Calculation of Linear Two-Fluid Plasma Response to Applied Non-Axisymmetric Fields

by
N.M. Ferraro
General Atomics

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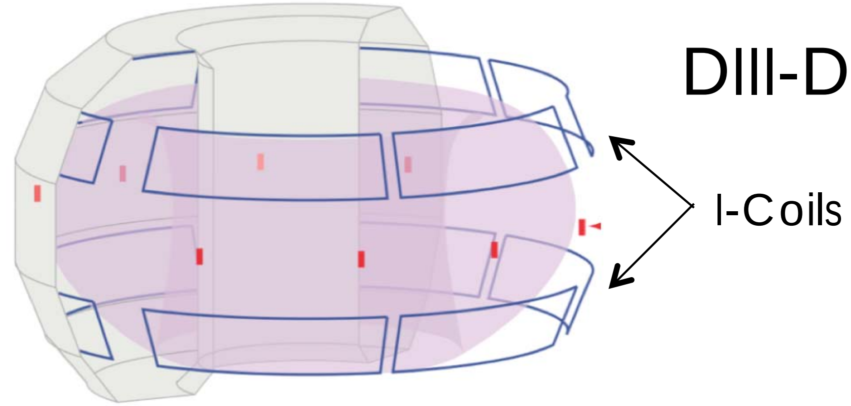
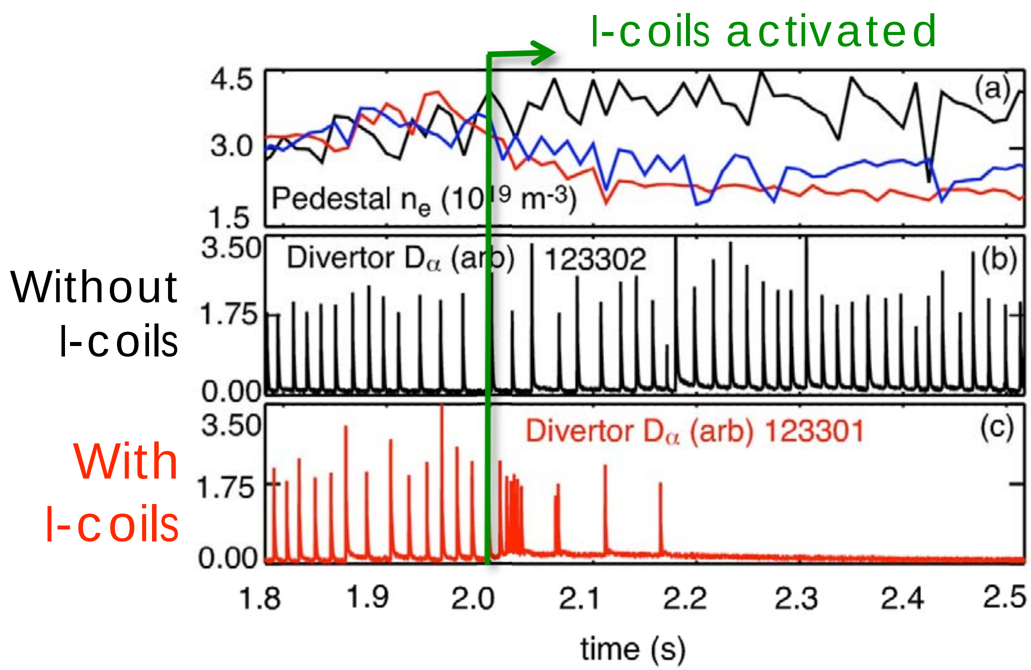
GENERAL ATOMICS

3D Fields Significantly Affect Tokamak Performance

- **Edge-localized modes**
  - Mitigate/suppress ELMs in H-mode
- **Transport**
  - Density pump-out
- **Drive/brake rotation**
  - Affects RWM stability
  - Affects tearing mode stability (Buttery talk this session)
  - Allow access QH-mode without NBI (Burrell talk Friday)





Evans, *et al. Phys. Plasmas* **13** (2006)

A Predictive Capability Requires Understanding Plasma Response

- **Predictive capability is challenging because plasma response is complicated**
  - Plasma may strongly enhance/suppress spectral components of applied field
  - New fields affect transport and rotation
  - Rotation strongly affects plasma response

- **New tools are being developed and applied to gain predictive understanding (M3D-C1)**

Our Models of Plasma Response Are Evolving

- **“Vacuum” Fields (TRIP3D)**
  - Plasma does not respond to applied fields
  - Tells us degree to which applied fields are “resonant”
  - Doesn’t tell us dependence on plasma parameters

- **Ideal (IPEC, MARS-F, VMEC)**
  - Plasma responds such that magnetic surfaces remain intact → no islands
  - Tells us how strongly ideal modes respond to applied fields
  - Doesn’t explain dependence of plasma response on rotation; doesn’t directly determine island size

Our Models of Plasma Response Are Evolving

- **Resistive, Single-Fluid (MARS-F, JOREK)**
  - Describes tearing response → islands
  - Tells us how response depends on plasma parameters, especially **rotation**
  - When is response more like ideal? When is it more like vacuum? Does it smoothly transition between the two?

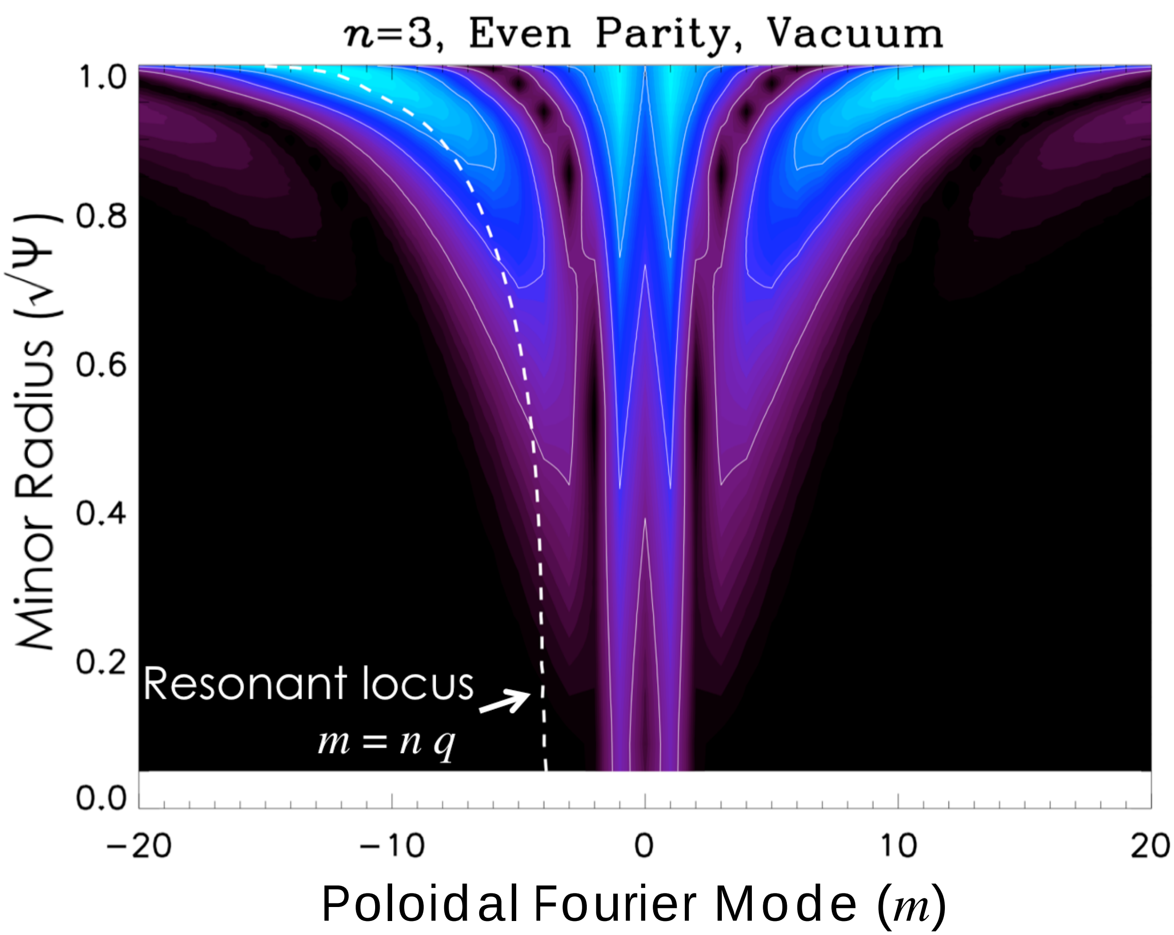
- **Two-fluid & “Extended” MHD (M3D-C1, NIMROD)**
  - Ion rotation (Ω) and electron rotation (Ω^e) are distinct
  - FLR effects, NTV, etc.

