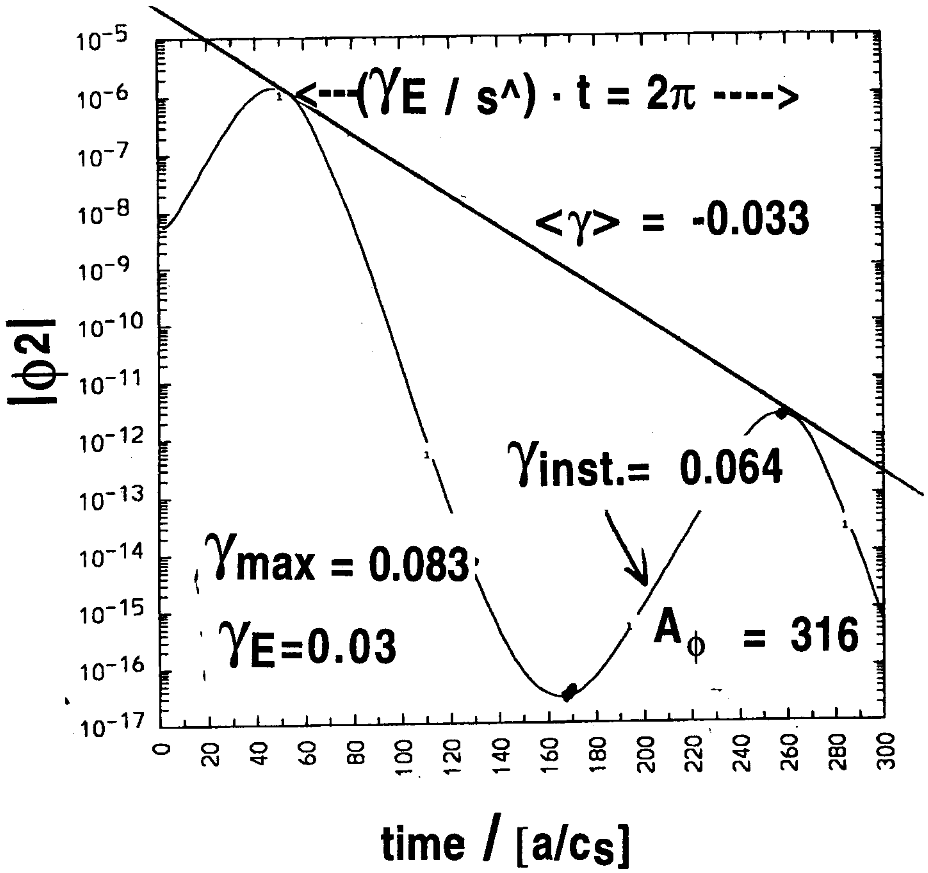

_Miller and Waltz (1994) showed that discrete $\theta_0$ could lead to "box modes" which require a finite $\gamma_E$ to stabilize, even when "true" continuum eigenmodes were stable

$$\langle\gamma\rangle_0 = \oint \gamma_0(\theta_0)\, d\theta_0 / 2\pi < 0$$

_The quench rule $\gamma_{E\_crit} \approx \gamma_{max}$ in the discrete $\theta_0$ nonlinear ballooning mode code was later found to be associated with linear stabilization of "box modes".

## • Numerical illustrations with new centered Floquet ballooning mode representation which avoids numerical "box modes"

## but still recovers approximate quench rule $\gamma_{E\_crit} \approx \gamma_{max}$




- ITG with adiabatic electrons

$R / a = 3$, $a/L_n=1$, $a/L_T=3$, $q=2$, $\hat{s}=1$, $\alpha=0$, $\gamma_P=0$

- At $\gamma_E=0.03$, least stable mode $k_y\rho_s=0.3$ time average growth rate or eigenmode rate same as ballooning mode angle average rate

$$\langle\gamma\rangle_0 = \oint \gamma_0(\theta_0) d\theta_0 / 2\pi = -.033$$

which is negative.

