Comparisons of Linear and Nonlinear Plasma Response Models for Non-axisymmetric Perturbations

by
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Understanding Plasma Response to Non-Axisymmetric Perturbations is a Vital Area of Fusion Research

• Plasma response to 3-D perturbations is a major focus of the experimental tokamak program
  – ELM suppression from internal non-axisymmetric coils (DIII-D I-coils)

• Plasma response is a key ingredient in determining the consequences of non-axisymmetric perturbations

  Plasma can amplify, suppress or otherwise modify perturbation!

  First attempt at comparing and documenting applicability of the predicted detailed internal response from different approaches

• Main goal is to identify the issues limiting each approach
  – Which is right and when is it right (i.e. the experimental plasma response)
    • Answer depends on conditions
There Are Two Key But Interrelated Responses to an External Non-axisymmetric Field

- **Equilibrium response**
  - Magnetic geometry
  - Flux surface displacement
  - Changes in topology
  - Changes in profiles responding to force balance

- **Transport response due to topology and equilibrium changes**
  - Changes in profiles from changed local transport

**MHD Response**

**Beyond MHD**

- Transport response can be thought of as part of nonlinear response

In principle, this can all be captured within an Extended MHD framework

**Final state is a new MHD force balance equilibrium**
Four Conventional Approaches Are Traditionally Used to Find the Steady State Plasma Response

Dynamic Evolution
- Follow time evolution to determine final nonlinear saturated state using Extended MHD stability code

Perturbed Equilibrium
- Find the nearby stable non-axisymmetric equilibrium
- Both viewpoints hold for both linear and nonlinear formulations
Each Approach Has Relative Advantages

<table>
<thead>
<tr>
<th>Dynamic evolution viewpoint:</th>
<th>Linear:</th>
<th>Nonlinear:</th>
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<tbody>
<tr>
<td>Forced eigenvalue (MARS-F, M3D-C(^1))</td>
<td>Fast turn around</td>
<td>3D Extended MHD stability (NIMROD, M3D, M3D-C(^1))</td>
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<tr>
<td>Valid only for sufficiently small response</td>
<td>Requires complete physics and realistic parameters</td>
<td>Time consuming</td>
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<tr>
<th>Nearby equilibrium viewpoint:</th>
<th>Basis expansion (IPEC)</th>
<th>Nearby equilibrium (VMEC, PIES, HINT, SPEC, SIESTA)</th>
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<tr>
<td></td>
<td></td>
<td>Computed state may not be physically accessed</td>
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- Each approach has significant past success in predicting external magnetic response
Different Approaches Bring Complementary Insights for Predicting Detailed Internal Responses

- All four approaches have yielded past successes
- Comparison of predictions of internal plasma response to I-coil perturbations in DIII-D
- Resolution of discrepancies
- Magnetic helicity as a constraint
- Conclusions
M3D-C$^1$ Predicts Observed Pedestal Temperature and Oscillation of Edge Thomson Location With $I$-coil Phasing

- Edge location inferred from Thomson oscillates in phase with $n = 3$ $I$-coil current in DIII-D

- M3D-C$^1$ includes linear plasma response

(Ferraro IAEA 2012, this meeting)
MARS-F Calculations of Ideal Plasma Response Agree With Measured Response in DIII-D at Sufficiently Low $\beta$

- Amplitude and phase agree for $\beta_N < 1.8$

- Disagreement for $\beta_N > 1.8$
  - Ideal model over-estimates response just below no wall limit
  - No agreement above no-wall limit

(Lanctot Phys. Plasmas 2011)
DIII-D Discharge #142603 Provides Well Documented Case for Comparing Response Predictions

- DIII-D discharge #142603 at 3519 ms
  - Up-down symmetric 2-D configuration
  - Applied internal l-coil (even) \( n = 3 \) field:
    - \( \delta B^{\text{ext}} \sim 10^{-3} B\phi \)

- How well can the different approaches predict detailed response?
  \( \Rightarrow \) Compare the predictions against each other
Linear Response Calculated for DIII-D Discharge #142603 Including Plasma Rotation Using MARS-F

- MARS-F is a linear eigenvalue code modified to find the response due to an inhomogeneous forcing function representing an external field

\[ \rho \ddot{\xi} + L \dot{\xi} = \omega^2 \xi + L \dot{\xi} = 0 \rightarrow \omega_0^2 \xi + L \dot{\xi} = F \]

