Measurement and Modeling of Three-dimensional Equilibria in DIII-D

By

M.J. Lanctot[1,2]

In collaboration with
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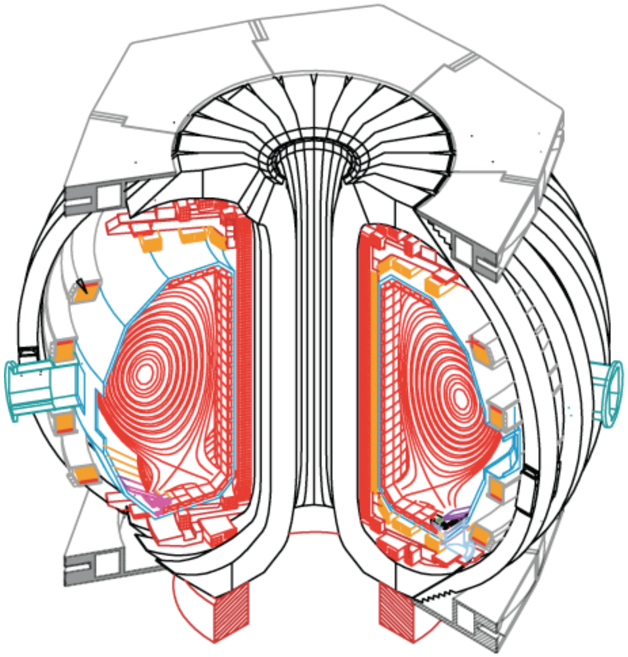
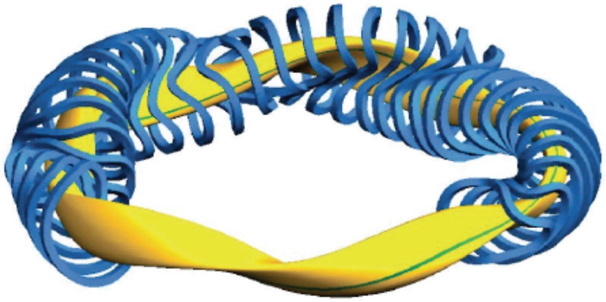
All Toroidal Confinement Devices Have Non-axisymmetric (3D) Magnetic Fields