Outline

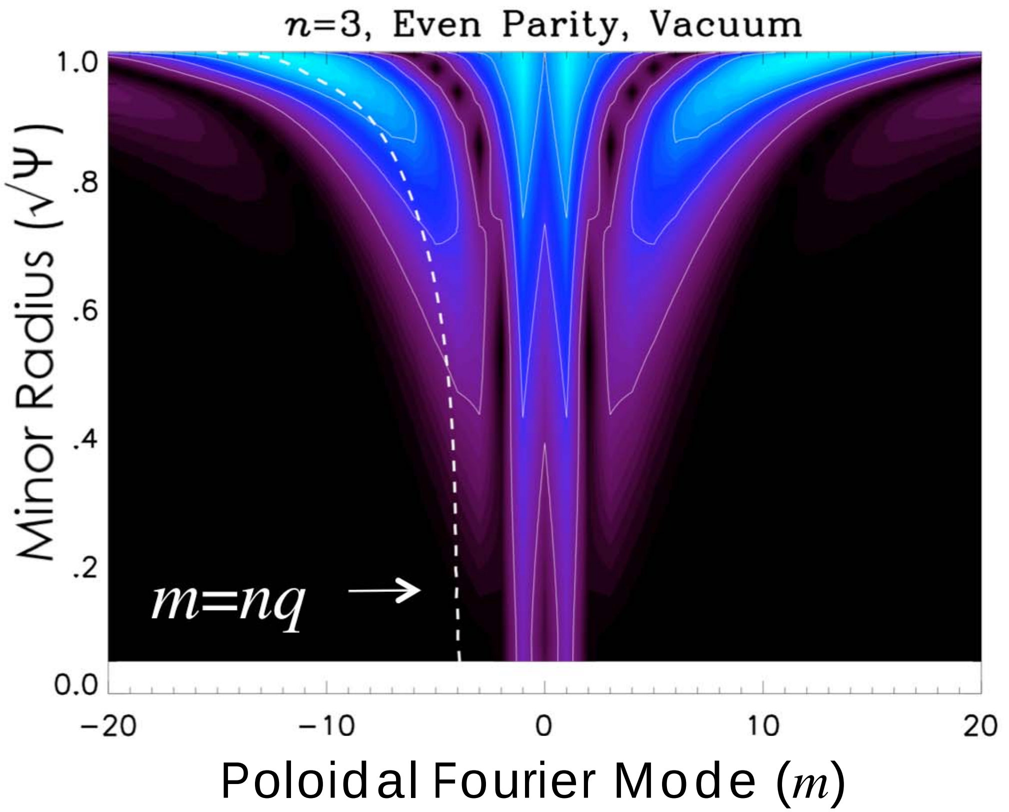
- **Basics of 3D response**
- **Introduction to M3D-C1**
- **Linear single-fluid results**
  - Rotation has strong effect on plasma response
- **Linear two-fluid results**
  - Electron rotation screens core islands
  - Sheared ion rotation affects edge
- **Linear vs. nonlinear**
  - Nonlinear calculations are required for some phenomena

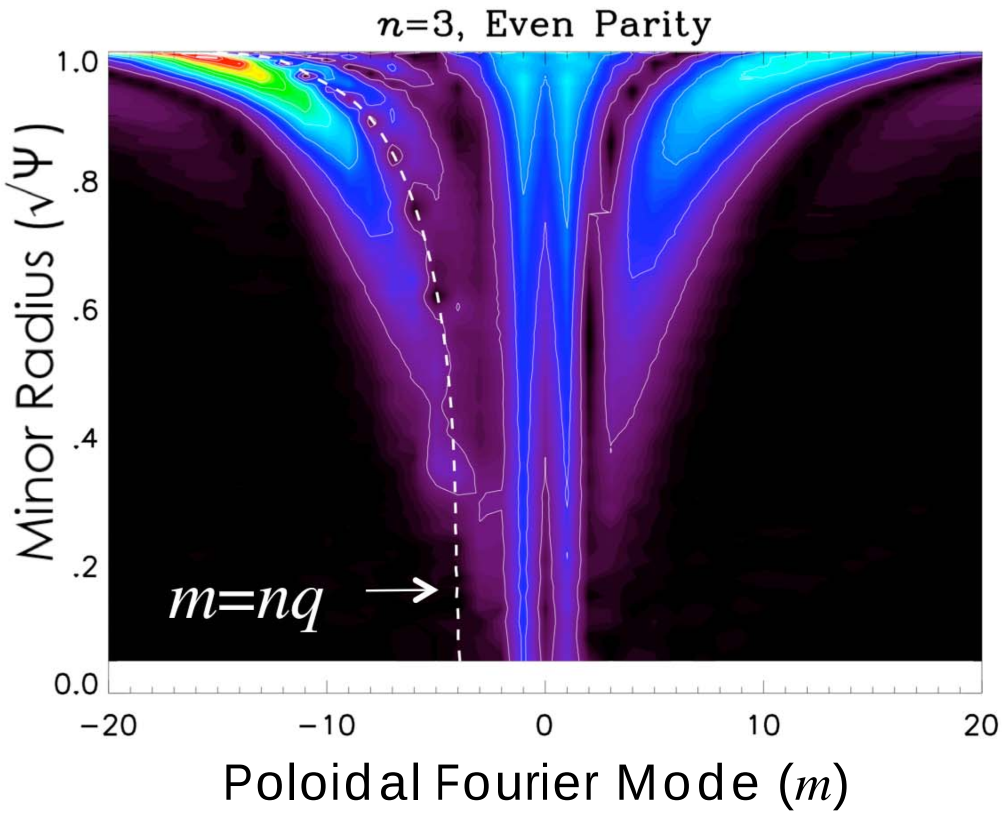
“Vacuum” Model Tells Us a Lot

- **Plot shows Fourier spectrum of B_n**
- **B_n = component of applied field normal to equilibrium magnetic surfaces**
- **Resonant components (along dashed line) cause islands**
- **Non-resonant components cause bending of surfaces**
- **Poloidal spectrum of B_n depends on Ψ**



Plasma Response Modifies Spectrum





- **Ideal response → no islands → reduction in resonant components**
- **Excited ideal modes → enhancement of non-resonant components**

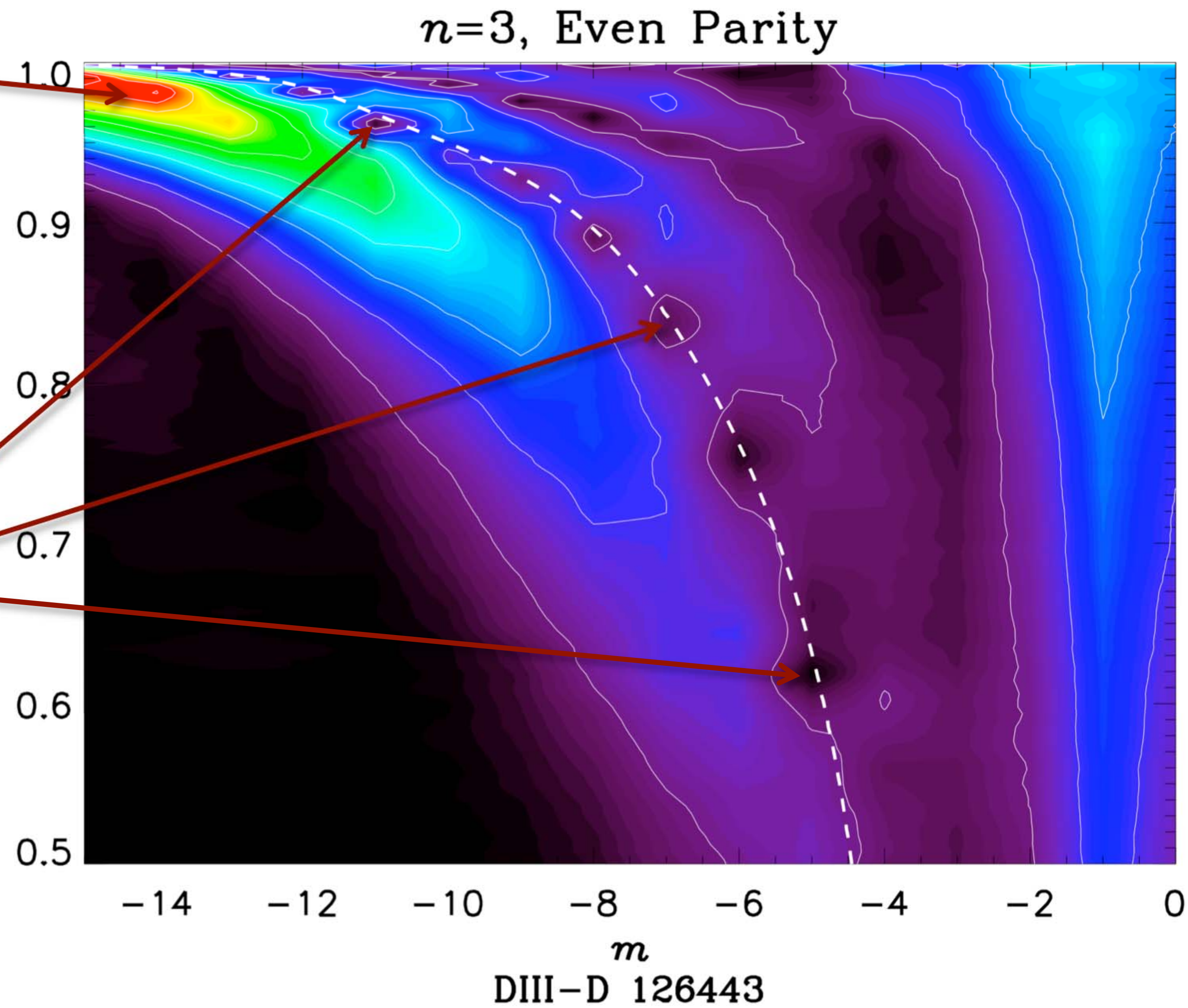
Plasma Can Kink and Screen

"Kinking"