- Includes rotation and resistivity profiles

![Diagram showing the response for n=3](image)
MARS-F, M3D-C¹ and IPEC Predict Qualitatively Similar Linear Responses With Significant Inboard Oscillations

- Oscillations follow
  \[ \xi \sim \xi_0 e^{im\chi_{pol}} \]
  with \( m \sim nq - 1 \)

  oscillations down the inboard side and one or two large oscillations on outboard side

  (Kink-like response)

- MARS-F

Scaled displacements factor 56

\( q = 8.0/3 \)

Scale: 200 kA
Nonlinear VMEC Response is Significantly Different from Ideal Linear Response Especially on the Inboard Side

- Equilibrium calculation with non-axisymmetric I-coil fields
- Profiles taken from reconstructed 2-D equilibrium

n=3

- Inboard response is quite different from linear ideal response predictions
- Oscillations do not follow 
  \[ \xi \sim \xi_0 e^{im\chi_{pol}} \]  
  (Non kink-like response)
- Similar disagreement for non-resonant surfaces

Scaled displacements
Possible Sources for Discrepancy Can be Identified in Each Approach

Linear Dynamic Approach
- Response depends on what physics is included in dynamic evolution
  - Response is sensitive to marginal “near internal” eigenmodes
- Linear model can break down for finite perturbations
  - Response can be large even when applied external field is small

Nonlinear Dynamic Evolution Approach
- Physics required to obtain saturated state is case dependent

Nonlinear Perturbed Equilibrium Approach
- Convergence issues arise for resolving singular currents or islands
- Equilibrium code can find the “wrong” equilibrium
  - Constraints imposed to define new equilibrium may be inappropriate
Small Boundary Distortions Can Excite Near-Internal Modes That Dominate the Response

- Nominally internal normal modes like the 1/1 kink have some small boundary perturbation if the wall is removed. Conversely, a small imposed boundary perturbation can excite these normal modes yielding a large internal response.

- In practice, these may or may not be suppressed by non-ideal effects.

- Ideal response from MARS-F with experimental rotation has a near-internal 3/3 component inside \( \rho < 0.3 \) that dominates the response.
  - Slightly hollow q profile with \( q_{\text{min}} = 1.01 \)

- This has even more serious implications for nonlinear dynamic evolution approach.
Criterion for Flux Surface Crossing Shows Break Down of Linear Model for Finite Displacements

- Sufficient criterion for crossing
  \[ \left| \frac{\partial \xi_n}{\partial s} \right| > +1 \]

- Extensive crossings occur on inboard side and very edge

Infinitesimal: \( \xi_n \to 0 \Rightarrow \left| \frac{\partial \xi_n}{\partial s} \right| \to 0 \)

Forced: \( \xi_n \not\to 0 \Rightarrow \left| \frac{\partial \xi_n}{\partial s} \right| \not\to 0 \)

Additional breakdown in core from large internal 3/3 mode
Nonlinear Dynamic Approach Requires Correct Saturation Physics for Steady State

- Required saturation physics is case dependent
  - Saturation mechanism needed for each normal mode in response

- Internal mode appearing in linear MARS-F calculation for 142603 is also present in the M3D-C\(^1\) nonlinear evolution
  - Nonlinear run failed to reach steady state as the required saturation mechanism for the 3/3 core mode is not correct

- Near steady state can be obtained in some cases
  - 3/3 mode not present for discharge #126006
  - Approximate steady state reached early
  - Nonlinear mode appears to grow later in the evolution

Nonlinear evolution Poincare Plot

Final state before mode grows looks qualitatively like the linear result

q=2.6
Perturbed Equilibrium Approach Requires Constraints to Guarantee State is Accessible

- Multiple nearby 3-D equilibria typically exist
  - How can the unique accessible state be selected?
- Need constraints or invariants relating initial axisymmetric state with the unique nearby final non-axisymmetric state

Imposed constraints need to account for topological transformations but restrict physically inaccessible changes.

Cooper IAEA 2012

How can the unique accessible state be selected?