Non-axisymmetric Shaping

2D → DIII-D → W-7X → 3D





Certain 3D Magnetic Fields Can Have Adverse Effects

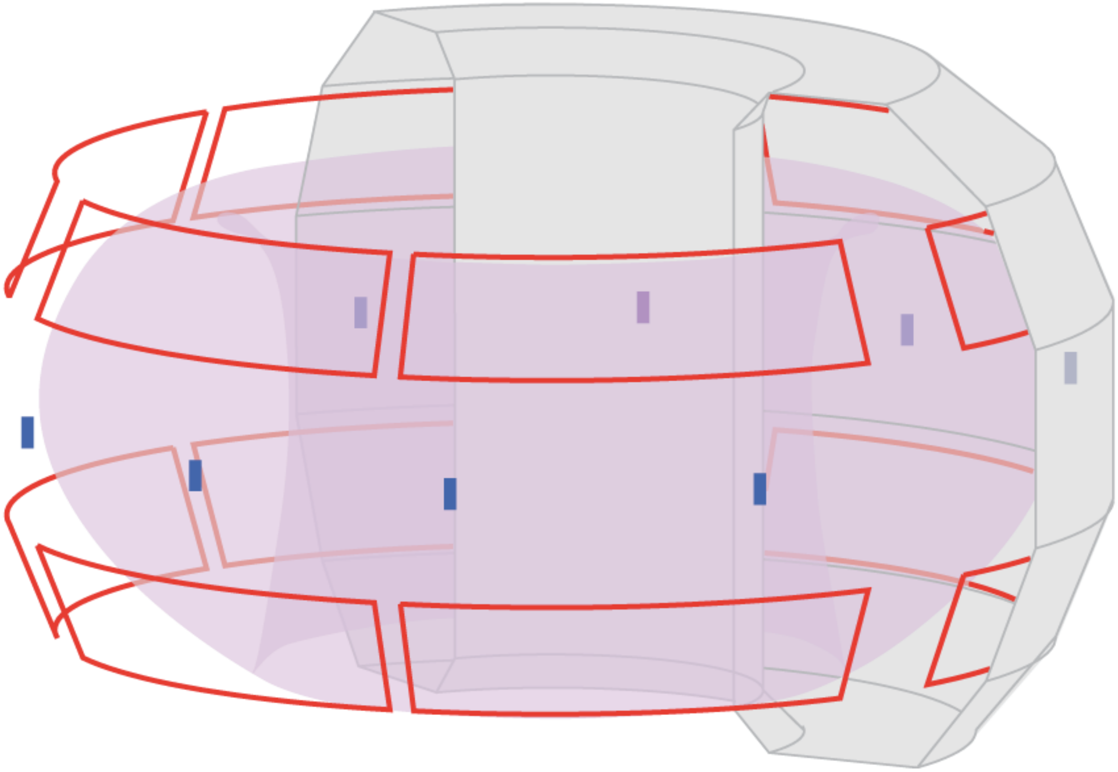
Non-axisymmetric Shaping

2D

DIII-D

3D

Error Field Optimization

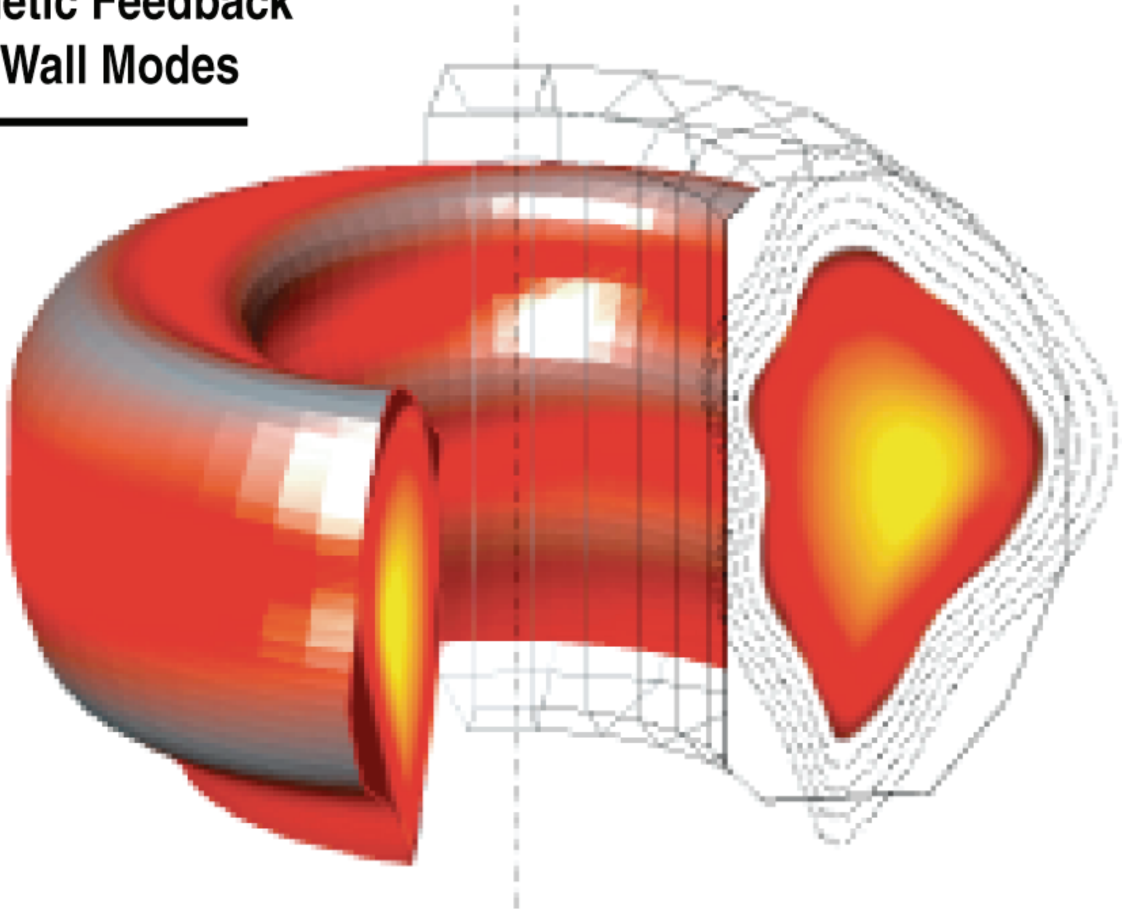


3D Fields Can Suppress Unstable Ideal MHD Instabilities

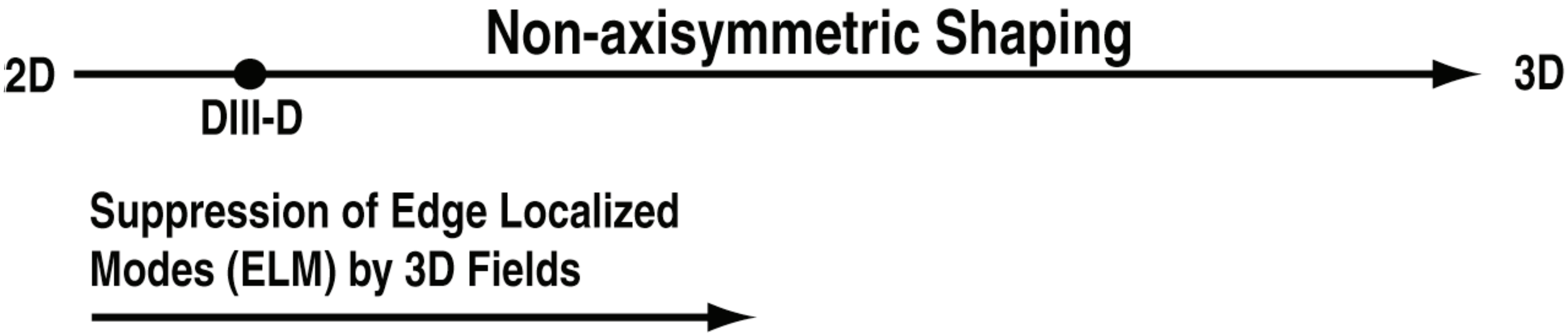
Non-axisymmetric Shaping

2D ● DIII-D → 3D

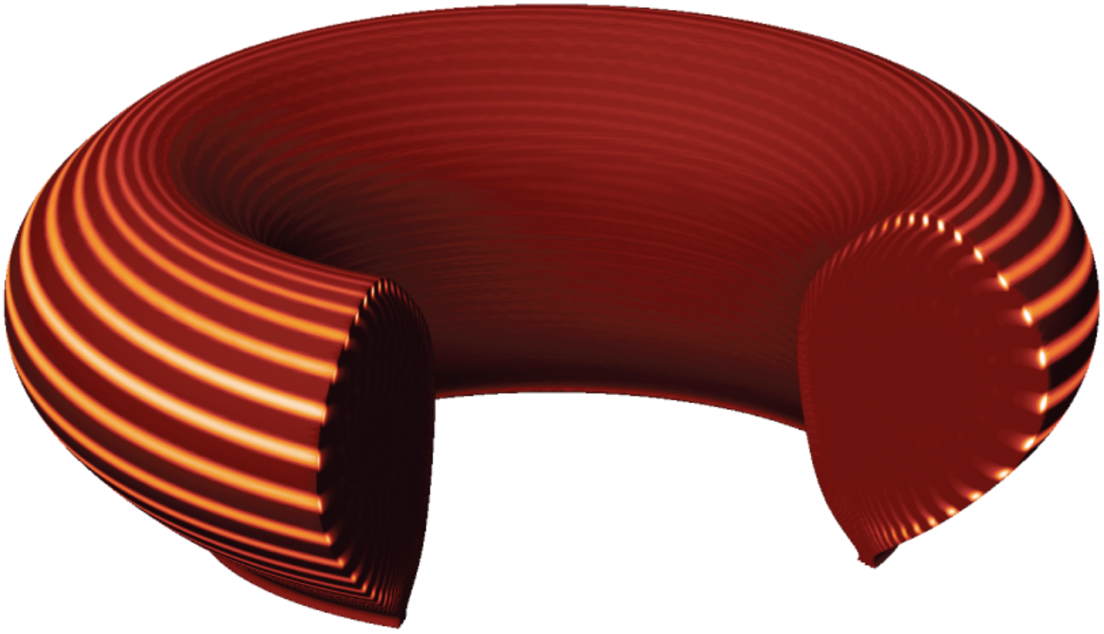
Active Magnetic Feedback
of Resistive Wall Modes ←



3D Fields Can Affect Edge Transport

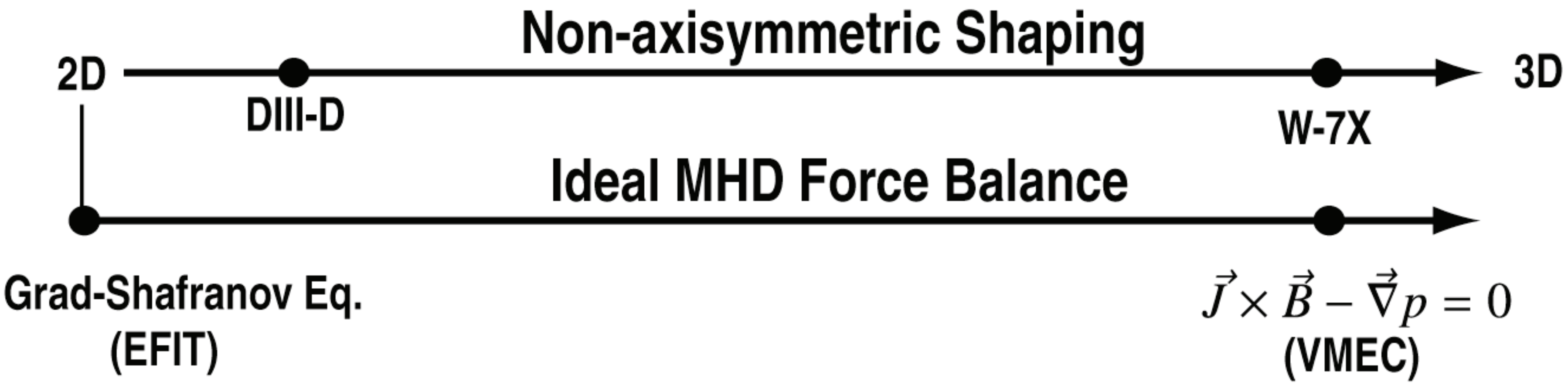


Suppression of Edge Localized Modes (ELM) by 3D Fields



ELM Pressure Eigenfunction (n=7)

3D Equilibria Must Satisfy Force Balance



Non-axisymmetric Shaping

2D

3D

DIII-D

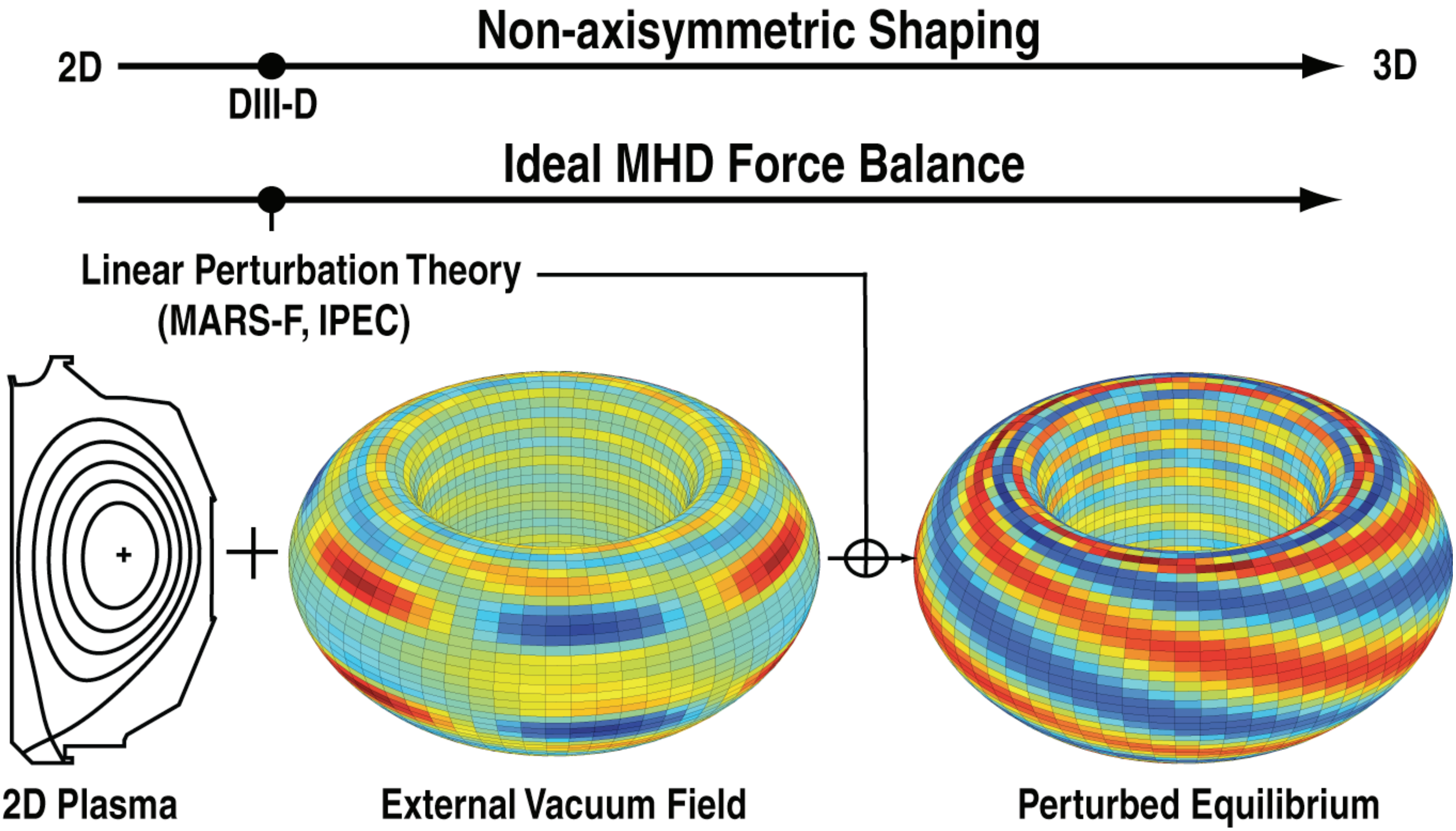
W-7X

Ideal MHD Force Balance

Grad-Shafranov Eq.
(EFIT)

$\vec{J} \times \vec{B} - \vec{\nabla} p = 0$
(VMEC)