- **Distorts surfaces**

Screening

- **Eliminates islands**



M3D-C1 Can Calculate Two-Fluid Response

- **M3D-C1 is a two-fluid resistive finite element code**
  - Shares some design principles with M3D
  - (R,Z) coordinates (not spectral in poloidal angle)
- **Computational domain includes plasma, separatrix, and open field-line region**
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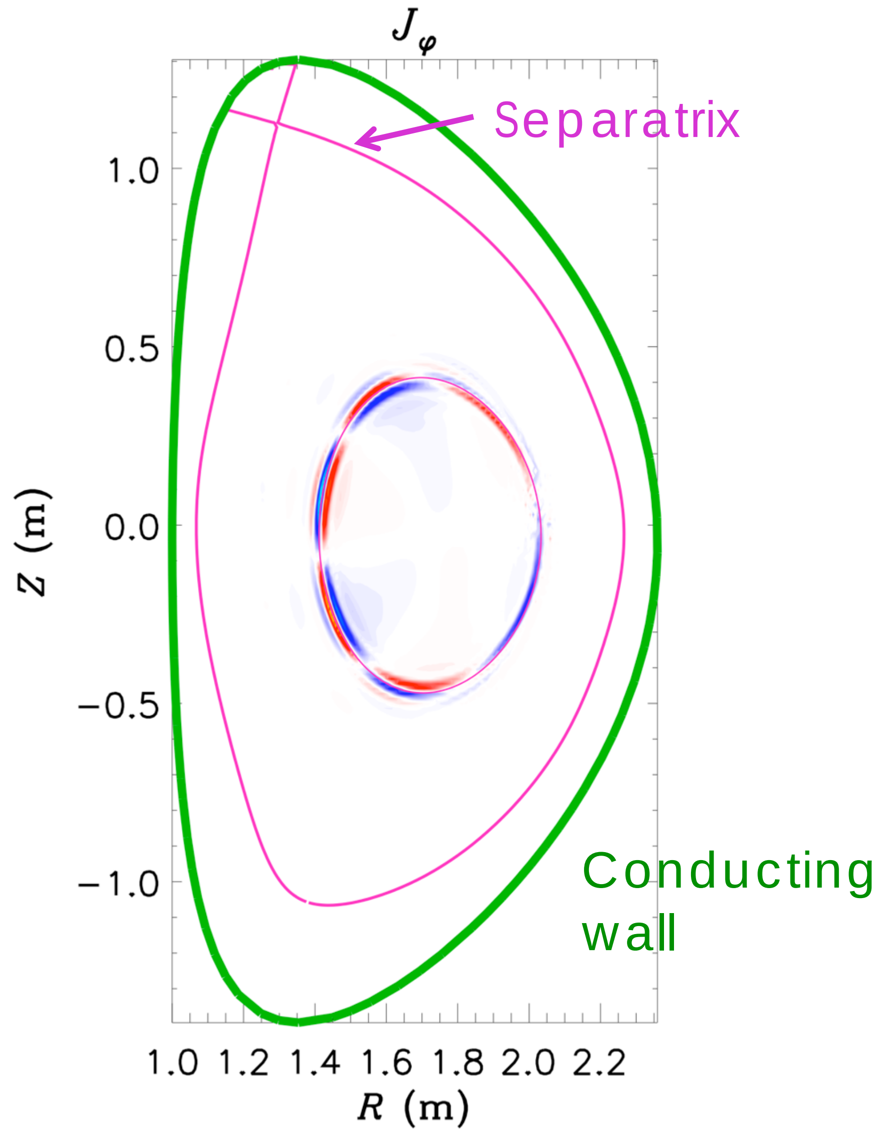

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  - Shares some design principles with M3D
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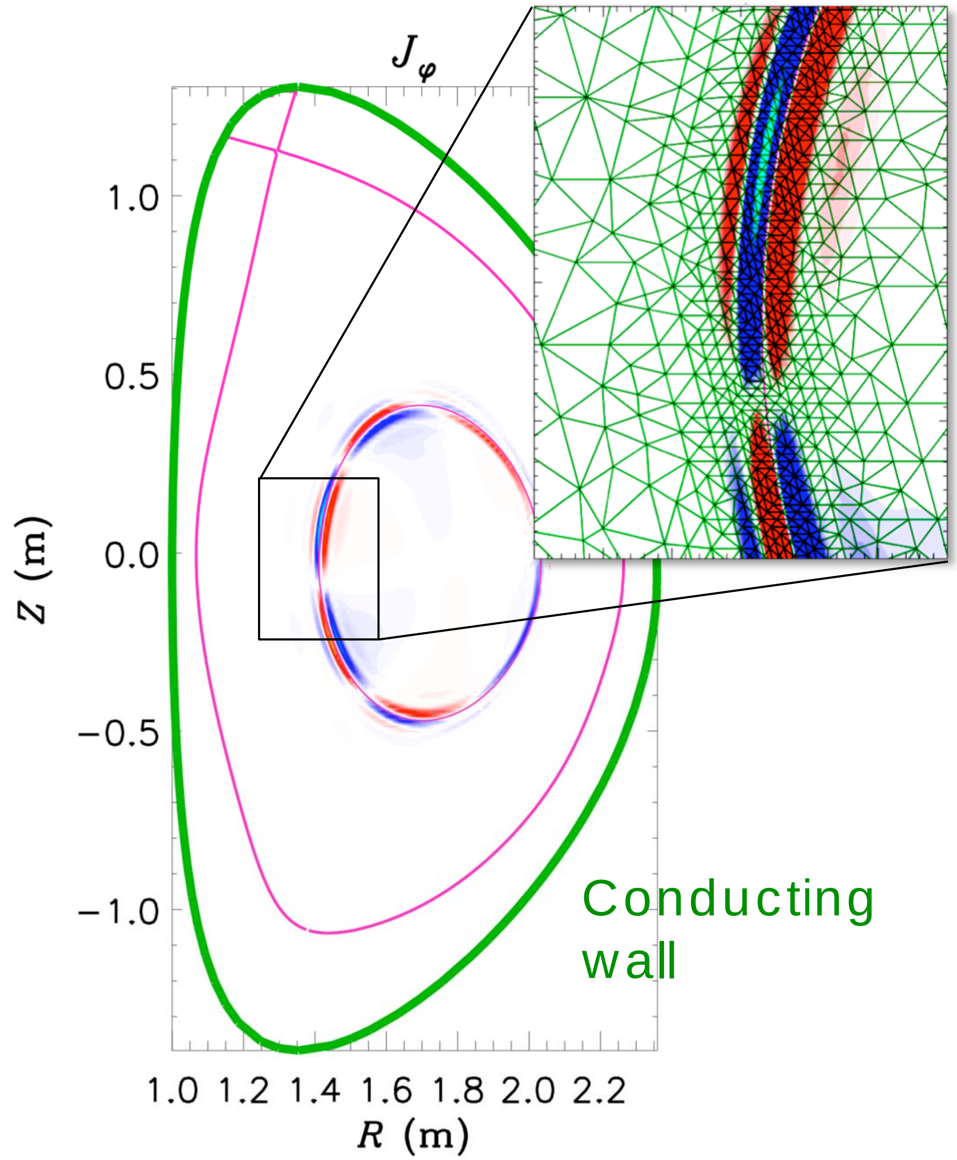
- **Computational domain includes plasma, separatrix, and open field-line region**
- **Unstructured mesh allows resolution packing at rational surfaces**
- **Both linear and nonlinear models are implemented**



Two-Fluid Model Implemented in M3D-C1

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$$

$$n\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi$$

$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p = -\Gamma p \nabla \cdot \mathbf{u} - \boxed{\frac{d_i}{n}\mathbf{J} \cdot \left(\Gamma p_e \frac{\nabla n}{n} - \nabla p_e\right)}$$

$$-(\Gamma - 1)\nabla \cdot \mathbf{q}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{J} + \boxed{\frac{d_i}{n}(\mathbf{J} \times \mathbf{B} - \nabla p_e)}$$

$$\Pi = -\mu\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right]$$

$$\mathbf{q} = -\kappa \nabla\left(\frac{p}{n}\right) - \kappa_{\parallel} \mathbf{bb} \cdot \nabla\left(\frac{p_e}{n}\right)$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