- Perturbed Equilibrium
- Initial guess
- Dynamic Evolution
- Perturbed equilibrium (Iterations)
- Non-axisymmetric
- Wrong State

\[ \delta B^{\text{tot}} \]
\[ \delta B^{\text{ext}} \]

(Time steps)
Perturbed Require Constraints Relating Initial 2-D System Equilibrium With Dynamically Accessed 3-D Equilibrium

- Different 3-D equilibrium codes invoke different implicit constraints
  - VMEC imposes nested surfaces but not stellarator symmetry
  - PIES, SIESTA, SPEC currently impose stellarator symmetry

None are necessarily the constraints exhibited by the actual dynamics

- Equilibrium codes require specification of two independent functions
  - $s_1(\psi) = p(\psi)$ (pressure) or $s_1(\psi) = dp(\psi)/d\psi$
  - $s_2(\psi) = \iota(\psi)$ (rotational transform) or equivalently current density
  - For 2-D equilibria these are measured routinely

- In absence of 3-D reconstruction
  Require a relation between 2-D profiles and the subsequent dynamically accessible profiles in the 3-D state
  - i.e. a set of “constraints”

- Simplest and most convenient approach is to set profiles for 3-D same as measured initial 2-D profiles
There is No Guarantee The Simple Approach Yields Dynamically Accessible State

- Ambiguity exists even if profiles are set to be the same as in 2-D state
  - Should \( \psi \) be taken as the poloidal flux \( \Psi \) or toroidal flux \( \Phi \)?

- Changes in local or global transport may also modify pressure profile
  - Profiles can change as response to perturbation ("transport response")
  - Density pumpout is usually observed in experiments
    - Islands produce new regions where profiles need to be specified

- In 3-D with non-nested surfaces \( \mathbf{p} = \mathbf{p}(\psi, \Gamma_i) \), where \( \Gamma_i \) represents a simply connected region isolated from other regions by a separatrix
  - In the intact region
    - Specify \( \mathbf{p}(\psi) \) as in the 2-D equilibrium or
    - Evolve \( \mathbf{p}(\psi) \) via 1½-D transport
  - Within new island regions, \( \Gamma_i \), assumptions need to be imposed on \( \mathbf{p}(\psi, \Gamma_i) \)

How should the current density profile be determined?

- Keeping \( \mathbf{i}(\psi) \) or \( \mathbf{q}(\psi) \) fixed from the initial 2-D state implies no topological changes:
  - Only ideal motions are allowed
Specifying Magnetic Helicity Has Advantages as a Constraint to be Imposed on Current Profile

- Magnetic helicity is also conserved by ideal motions for every flux surface $\psi$
  \[ K(\psi) = \int_{\psi_0}^{\psi} \oint A \cdot B \, d\tau \]
  $A = \text{Vector potential}$
  $B = \nabla \times A$

- Magnetic helicity is sum over pairs of linked flux tubes
  \[ K = \sum_{i,j} l_{ij} \Phi_i \Phi_j \]
  $l_{ij}$ is the number of times one flux tube is threaded through the other

- Linking number can change when islands or stochastic regions form

$\Rightarrow$ A set of annular helicities can be defined between flux surfaces $\psi_v$

\[ M_v \equiv M(\psi_v, \psi_{v+1}) = \int_{\psi_v}^{\psi_{v+1}} \oint A \cdot B \, d\tau \quad (v = 0, 1, 2, ..., N) \]
Intuitively expect annular helicity changes around newly formed islands and unchanged in the intact flux surface regions.

Helicity profile is expected to undergo jumps with constant offset from each island region.

This can be tested in a dynamic simulation from an extended MHD code.

SPEC code (S. Hudson) specifies helicity in discrete regions.

A Finite Set of Helicity Integrals Between KAM Surfaces is Expected to be Conserved Up to a Constant.
NIMROD Calculation Shows Approximately Expected Behavior for Annular Helicities

- 3/2 island region shows up clearly in helicity profile

Instability generated perturbation

- Result is not simply an offset
  - Additional islands contribute

- Transitions through islands generally appear to be smoother than expected
  ⇒ Additional work needed

Initial 2-D equilibrium

Final 3-D saturated state

Separatrix
Linear and Nonlinear Calculations for DIII-D discharge #142603 Yield Qualitatively Different Responses

• Linear model predictions agree semi-quantitatively .... but
  Linear models break down for finite perturbations if surfaces cross

  Surprise is that linear theory breaks down at the level of $10^{-3}$ perturbations
  - Local breakdown of ideal model can be predicted

• Nonlinear dynamic approach is time consuming and requires all
  essential physics to obtain correct saturation
  - Calculations so far suggest final state is similar to linear response

• Nearby equilibrium approach can find the right final state in principle
  if constraints are imposed

  ⇒ Hypothesis of “invariant” annular helicity appears to be
  approximately right but requires further work