Comparisons of Linear and Nonlinear Plasma Response Models for Non-axisymmetric Perturbations

by
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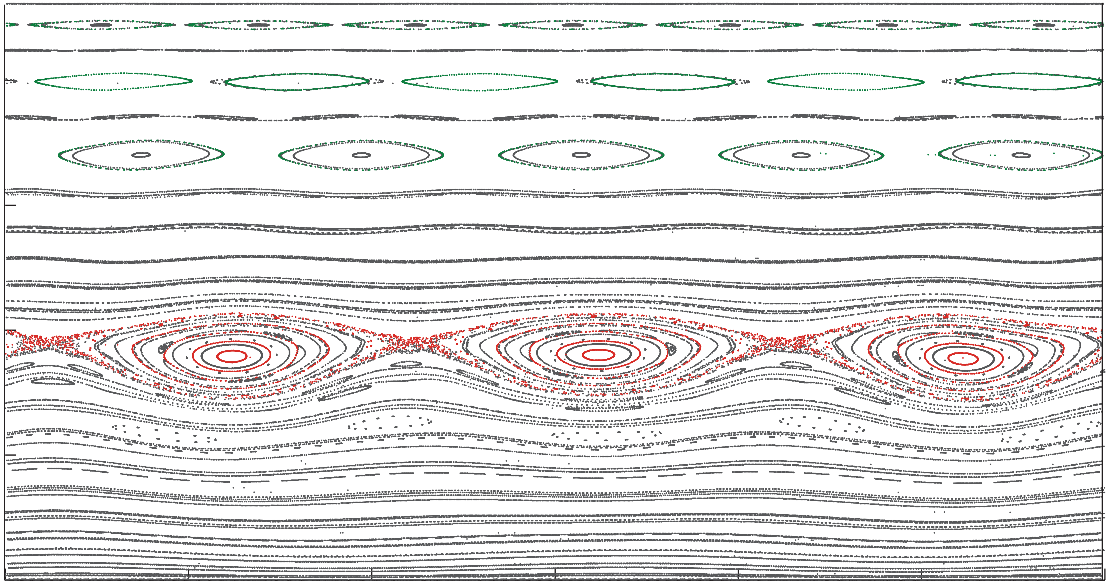


Presented at the
54th Annual APS Meeting
Division of Plasma Physics
Providence, Rhode Island

October 29 — November 2, 2012

Acknowledgements:
A. Reiman, S. Lazerson (PPPL)
S. Hirshman (ORNL)

Understanding Plasma Response to Non-Axisymmetric Perturbations is a Vital Area of Fusion Research

- **Plasma response to 3-D perturbations is a major focus of the experimental tokamak program**
  - ELM suppression from internal non-axisymmetric coils (DIII-D I-coils)

- **Plasma response is a key ingredient in determining the consequences of non-axisymmetric perturbations**

Plasma can amplify, suppress or otherwise modify perturbation!

First attempt at comparing and documenting applicability of the predicted detailed internal response from different approaches

- **Main goal is to identify the issues limiting each approach**
  - Which is right and when is it right (i.e. the experimental plasma response)
    - Answer depends on conditions

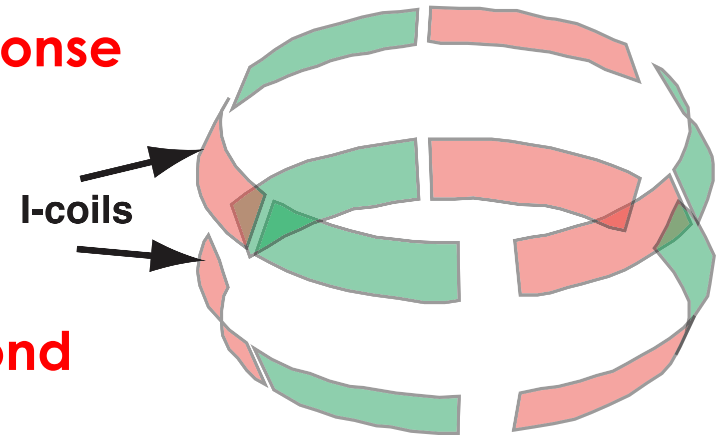
There Are Two Key But Interrelated Responses to an External Non-axisymmetric Field

- **Equilibrium response**
  - Magnetic geometry
    - Flux surface displacement
    - Changes in topology
  - Changes in profiles responding to force balance

MHD Response

- **Transport response due to topology and equilibrium changes**
  - Changes in profiles from changed local transport

Beyond MHD



- **Transport response can be thought of as part of nonlinear response**

In principle, this can all be captured within an Extended MHD framework

Final state is a new MHD force balance equilibrium

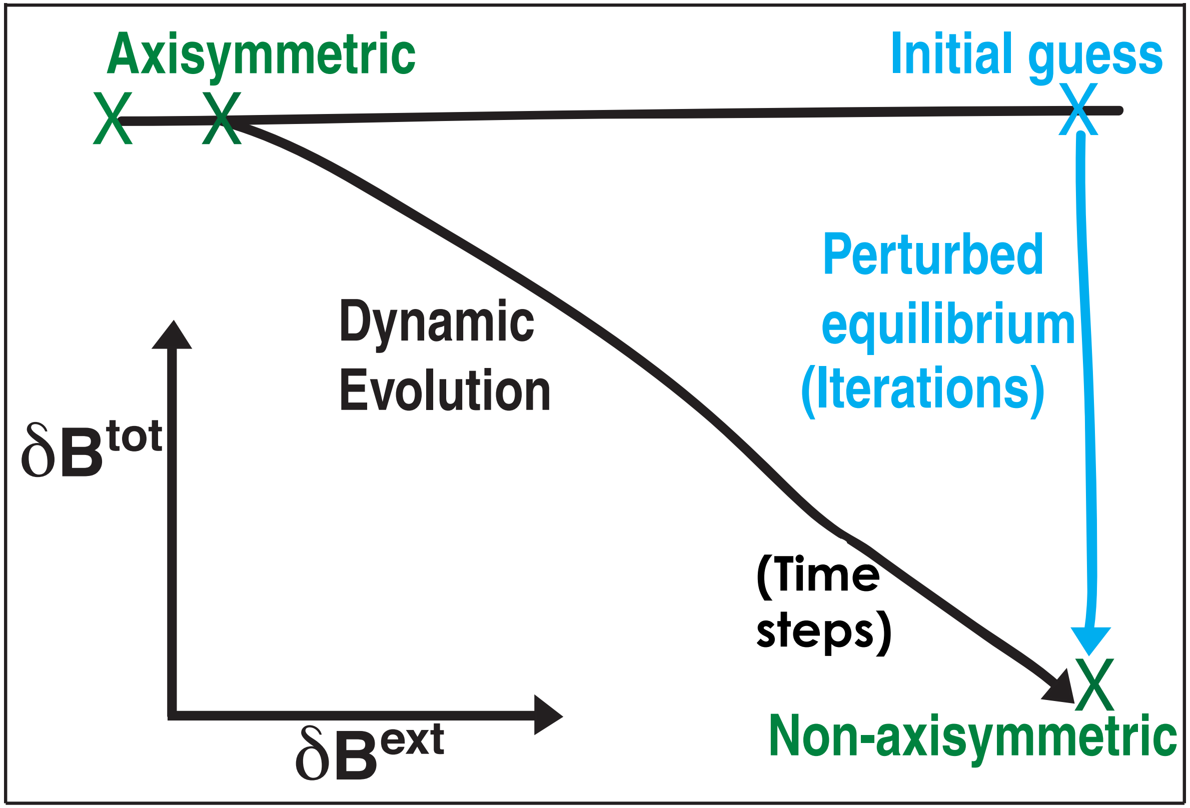
Four Conventional Approaches Are Traditionally Used to Find the Steady State Plasma Response