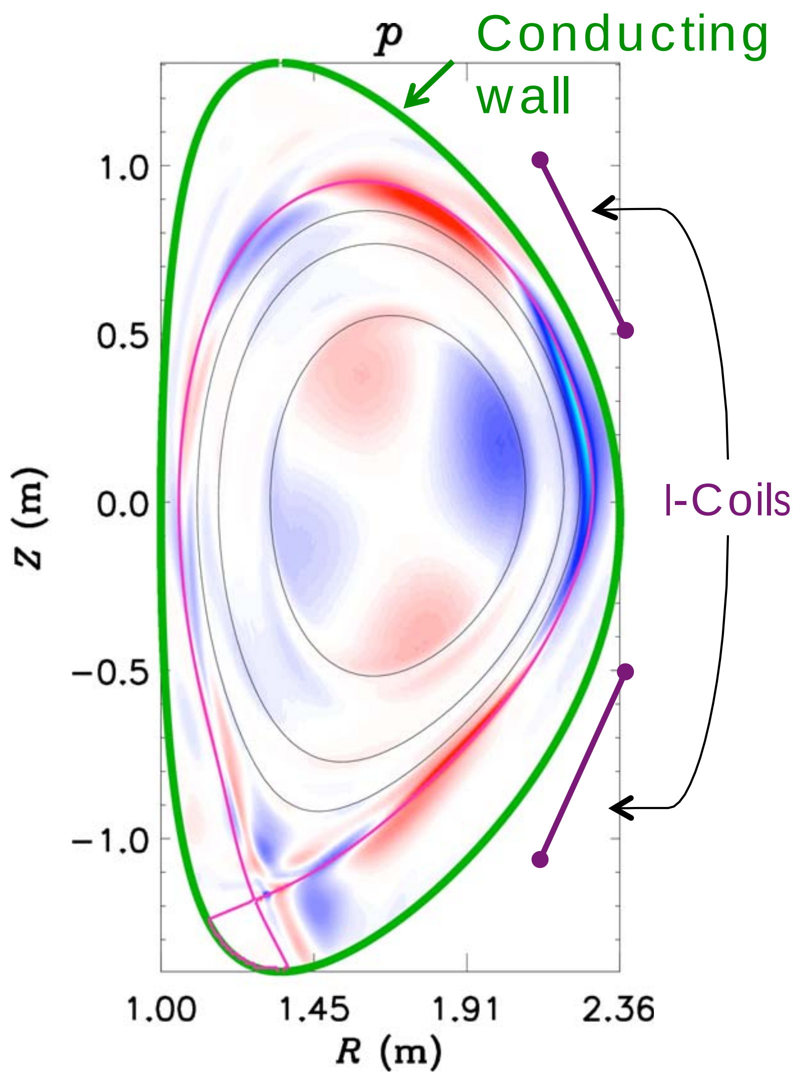
$$\Gamma = 5/3$$

$$p_e = p/2$$

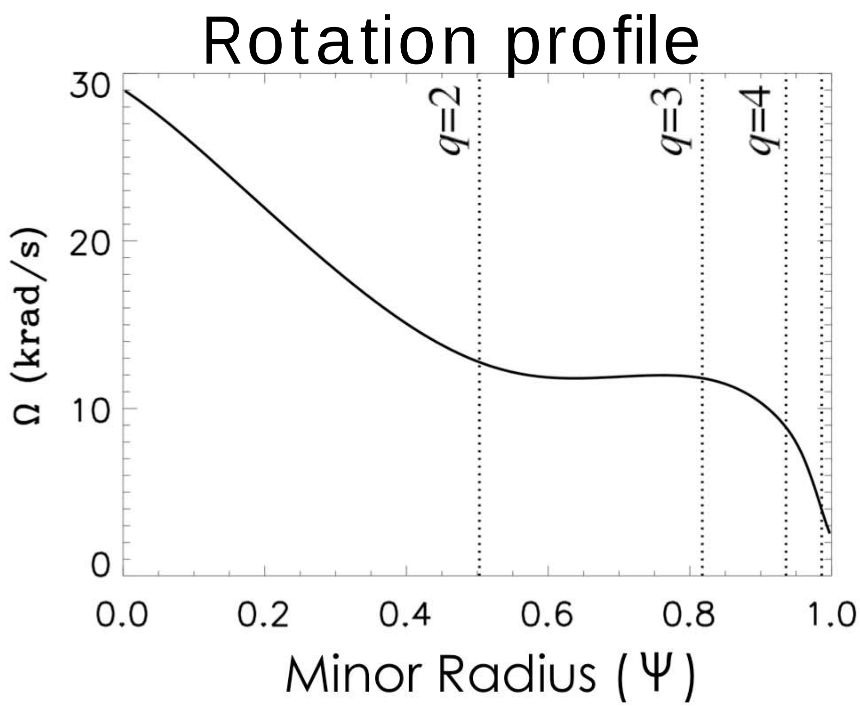
- **Complete (not reduced) two-fluid model is implemented**
- **Time-independent equations may be solved directly for linear response**

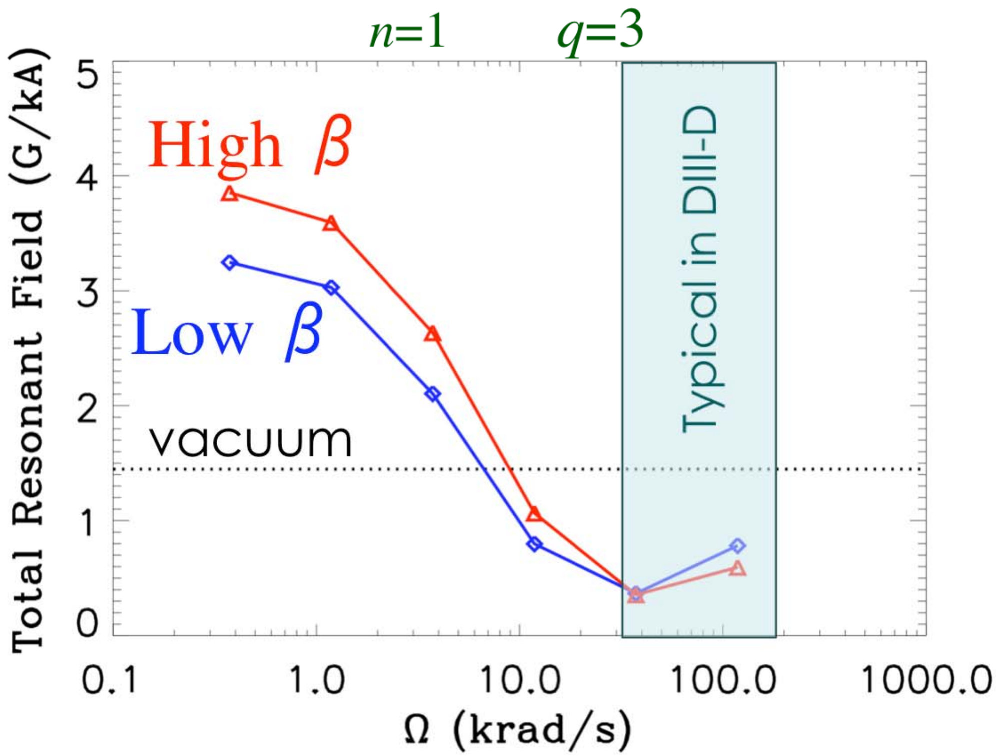
Analysis Considers Reconstructed DIII-D Equilibria

- **Vacuum fields generated by DIII-D I-coils**
- **Boundary conditions**
  - Vacuum B_n is held constant at the boundary
  - No-slip ($\mathbf{v}=0$)
- **Realistic transport parameters**
  - Lundquist number ~10^9
- **Toroidal rotation**
  - Rotation is added self-consistently: $p \neq p(\psi)$



