Discoveries from the Exploration of Gyro-kinetic Momentum Transport

by
G.M. Staebler

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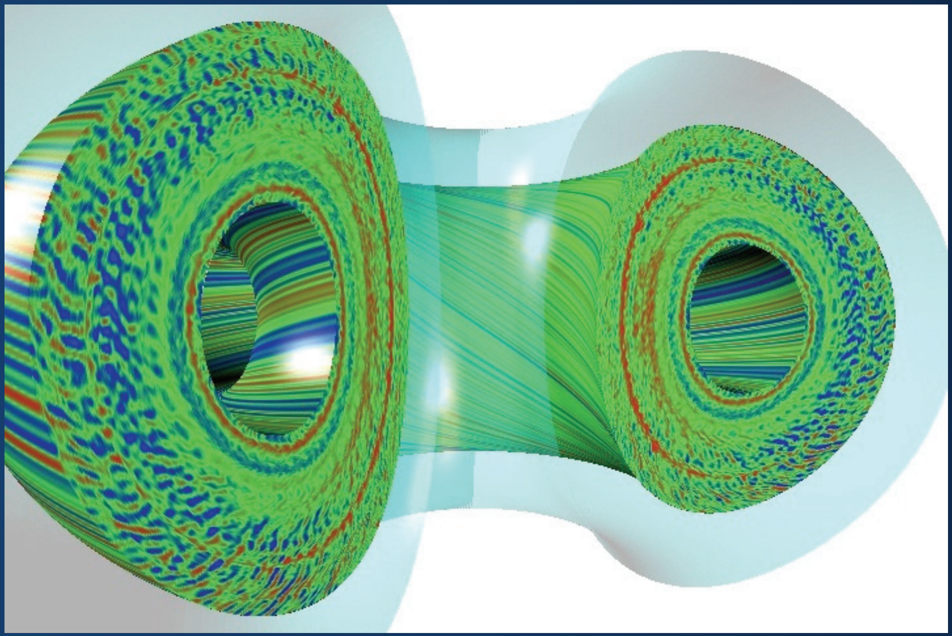

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The Momentum Transport Discoveries Reported Here Were Made While Verifying TGLF with GYRO

- **New physics was added to the Trapped gyro-Landau fluid transport model (TGLF) for momentum transport**
  - ExB velocity Doppler shift shear is included in the linear eigenmodes using a "generalized quench rule"
  - Parallel flow and flow shear
  - TGLF is a quasilinear model: linear eigenmodes + saturation model fit to nonlinear GYRO turbulence simulations
- **During the process of verification of TGLF with GYRO a deeper understanding of momentum transport was gained and a number of new and interesting properties were discovered that will be reported in this talk**

Gyro-kinetic Momentum Transport is Caused By Poloidal Parity Breaking

- **Poloidal parity of the gyro-kinetic equations refers to the combined operations:** $\theta \to -\theta, \quad v_{\|} \to -v_{\|}$
- **If the linear gyro-kinetic equation is invariant under this parity operation then the linear eigenmodes will have a definite poloidal parity (even or odd) and there will be no momentum transport due to the turbulence**

$$\Pi_{a\|} = m_a n_a \left\langle \tilde{u}_{ExB} \tilde{u}_{a\|} \right\rangle \qquad \Pi_{a\perp} = m_a n_a \left\langle \tilde{u}_{ExB} \; i \frac{c\tilde{p}_{a\perp}}{e_a n_a} \frac{\partial S}{\partial \psi} \right\rangle$$

The ballooning eikonal S is the general solution of $\vec{B} \cdot \vec{\nabla} S = 0$

$$S = n\varphi - 2\pi n \tilde{q}(\psi,\theta) + S_{\psi}(\psi) \qquad \tilde{q}(\psi,\theta) = \int_0^{\theta} \frac{\vec{B}\cdot\vec{\nabla}\varphi}{\vec{B}\cdot\vec{\nabla}\theta'} \frac{d\theta'}{2\pi}$$

hence $$\frac{\partial S}{\partial \psi} = \frac{dr}{d\psi}\left[-2\pi n \frac{\partial \tilde{q}(\psi,\theta)}{\partial r} + k_r\right] \qquad k_r = \frac{dS_{\psi}}{dr}$$

There Are Three Types of Poloidal Parity Breaking

- **Poloidal parity breaking by a finite radial wavenumber**
  - The eikonal breaks parity through the mixed parity term.

$$\frac{\partial S}{\partial r} = -2\pi n \frac{\partial \tilde{q}(\psi,\theta)}{\partial r} + k_r$$

  - It will be shown that a finite radial wavenumber is induced by shear in the ExB velocity Doppler shift

- **Direct poloidal parity breaking by up/down asymmetry of flux surface shape (e.g single null divertors)**
  - The eikonal breaks parity: $\tilde{q}(\psi,-\theta) \neq -\tilde{q}(\psi,\theta)$

