RMP Effects on Pedestal Structure and ELMs

by

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with
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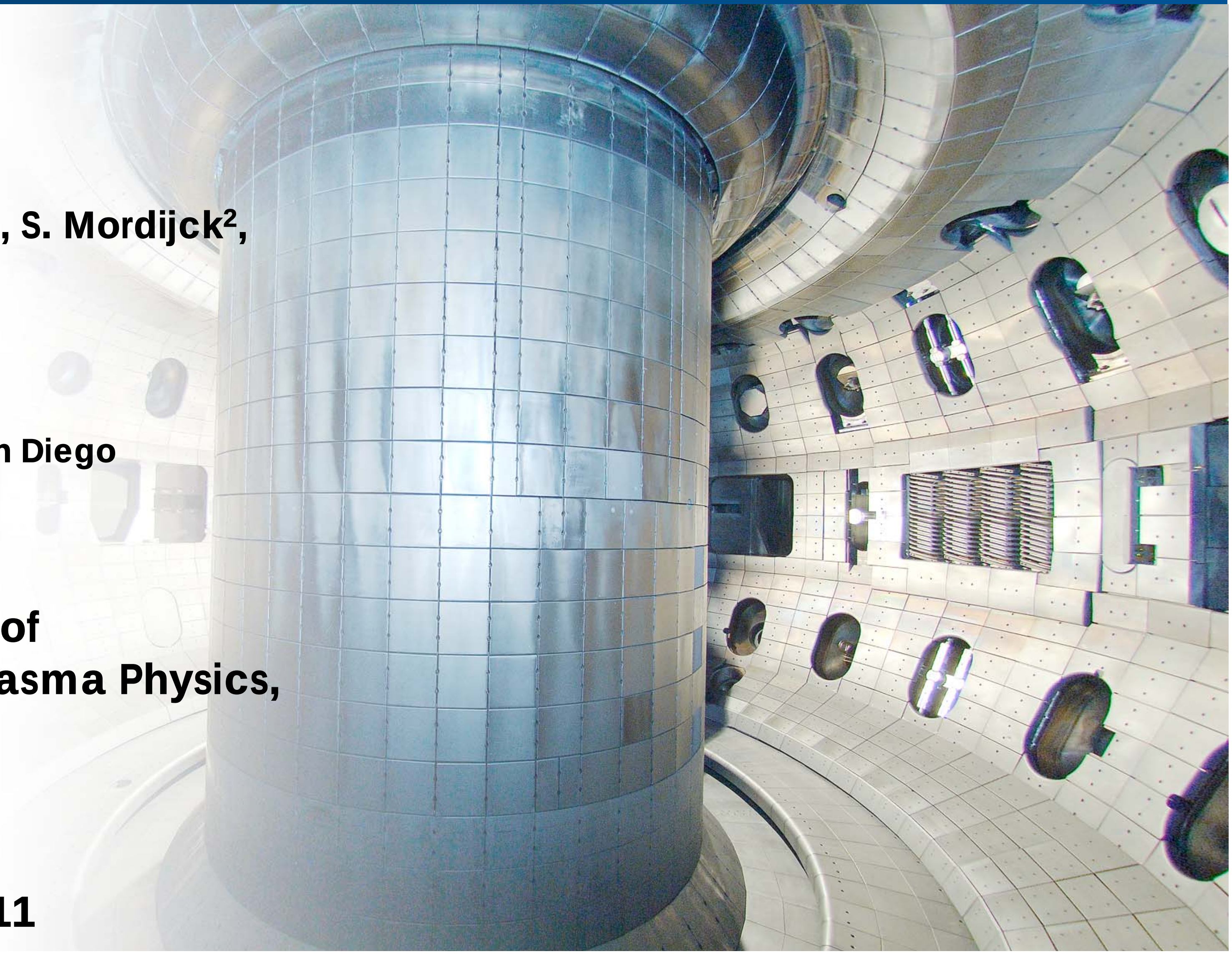

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Issue to be Addressed

- **If flow screening of resonant magnetic perturbations (RMPs) prevents island and stochasticity, how do RMPs suppress ELMs?**
- **Theses**
  - Reduction of $|\vec{\nabla} P|$ at top of pedestal is key
  - RMPs reduce $|\vec{\nabla} T_e|$ more than $|\vec{\nabla} n_e|$ at pedestal top
  - Reductions are proportional to I_{coil}^2
  - RMP-induced “magnetic flutter” transport might do this