- Convective amplification of 316x



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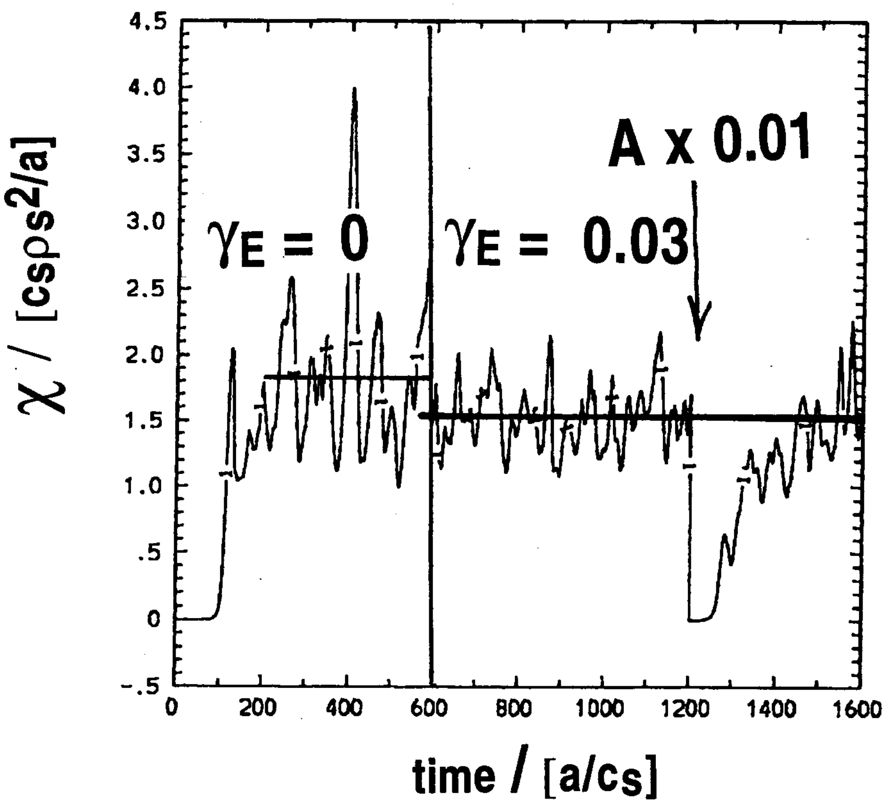
## • At small $\gamma_E$ turbulence persists from convective amplification and nonlinear coupling **even with stable eigenmodes.**

ITG with adiabatic electrons




- At $\gamma_E$=0.03 diffusion is only slightly suppressed.
- After multiplying all amplitudes by 0.01X, the turbulence is able recover but it does not recover after 0.0001X.
- Implies that Floquet modes passing bad curvature region are convectively amplified and nonlinearly couple energy to convectively decaying modes passing good curvature region preventing eigenmode decay.



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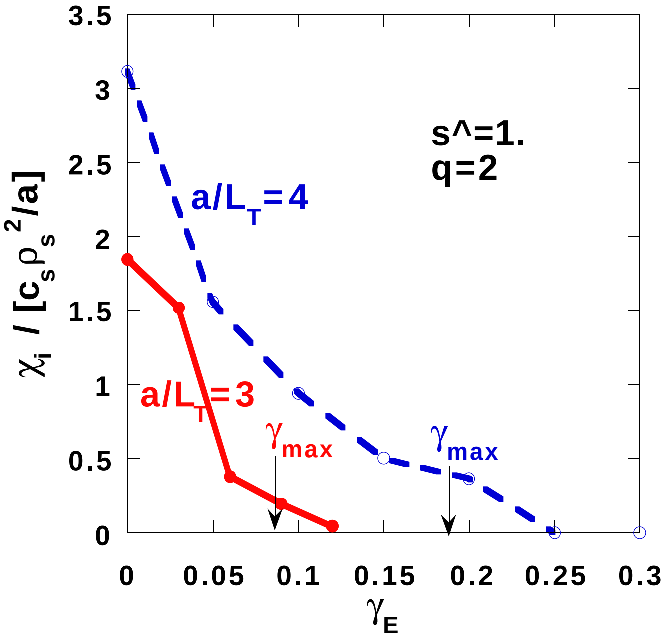
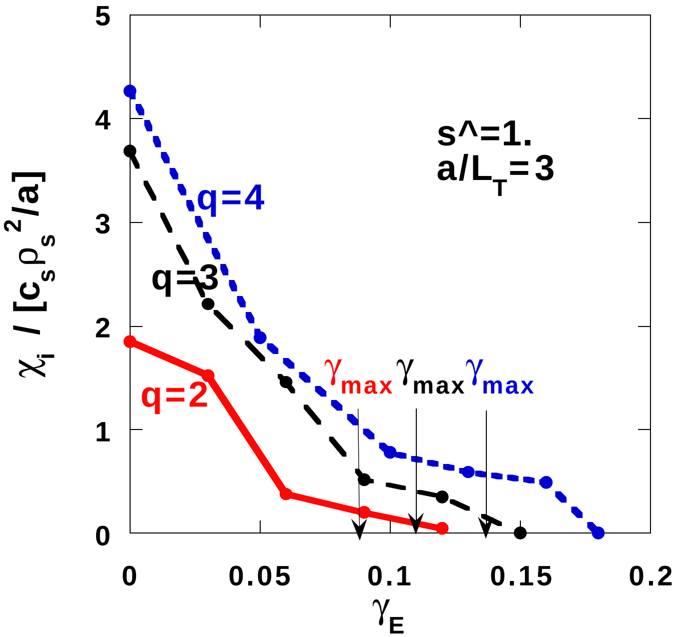
## • At higher shear rates turbulence quenched at near $\gamma_{E\_crit} \approx \gamma_{max}$