- **Parallel velocity space breaking by parallel flow and parallel flow shear**
  - Both flow terms explicitly break the invariance of the gyro-kinetic equation with respect to: $v_{\parallel} \rightarrow -v_{\parallel}$ (odd in $v_{\parallel}$)

Gyro-kinetic Momentum Transport Is Complex

- In this talk the properties of the three poloidal parity breaking types will be explored as an aid to understanding future momentum transport modeling of experiments

Radial wavenumber

Shear in the ExB velocity Doppler shift induces a radial wavenumber and yields a net stress

Diamagnetic velocity shear produces stress

Up/down asymmetry of flux surfaces

Momentum transport depends upon the ion grad-B drift direction for single null divertors

Parallel flow and flow shear

Parallel flow shear drives an energy pinch

Parallel flow self-amplifies

Parallel current drift shear lowers KBM threshold

1st Type of Poloidal Parity Breaking

Radial wavenumber

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Shear in the ExB velocity Doppler shift induces a radial wavenumber and yields a net stress

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Diamagnetic velocity shear produces stress

Momentum Transport Due to the E×B Doppler Shear is A Signature of the Radial Wavenumber

- **Including the shear in the ExB Doppler shift in toroidal eigenmodes has been a theoretical challenge**
  - Review by J.W. Connor, R.J. Hastie and J.B. Taylor, 2004 EPS
  - There are travelling wave solutions (Floquet modes)
  - Solving the 2-D eigenmode problem requires finding a radial wavenumber envelope that keeps the Doppler shifted frequency constant across flux surfaces
- **The shear in the ExB velocity Doppler shift does not directly break the poloidal parity**

$$-i\omega + in\left(\omega_{ExB} - \gamma_{ExB}\frac{q}{r}(r - r_s)\right) \quad \omega_{ExB} = -\left.\frac{c\,\partial\phi_0}{\partial\psi}\right|_{r_s}, \quad \gamma_{ExB} = -\frac{r}{q}\left.\frac{\partial\omega_{ExB}}{\partial r}\right|_{r_s}$$

- **A radial wavenumber induced by the Doppler shear will break the parity and yield a viscous stress**

$$\frac{\partial S}{\partial r} = -2\pi n\frac{\partial\tilde{q}(\psi,\theta)}{\partial r} + k_r$$

Model for the Impact of Exb Doppler Shear on Linear Ballooning Eigenmodes

- **The impact of the ExB Doppler shear on energy and particle fluxes computed in non-linear turbulence simulations is well modeled by the "quench rule". (R.E. Waltz 1995)**

$$\gamma^{\mathrm{net}} = \gamma^{0} - \alpha_{\mathrm{ExB}}\left|\gamma_{\mathrm{ExB}}\right|, \quad \text{with} \quad \alpha_{\mathrm{ExB}} = 0.3\sqrt{\kappa}, \ \text{for} \ \ \mathrm{TGLF}$$

- **The quench rule can be motivated by the transformation**

$$-i k_{\theta} \gamma_{\mathrm{ExB}} \left(r - r_s\right) = \gamma_{\mathrm{ExB}} k_{\theta} \frac{\partial}{\partial k_r} \rightarrow \alpha_{\mathrm{ExB}} \left|\gamma_{\mathrm{ExB}}\right| \qquad k_{\theta} = \frac{nq}{r}$$

- **The "generalized quench rule" adds a model for the radial wavenumber induced by the sheared Doppler shift**

$$k_r = k_{\theta} \alpha_{k_r} \gamma_{\mathrm{ExB}} / \omega_{di} \qquad \omega_{di} = \left|\frac{k_{\theta} c T_i}{R_s e_i B_0}\right|$$

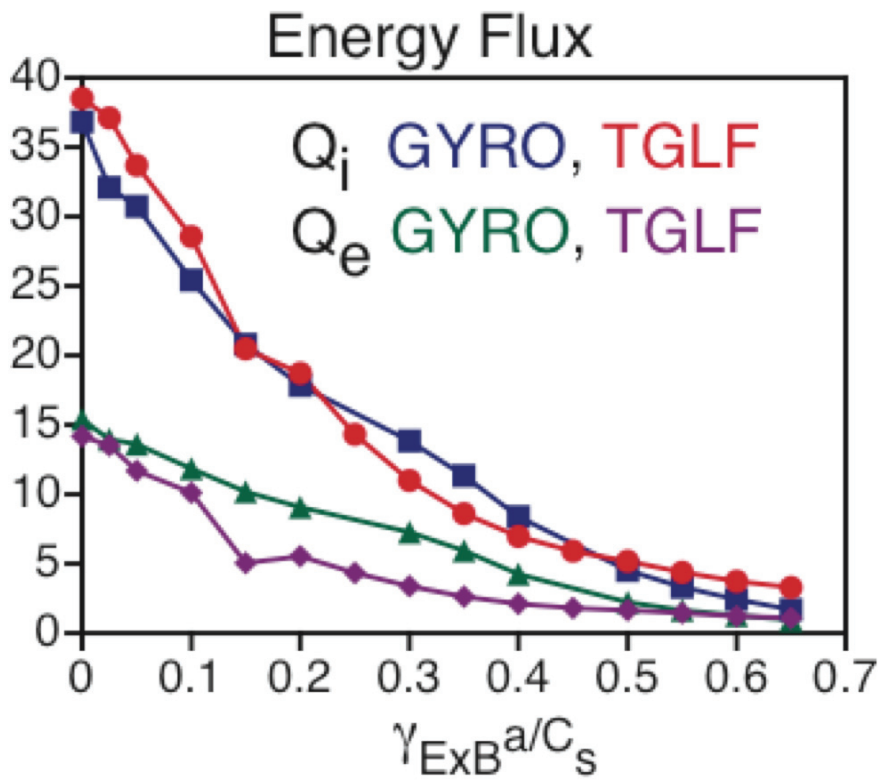
- **The Doppler shift damping term and the radial wavenumber model are both included in the linear eigenmode solution in TGLF so the poloidal parity breaking is in the wavefunction**