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RMPs Reduce Pressure Gradient at Pedestal Top

- RMP-induced $|\vec{\nabla} P|$ reductions are:

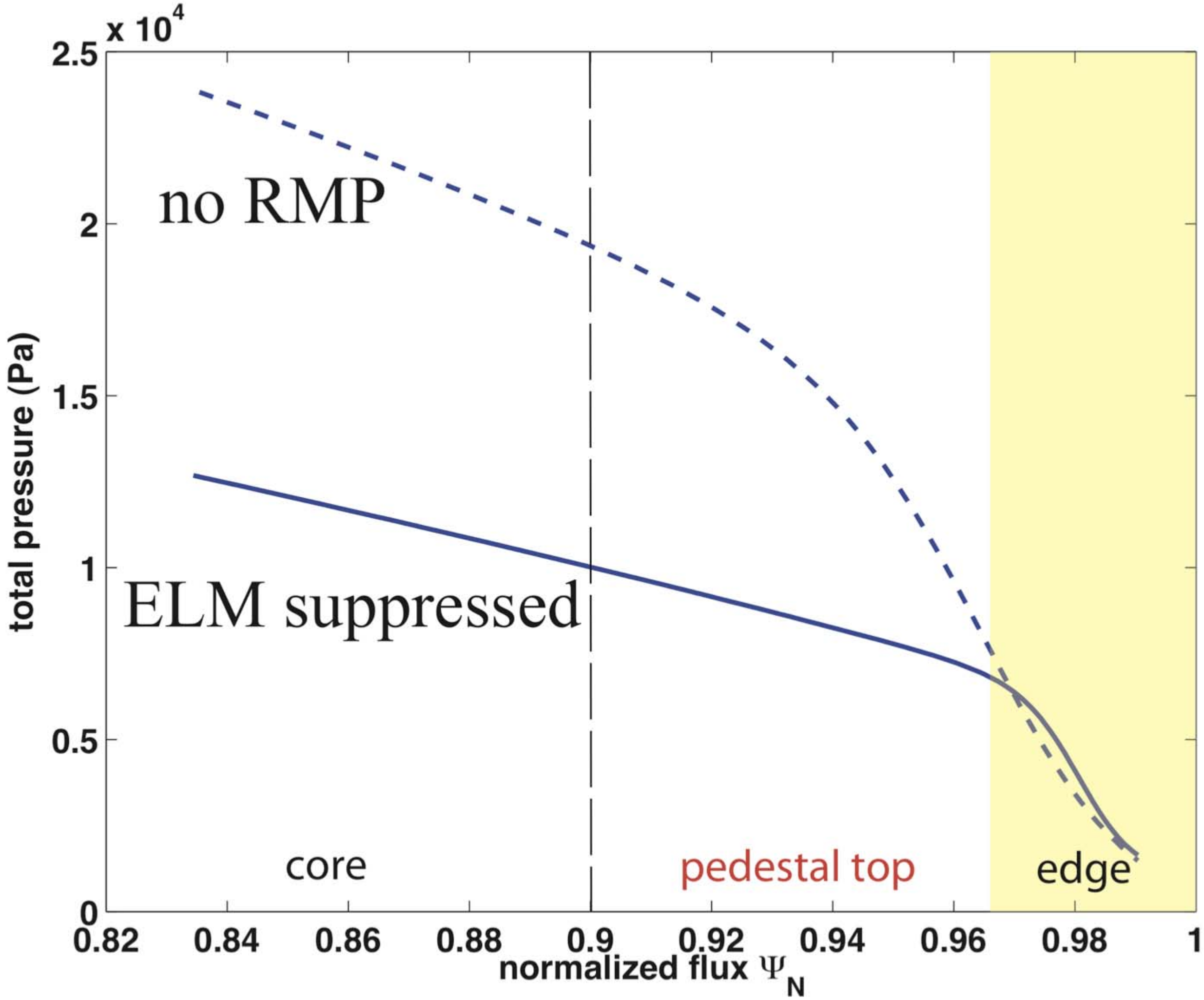
  small in core,

  largest at top of the pedestal, and

  small (increase) at the edge.