R / a = 3, a/Lη=1, s^=1, α=0, $\gamma_P$=0

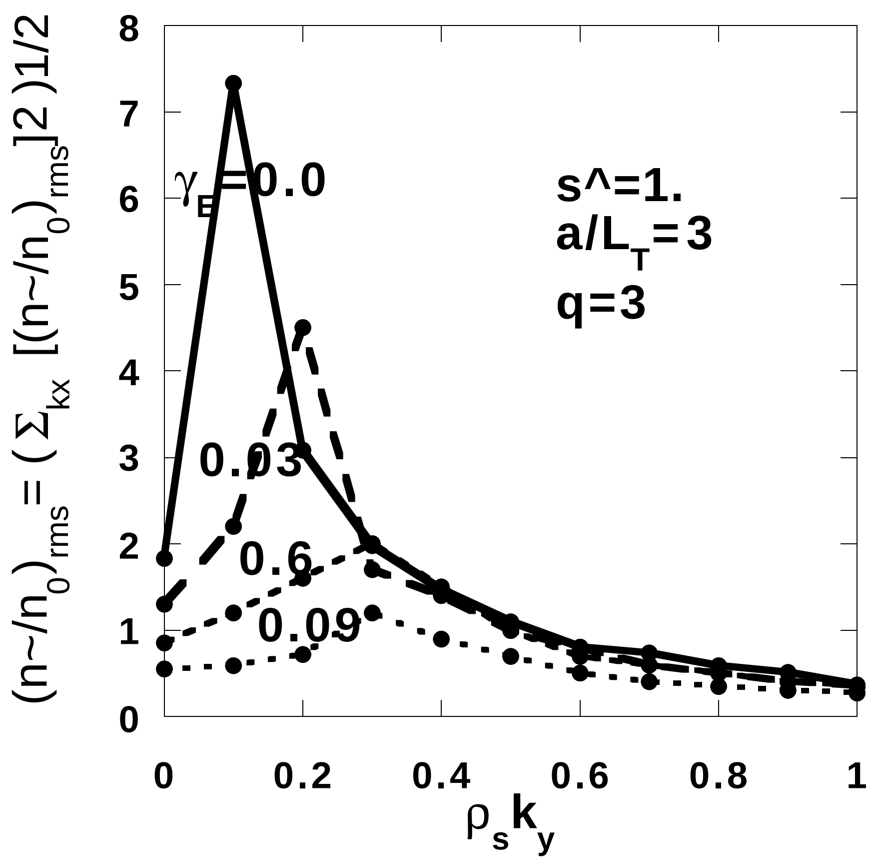


## • $k_y\rho_s$-spectrum as a function of $\gamma_E$

R / a = 3, $a/L_n$=1, $a/L_T$=3, q=2, s^=1, $\alpha$=0, $\gamma_P$=0

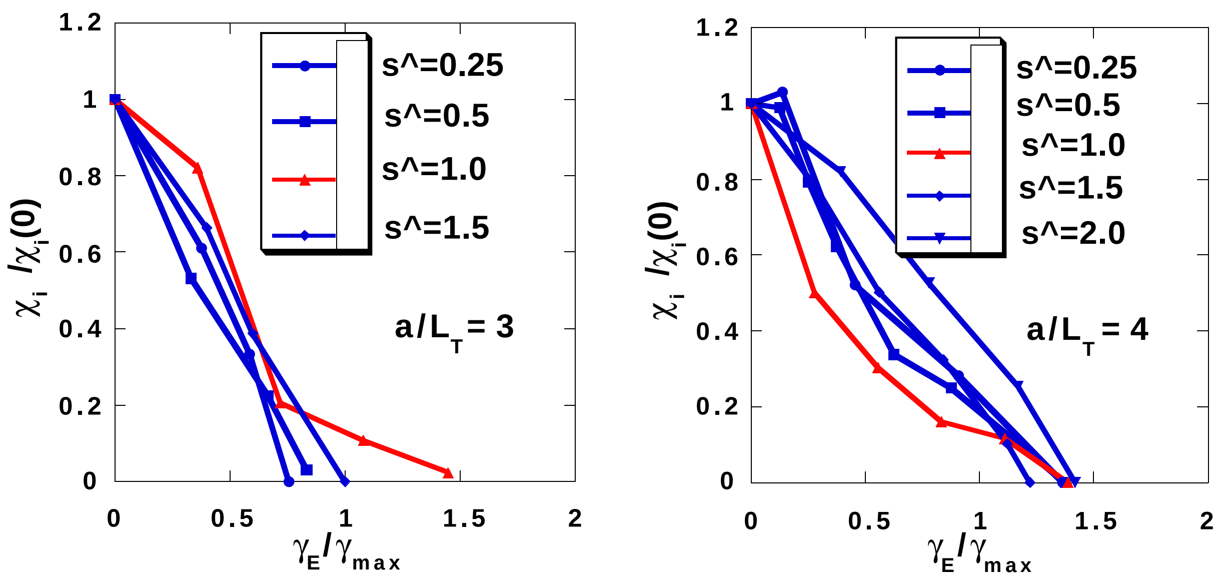
• Depletion at low-ky suggests local rule $\gamma_{E\_crit} \approx \gamma$ (ky) may be justified.