Dynamic Evolution

- Follow time evolution to determine final nonlinear saturated state using Extended MHD stability code

Perturbed Equilibrium

- Find the nearby stable non-axisymmetric equilibrium



- Both viewpoints hold for both linear and nonlinear formulations

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Each Approach Has Relative Advantages

	Linear:	Nonlinear:
Dynamic evolution viewpoint:	Forced eigenvalue (MARS-F, M3D-C[1]) Fast turn around Valid only for sufficiently small response	3D Extended MHD stability (NIMROD, M3D, M3D-C[1]) Requires complete physics and realistic parameters Time consuming
Nearby equilibrium viewpoint:	Basis expansion (IPEC) Fast turn around Valid only for sufficiently small response	Nearby equilibrium (VMEC, PIES, HINT, SPEC, SIESTA) Faster turn around Computed state may not be physically accessed

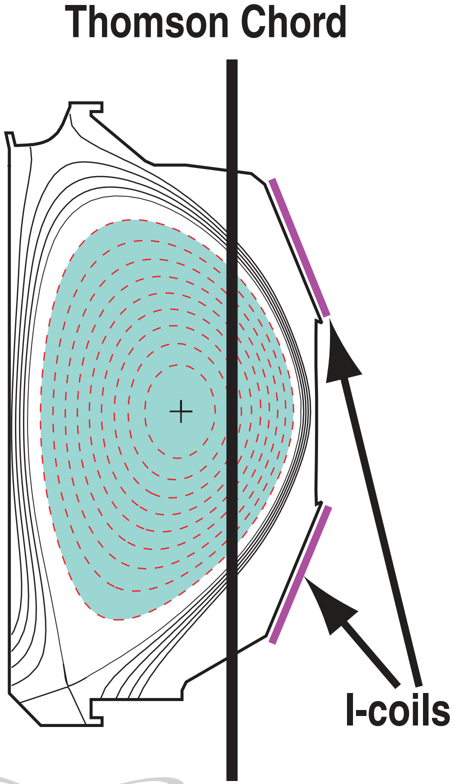
- **Each approach has significant past success in predicting external magnetic response**

Different Approaches Bring Complementary Insights for Predicting Detailed Internal Responses

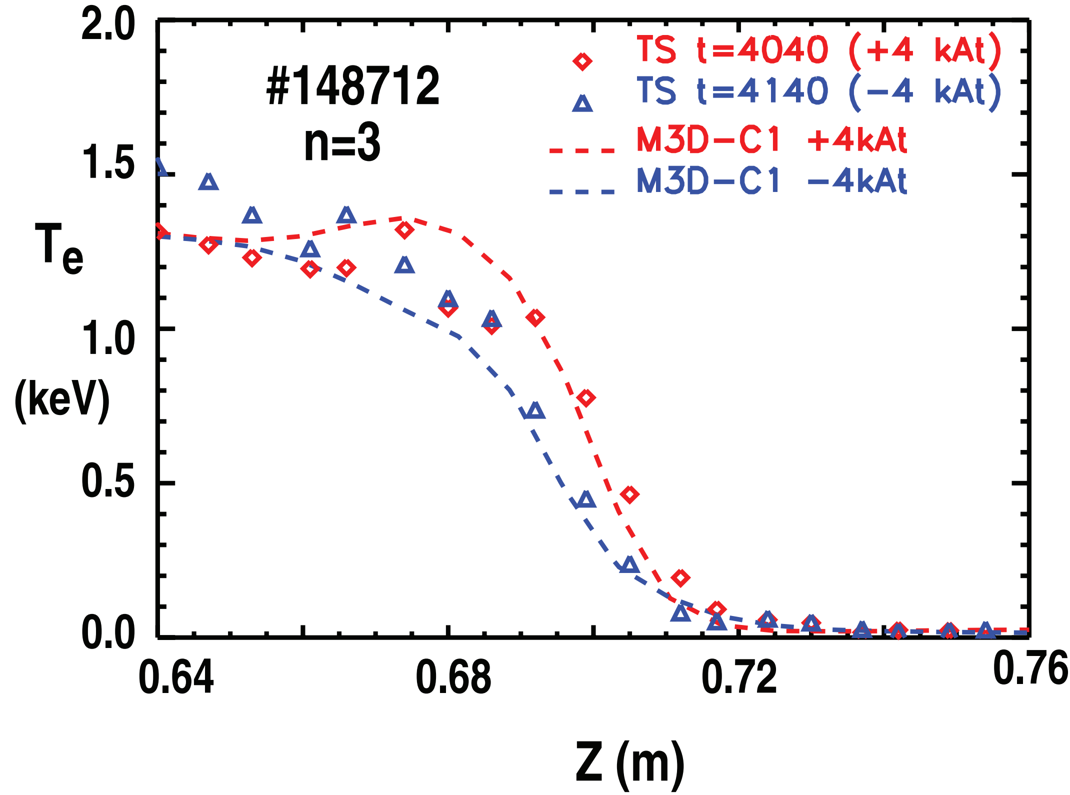
- **All four approaches have yielded past successes**
- **Comparison of predictions of internal plasma response to I-coil perturbations in DIII-D**
- **Resolution of discrepancies**
- **Magnetic helicity as a constraint**
- **Conclusions**

M3D-C¹ Predicts Observed Pedestal Temperature and Oscillation of Edge Thomson Location With I-coil Phasing

- **Edge location inferred from Thomson oscillates in phase with n = 3 I-coil current in DIII-D**



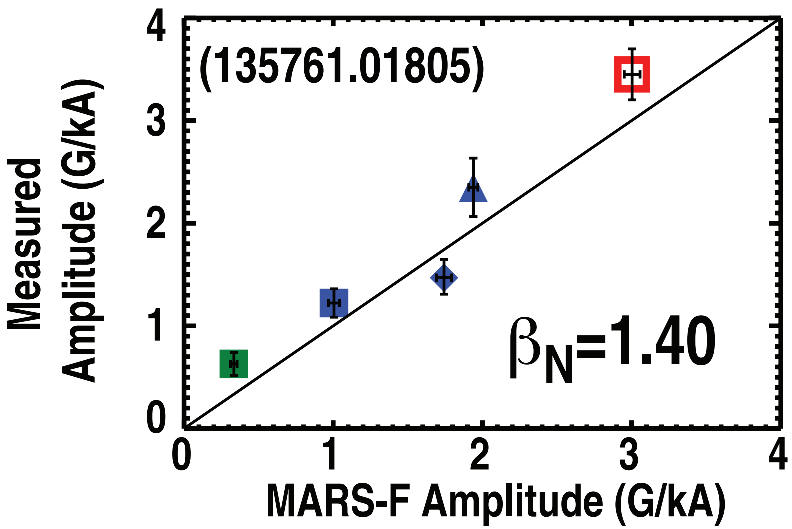
- **M3D-C¹ includes linear plasma response**