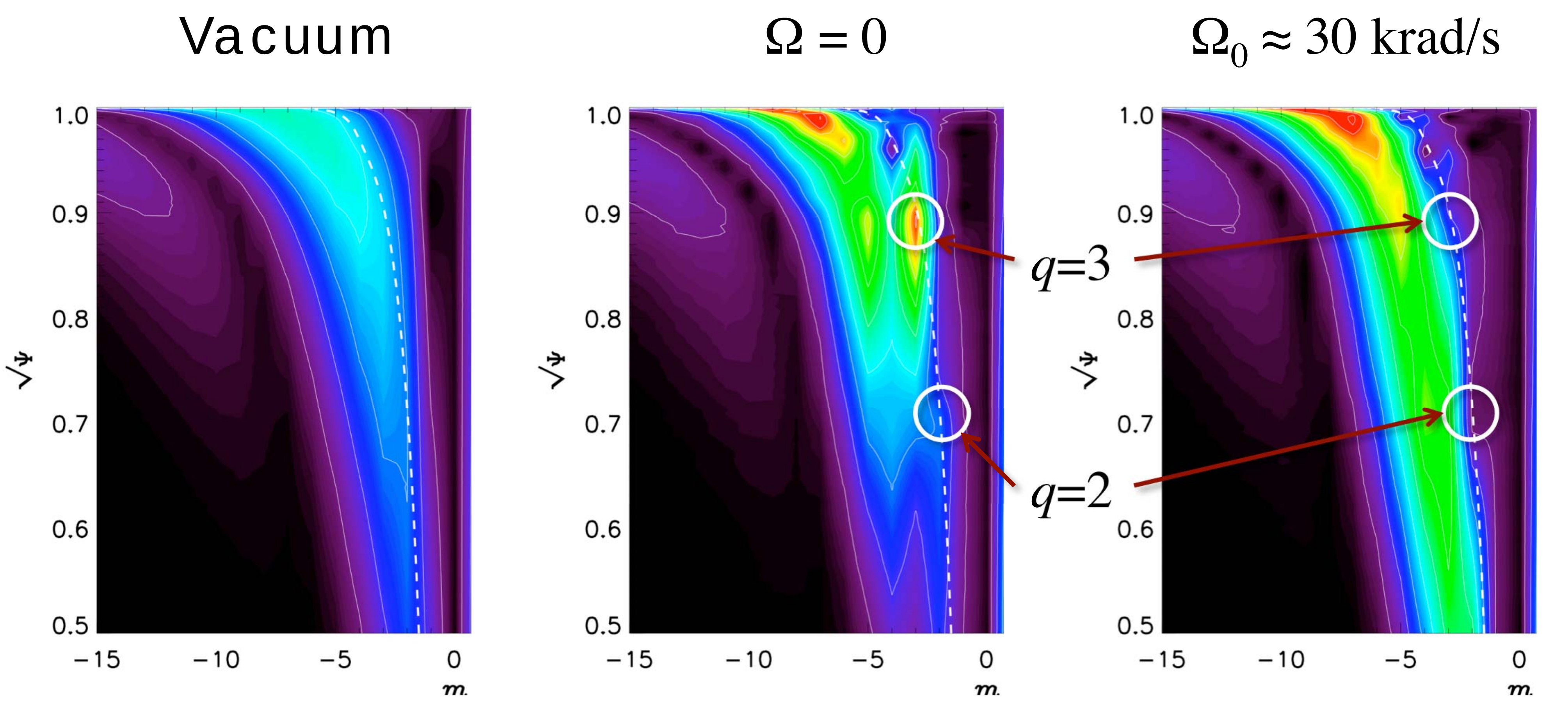
Single-Fluid Result —
Rotation (Usually) Improves Screening





- **Plasma may enhance resonant fields at low rotation**
- **Large rotation screens resonant fields**
- **Response depends on beta**

Tearing of 3/1 and 2/1 Surfaces is Driven by External Fields, But Suppressed by Rotation



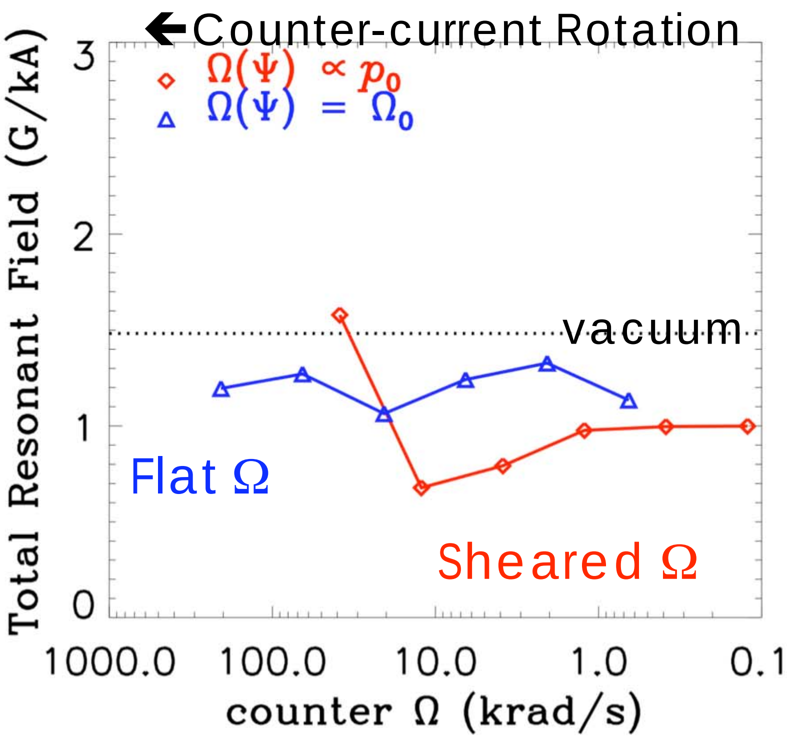
n=1

Why is Plasma Response Sensitive to Rotation? Why is it Sensitive to Beta?

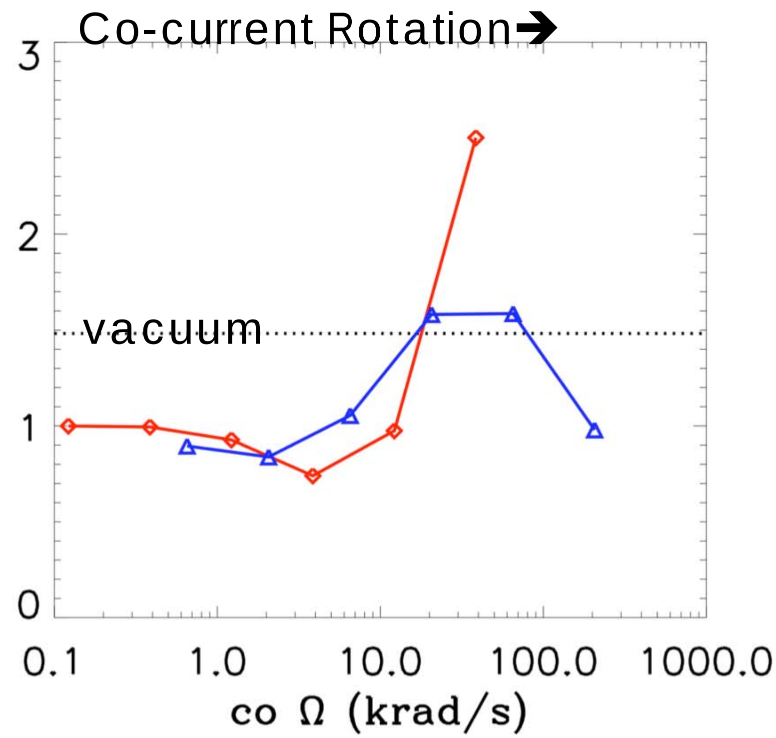
- **From a (rotating) plasma's perspective, the static external fields are oscillating**
  - If field is oscillating faster than tearing response, plasma won't tear
- **Rotation drives static tearing modes away from marginal stability**
- **Higher beta moves modes closer to marginal stability**
  - At marginal stability, an infinitesimal perturbation yields an infinite response