- Key transport issue for ELM suppression is:

  What causes the reduced $|\vec{\nabla} P|$ at pedestal top?



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RMPs Decrease T_e, n_e Gradients at Pedestal Top

- Ratio of T_e, n_e gradients wo (sym) to with RMPs indicate increases in χ_e, D:

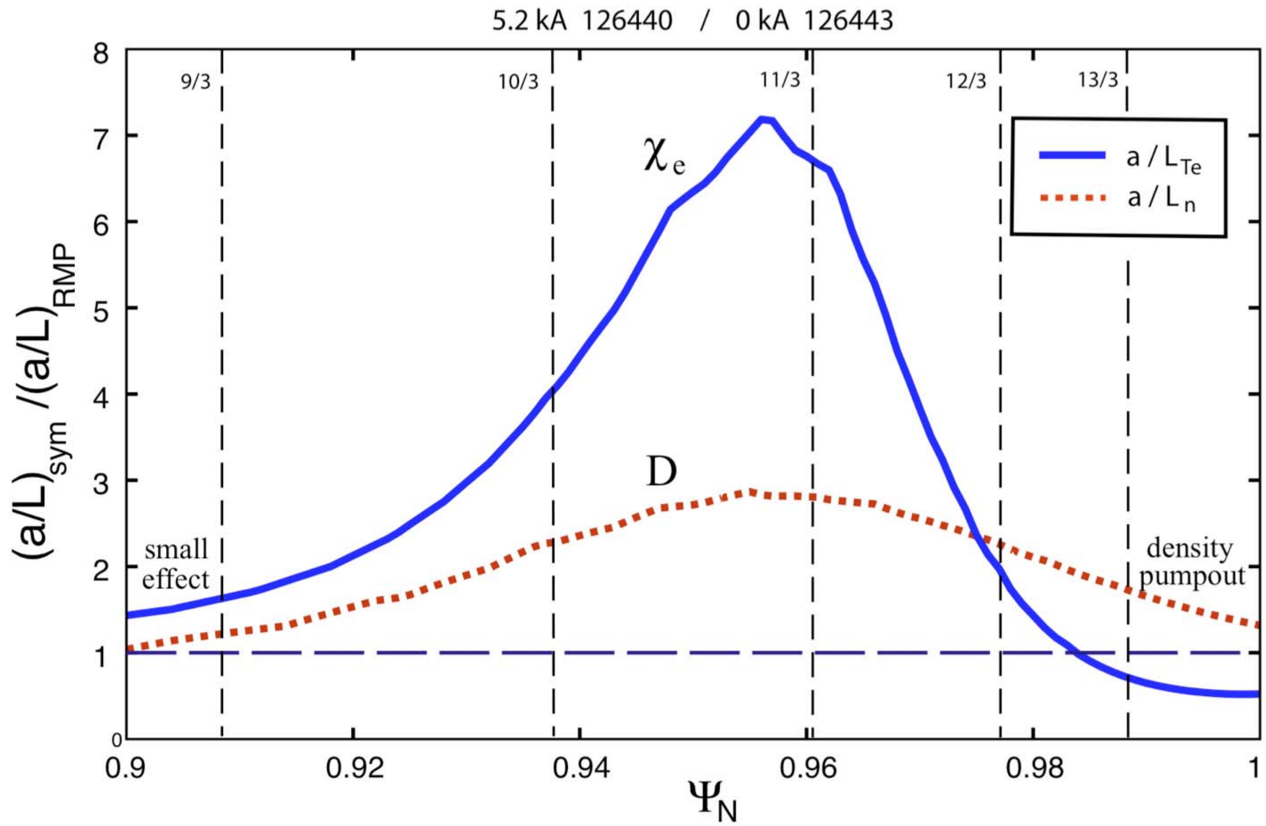
$$\boxed{\frac{|\vec{\nabla} T_e|_{\text{sym}}}{|\vec{\nabla} T_e|_{\text{RMP}}} = \frac{[a/L_{Te}]_{\text{sym}}}{[a/L_{Te}]_{\text{RMP}}} \simeq \frac{\chi_e^{\text{sym}} + \chi_e^{\text{RMP}}}{\chi_e^{\text{sym}}};}$$ similarly for D^{RMP}.