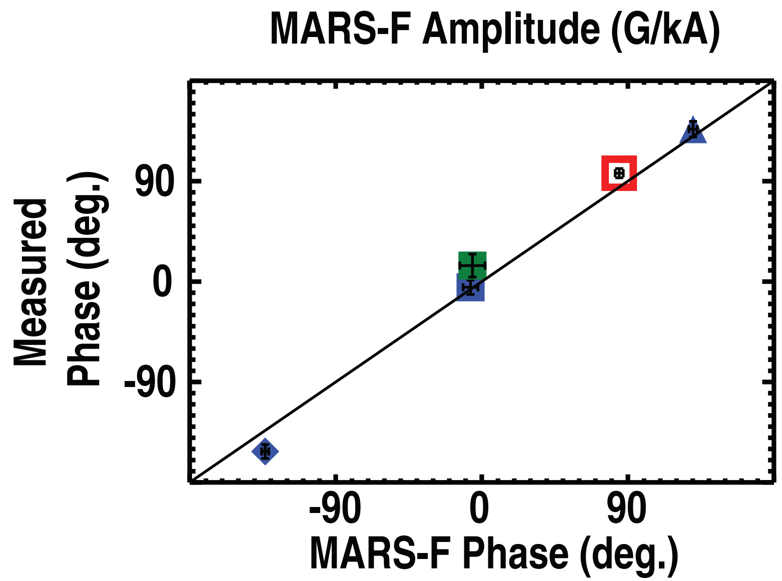


(Ferraro IAEA 2012, this meeting)

MARS-F Calculations of Ideal Plasma Response Agree With Measured Response in DIII-D at Sufficiently Low β

- **Amplitude and phase agree for β_N < 1.8**



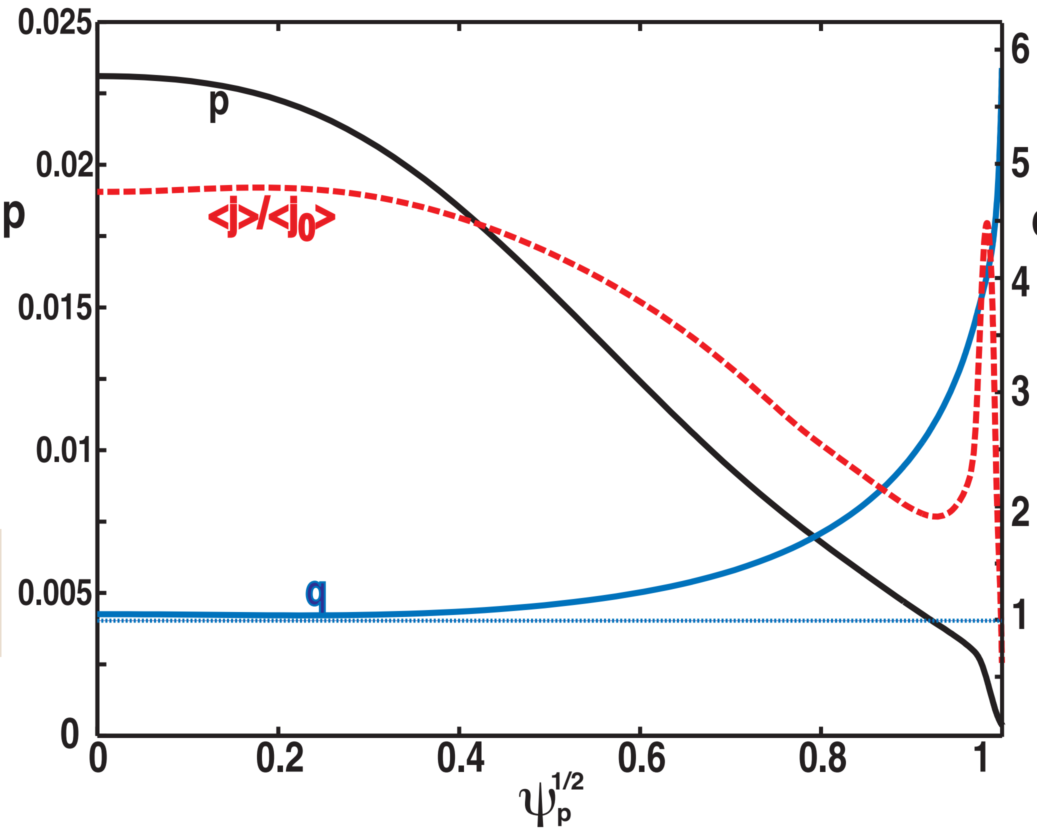


(Lanctot
Phys. Plasmas 2011)

- **Disagreement for β_N > 1.8**
  - Ideal model over-estimates response just below no wall limit
  - No agreement above no-wall limit

DIII-D Discharge #142603 Provides Well Documented Case for Comparing Response Predictions

- **DIII-D discharge #142603 at 3519 ms**
  - Up-down symmetric 2-D configuration
  - Applied internal I-coil (even) n = 3 field:
    - $\delta B^{ext} \sim 10^{-3} B\phi$



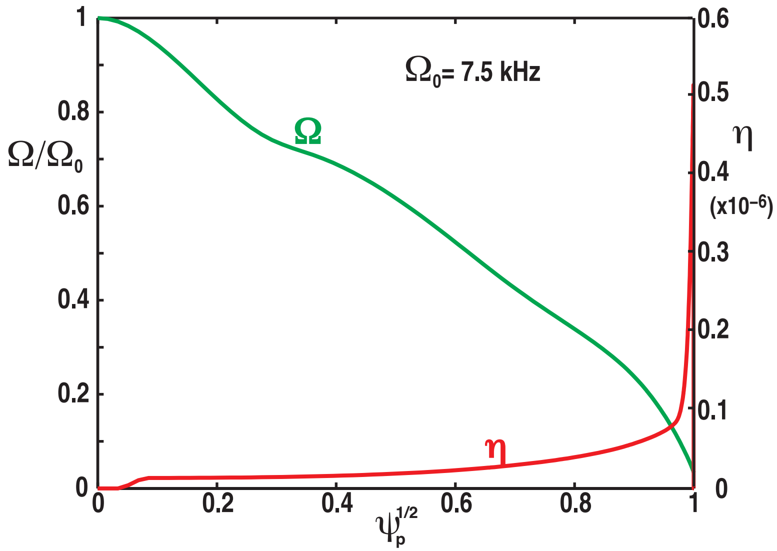
- **How well can the different approaches predict detailed response?**
  - ⇒ **Compare the predictions against each other**