# • Quench rule $\gamma_{E\_crit} \approx \gamma_{max}$ persists at low s^ = (r/q ) (d q/ dr)

- **Even though the Floquet $\theta_0$ rotation rate is $\gamma_E$/s^,**

  **there is no evidence that the critical $\gamma_E \propto$ s^ $\gamma_{max}$.**
- **Hence no evidence here that s^=0 is the "seat" of core reversed shear transport barrier.**




- **$(\gamma_E$ /s^)/$\gamma_{max}$ scaling does not give a good invariance with s^.**



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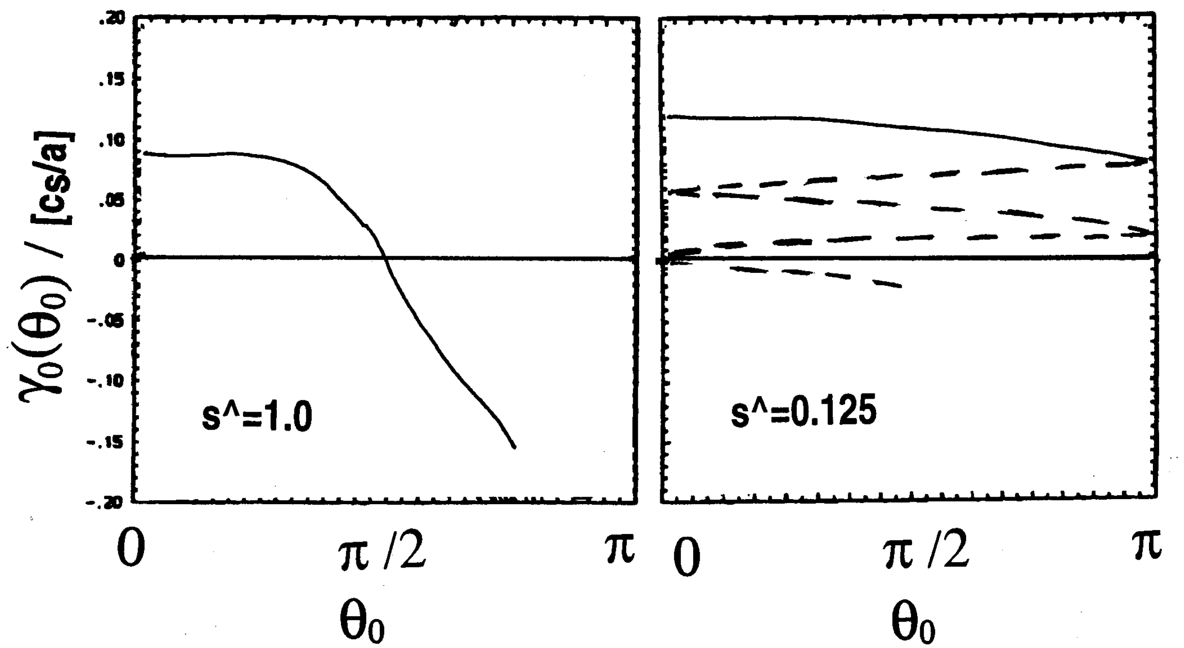
- **At low s^ convection speed $\gamma_E$/s^ goes up, but most unstable mode at $\theta_0 = 0$ must convect 1/s^ times farther in $\theta_0$ to reach stability.**




$\Delta\theta_0 \approx \pi/2$ $\Delta\theta_0 \approx 4\pi$

- Heuristic argument: Balancing growth time with convection time implies

$$\gamma_{E\_crit} \approx s\text{^}\Delta\theta_0 \ \gamma_{max} \approx O(1)\ \gamma_{max}$$