- RMPs increase:

  χ_e by $\lesssim \times 7$,

  D by $\lesssim \times 3$.

- Peak change is at $\Psi_N \sim 0.95$.



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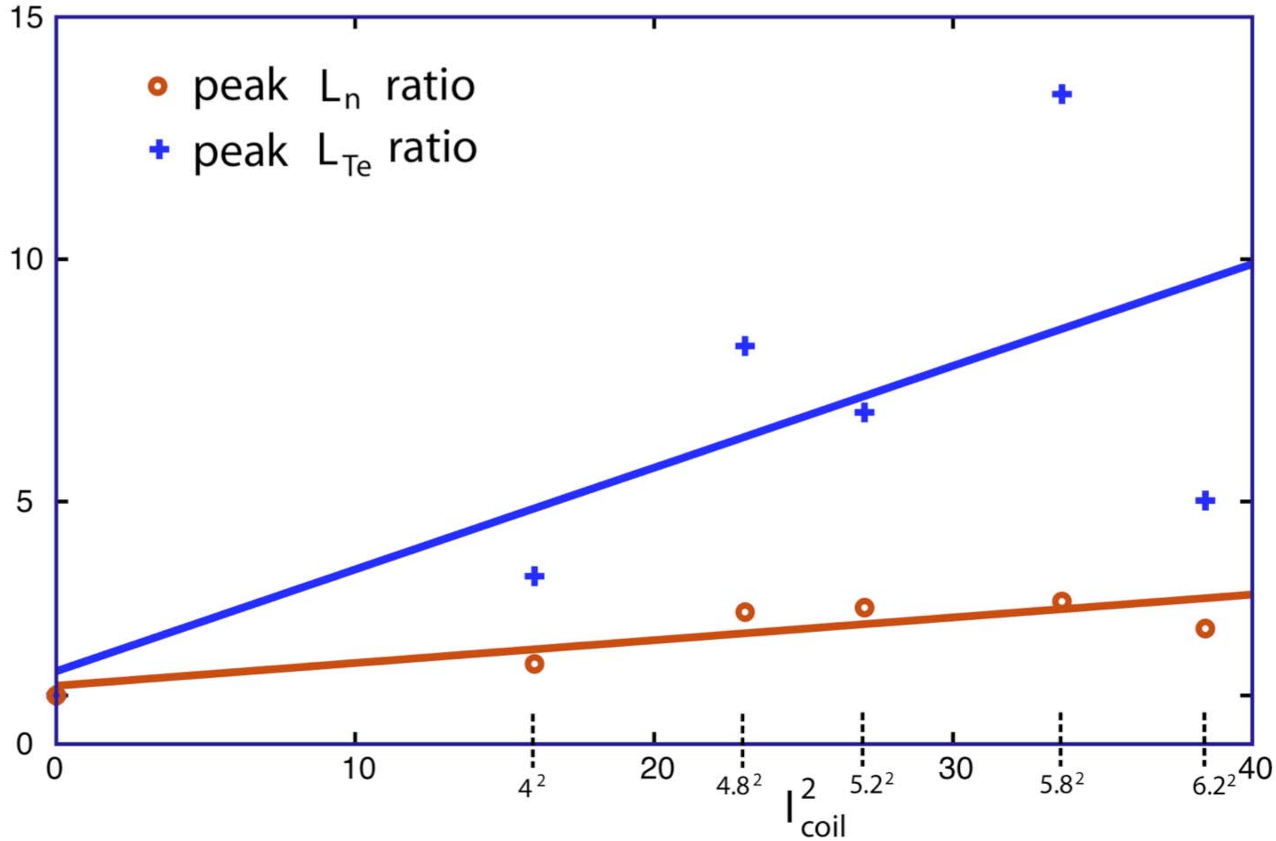
Peak of RMP-Induced Extra Transport $\propto I_{\mathrm{coil}}^2$

- Peak T_e, n_e gradient ratios scale approximately with I_{coil}^2.
- Peak L_{Te} ratio increases $\sim 3\times$ more than peak L_n ratio does, which indicates $D^{\mathrm{RMP}}/\chi_e^{\mathrm{RMP}} \sim 1/3$.