Single-Fluid Result —
Rotation Shear Increases Edge Response

DIII-D 135762



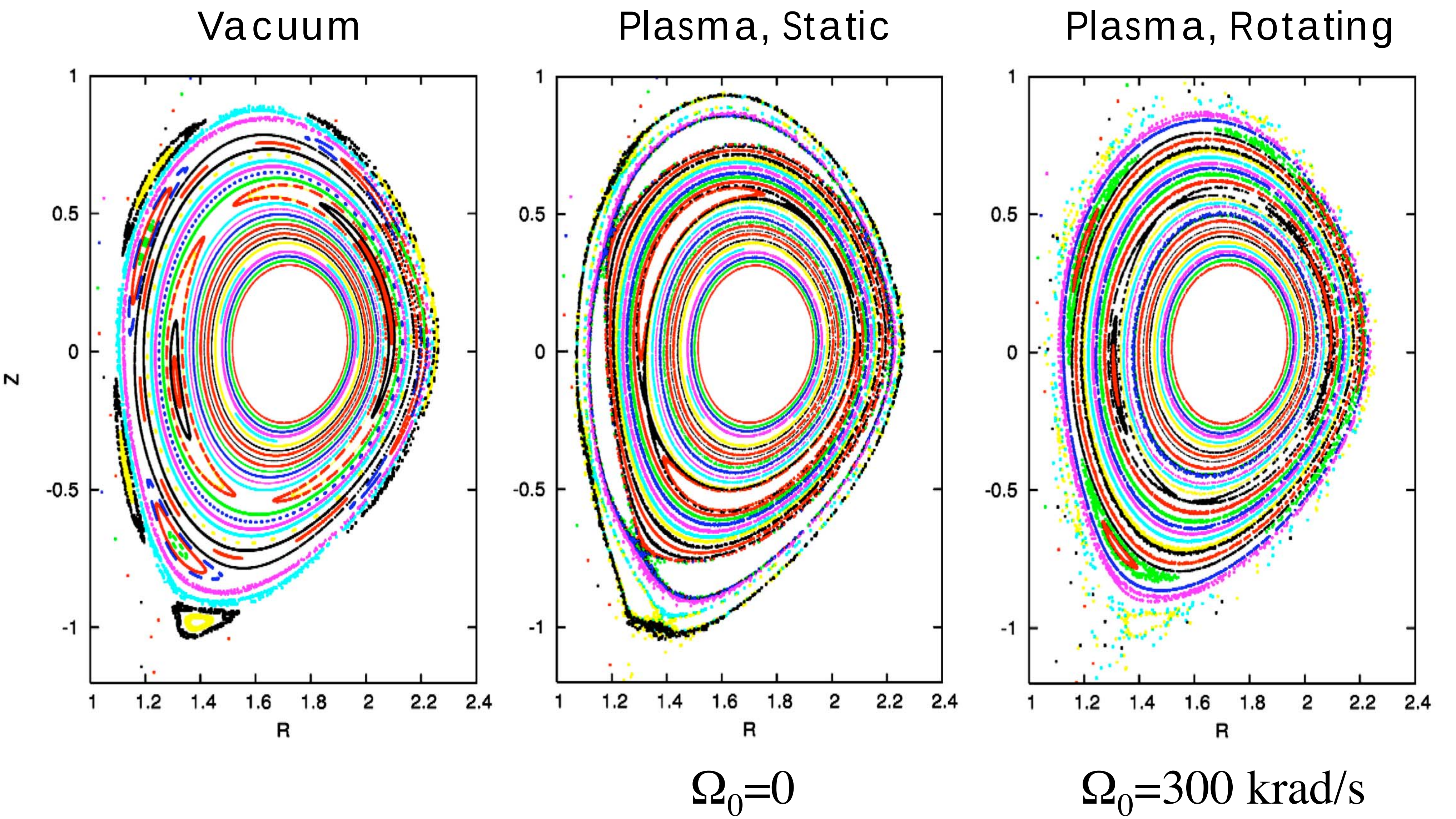
q=5



- **Large rotation shear seems to increase edge response**
- **Why? Theory predicts Ω' is destabilizing to low-*n* peeling-ballooning modes***

*Snyder, *et al., Nucl. Fusion* **47** (2007); Aiba, *et al., Nucl. Fusion* **50** (2010); Ferraro, *et al., Phys. Plasmas* **17** (2010)

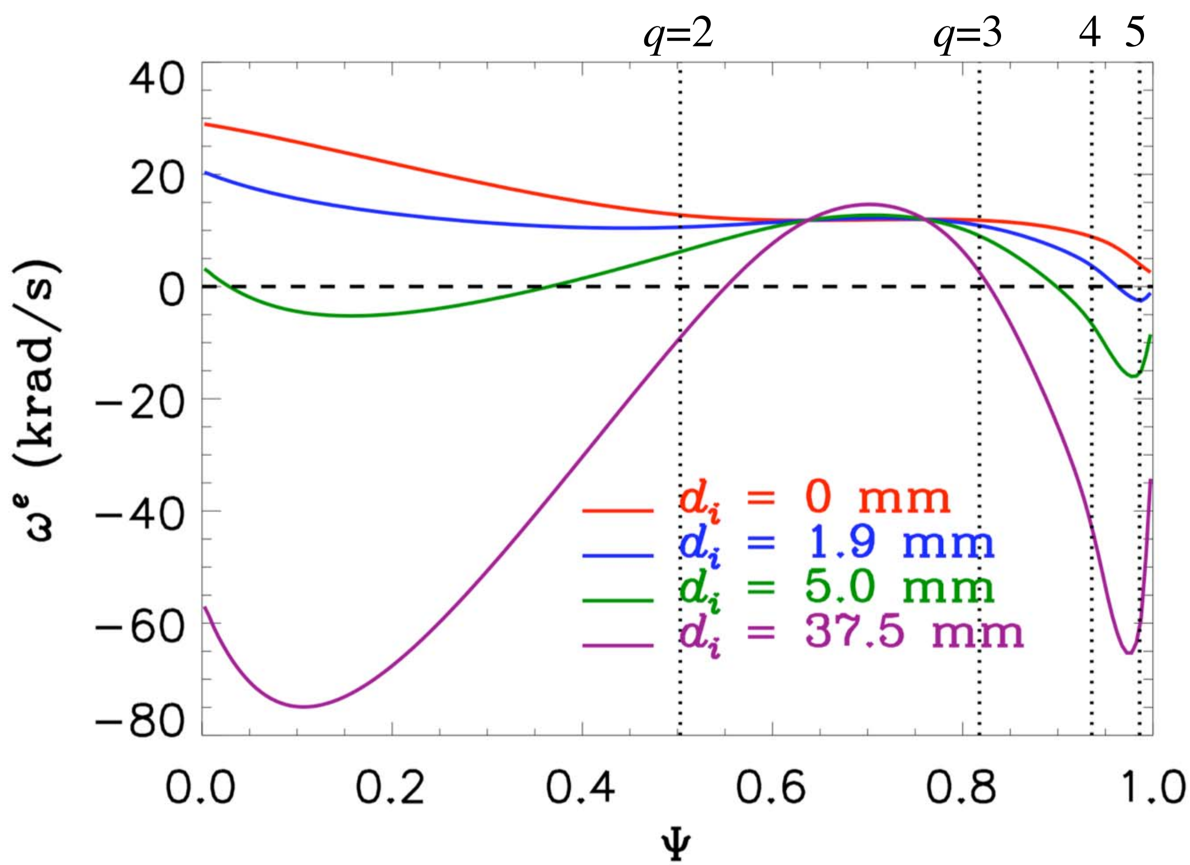
Rotation Improves Core Screening; But Sheared Rotation Stochasticizes Edge



Two-Fluid Results

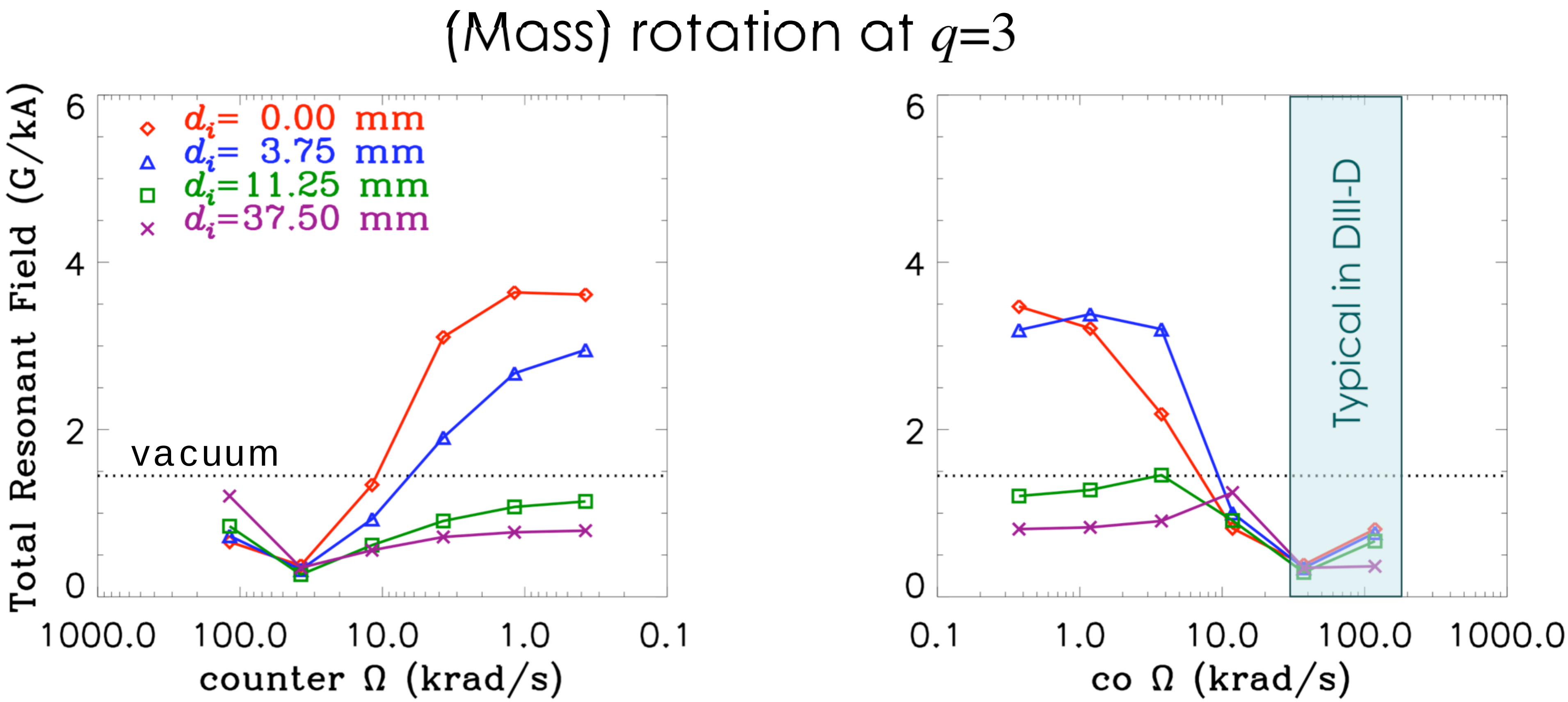
Two-Fluid Results — Ion and Electron Rotations are Distinct