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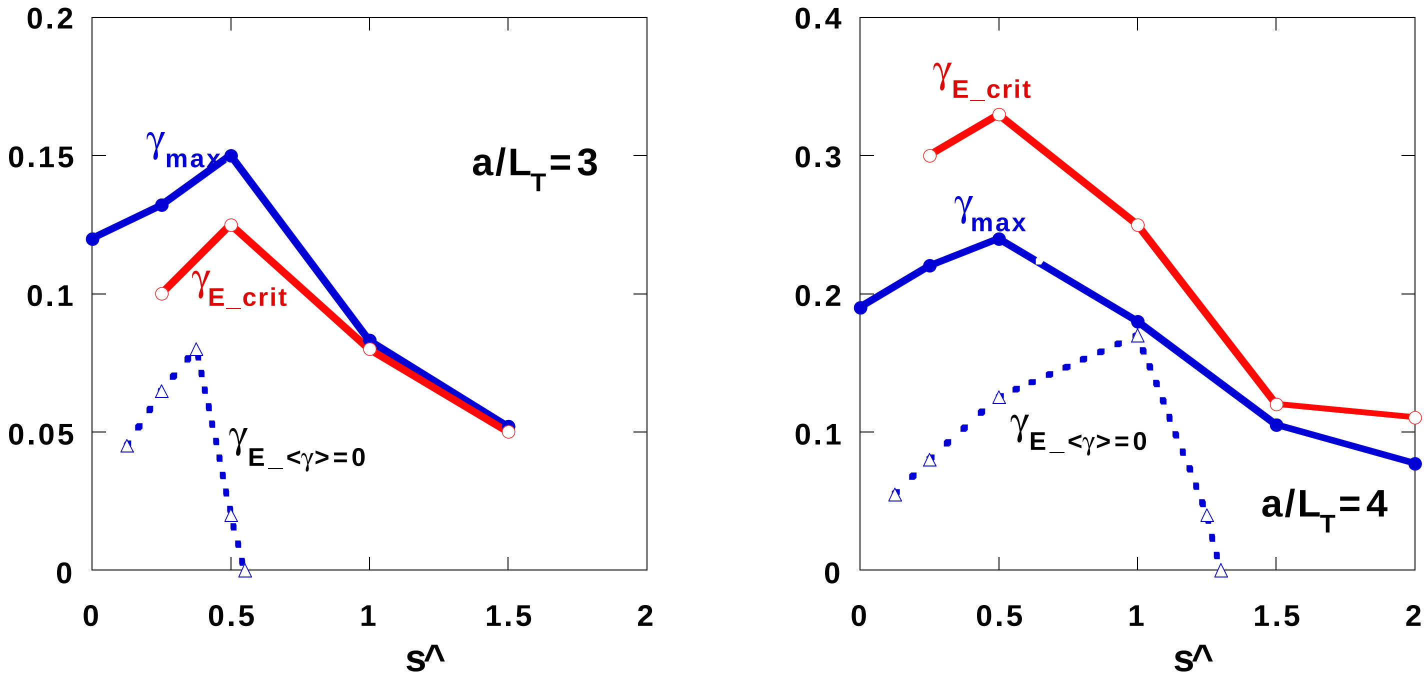
## • $\gamma_{E\_crit}$ does not follow eigenmode stability in toroidal geometry

R / a = 3, $a/L_n$=1, q=2, $\alpha$=0, $\gamma_P$=0

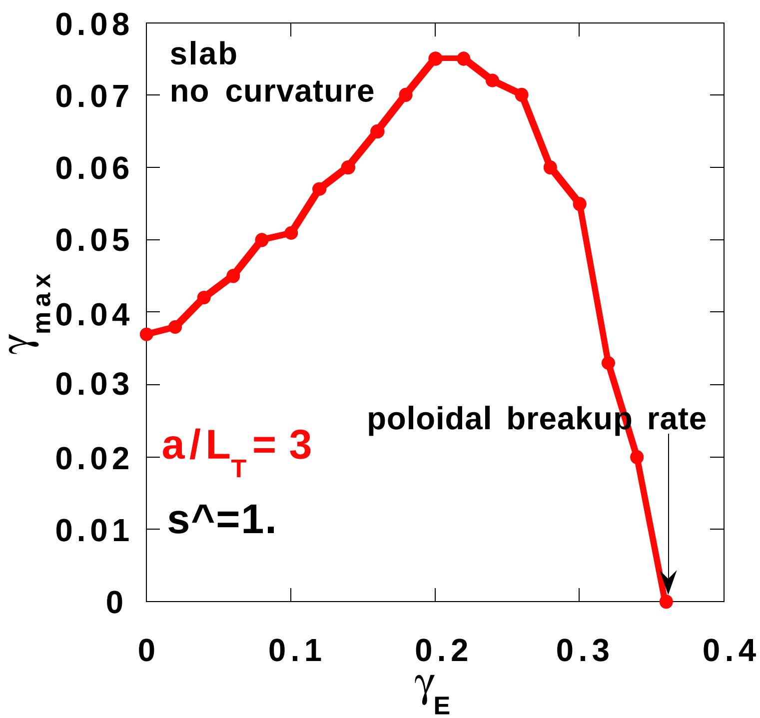
$\gamma_{E\_<\gamma>}$ is the ExB shear rate for eigenmode stability $\langle\gamma\rangle$ =0.