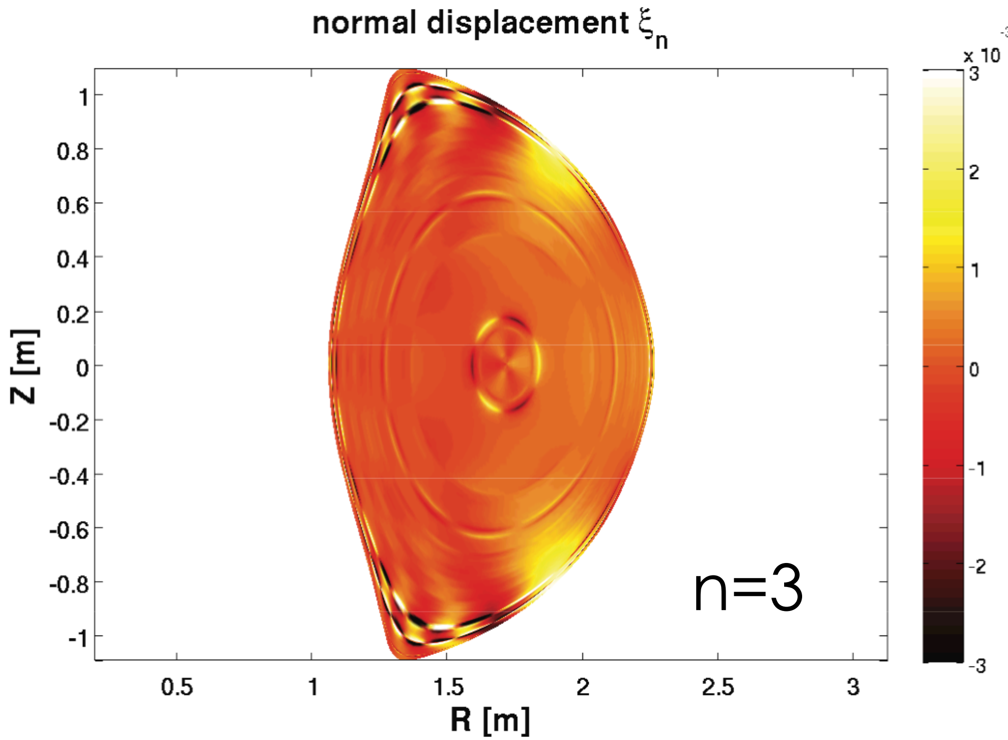
Linear Response Calculated for DIII-D Discharge #142603 Including Plasma Rotation Using MARS-F

- **MARS-F is a linear eigenvalue code modified to find the response due to an inhomogeneous forcing function representing an external field**

$$\rho\ddot{\xi} + L\xi = \omega^2\xi + L\xi = 0 \quad \rightarrow \quad \omega_0^2\xi + L\xi = F$$

- **Includes rotation and resistivity profiles**





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MARS-F, M3D-C¹ and IPEC Predict Qualitatively Similar Linear Responses With Significant Inboard Oscillations

- **Oscillations follow**

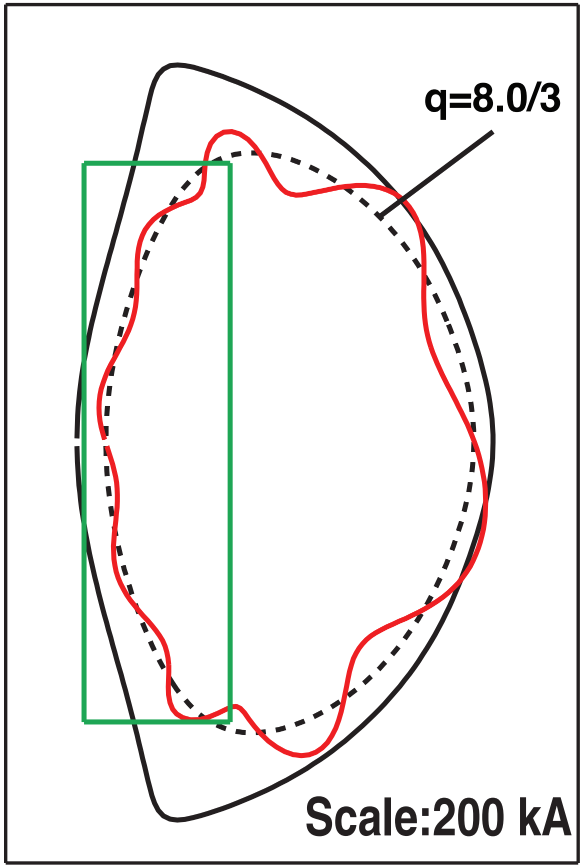
$$\xi \sim \xi_0 e^{im\chi_{pol}}$$

with $m \sim nq - 1$

oscillations down the inboard side and one or two large oscillations on outboard side

(Kink-like response)

- **MARS-F**



Scaled displacements factor 56

Nonlinear VMEC Response is Significantly Different from Ideal Linear Response Especially on the Inboard Side

- **Equilibrium calculation with non-axisymmetric I-coil fields**
- **Profiles taken from reconstructed 2-D equilibrium**

n=3

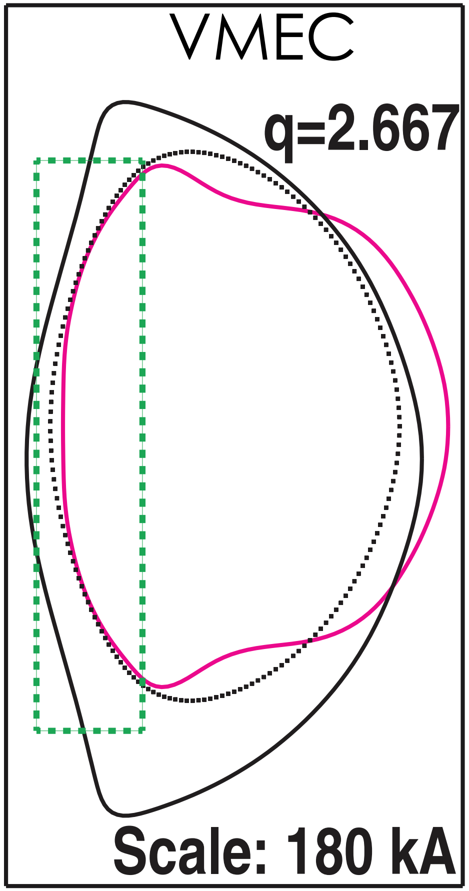
- **Inboard response is quite different from linear ideal response predictions**
- **Oscillations do not follow**