Transport Effects of RMPs: Flutter or Stochasticity?

- RMP-induced radial (ρ) magnetic perturbations δB_ρ:

  mostly just non-resonantly spatially "flutter" the field lines, flux surfaces,

  but can induce stochasticity if islands are created and overlap (Chirikov).

- Transport can be induced by magnetic flutter and stochasticity:

  flutter causes[1] $\chi_e^{\mathrm{RMP}} \sim v_{Te}\,\lambda_e(\delta B_\rho/B_0)^2$ via collisional $\|$ T_e transport,

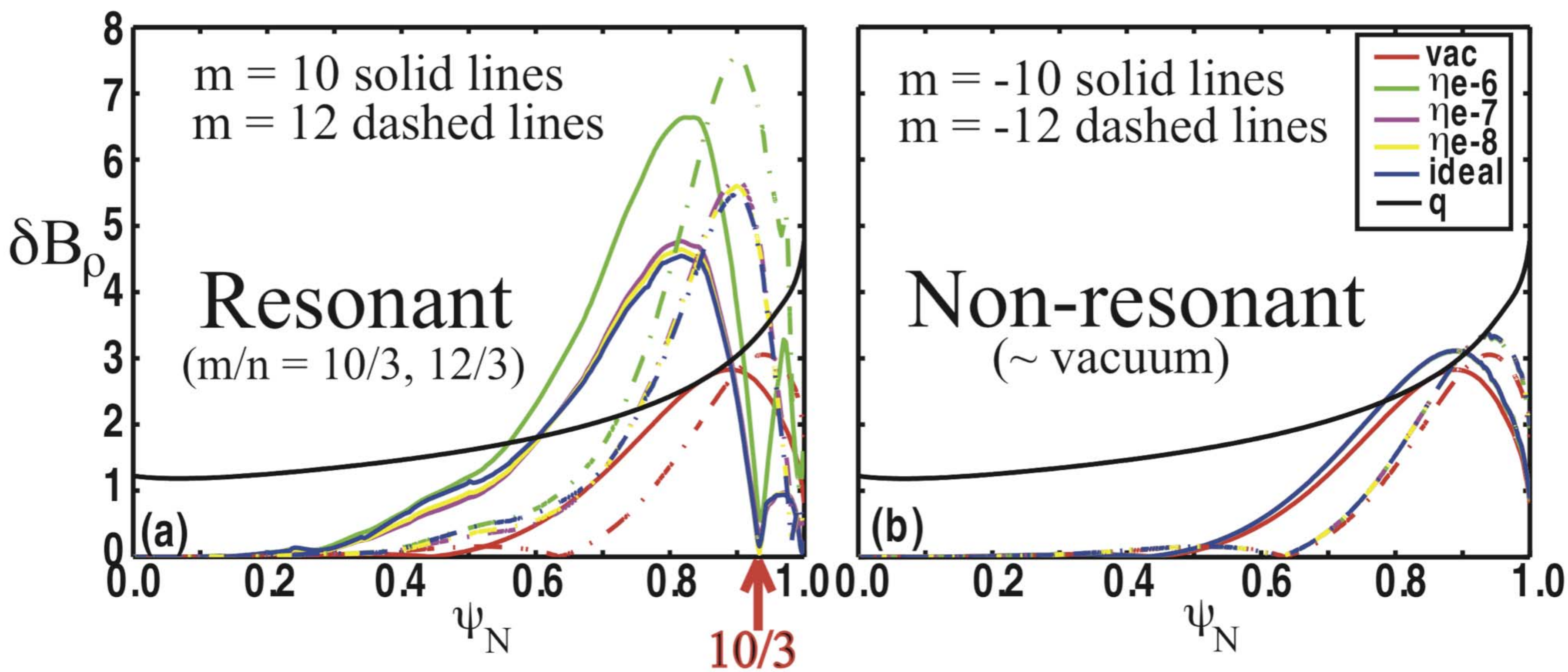
  stochasticity causes (RR[2]) $\chi_e^{\mathrm{RMP}} \sim v_{Te}\,L_{\|c}(\delta B_{\mathrm{st}}/B_0)^2$ via $\|$ motion along $\vec{B}_{\mathrm{st}}$.