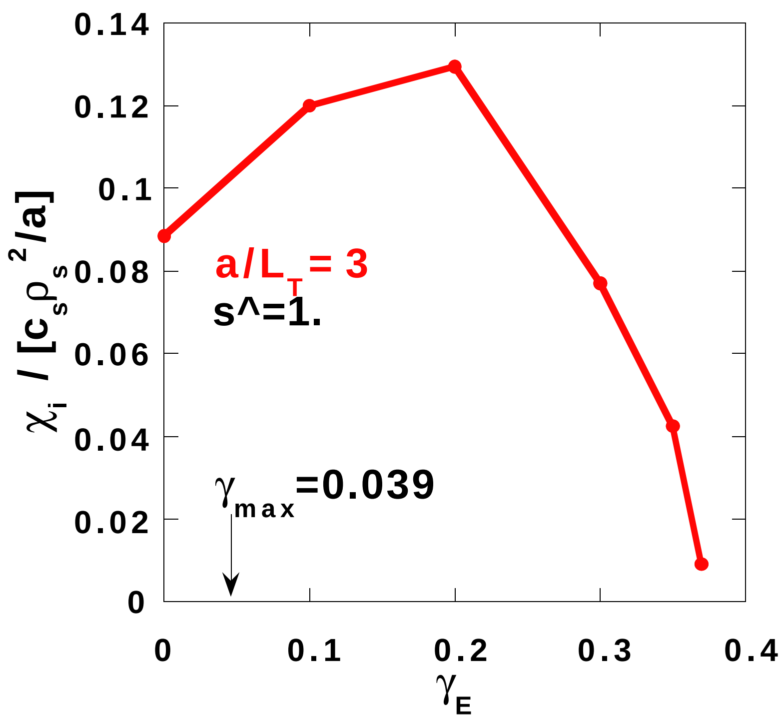
Due to the poloidal breakup effect, actual rate $\langle\gamma\rangle \le \langle\gamma\rangle_0$ Connor,Taylor,& Wilson rate



## • $\gamma_{E\_crit}$ does follow eigenmode stability in slab geometry $\gamma_{E\_crit}$ >> $\gamma_{max}$

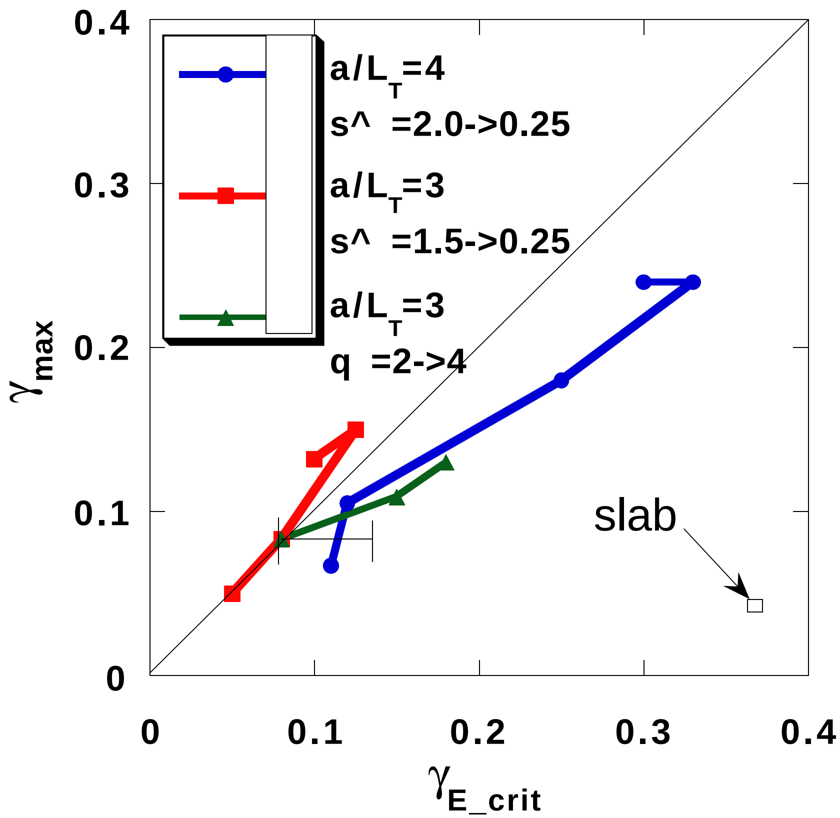
R / a = 3, a/Lη=1, a/LT=3, q=2, s^=1, α=0, γP=0   toroidal curvature terms "turned off"



# • Summary of ITG simulation cases for quench rule

$R / a = 3$, $a/L_\eta=1$, $\alpha=0$, $\gamma_p=0$




• In summary, for the vanishing ρ∗ limit, it appears that $\gamma_{E\_crit} \approx 4/3\ \gamma_{max}$ is the best approximate description of the critical ExB shear rate for quenching transport in toroidal geometry.