- **Given Ω, we can change $\Omega^e=\Omega+\omega_*$ by adjusting** $\omega_*=d_i\, p'/n$



For this equilbrium, $d_i = 37.5$ mm is the physical value

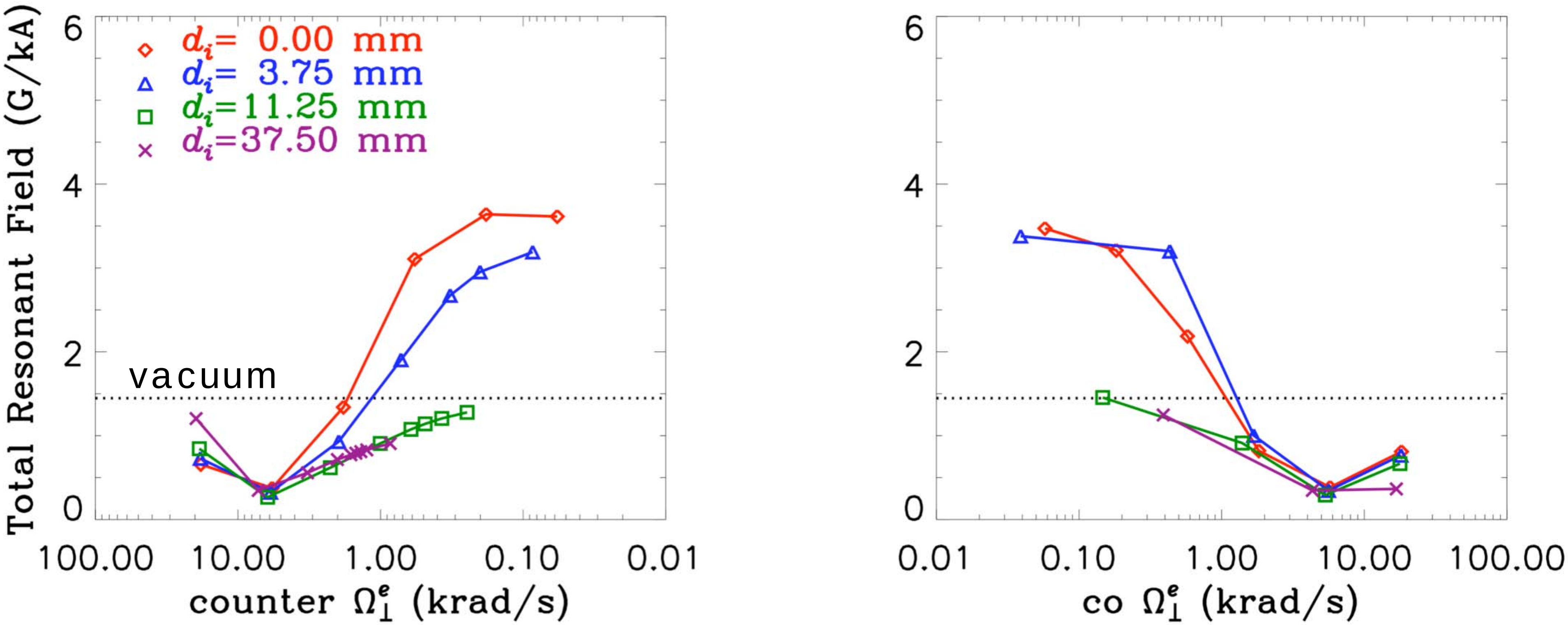
Two-Fluid Effects Shift Resonance



- **Strongest tearing no longer occurs at** $\Omega = 0$

Penetration In Core Depends on Electron Rotation

Perpendicular electron rotation at $q = 3$



- **Screening of q=3 island clearly depends more on Ω^e than Ω**

What is “Perpendicular” Electron Velocity?

- **The perpendicular angular velocity is defined as**

$$\Omega_\perp^{e,i} = \frac{\mathbf{v}^{e,i}}{R} \cdot \frac{\mathbf{B} \times \nabla\psi}{|\mathbf{B} \times \nabla\psi|}$$

- **To lowest order,** $\mathbf{v}^{e,i} = R^2\omega^{e,i}(\psi)\nabla\varphi + \lambda^{e,i}\mathbf{B}$**. Thus:**

$$\Omega_\perp^{e,i} = \frac{|\nabla\psi \times \nabla\varphi|}{|B|}\omega^{e,i}(\psi)$$

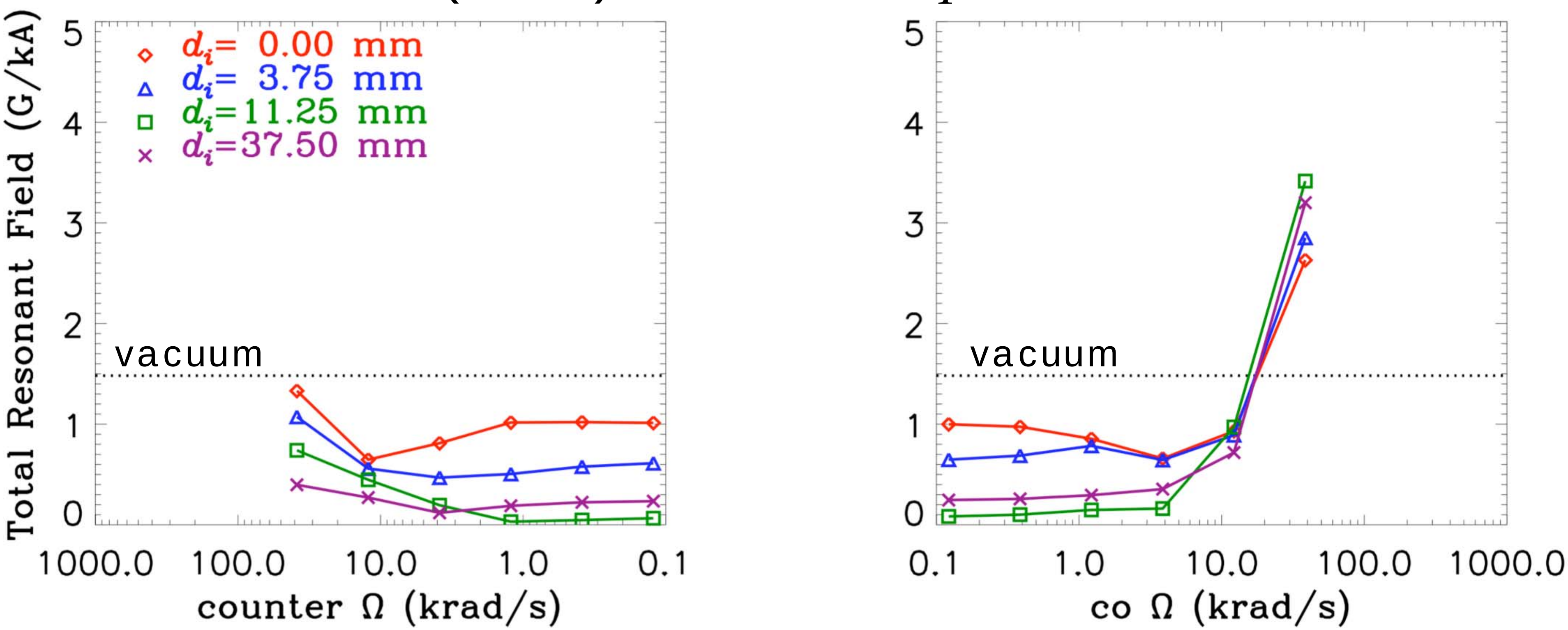
- **From radial force balance:** $\omega^{e,i}(\psi) = \phi'(\psi) + \dfrac{p_{e,i}{}'(\psi)}{n_{e,i}q_{e,i}}$

- $\Omega_\perp^{e}$ **vanishes wherever** ω^{e} **vanishes, but also at** $\mathbf{B}_{pol}$ **nulls**

Edge Response Depends on Mass Rotation Shear

- **Tearing of edge modes is dependent on ion, not electron, rotation shear**

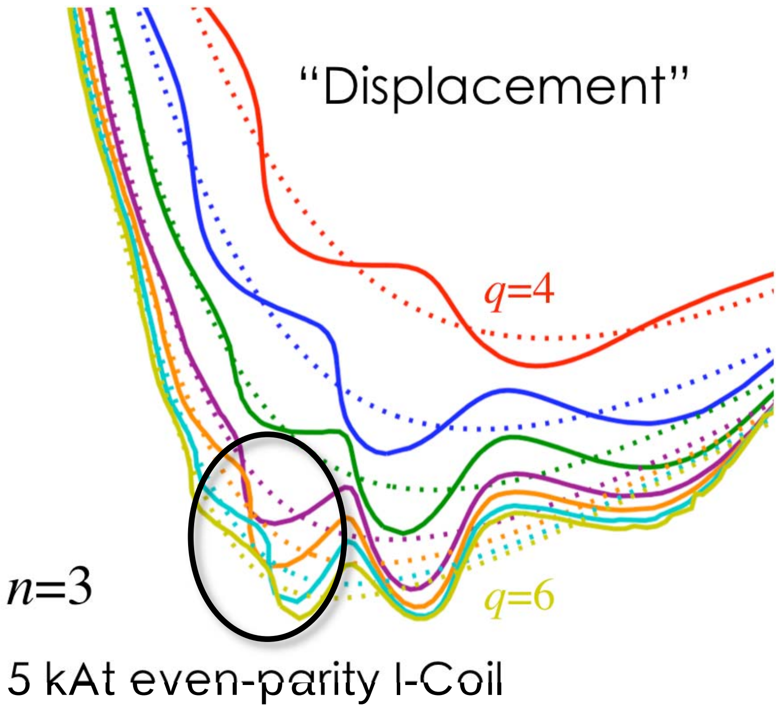
(Mass) rotation at q=5



Linear vs. Nonlinear

Is Linear Response Appropriate?