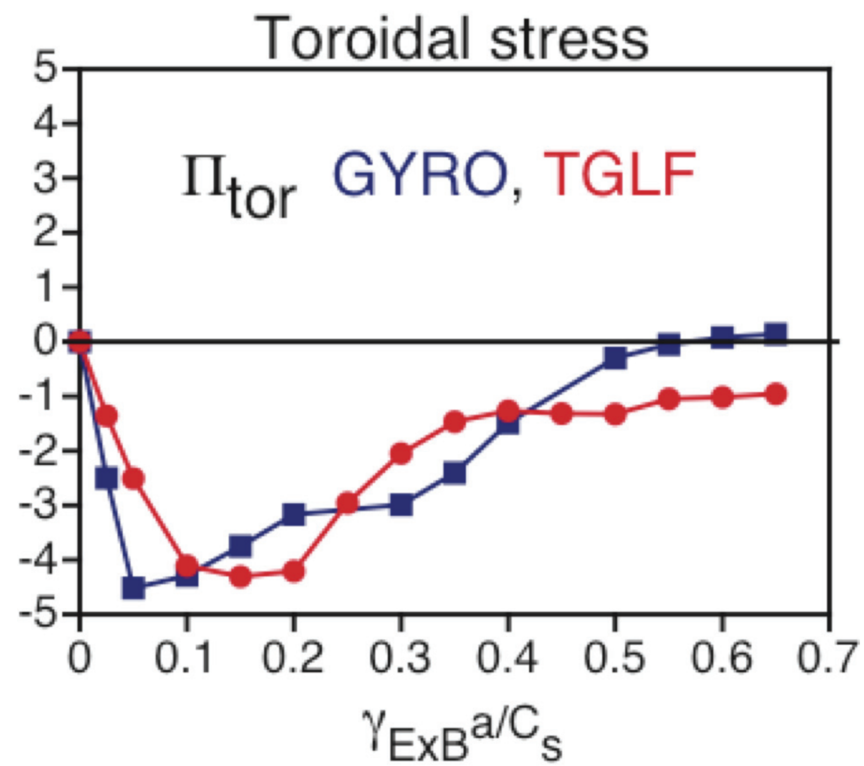
The Generalized Quench Rule in TGLF Fits the Toroidal Stress and Energy Fluxes from GYRO

- **The two coefficients in the generalized quench rule are determined by fitting TGLF to GYRO for the GA-STD-Miller case**

$$\alpha_{ExB} = 0.12, \ \alpha_{kr} = 0.13$$

$$\Pi_{tor} = \left\langle \frac{RB_T}{B}\Pi_{\parallel} - \frac{RB_p}{B}\Pi_{\perp} \right\rangle$$