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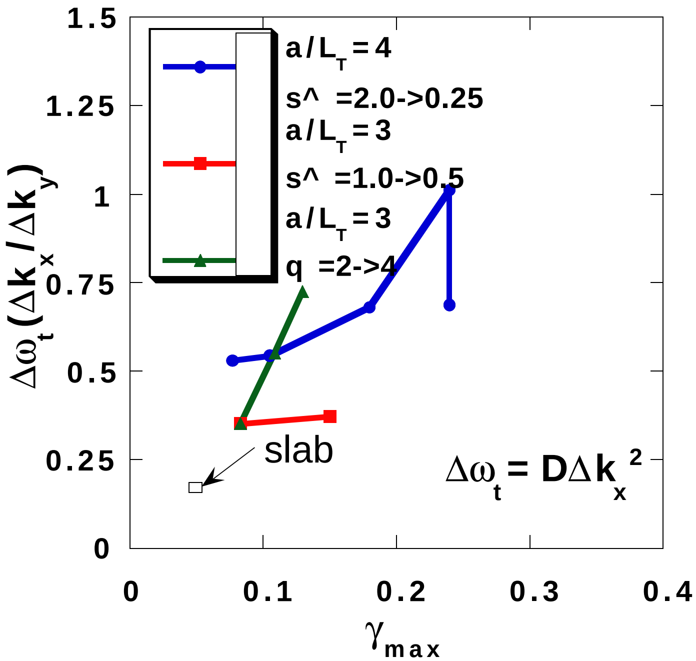
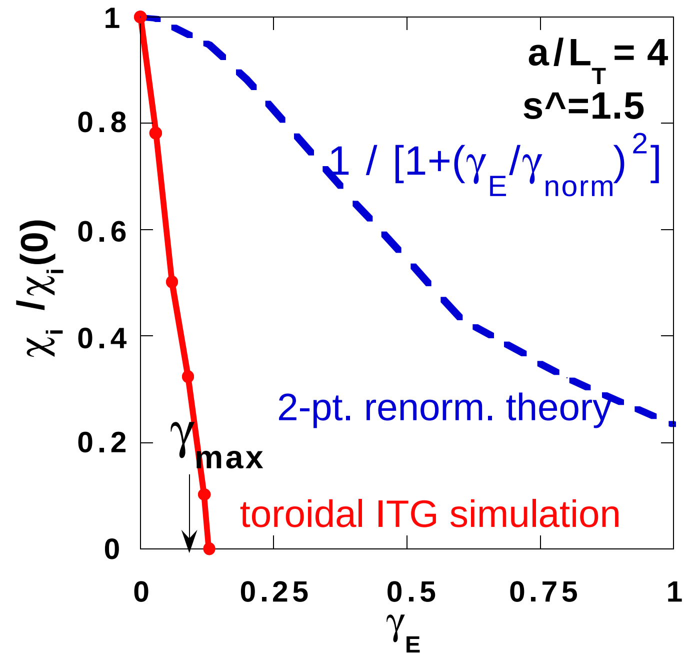
# • Quantitative test of 2-point nonlinear renormalization theories

_Theory: nonvanishing suppression factor $\chi/\chi_0 = 1/[1 + (\gamma_E/\gamma_{E\_norm})^2]$

$\gamma_{E\_norm} \approx \Delta\omega_t\ (\Delta k_x/\Delta k_y)$ $\Delta\omega_t = D\Delta k_x^2$ used interchangeably




• $\Delta\omega_t\ (\Delta k_x/\Delta k_y)$ does tracks $\gamma_{max}$ using $D = \chi$

• But supression formula gives less than 10% reduction at the actual quench point.

_Most importantly, this ($\rho*$->0) theory does not account for the difference between simulations in slab and toroidal geometry.

- **Likely modifications and limitations on the $\gamma_{E\_crit} \approx \gamma_{max}$ quench rule : lessons from the ballooning-Schrodinger eigenmode equation.**

**complex ballooning mode growth rate: $\gamma_0(x,\theta_0)$ expand in $x=r-r_0$**

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$\rho_*$ finite

$V_{ExB} = O\ (V_*)$

x-variation

$(d\gamma_0/dx) = \gamma_0'$

profile shear

$Im(d\gamma_0/dx) = k_y\ \gamma_{mode}$

$x^2$-variation

$(d^2\gamma_0\ dx^2) = \gamma_0''$

profile curvature

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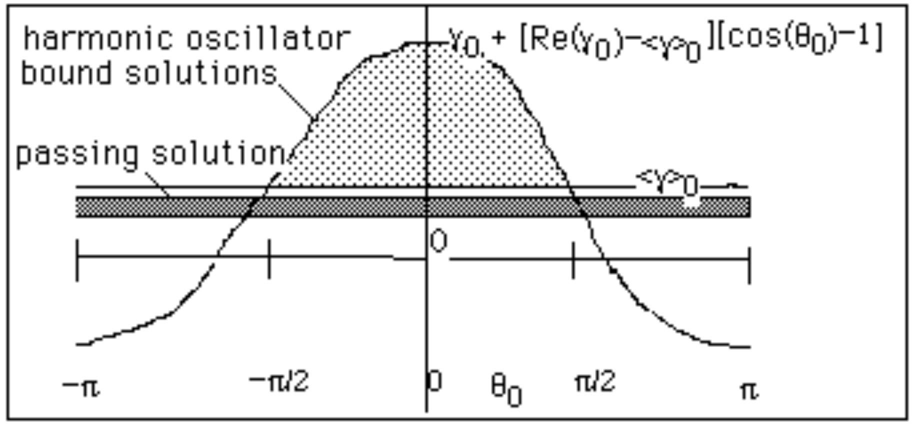