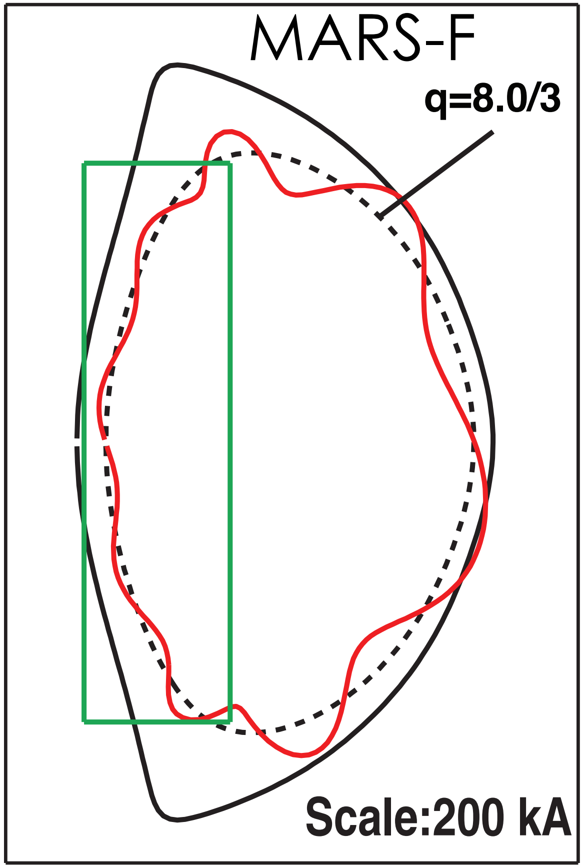
$$\xi \sim \xi_0 e^{im\chi_{pol}}$$

(Non kink-like response)

- **Similar disagreement for non-resonant surfaces**

Scaled displacements





Possible Sources for Discrepancy Can be Identified in Each Approach

Linear Dynamic Approach

- **Response depends on what physics is included in dynamic evolution**
  - Response is sensitive to marginal "near internal" eigenmodes
- **Linear model can break down for finite perturbations**
  - Response can be large even when applied external field is small

Nonlinear Dynamic Evolution Approach

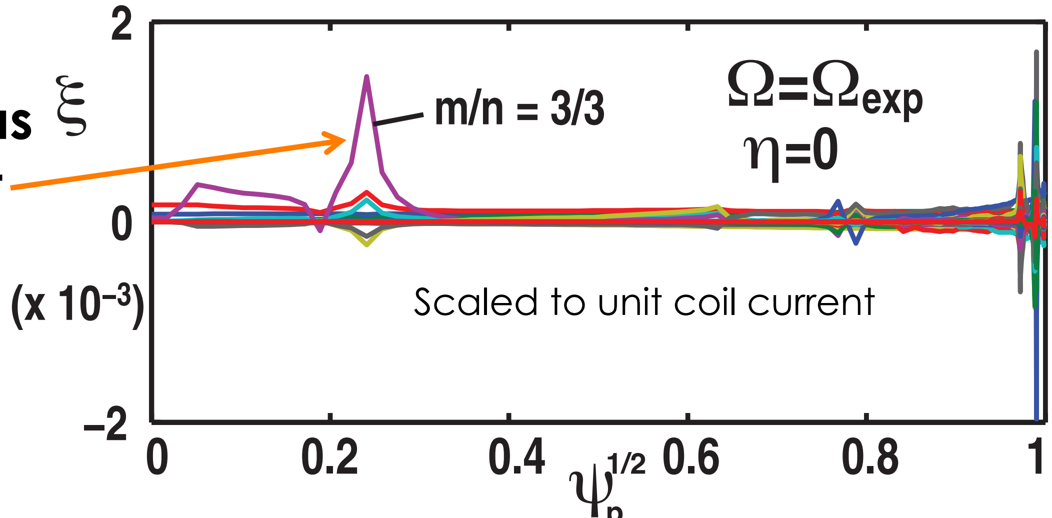
- **Physics required to obtain saturated state is case dependent**

Nonlinear Perturbed Equilibrium Approach

- **Convergence issues arise for resolving singular currents or islands**
- **Equilibrium code can find the "wrong" equilibrium**
  - Constraints imposed to define new equilibrium may be inappropriate

Linear Dynamic — Small Boundary Distortions Can Excite Near-Internal Modes That Dominate the Response

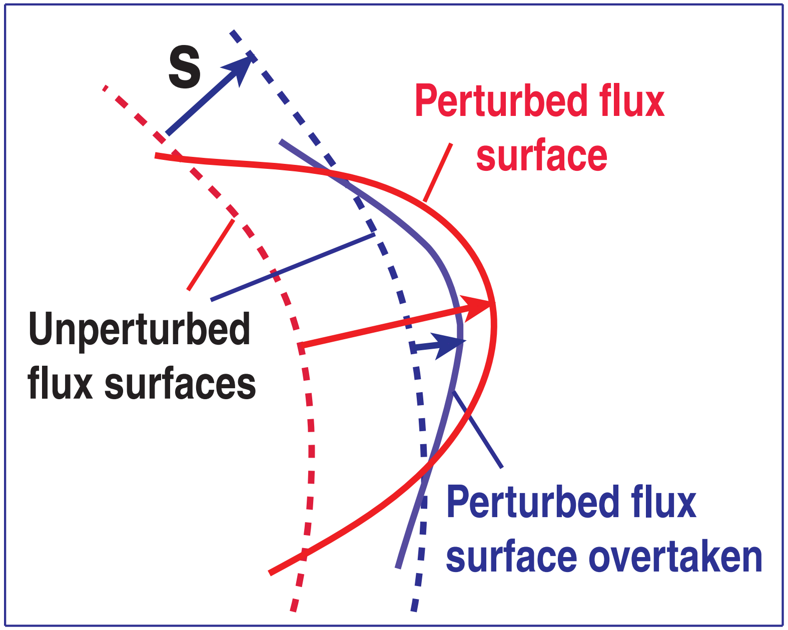
- **Nominally internal normal modes like the 1/1 kink have some small boundary perturbation if the wall is removed** Lazarus IAEA 2012
  - Conversely a small imposed boundary perturbation can excite these normal modes yielding a large internal response
- **In practice these may or may not be suppressed by non-ideal effects**
- **Ideal response from MARS-F with experimental rotation has near-internal 3/3 component inside $\rho < 0.3$ that dominates response**
  - Slightly hollow q profile with q_{min} = 1.01



- **This has even more serious implications for nonlinear dynamic evolution approach**