Linear Perturbation Theory Offers An Alternative To Solving the Full Ideal MHD Force Balance Equation



Non-axisymmetric Shaping

2D → 3D

DIII-D

Ideal MHD Force Balance

Linear Perturbation Theory
(MARS-F, IPEC)

2D Plasma + **External Vacuum Field** ⊕ → **Perturbed Equilibrium**

Outline

- **Below the no-wall limit, the linear ideal MHD perturbed equilibrium model**
  - Describes the plasma response in rotating plasmas
  - Predicts two types of response fields:

    “Resonant field screening” and “Kink mode excitation”

- **Above the no-wall limit, the measured rotation dependence of the plasma response reveals direct evidence of kinetic RWM stabilization**

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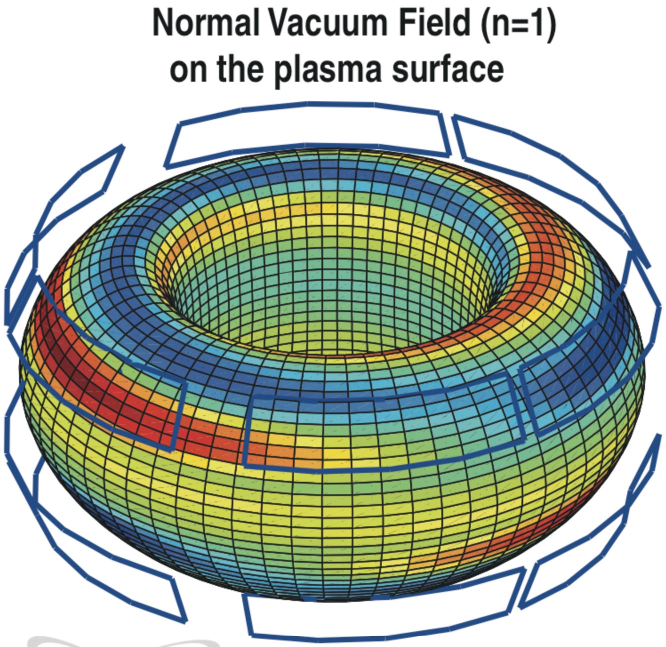
3D Equilibria Are Studied By Probing H-mode Discharges With The Internal Coil (I-coil)

- Toroidal (n) and poloidal (m) spectra of δB^{ext} can be varied (n=1,2,3)



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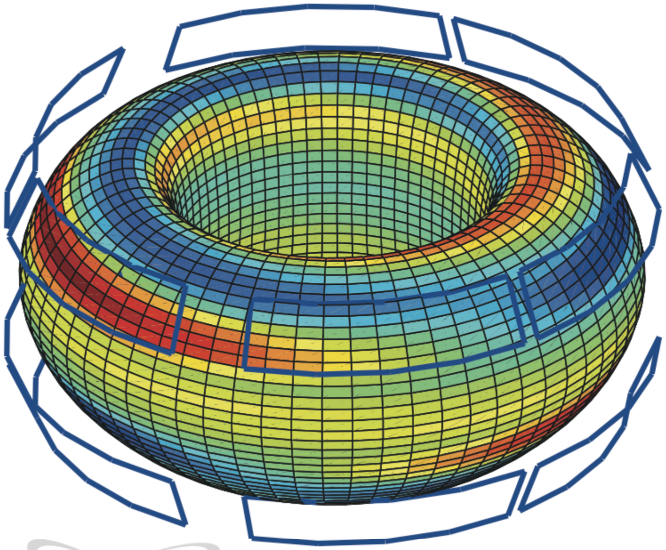
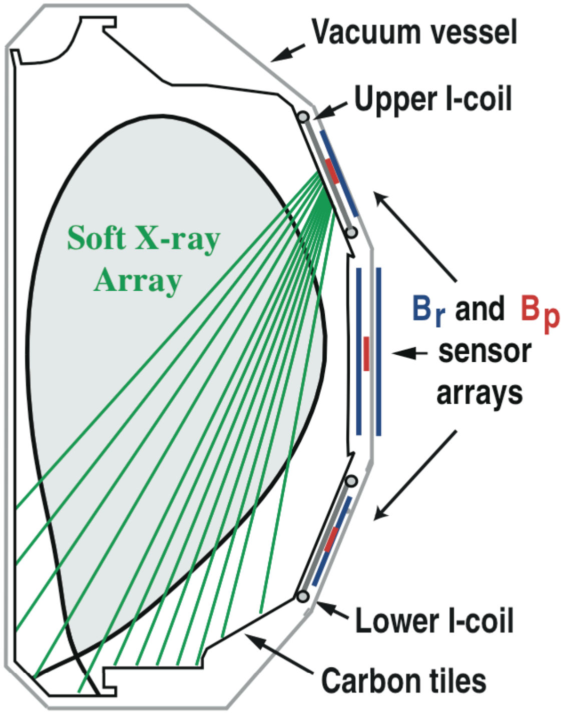
- Toroidal (n) and poloidal (m) spectra of δB^{ext} can be varied (n=1,2,3)
- Toroidally distributed magnetic and soft x-ray diagnostics resolve the toroidal spectrum of the 3D equilibrium


Normal Vacuum Field (n=1) on the plasma surface



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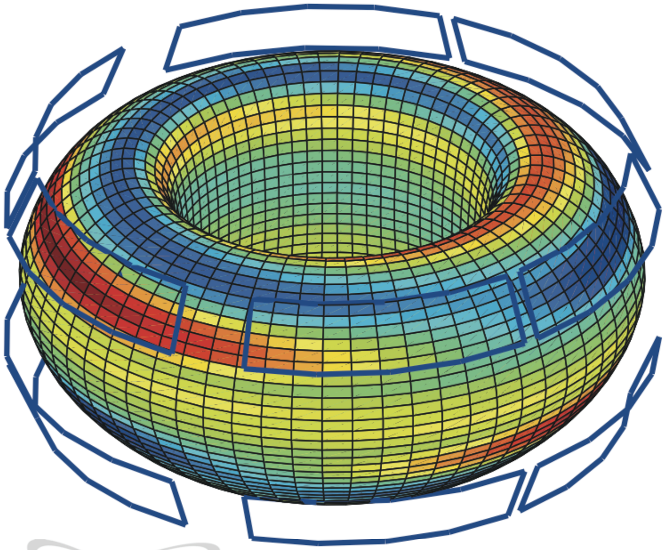
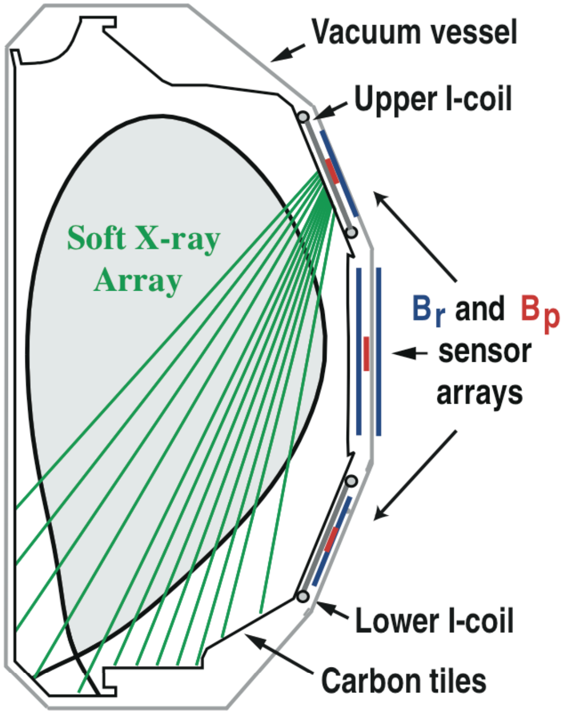
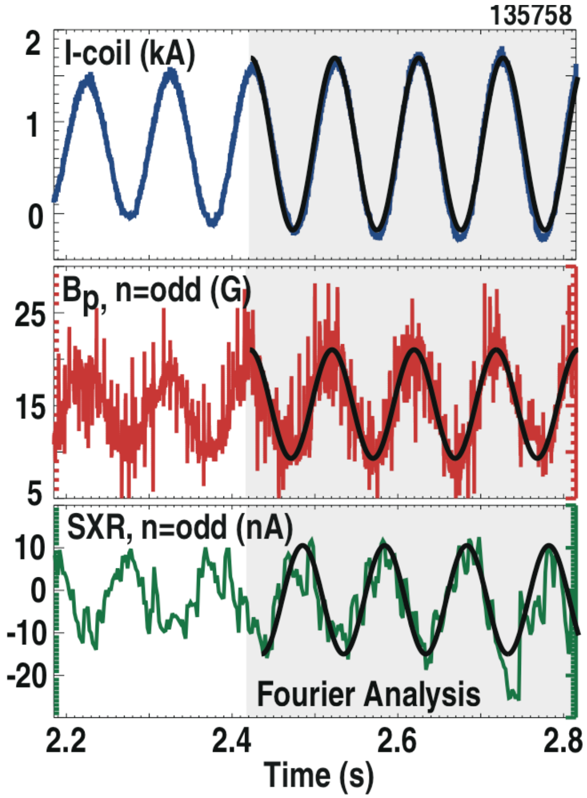
- Toroidally distributed magnetic and soft x-ray diagnostics resolve the toroidal spectrum of the 3D equilibrium
- Rotating the n=1 external field in the toroidal direction allows synchronous detection of the plasma response