- But flow screening of RMP fields inhibits reconnection, island formation & overlap, and hence stochasticity — next viewgraph.

[1]J.D. Callen, "Drift-Wave Turbulence Effects on Magnetic Structure and Plasma Transport in Tokamaks," Phys. Rev. Lett. **39**, 1540 (1977).
[2]A.B. Rechester and M.N. Rosenbluth, "Electron heat transport in a tokamak with destroyed magnetic surfaces," Phys. Rev. Lett. **40**, 38 (1978).

Flow Screening Reduces δB at Regional Surfaces, But RMPs Still Induce δBs at Pedestal Top

- RMP-induced δB_ρs peaked near pedestal top (Chu, NF 2011).
- Resonant δB_ρs "screened" at 10/3 surface $\Longrightarrow$ no island there.
- Ferraro paper JI2.002 at 2:30 pm will discuss 2-fluid screening.



Can Flutter Transport Reduce $|\vec{\nabla} P|$ at Pedestal Top?

- **Flow screening probably prevents islands and stochasticity.**
- **So how large could flutter-induced plasma transport be?**

Represent magnetic field by axiymmetric $\vec{B}_0$ plus RMP field: $\vec{B} = \vec{B}_0 + \delta\vec{B}$.

Radial perturbation $\delta\hat{B}_{\rho mn}\cos(m\theta - n\zeta)$ induces $\|$ heat flow along $\vec{B}_0$:

$$\delta q_{\| mn} \equiv -n\,\chi_{\|}\left(\frac{\vec{B}}{B}\cdot\vec{\nabla}T\right) = -n\,\chi_{\|}\,\frac{\delta\hat{B}_{\rho}}{B_0}\cos(m\theta - n\zeta)\,\frac{dT}{d\rho}.$$

Average radial $(\hat{\vec{e}}_\rho\,\cdot)$ heat flow induced by particle motion along $\vec{B}$ is

$$\langle\hat{\vec{e}}_\rho\cdot\vec{q}\rangle \equiv \langle\int d^3v\left(\frac{mv^2}{2T}-\frac{5}{2}\right)\left(v_{\|}\frac{\hat{\vec{e}}_\rho\cdot(\vec{B}_0+\delta\vec{B})}{B}\right)\delta f\,\rangle = \langle\,\delta\hat{B}_{\rho mn}\cos(m\theta - n\zeta)\,\delta q_{\| mn}\,\rangle.$$

This results in an effective radial electron heat diffusivity of[1]

$$\boxed{\chi_{mn} = \left(\frac{\delta\hat{B}_{\rho mn}}{B_0}\right)^2\frac{\chi_{\| mn}}{2},}$$

which is to be summed over all mn RMPs.

For one mn $\dfrac{\chi_{e\| mn}}{2} \lesssim 10^8\,\dfrac{\mathrm{m}^2}{\mathrm{s}}$, so need $\dfrac{\delta\hat{B}_{\rho mn}}{B_0} \gtrsim 10^{-4}$ to yield $\chi_{emn} \sim 1\,\dfrac{\mathrm{m}^2}{\mathrm{s}}$.

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RMP-Induced Flutter Modifies Electron Distribution

- Neglecting drifts, electron drift kinetic equation (DKE) for $\vec{B} \to \vec{B}_0 + \delta\vec{B}$ is[1,3]

$$\frac{\partial f_e}{\partial t} + \frac{v_\parallel}{B}(\vec{B}_0 + \delta\vec{B})\cdot\vec{\nabla} f_e + \frac{d\varepsilon}{dt}\frac{\partial f_e}{\partial \varepsilon} = \mathcal{C}\{f_e\}, \quad \text{in which } \varepsilon = \frac{m_e v_\parallel^2}{2} + \mu B - e\Phi.$$

- Lowest order solution is a Maxwellian constant along $\vec{B}_0$: $f_e = f_{\mathrm{M}e}(\rho, \varepsilon)$.