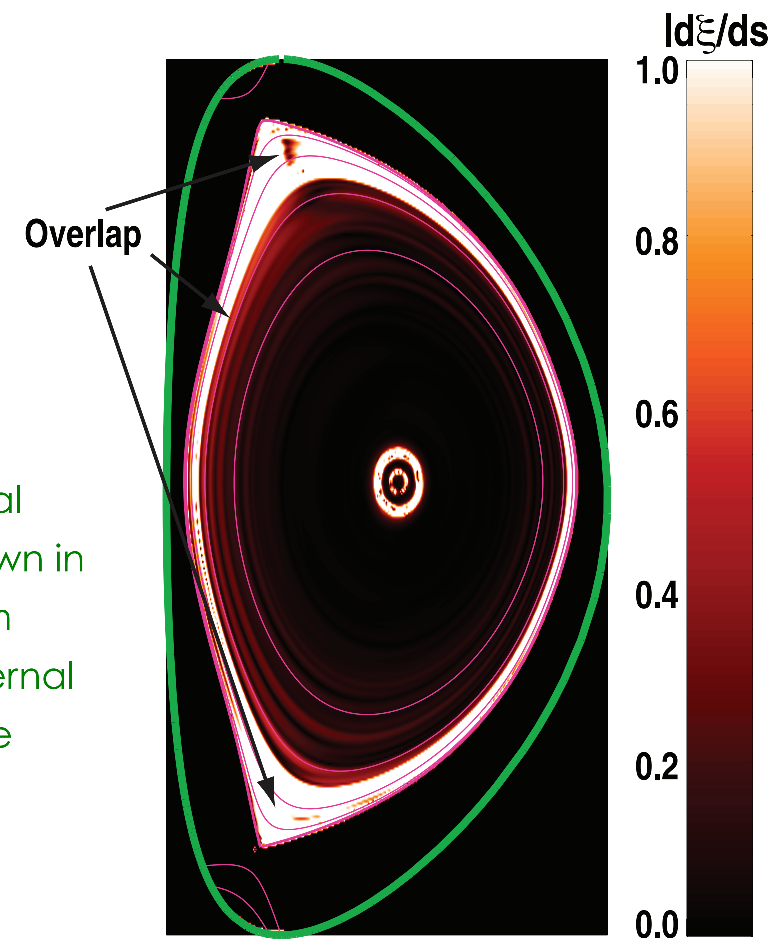
Linear Dynamic Criterion for Flux Surface Crossing Shows Break Down of Linear Model for Finite Displacements

- **Sufficient criterion for crossing** $\left|\partial \xi_n / \partial s\right| > +1$



Infinitesimal	$\xi_n \rightarrow 0 \quad \Rightarrow$	$\left|\partial \xi_n / \partial s\right| \rightarrow 0$
Forced	$\xi_n \not\rightarrow 0 \quad \Rightarrow$	$\left|\partial \xi_n / \partial s\right| \not\rightarrow 0$

- **Extensive crossings occur on inboard side and very edge**



Additional breakdown in core from large internal 3/3 mode

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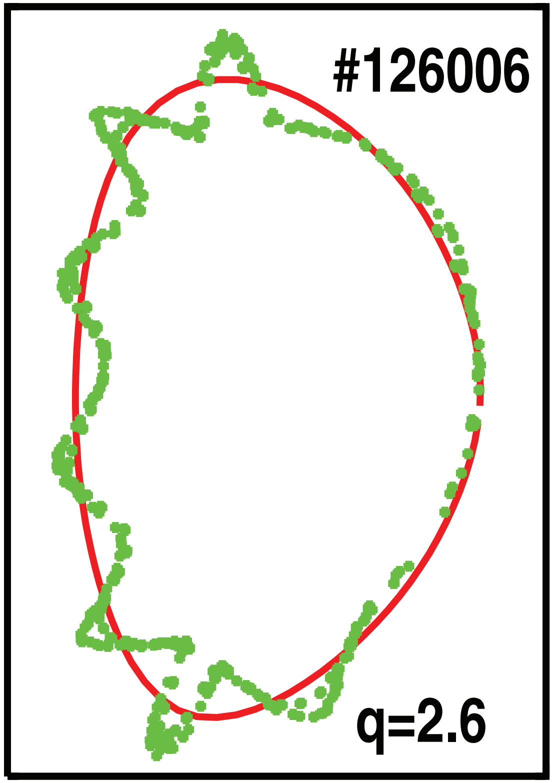
Nonlinear Dynamic — Nonlinear Dynamic Approach Requires Correct Saturation Physics for Steady State

- **Required saturation physics is case dependent**
  - Saturation mechanism needed for each normal mode in response
- **Internal mode appearing in linear MARS-F calculation for 142603 is also present in the M3D-C^1 nonlinear evolution**
  - Nonlinear run failed to reach steady state as the required saturation mechanism for the 3/3 core mode is not correct
- **Near steady state can be obtained in some cases**
  - 3/3 mode not present for discharge #126006
  - Approximate steady state reached early
  - Nonlinear mode appears to grow later in the evolution

Nonlinear evolution Poincare Plot

#126006

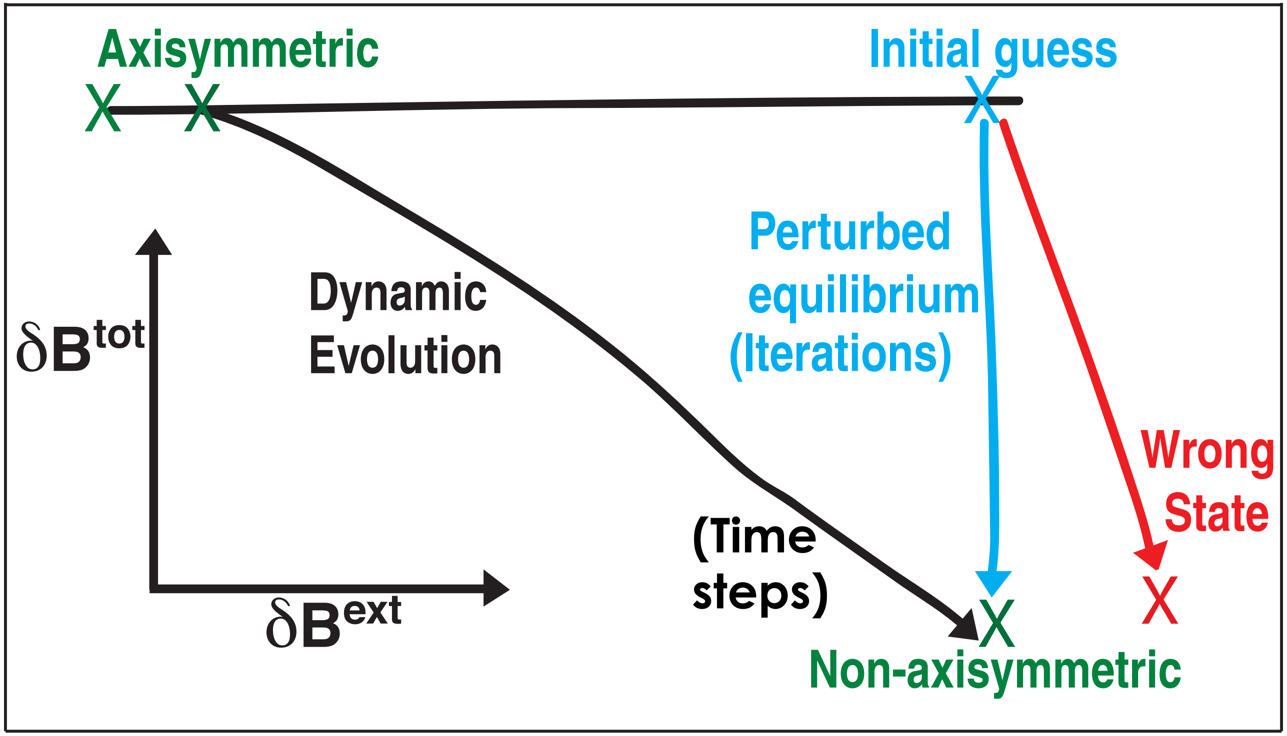
Final state before mode grows looks qualitatively like the linear result