$\gamma = \gamma_0 + \gamma_0'\ x + \gamma_0''/2\ x^2 + [Re(\gamma_0) - \langle\gamma\rangle_0][\cos(\theta_0) - 1]$

$x \leftrightarrow -i\,\Delta_{ss}\,\partial/\partial\theta_0 \qquad \theta_0 \leftrightarrow i\,\Delta_{ss}\,\partial/\partial x$

- generalize $\gamma_E \longrightarrow \gamma_{mode} = (r/q)\partial(qV_{mode}/r)/\partial r$ to include shear in intrinsic mode phase velocity not just ExB Doppler shift

$\Rightarrow\ \gamma_{mode\_crit} \approx \gamma_{max}$

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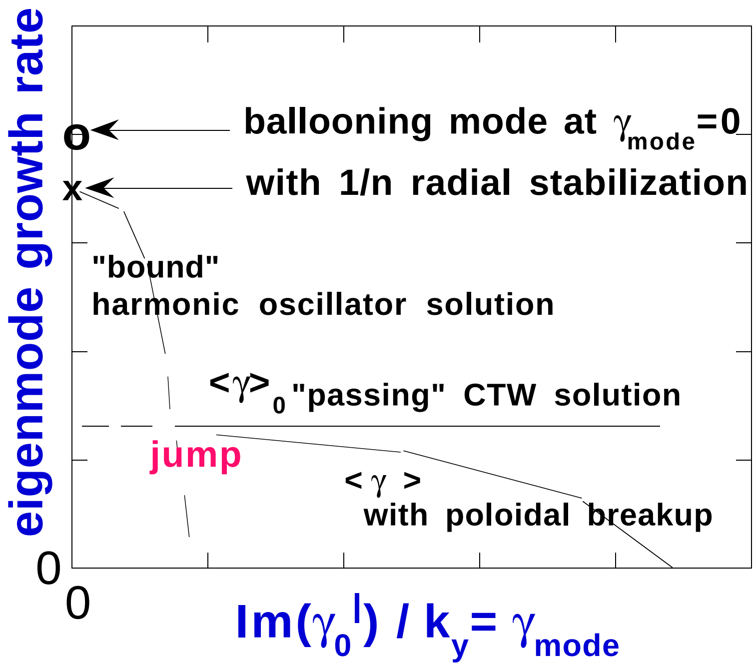
● Likely modifications and limitations on the $\gamma_{E\_crit} \approx \gamma_{max}$ quench rule : lessons from the ballooning-Schrodinger eigenmode equation....continued




$\gamma_{bound} = \gamma_0 - 1/2\,(-\gamma_0'' \Delta_{ss}^2 \gamma_0)^{1/2} - 1/2(\gamma_0'^2/\gamma_0'')$

$\gamma_{pass} = \langle\gamma\rangle_0 = \oint \gamma_0(\theta_0) d\theta_0 / 2\pi$

- $Re\ (\gamma) = max\,\{Re\ (\gamma_{bound}), Re\ (\gamma_{pass})\}$

- 1/n "radial correction" is small
- Ballooning-Schrodinger equation does not contain poloidal breakup
- Jump point likely criterion for minimum profile shear $\gamma_0'$ or $\gamma_{mode}$ to ignore profile curvature $\gamma_0''$ which prevents mode circulation and convective amplifcation:

$$Re\ [1/2\,(\gamma_0'^2/\gamma_0'')] > Re\ [\gamma_0 - \langle\gamma\rangle_0]$$

otherwise quench rule may not hold



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# CONCLUSIONS

- From ITG adiabatic electron simulations the quench rule for rotational shear stabilization $\gamma_E = (r/q)\partial(qV_{ExB}/r)/\partial r \approx \gamma_{max}$ results from a nonlinear convective amplification process unique to toroidal geometry. The rule holds over a wide parameter range including low s^= (r/q ) (d q/dr) at vanishing $\rho*$

  - ExB shear stabilization does not follow eigenmode stability in toroidal geometry but does follow eigenmode stability in slab geometry.

- From the ballooning-Schrodinger eigenmode equation at finite $\rho*$ we speculate:
  - The rule should be modified to include the total mode velocity not just the EXB Dopper shift.

  - The quench rule is likely to hold only if the profile shear $\gamma_0' = d\gamma_0/dx$ is large enough to overcome profile curvature $\gamma_0'' = d^2\gamma_0/dx^2$ which prevents full circulation in ballooning mode angle and convective amplifcation.

- Studies with 3D full radius nonlinear simulations are needed at finite but small $\rho*$