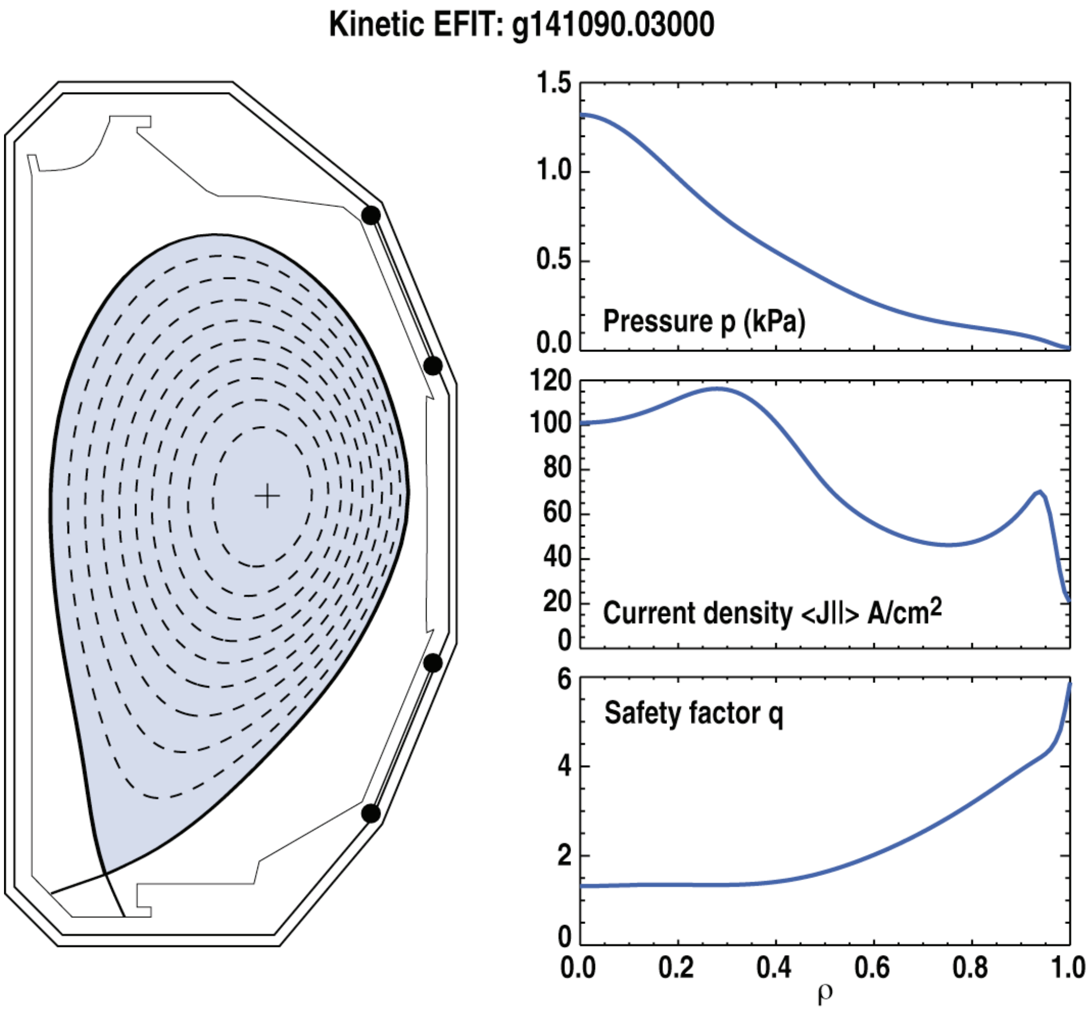
Normal Vacuum Field (n=1)
on the plasma surface







Linear Perturbation Theory Extends The 2D Ideal MHD Equilibrium Model to 3D



- **Axisymmetry (2D) reduces Ideal MHD force balance to Grad-Shafranov equation**
  - Solve using EFIT [Lao, FS&T, 2005]
- **Extend to 3D using linear perturbation theory** [Boozer, *Phys. Plasmas* 1999]

$$\vec{B}^{tot} = \vec{B}_0 + \delta\vec{B}^{tot}$$

$$\delta B^{tot} / B_0 \sim 10^{-4} - 10^{-3}$$

$$\delta\vec{B}^{tot} = \delta\vec{B}^{ext} + \delta\vec{B}^{plas}$$

- **Use MARS-F** code to solve the linearized single fluid MHD equations for stable equilibria and external current distributions [Liu et al., *Phys. Plasmas* 2000]

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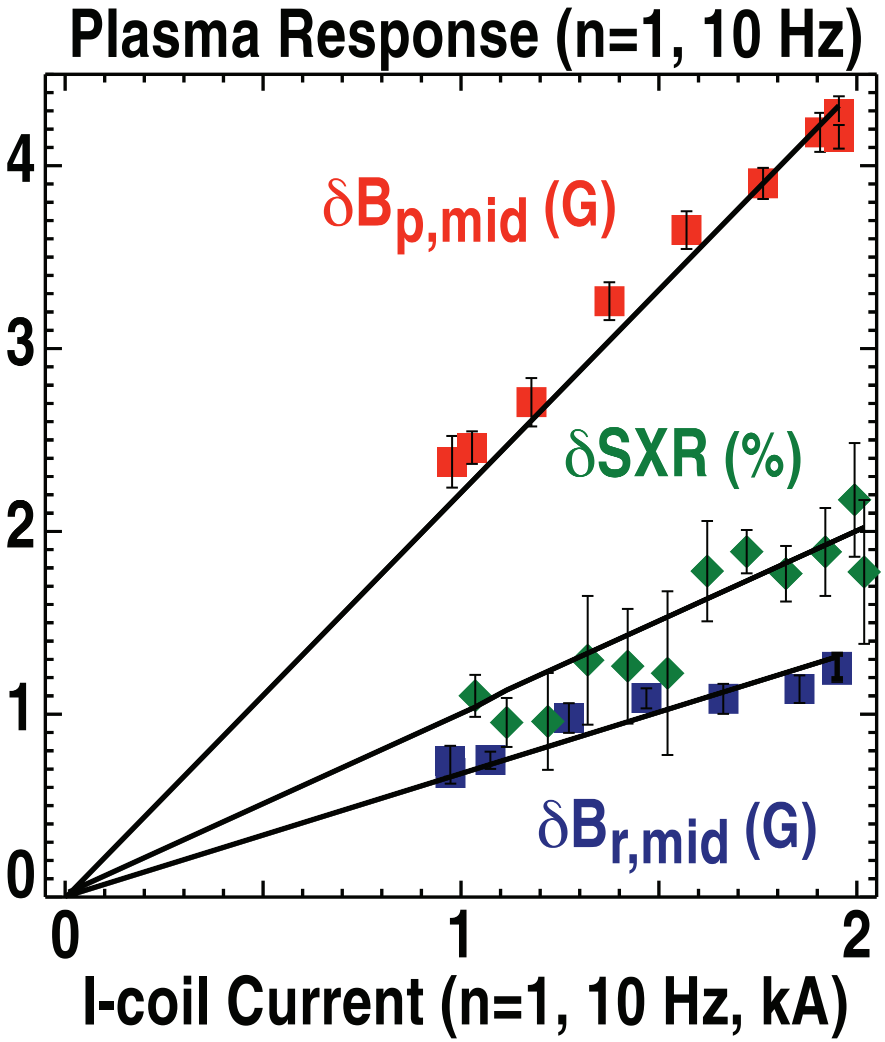
Measured Plasma Response in Rotating Discharges Is Linear in the Applied Magnetic Field



- **Vary n=1 I-coil amplitude at fixed plasma pressure**
  - Use closed feedback control of neutral beam power
- **Plasma response measurements show a linear scaling with I-coil amplitude**
- **Linear response is observed in rotating discharges**

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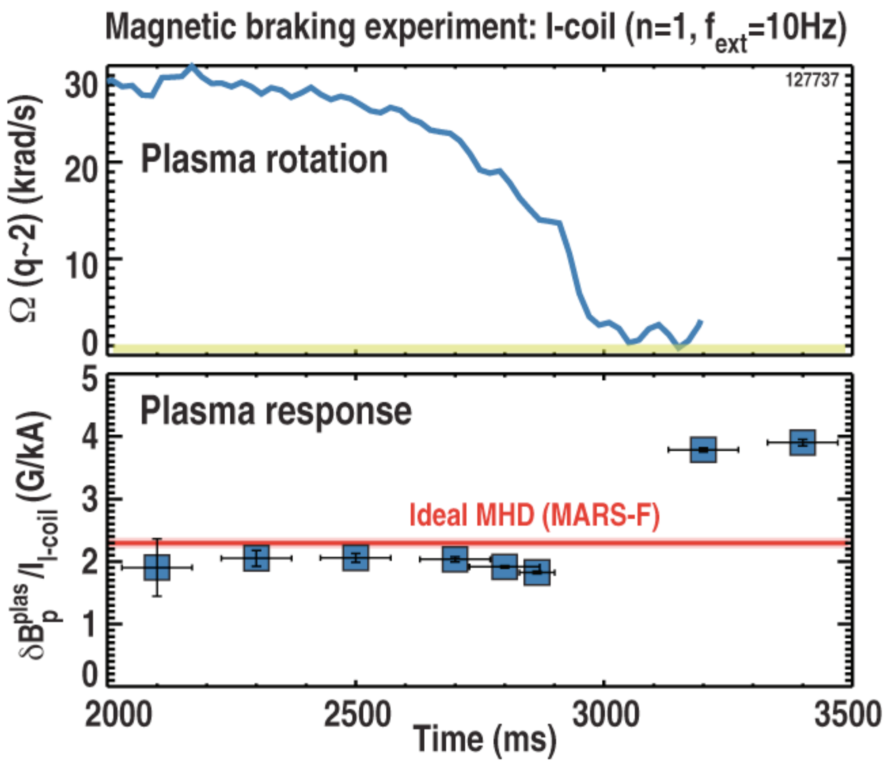
A Linear Response Is Observed As Long As The Plasma Rotation Is Sufficiently Large