Perturbed Equilibrium Approach Requires Constraints to Guarantee State is Accessible

Perturbed Equilibrium

- **Multiple nearby 3-D equilibria typically exist** Cooper IAEA 2012
  - How can the unique accessible state be selected ?
- **Need constraints or invariants relating initial axisymmetric state with the unique nearby final non-axisymmetric state**
- **Imposed constraints need to account for topological transformations but restrict physically inaccessible changes**



Perturbed Equilibrium Require Constraints Relating Initial 2-D System With Dynamically Accessed 3-D Equilibrium

- **Different 3-D equilibrium codes invoke different implicit constraints**
  - VMEC imposes nested surfaces but not stellarator symmetry
  - PIES, SIESTA, SPEC currently impose stellarator symmetry

None are necessarily the constraints exhibited by the actual dynamics

- **Equilibrium codes require specification of two independent functions**
  - $s_1(\psi) = p(\psi)$ (pressure) or $s_1(\psi) = dp(\psi)/d\psi$
  - $s_2(\psi) = \iota(\psi)$ (rotational transform) or equivalently current density
  - For 2-D equilibria these are measured routinely

- **In absence of 3-D reconstruction**
  Require a relation between 2-D profiles and the subsequent dynamically accessible profiles in the 3-D state
  - **i.e. a set of "constraints"**

- **Simplest and most convenient approach is to set profiles for 3-D same as measured initial 2-D profiles**

Perturbed Equilibrium: There is No Guarantee The Simple Approach Yields Dynamically Accessed State

- **Ambiguity exists even if profiles are set to be the same as in 2-D state**
  - Should ψ be taken as the poloidal flux Ψ or toroidal flux Φ ?
- **Changes in local or global transport may also modify pressure profile**
  - ⇒ Profiles can change as response to perturbation ("transport response")
  - ⇒ Density pumpout is usually observed in experiments
  - Islands produce new regions where profiles need to be specified
- **In 3-D with non-nested surfaces p = p(ψ,Γ_i), where Γ_i represents a simply connected region isolated from other regions by a separatrix**
  - In the intact region
    - Specify p(ψ) as in the 2-D equilibrium or
    - Evolve p(ψ) via 1½-D transport
  - Within new island regions, Γ_i, assumptions need to be imposed on p(ψ, Γ_i)

How should the current density profile be determined?

- **Keeping $\iota(\psi)$ or q(ψ) fixed from the initial 2-D state implies no topological changes:** ⇒ Only ideal motions are allowed

Specifying Magnetic Helicity Has Advantages as a Constraint to be Imposed on Current Profile

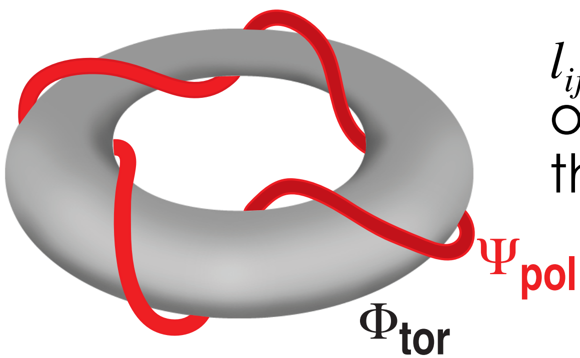
- **Magnetic helicity is also conserved by ideal motions for every flux surface ψ**

$$K(\psi) = \int_{\psi_0}^{\psi} \oiint A \bullet B \, d\tau$$

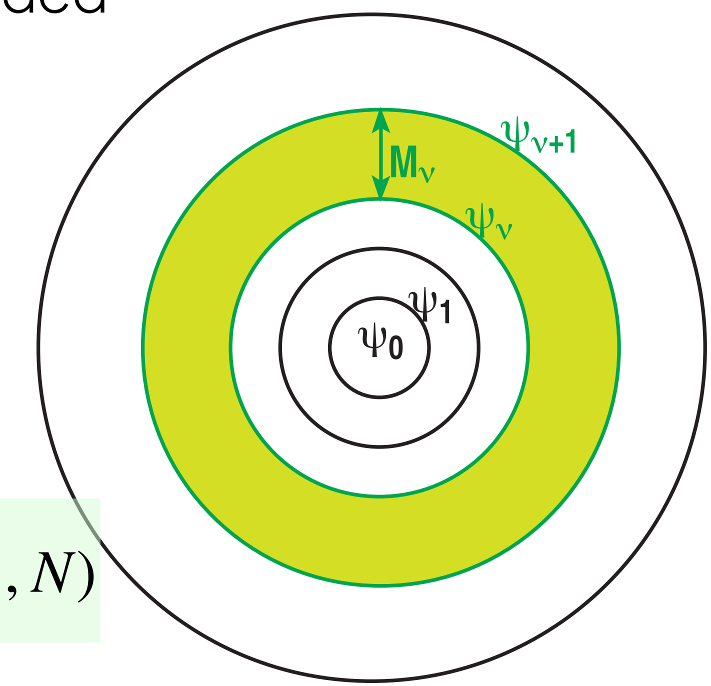
A = Vector potential

$B = \nabla \times A$

- **Magnetic helicity is sum over pairs of linked flux tubes** $K = \sum_{i,j} l_{ij} \Phi_i \Phi_j$



l_{ij} is the number of times one flux tube is threaded through the other



- **Linking number can change when islands or stochastic regions form**

⇒ **A set of annular helicities can be defined between flux surfaces ψ_ν**

$$M_\nu \equiv M(\psi_\nu, \psi_{\nu+1}) = \int_{\psi_\nu}^{\psi_{\nu+1}} \oiint A \cdot B \, d\tau \qquad (\nu = 0, 1, 2, \ldots, N)$$