- Equation for first order non-adiabatic distribution $\delta\check{h} \equiv \delta\check{f}_e - (e/T_e)\,\delta\check{\Phi}\, f_{\mathrm{M}e}$ induced by a magnetic perturbation $\delta\vec{B}_{mn} \equiv \delta\check{\vec{B}}_{mn}(\rho,\theta)\, e^{-i(n\zeta - m\theta + \varphi_{mn})}$ is[3]

$$v_\parallel\left(\frac{\partial}{\partial \ell} + i(m - nq)\frac{d\theta}{d\ell}\right)\delta\check{h} - \mathcal{C}\{\delta\check{h}\} = -\frac{v_\parallel}{B_0}\,\delta\check{\vec{B}}_{mn}\cdot\vec{\nabla} f_{\mathrm{M}e}, \qquad \text{in which } d\ell/B_0 = d\theta/\vec{B}_0\cdot\vec{\nabla}\theta.$$

- Trapped particle solution $\delta\check{h}_t$ solution vanishes — bounce average yields no drive since trapped particles don't carry any parallel flow over $\ell \gg 2\pi R_0 q$.

- Magnetic flutter (in space) drive on right causes untrapped electron $\delta\check{h}_u$ which induces electron parallel density and heat flows, radial fluxes ($\Gamma_e \sim n_e \delta V_{e\parallel} \delta B_\rho$).

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[3]J.D. Callen, A.J. Cole and C.C. Hegna, "RMP effects on pedestal plasma transport," UW-CPTC 11-13, to be at `http://www.cptc.wisc.edu` "soon."

Flutter Induces || Flows and Radial Transport Fluxes

- Parallel flows induced by mn RMP field is ($L_0^{3/2} = 1,\ L_1^{3/2} = m_e v^2/2T_e - 5/2$)

$$\begin{bmatrix} \langle n_e\,\delta\hat{V}_{e\|mn}\rangle \\ \langle \delta\hat{q}_{e\|mn}\rangle \end{bmatrix} = -\,f_c\frac{v_{Te}^2}{\nu_e}\frac{\delta\hat{B}_{\rho mn}(x)B_{\max}}{\langle B_0^2\rangle}\frac{1}{3\sqrt{\pi}}\left(\int_0^{1/X^{1/2}}\!dy\,y^3e^{-y}+\frac{c_{\|t}}{X^{3/2}}\int_{1/X^{1/2}}^{\infty}\!dy\,e^{-y}\right)\begin{bmatrix} L_0^{3/2} \\ T_e L_1^{3/2}\end{bmatrix}\frac{d\ln f_{Me}}{d\rho}$$

$$\equiv -\,\frac{\delta\hat{B}_{\rho mn}(\Delta_{\rm t})\,B_{\max}}{\langle B_0^2\rangle}\,n_e\begin{bmatrix} D_{mn}^p & D_{mn}^T \\ \chi_{mn}^p & \chi_{mn}^T\end{bmatrix}\cdot\begin{bmatrix} d\ln\hat{p}_e/d\rho \\ dT_e/d\rho\end{bmatrix},$$

in which $f_c \equiv \dfrac{3}{4}\dfrac{\langle B_0^2\rangle}{B_{\max}^2}\displaystyle\int_0^1\frac{\lambda\,d\lambda}{\langle\sqrt{1-\lambda\,B_0/B_{\max}}\,\rangle},\ c_{\|t}\equiv\frac{3\,\langle B_0^2\rangle/B_{\max}^2}{16 f_c\langle v_\| |_{\lambda=1}/v\rangle},\ \frac{d\ln\hat{p}_e}{d\rho}=\frac{d\ln p_e}{d\rho}-\frac{e}{T_e}\frac{d\Phi_0}{d\rho}.$

- The parallel flow and heat flow cause corresponding radial transport fluxes:[3]

$$\begin{bmatrix}\Gamma_e \\ \Upsilon_e\end{bmatrix}\equiv\begin{bmatrix}\langle\Gamma_e\cdot\vec{\nabla}\rho\rangle \\ \langle\vec{q}_e\cdot\vec{\nabla}\rho\rangle\end{bmatrix}=\sum_{mn}\frac{\delta\hat{B}_{\rho mn}}{B_{\max}}\begin{bmatrix}\langle n_e\delta\hat{V}_{e\|mn}\rangle \\ \langle\delta\hat{q}_{e\|mn}\rangle\end{bmatrix}=-\sum_{mn}\frac{\delta\hat{B}_{\rho mn}^2}{\langle B_0^2\rangle}\,n_e\begin{bmatrix} D_{mn}^p & D_{mn}^T \\ \chi_{mn}^p & \chi_{mn}^T\end{bmatrix}\cdot\begin{bmatrix} d\ln\hat{p}_e/d\rho \\ dT_e/d\rho\end{bmatrix}.$$