- **Measure response to *n*=1 I-coil field in magnetic braking experiment**

- **For "large" rotation**
  - δB^{plas} *is* independent of rotation
  - δB^{plas} *is* consistent with ideal MHD



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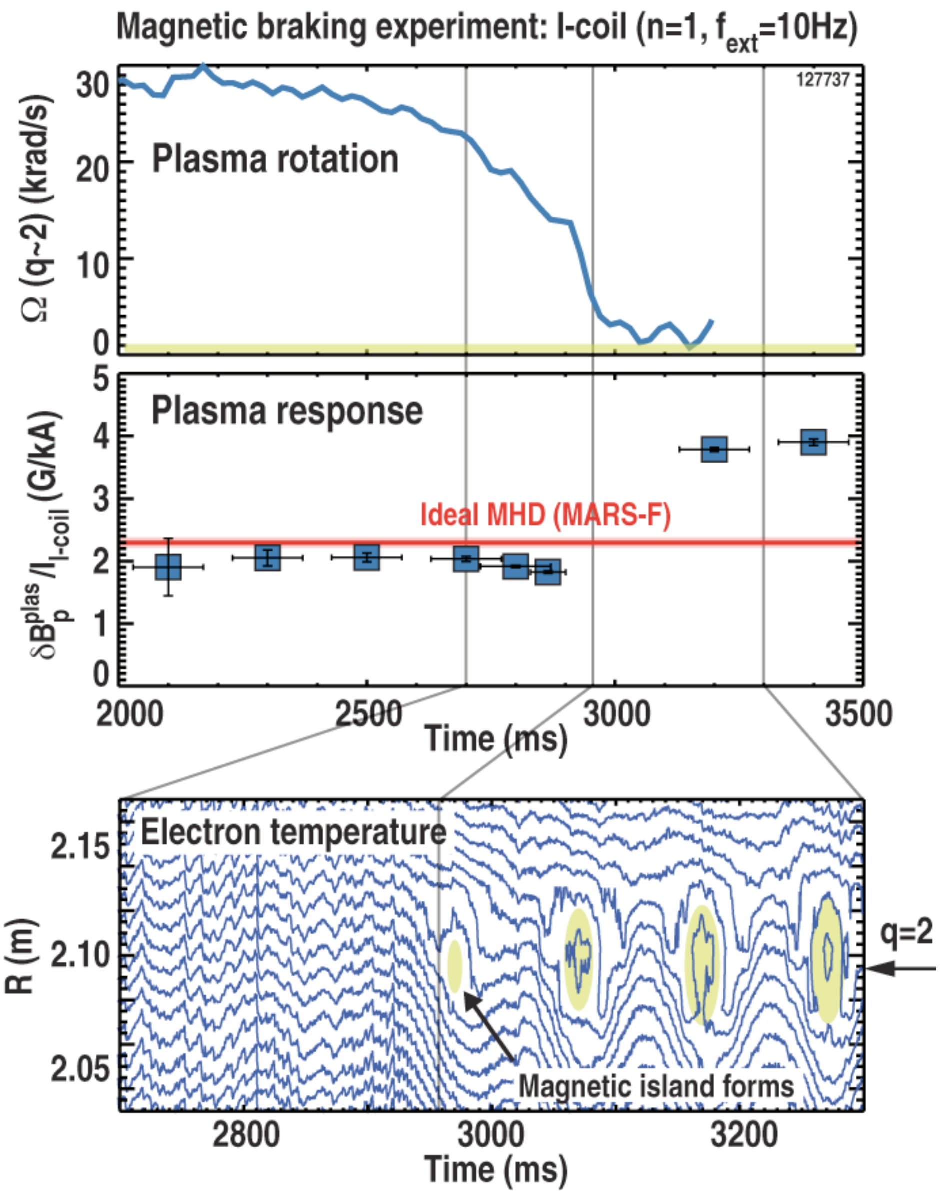
- **After the rotation has collapsed**
  - δB^{plas} deviates from ideal MHD
  - A magnetic island forms



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- **After the rotation has collapsed**
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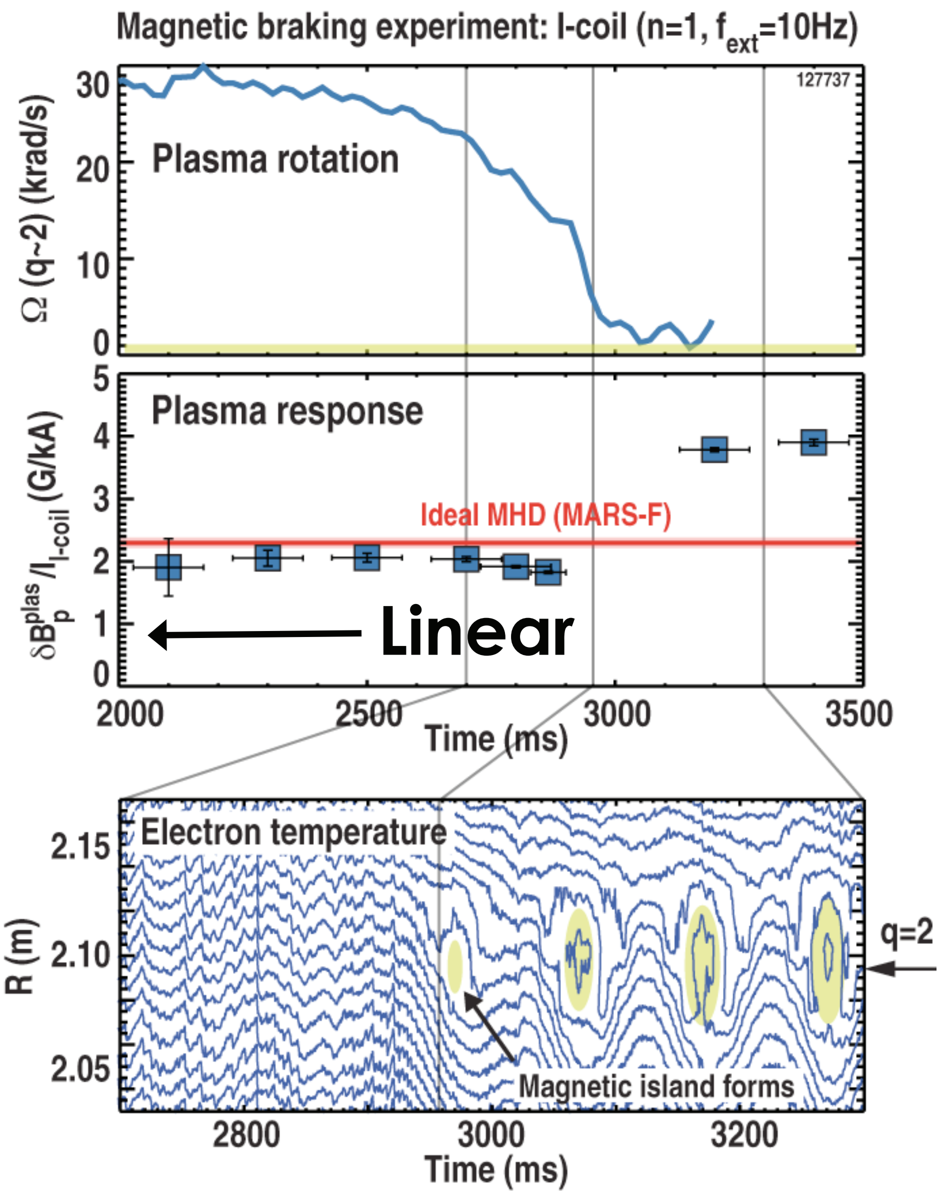
  - A magnetic island forms
- **Recent focus has been on the linear response prior to island formation**