- **For typical experimental parameters, linear response may not be strictly valid in some regions**
  - Large current density near rational surfaces
  - Back-reaction on rotation is important

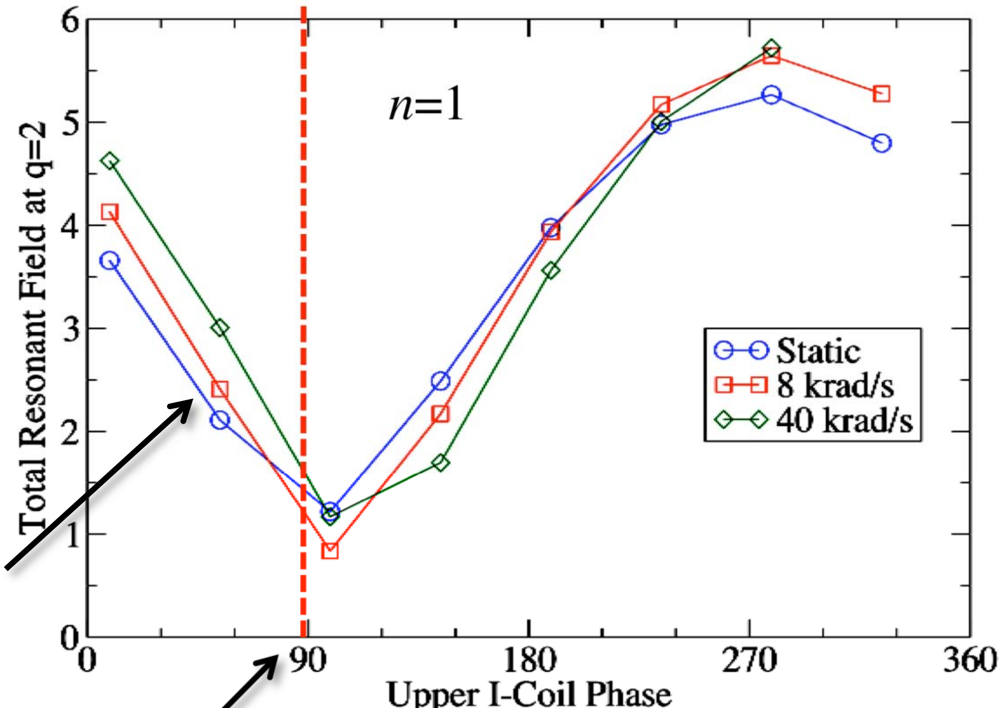


  - "Displacement" shows overlapping surfaces near separatrix!

- **Quantitative predictions of island size, stochasticity from linear calculations are suspect**

Linear Response Gets Some Things Right

- **Which modes are most sensitive**
- **How parameters (rotation, viscosity, etc.) affect sensitivity**
- **How to optimize coil design**



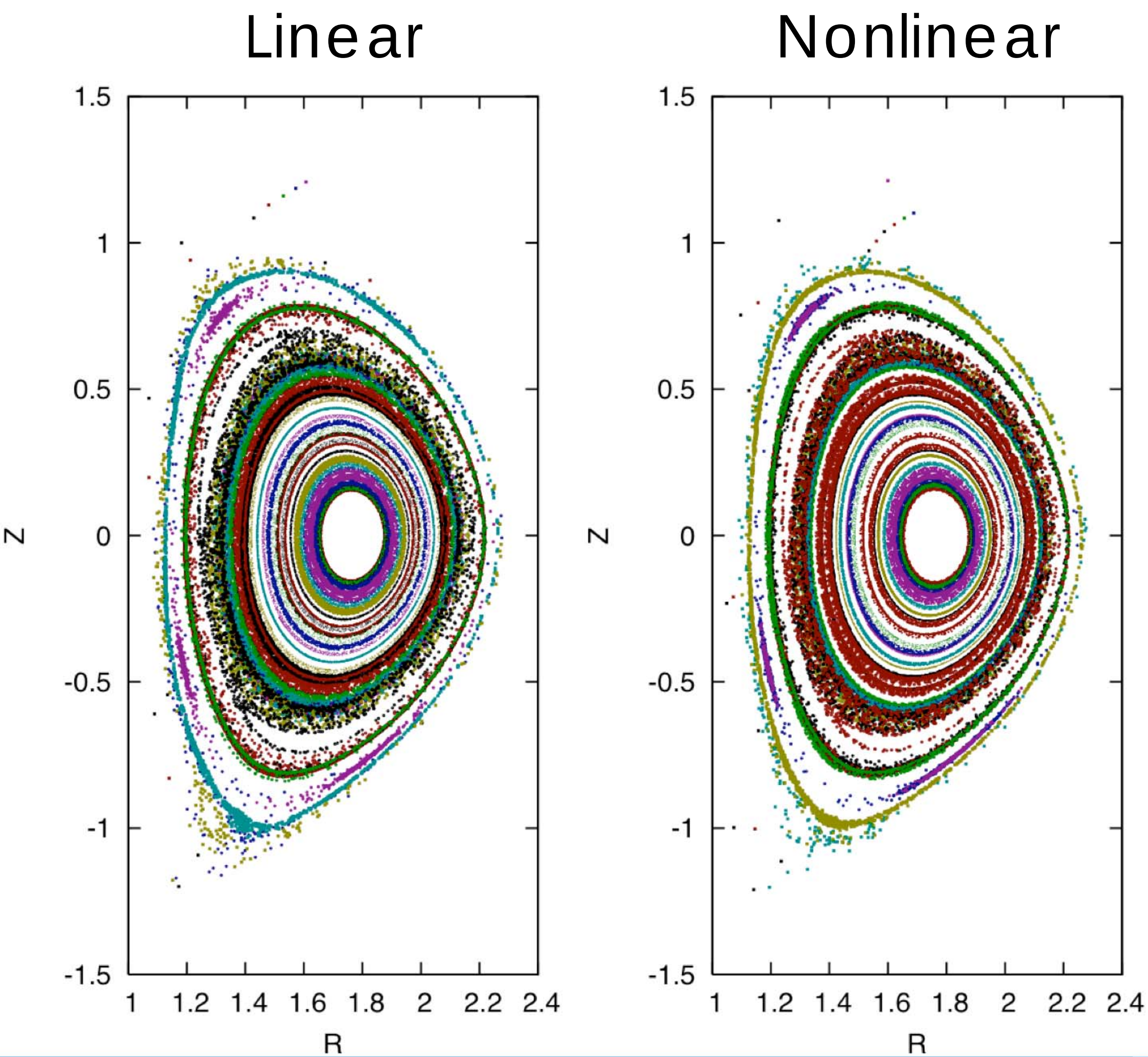
Calculated resonant field (proxy for resonant torque)

Empirical phase least prone to locking

DIII-D 144182

Nonlinear Calculations are Underway

- **Nonlinear calculations are necessary for some things**
  - Rotation/locking
  - Transport
  - Large islands
- **Preliminary non-linear results agree with linear results for non-rotating plasma**



Summary

- **We can calculate resistive two-fluid plasma response for diverted equilibria with realistic transport parameters**

- **Rotation (usually) improves screening**

- **Perpendicular electron velocity is the most relevant rotation for core islands**
  - ELM suppression may correlate with intersection of $\omega^e \approx 0$ and rational surface (q_{95} windows?)

- **(Mass) rotation seems to enhance edge response**
  - Edge rotation may be crucial to ELM suppression (depending on mechanism)

Challenges to Understanding Plasma Response Remain

- **Mode locking depends sensitively on scaling of viscous torques**
- **ELM suppression requires interplay between field response and transport (probably)**
- **Need better understanding of transport in 3D fields**
- **3D equilibrium properly requires nonlinear calculation**
  - $n = 0$ rotation and $n \neq 0$ response are strongly coupled
  - Island saturation is nonlinear
  - There is healthy debate how to do this efficiently!
- **We are moving quickly to overcome these challenges**