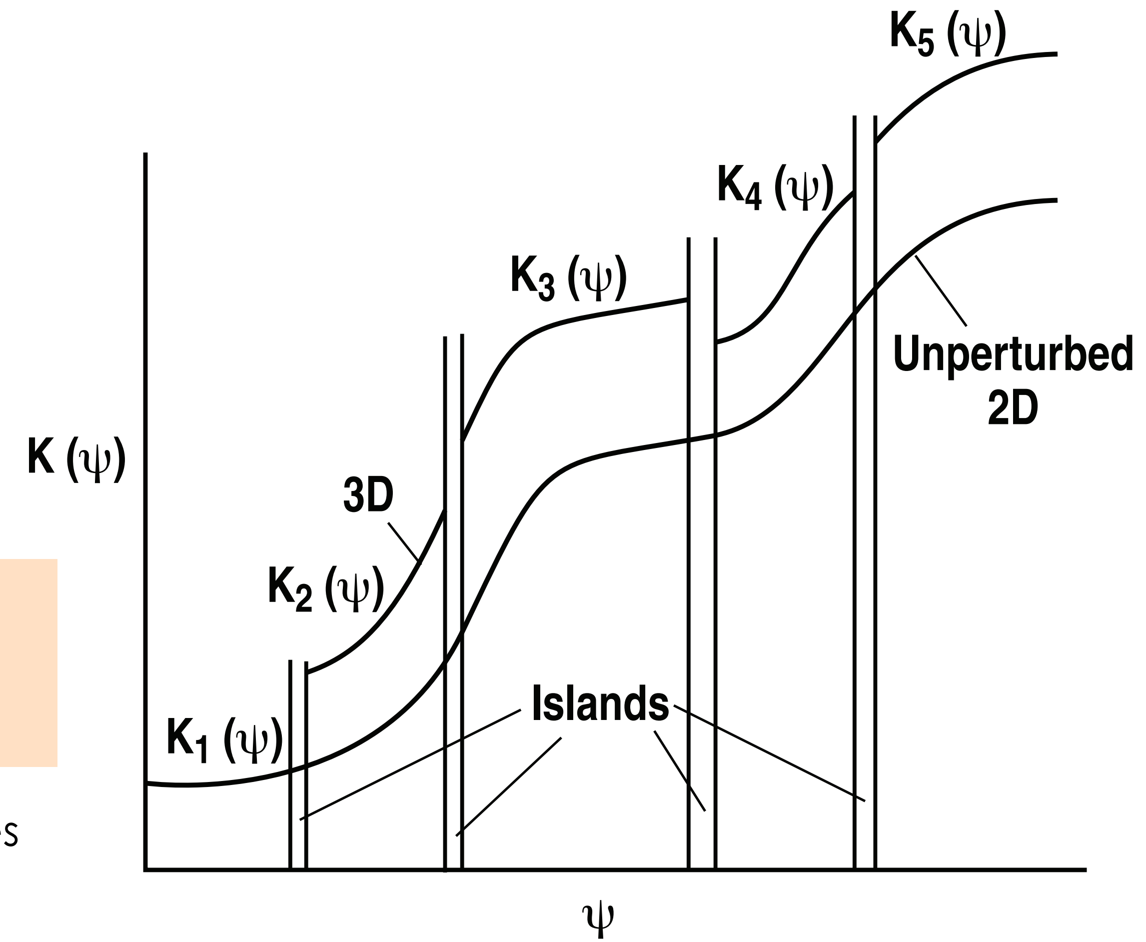
A Finite Set of Helicity Integrals Between KAM Surfaces is Expected to be Conserved Up to a Constant

- **Intuitively expect annular helicity changes around newly formed islands and unchanged in the intact flux surface regions**

⇒ Helicity profile is expected to undergo jumps with constant offset from each island region

This can be tested in a dynamic simulation from an extended MHD code

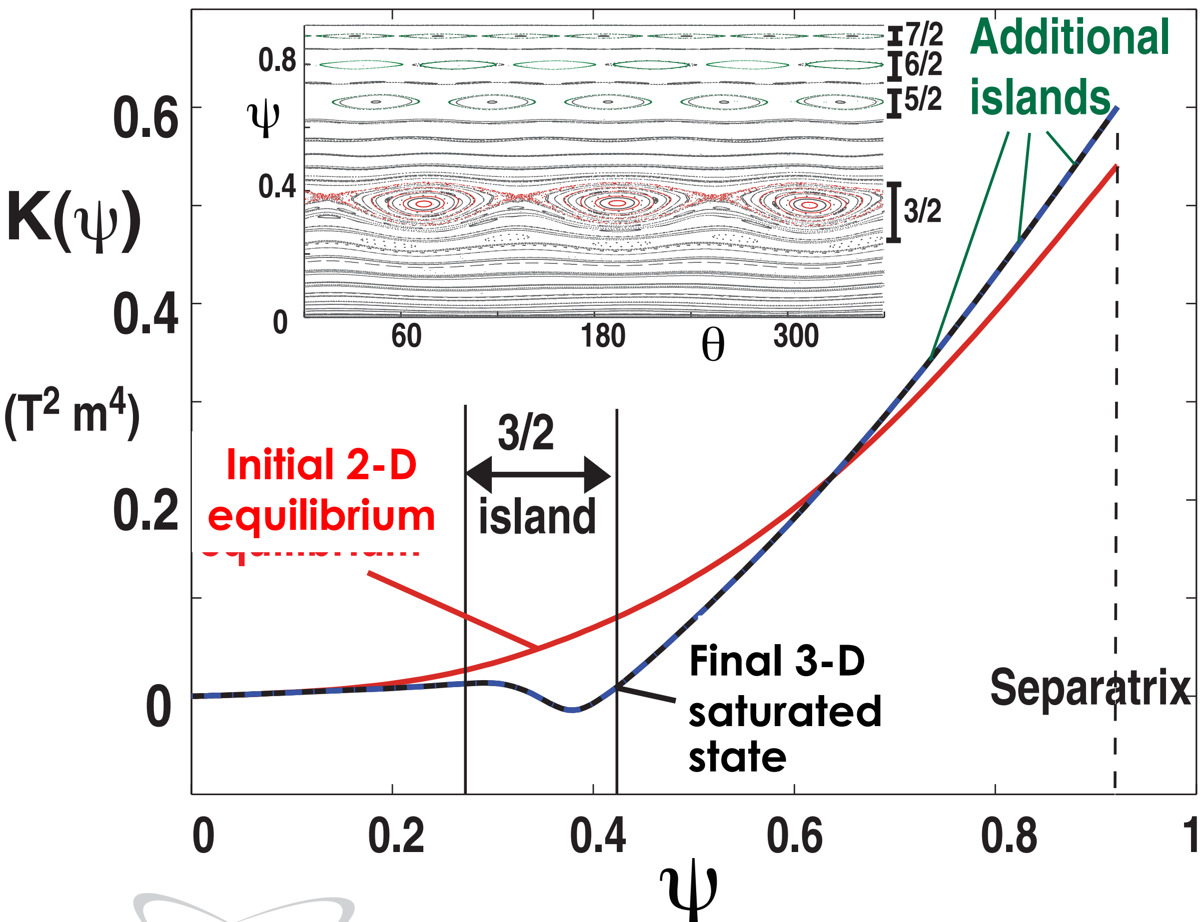
SPEC code (S. Hudson) specifies helicity in discrete regions



NIMROD Calculation Shows Approximately Expected Behavior for Annular Helicities

- **3/2 island region shows up clearly in helicity profile**

Instability generated perturbation



- **Result is not simply an offset**
  - Additional islands contribute

- **Transitions through islands generally appear to be smoother than expected**
  - ⇒ Additional work needed

Linear and Nonlinear Calculations for DIII-D discharge #142603 Yield Qualitatively Different Responses

- **Linear model predictions agree semi-quantitatively …. but Linear models break down for finite perturbations if surfaces cross**

Surprise is that linear theory breaks down at the level of 10^{-3} perturbations

  - Local breakdown of ideal model can be predicted

- **Nonlinear dynamic approach is time consuming and requires all essential physics to obtain correct saturation**
  - Calculations so far suggest final state is similar to linear response

- **Nearby equilibrium approach can find the right final state in principle if constraints are imposed**

⇒ Hypothesis of "invariant" annular helicity appears to be approximately right but requires further work