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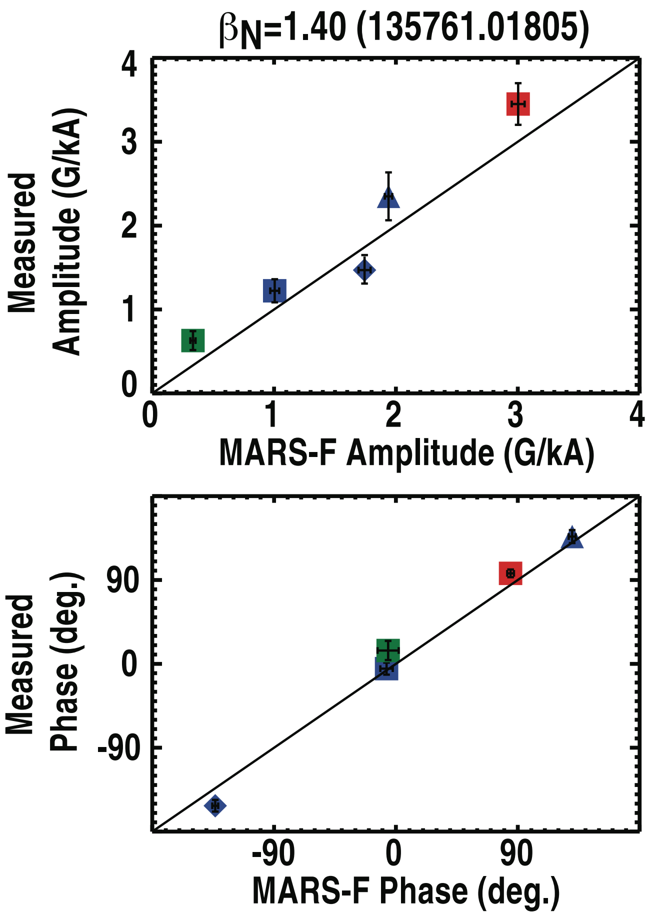
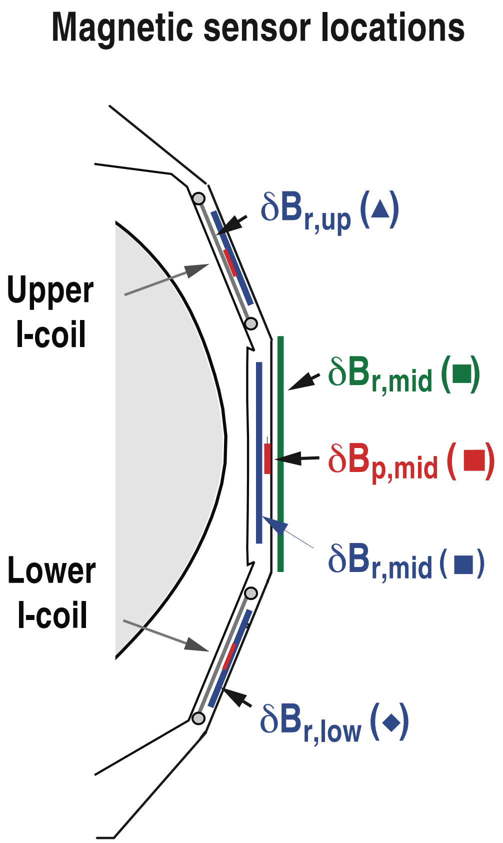
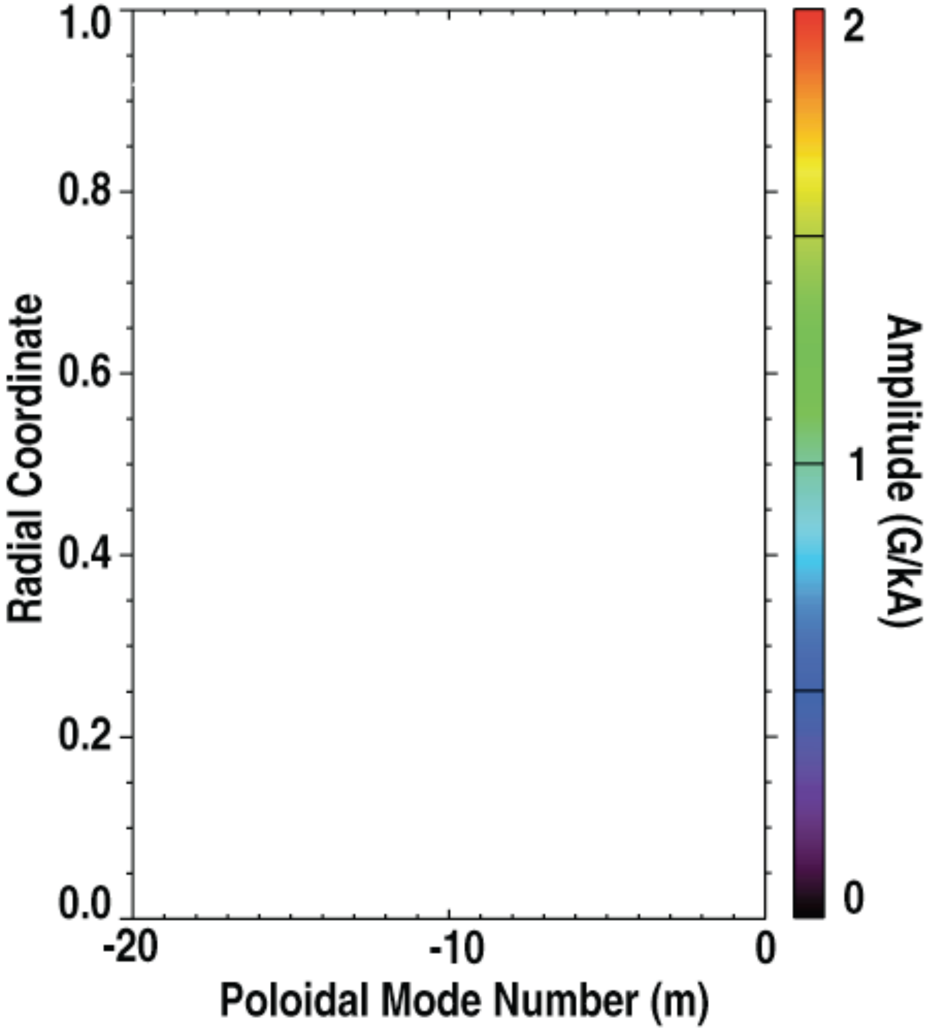
Linear Ideal MHD Theory Gives a Quantitative Description of 3D Equilibria

- **Probe plasma with 10 Hz n=1 field at fixed plasma pressure**
- **Linear ideal MHD calculations (MARS-F) based on detailed equilibrium reconstructions are in good agreement with experiment**[M.J. Lanctot et al., *Phys. Plasmas* 2010]

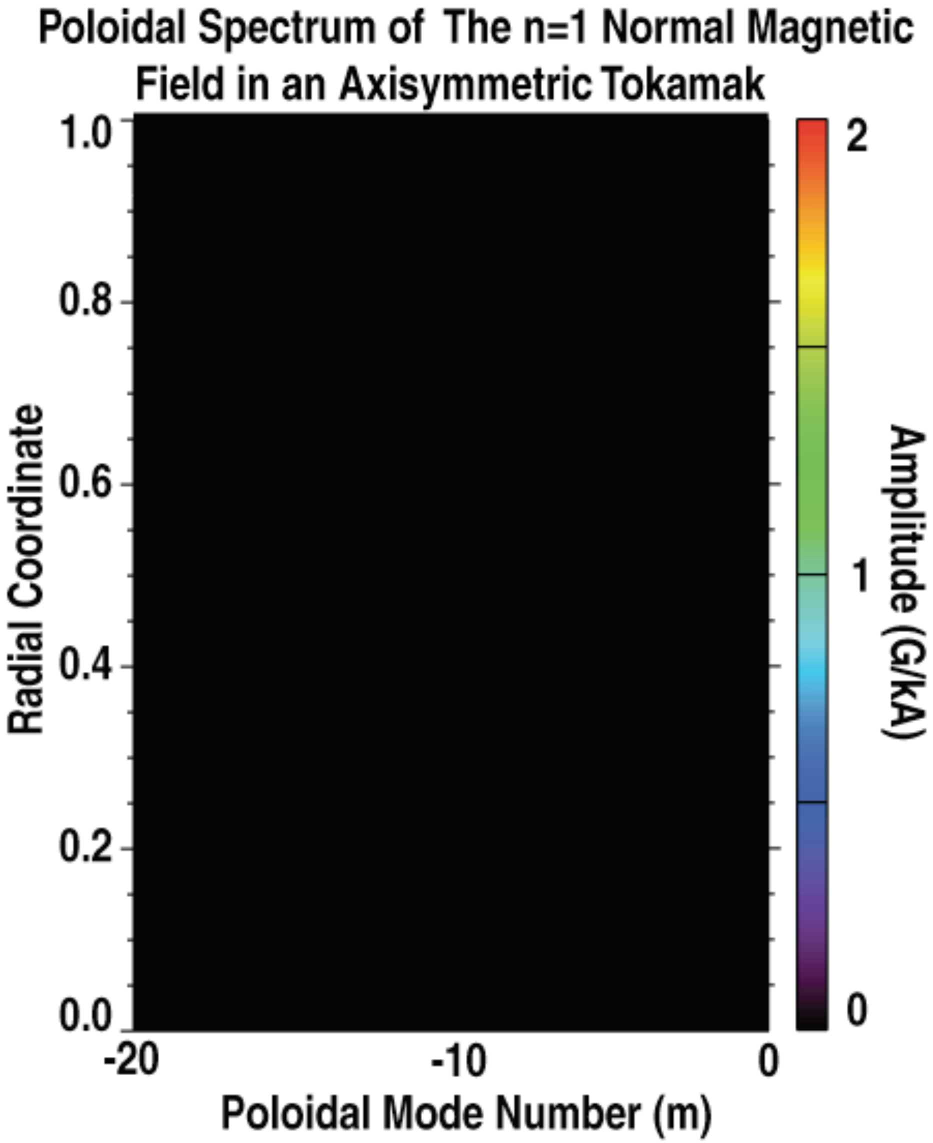




Characterize 3D Fields Using Poloidal Harmonics of the Normal δB in Straight Field Line Coordinates