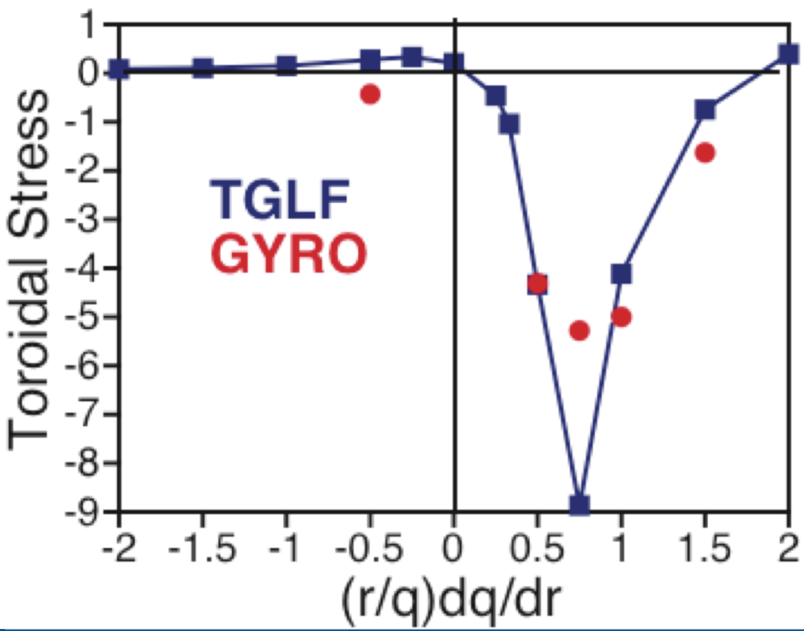




The Magnetic Shear Dependence Of The TGLF Model Agrees With GYRO Simulations

- **There is no magnetic shear dependence in the radial wavenumber model. The magnetic shear dependence is a result of the poloidal shift of the wavefunction in response to k_r**
  - The peak near 0.75 is stronger in TGLF than the non-linear GYRO run This is not surprising since GYRO has a spectrum of radial wavenumbers not a single value

$\gamma_{ExB}=0.1c_s/a$



GA-STD-Milller case:
$a/L_n=1$, $a/LT=3$
$n_e=n_i$, $T_e=T_i$, $R/a=3$, $r/a=0.5$
$q=2$, shear=1, collisionless, Electrostatic, kinetic ion and electrons
Miller geometry model with:
$\kappa=1$, $\delta=0$, $\kappa_{shear}=0$
$\delta_{shear}=0$, $p_{prime}=0$

Diamagnetic Velocity Shear is Important in Transport Barriers

- **In the transport barrier region the radial electric field is nearly balance by the ion diamagnetic velocity giving a small net perpendicular velocity**

$$\vec{\nabla} S \cdot \vec{U}_{\perp} = \frac{c}{B^2} \vec{\nabla} S \cdot \left(\vec{B} \times \left[-\vec{E} + \frac{T_a}{e_a n_a} \vec{\nabla} n_a + \frac{1}{e_a} \vec{\nabla} T_a \right] \right) = \omega_E + \omega^*_{n_a} + \omega^*_{T_a}$$

$$\left(-i\omega + i\omega_E + i v_{\|} k_{\|} + i\omega_{da} \left(\frac{v_{\|}^2 + v^2}{v_{ta}^2} \right) \right) \tilde{g}_a = \left(-i\omega + i\omega_E + i\omega^*_{n_a} + i\omega^*_{T_a} \left(\frac{v^2}{v_{ta}^2} - \frac{3}{2} \right) \right) \frac{e_a}{T_a} \tilde{\phi} J_0 F_0$$

- **Hence, shear of the diamagnetic velocities are of the same size as the ExB Doppler shift shear within the barrier region**
  - GYRO simulations have demonstrated that these “profile shear” effects produce momentum transport
  - R.E. Waltz Thursday 2PM poster session UP9.00052

2nd Type of Poloidal Parity Breaking

Radial wavenumber

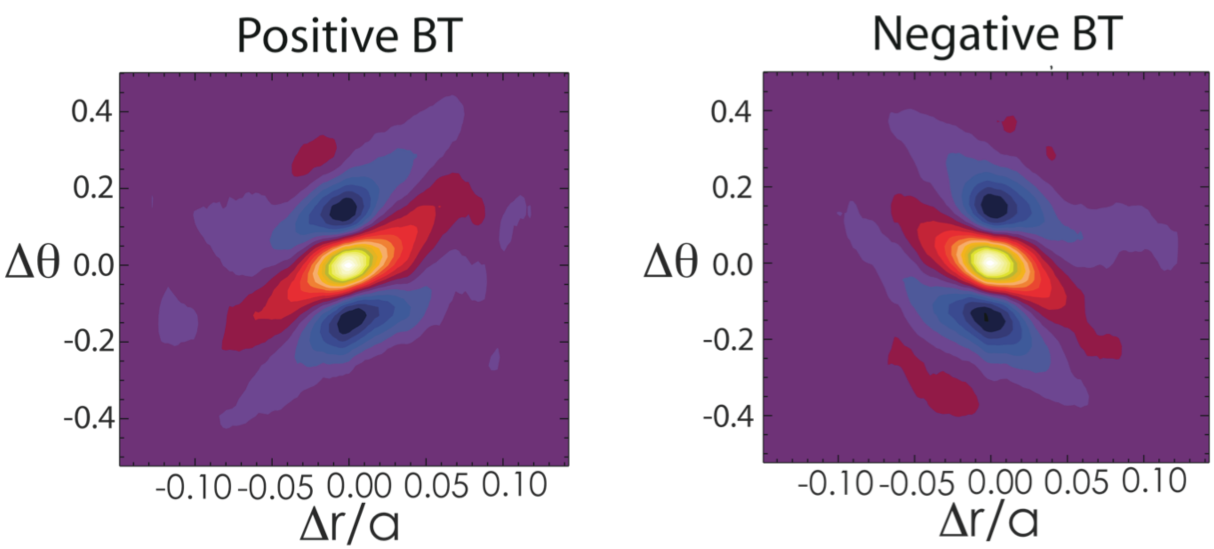
Up/down asymmetry of flux surfaces

Parallel flow and flow shear

Momentum transport depends upon the ion grad-B drift direction for single null divertors