- Some key quantities and magnitudes in these results are:

$$\boxed{\chi_e^{\rm RMP}\equiv\sum_{mn}\frac{\delta\hat{B}_{\rho mn}^2}{\langle B_0^2\rangle}\chi_{\rm mn}^T,\quad \chi_{mn}^T\equiv\chi_{\rm ref}\,F_{mn}(x/\Delta_{\rm t}),}\qquad \boxed{\frac{D_{mn}^p}{\chi_{mn}^T}\lesssim\frac{1}{3},}\qquad \boxed{\chi_{\rm ref}=f_c\frac{v_{Te}^2}{\nu_e},}$$

$$F_{mn}(X)\equiv\frac{1}{3\sqrt{\pi}}\left(\int_0^{1/X^{1/2}}\!dy\,y^3e^{-y}\left[y-\frac{5}{2}\right]^2+\frac{c_{\|t}}{X^{3/2}}\int_{1/X^{1/2}}^{\infty}\!dy\,e^{-y}\left[y-\frac{5}{2}\right]^2\right)\overset{X\gg1}{\simeq}\frac{0.4}{X^{3/2}},\qquad X\equiv\frac{x}{\Delta_{\rm t}}.$$

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Discussion: Comparing Flutter Theory to Experiment

- **Flutter theory predictions in rough agreement with experiment:**

  scaling of $\chi_e^{\rm RMP}$ and $D^{\rm RMP}$ with $\delta B_\rho^2 \sim I_{\rm coil}^2$, and

  ratio of $D^{\rm RMP}/\chi_e^{\rm RMP} \lesssim 1/3$.

- **Peak effects may be occurring in $\Psi_N \sim 0.95$ region because**

  for $\Psi_N > 0.97$ $\|$ heat diffusivity $\chi_{e\|} \sim v_{Te}^2/\nu_e \sim T_e^{5/2}/n_e$ decreases strongly,

  for $\Psi_N < 0.93$ there may be more flow screening and reduced $\delta B_{\rho mn}(x)$?

- **But flutter $\chi_e^{\rm RMP}$ may be smaller than experiment:**

  in DIII-D $\chi_e^{\rm RMP} \sim 4.6$ m^2/s (#126440) while $\chi_e^{\rm sym} \sim 0.6$ m^2/s (#126443),

  $$\text{whereas } \chi_e^{\rm RMP} \equiv \sum_{m,n} \frac{\delta \hat{B}_{\rho mn}^2}{B_0^2} \chi_{\rm ref} F_{mn}(x) \sim (10^{-8})\,(1.75\times 10^9)(0.04) \lesssim 1\,{\rm m^2/s};$$

  however, self-consistent flow-screened $\delta \hat{B}_{\rho mn}(x)$ are needed, and summed.

Summary

- **Key effects of RMPs on pedestals are that they:**

  reduce $|\vec{\nabla} P|$ at the pedestal top $(0.9 < \Psi_N < 0.97)$,

  reduce T_e gradient there by $\lesssim \times 7$,

  reduce n_e gradient there by $\lesssim \times 3$,

  and the gradient reductions are proportional to $I_{\rm coil}^2$.

- **Flow screening keeps RMPs from forming islands, stochasticity.**

- **RMP-flutter induces radial plasma transport at pedestal top:**

$$\chi_e^{\rm RMP} \equiv \sum_{m,n} \frac{\delta \hat{B}_{\rho mn}^2}{B_0^2} \chi_{\rm ref} F_{mn}(x) \text{ too small?}, \quad \text{but} \propto I_{\rm coil}^2, \quad \frac{D^{\rm RMP}}{\chi_e^{\rm RMP}} \lesssim \frac{1}{3}.$$

- **Further development of this model is needed:**

  preliminary, simplifications, only qualitatively consistent "screening;"

  reducing T_e, n_e gradients helps to avoid P-B modes — is it enough?