A Trivial Example: Axisymmetric Tokamak



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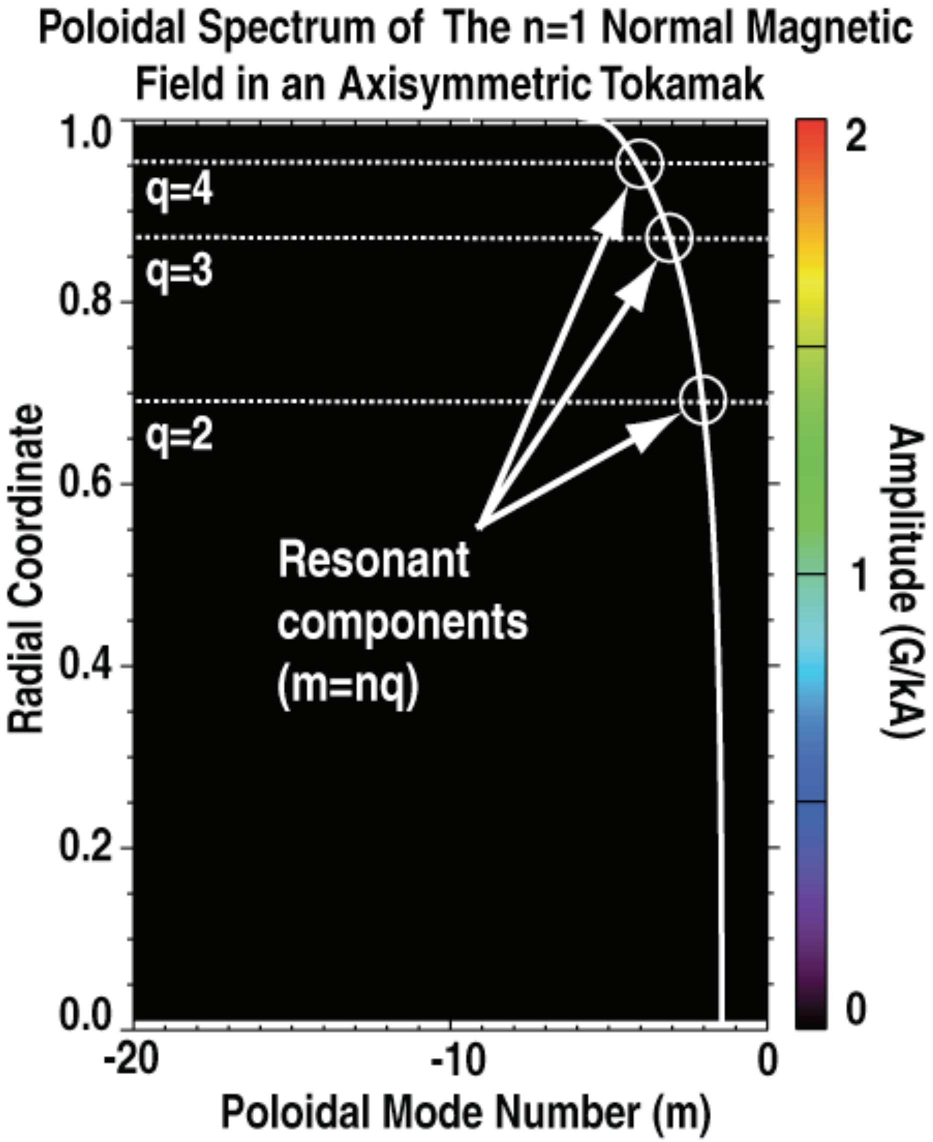
In Ideal MHD, Pitch Resonant Magnetic Fields Can Drive Screening Currents At the Rational Flux Surfaces



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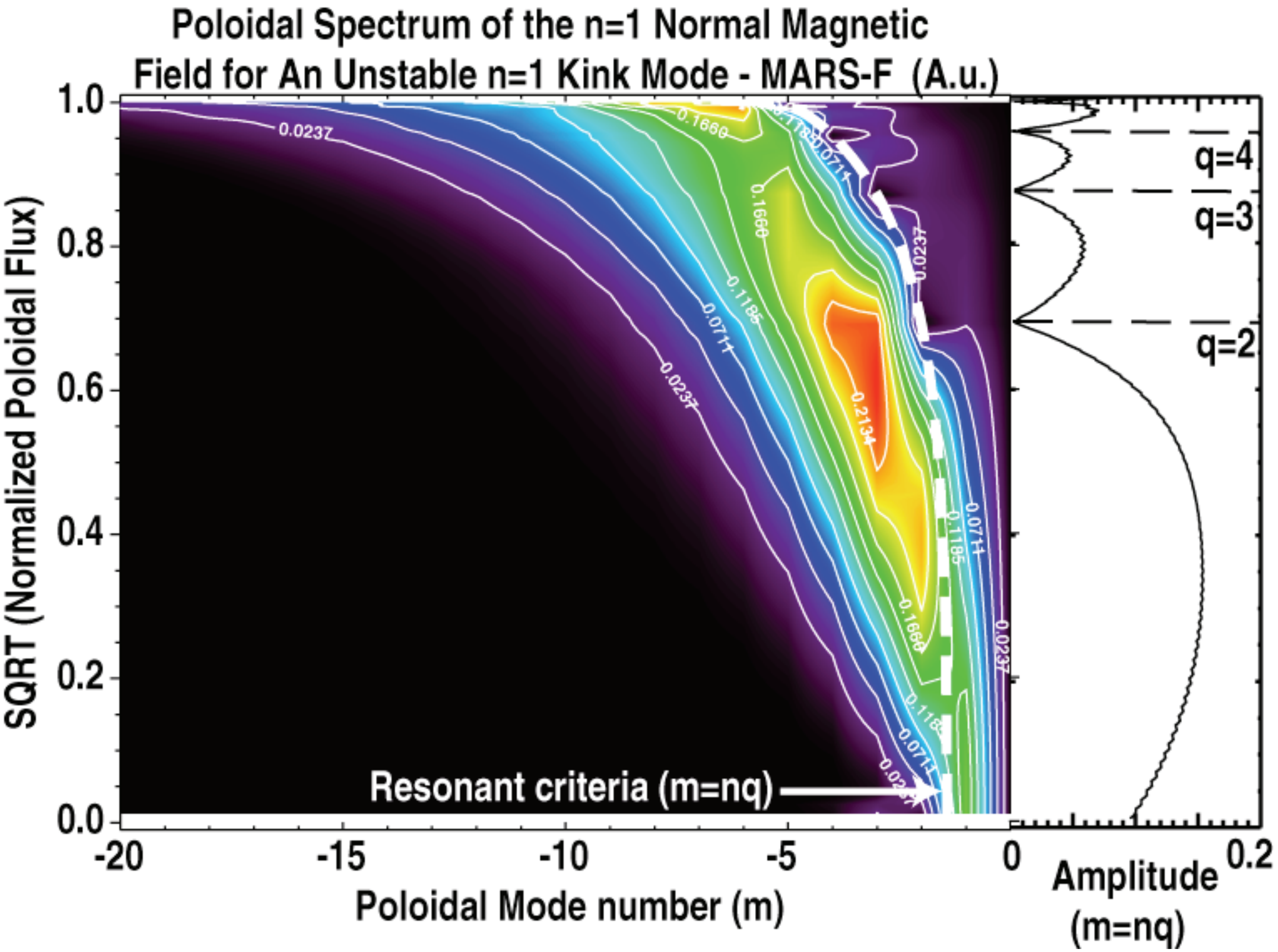
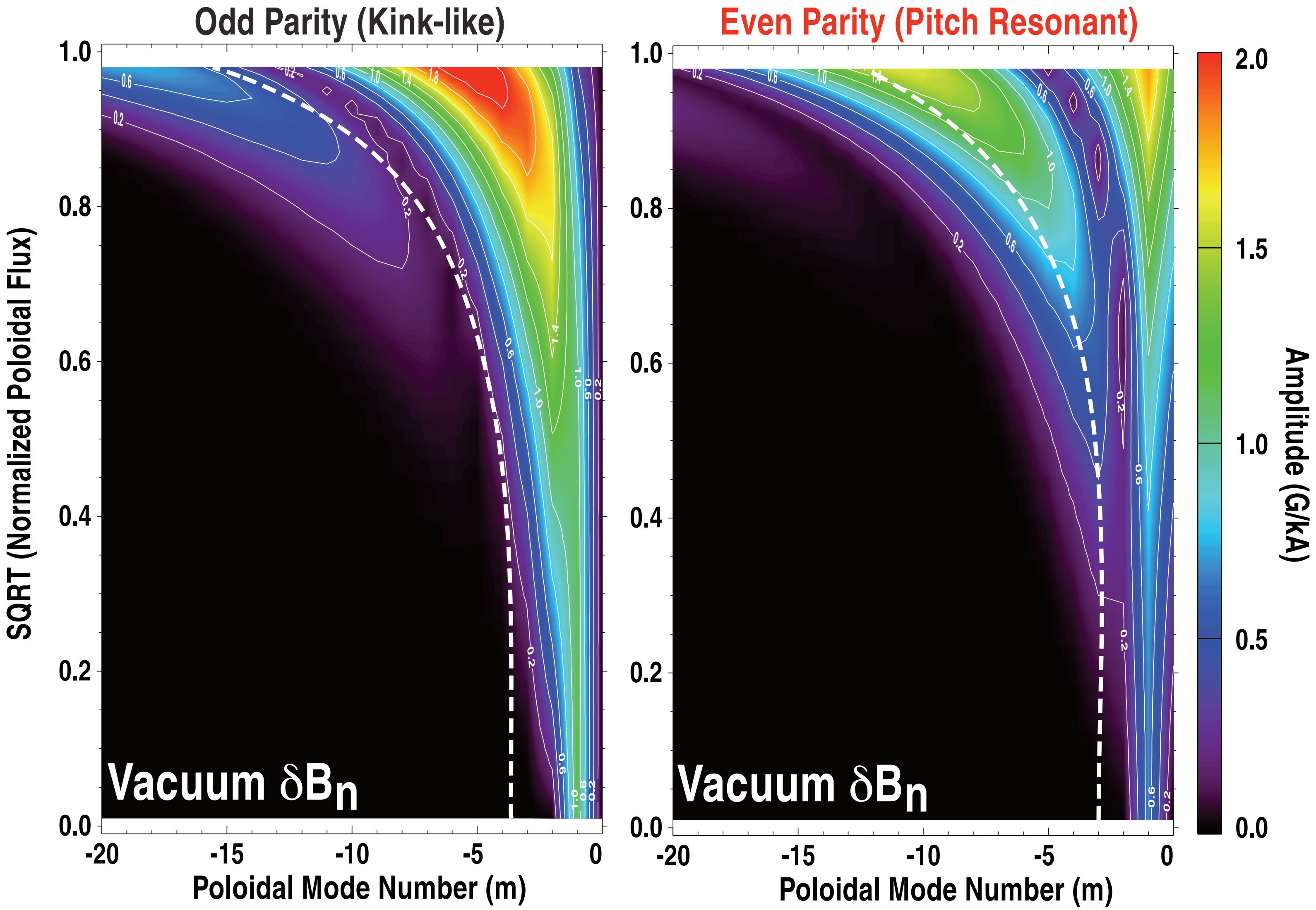
"Global" Kink Mode Is An Ideal MHD Instability With Extensive Poloidal Mode Coupling