The Ion Grad-B Drift Direction Matters for Single Null Divertors

- **The direction of the poloidal ion grad-B drift flips with B_T. The sign of the local safety factor also flips but not the radial wavenumber. This changes the poloidal tilt of the wavefunction in response to the radial wavenumber** $\frac{\partial S}{\partial r} = -2\pi n \frac{\partial \tilde{q}(\psi,\theta)}{\partial r} + k_r$ $\quad \tilde{q}(\psi,\theta) = \int_0^{\theta} \frac{\vec{B}\cdot\vec{\nabla}\varphi}{\vec{B}\cdot\vec{\nabla}\theta'} \frac{d\theta'}{2\pi}$
- **For single null divertors, the up/down asymmetry will interact with the eddy tilt producing a dependence of momentum transport on the direction of the ion grad-B drift (B_T) relative to the x-point**



GYRO correlation functions for GA-STD-Miller case with

$\gamma_{ExB}=0.1c_s/a$

3rd Type of Poloidal Parity Breaking

Parallel flow and flow shear

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Parallel flow shear drives an energy pinch

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Parallel flow self-amplifies

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Parallel current drift shear lowers KBM threshold

Parallel Velocity Shear Dives an Instability

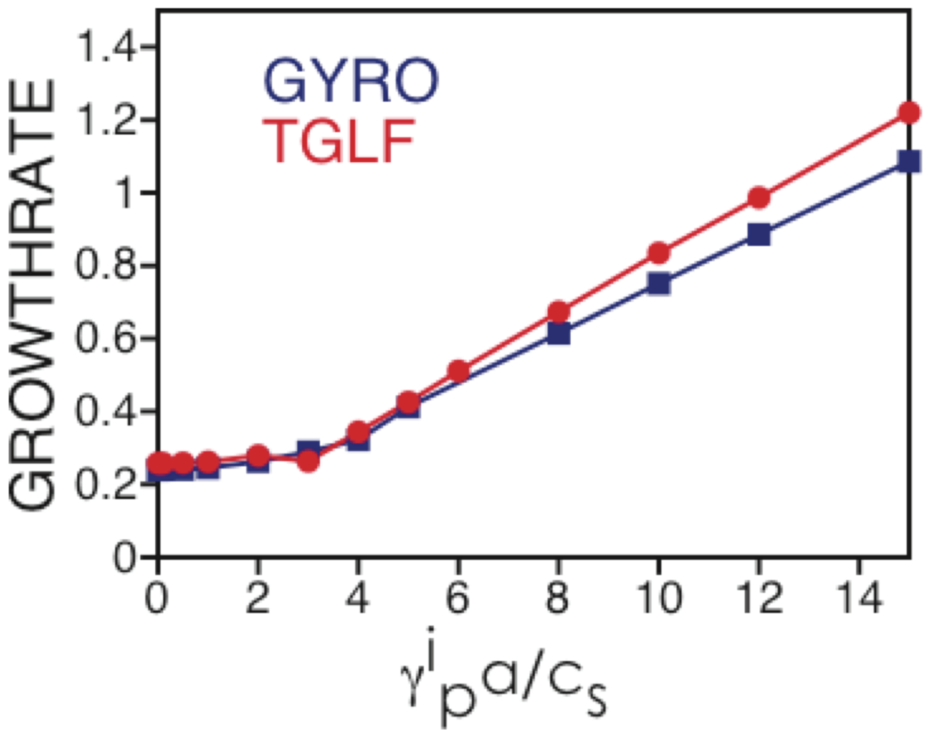
- It has long been known that parallel velocity shear drives an instability of the Kelvin-Helmholtz (KH) type
- A simple 2-moment Landau-fluid model shows that the KH mode has a linear threshold due to finite parallel wavenumber

$$\left|\gamma_p^a\right| \geq \left|\frac{\Omega_a k_{\|}}{k_\theta}\right|, \text{ where } \gamma_p^a \approx -\frac{du_{\|}^a}{dr}, \quad \Omega_a = \frac{e_a B}{m_a c}$$

- The linear growth rate is maximized at $k_{\|} = \dfrac{\gamma_p^a k_\theta}{2\Omega_a}$
- Accurately computing the KH mode growth rate at high drive requires higher parallel wavenumber resolution than for drift-waves
- The pure KH mode has an odd poloidal parity electrostatic potential

TGLF Matches Linear GYRO KH Eigenvalues Well

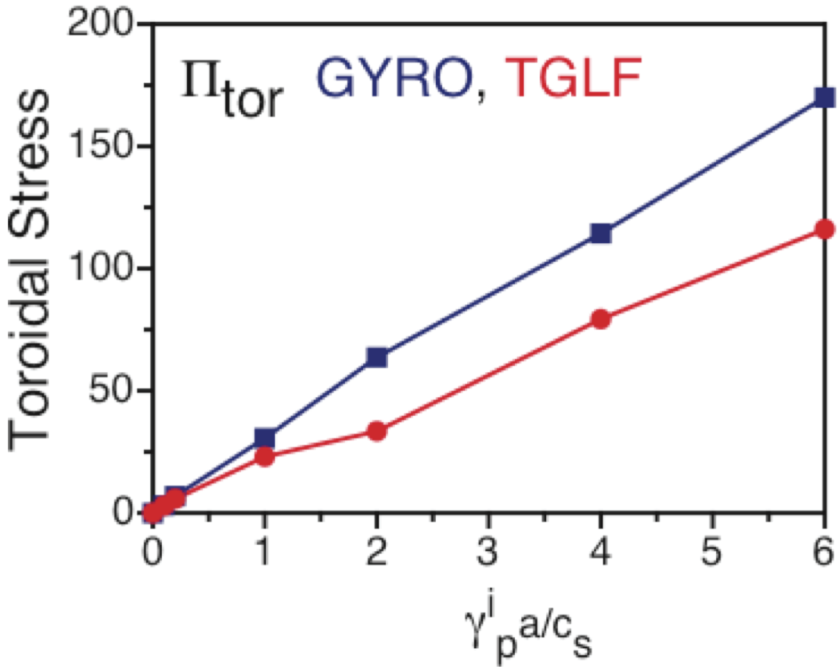
- **TGLF Linear growth rates for the GA-STD case match GYRO results with high-parallel resolution**
  - Frequencies also agree
  - Kotschenreuther's linear gyro-kinetic code GKS matches also

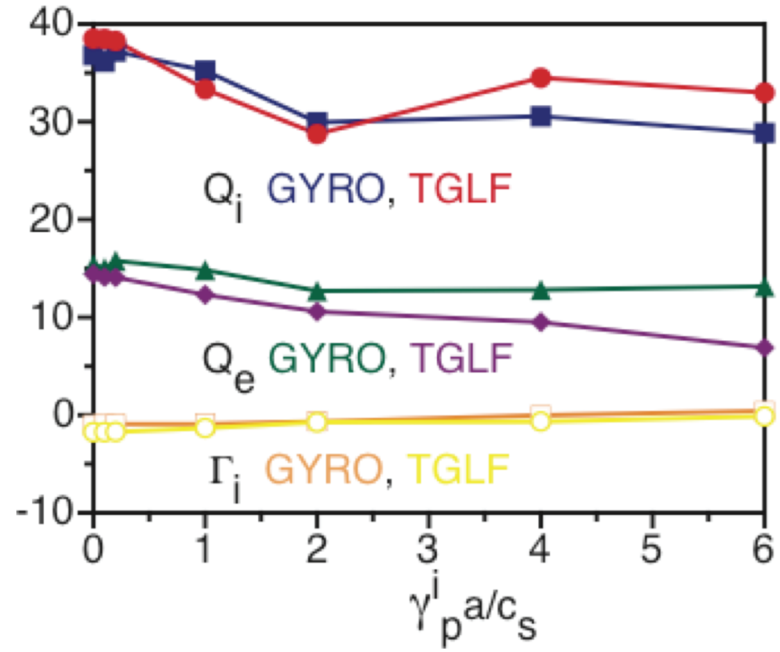


GA-STD case with k_y=0.3:
a/L_n=1, a/L_T=3
$n_e=n_i$, $T_e=T_i$, R/a=3, r/a=0.5
q=2, s-alpha geometry with
shear=1, alpha=0
collisionless,
Electrostatic,
kinetic ions and electrons

TGLF Needs Adjustment to Match Nonlinear Toroidal Stress Due to Parallel Velocity Shear

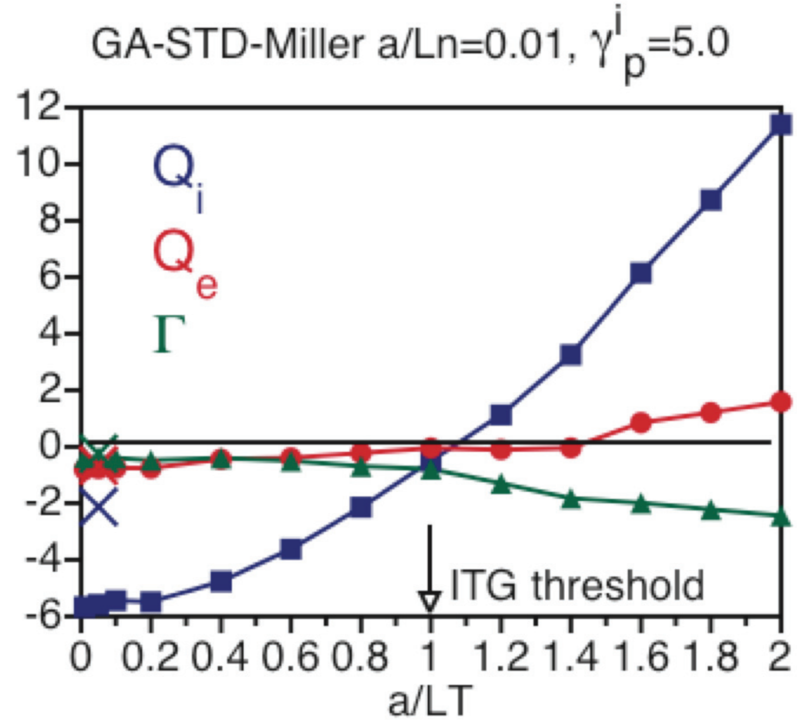
- **Using the TGLF saturation rule developed for drift-waves the toroidal stress for the GA-STD-Miller case is lower than GYRO**
  - It is not simple to fix since there is good agreement at low parallel velocity shear and the energy and particle fluxes are good





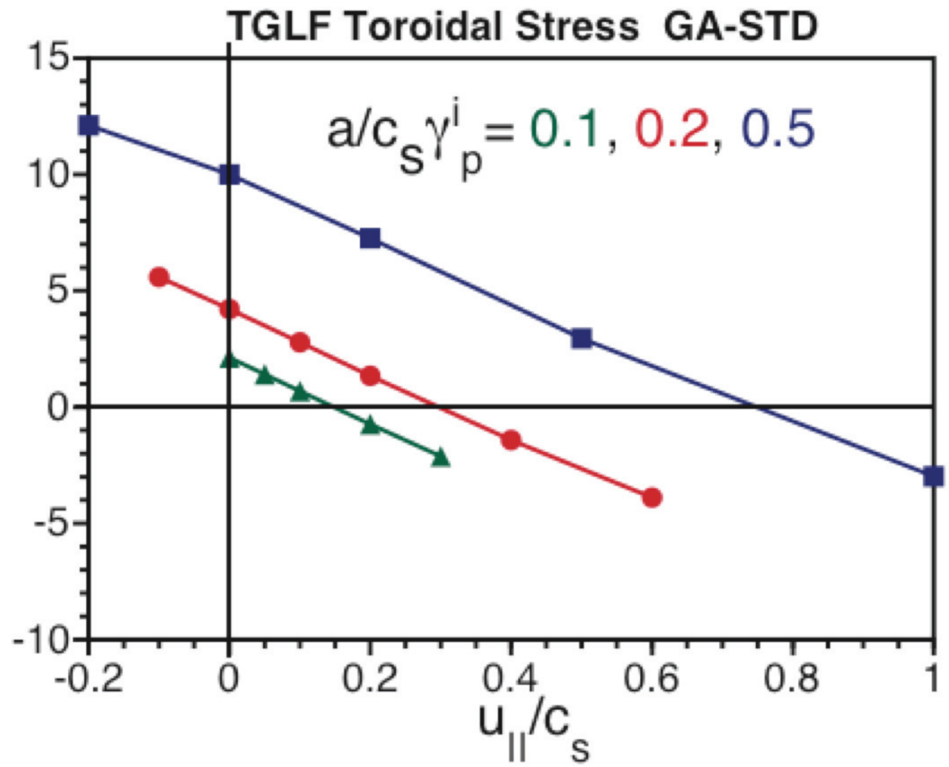
The KH Mode Can Drive A Net Energy Pinch

- When the ion temperature gradient is below the ITG mode threshold the KH mode yields a net negative energy flux
- This energy pinch could result in an effective experimental ion energy diffusivity below the neoclassical level in the deep core region



Parallel Ion Flow Self-amplifies

- The toroidal stress driven by the parallel ion flow term opposes the parallel flow shear drive
- This is more that just inward convection due to the particle flux which is small for this case
- A seed flow at the boundary is amplified towards the center even if there is no external torque

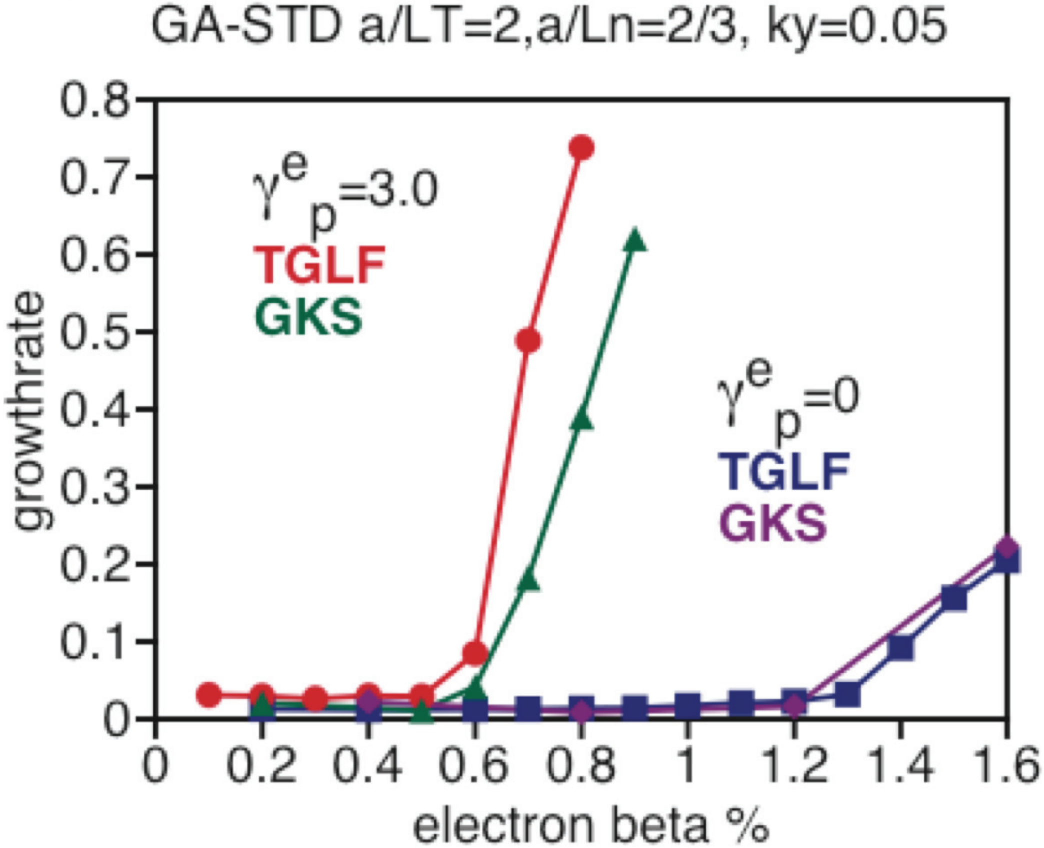


$$\Pi_{tor} = 0 \quad \text{for} \quad -\frac{du_{||}}{dr} = \frac{2}{3}u_{||}$$

$$\text{gives} \quad u_{||}(r) = u_{||}(a)\mathrm{Exp}\left[-\frac{2}{3a}(r-a)\right], \qquad u_{||}(0) = 1.94u_{||}(a)$$

Parallel Current Drift Shear Lowers the Kinetic Ballooning Mode Threshold

- **A parallel velocity shear that is the same for ions and electrons has no effect on the kinetic ballooning mode threshold**



$$u_{e\parallel} = u_{i\parallel} - \frac{J_{\parallel}}{en_e}$$

$$\gamma_p^e = -\frac{du_{e\parallel}}{dr}$$

Summary of Momentum Transport Discoveries

- **Interesting properties of gyro-kinetic momentum transport have been discovered by theoretical exploration with TGLF**
  – Shear in the ExB velocity Doppler shift induces a radial wavenumber that breaks the poloidal mode parity. This can be modeled with a "generalized quench rule"
  – Diamagnetic flow shear can cause momentum transport.
  – The momentum transport depends upon the ion grad-B drift direction in single null divertor geometry
  – Parallel flow shear can drive a negative energy flux in the deep core giving a total effective thermal diffusivity below neoclassical.
  – Parallel flow self-amplifies from a boundary seed
  – Parallel current drift gradients reduce the kinetic ballooning mode threshold
- **Once verification of the momentum transport physics in TGLF has been completed with GYRO a campaign to validate momentum transport with experiments will begin**

References

- **Early work on gyro-kinetic momentum transport (linear)**
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    - G.M. Staebler and R. R. Dominguez, Nuclear Fusion 33, 77 (1993)
    - R.R. Dominguez and G. M. Staebler, Phys. Fluids B5, 3876 (1993)
- **A number of parity breaking effects have been explored with non-linear gyro-kinetic simulations in recent years**
  - Parallel and ExB velocity shear
    - R.E. Waltz, G. M. Staebler, J. Candy and F. L. Hinton, Phys. Plasmas 14(2007) 122507
    - F. J. Casson, A. G. Peeters, Y. Camenen, et al., Phys. Plasmas 16 (2009) 092303
  - Parallel ion flows (Corriolis and centrifugal effects)
    - A. G. Peeters, C. Angioni, and D. Strintzi, Phys. Rev. Lett. 98(2007) 265003
  - Up/down asymmetry of magnetic flux surfaces.
    - Y. Camenen, A. G. Peeters C. Angioni, et al., Phys. Rev. Lett. 102 (2009) 125001
- **Diamagnetic velocity shear**
  - R.E. Waltz Thursday 2PM poster session UP9.00052