Vacuum Calculations Show Coupling to the Resonant Field and n=3 Kink Mode and Can Be Achieved With the I-coil



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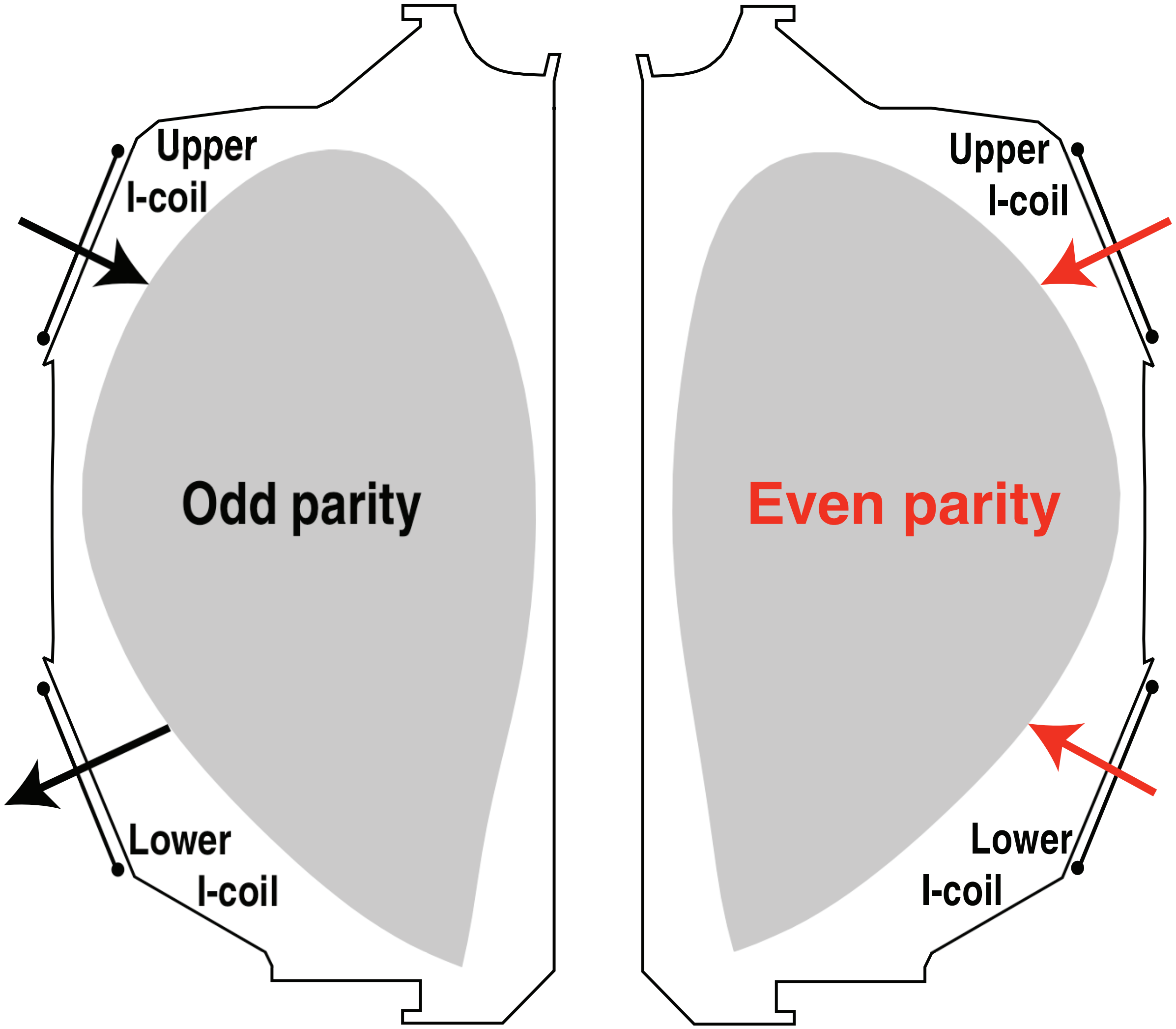
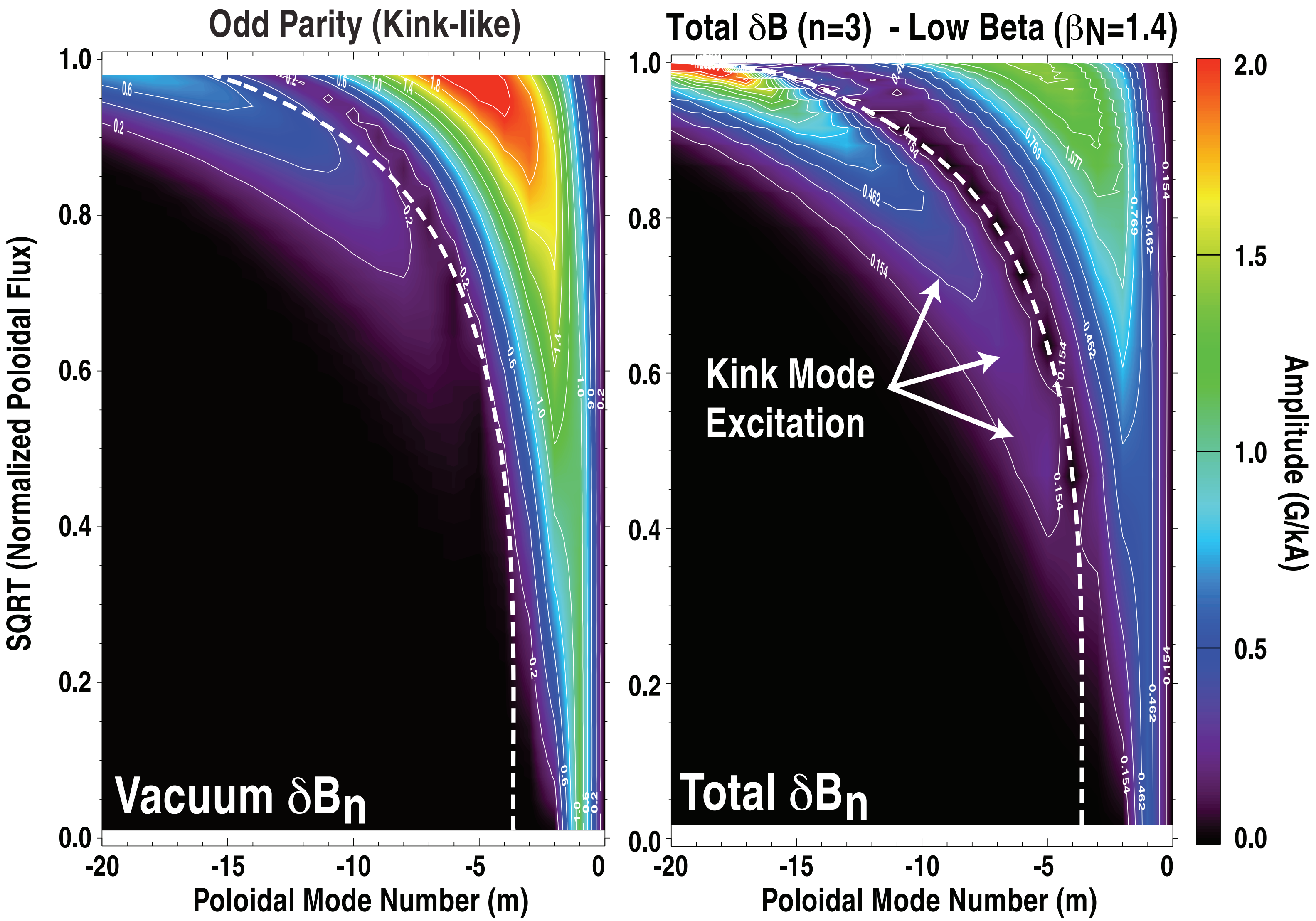
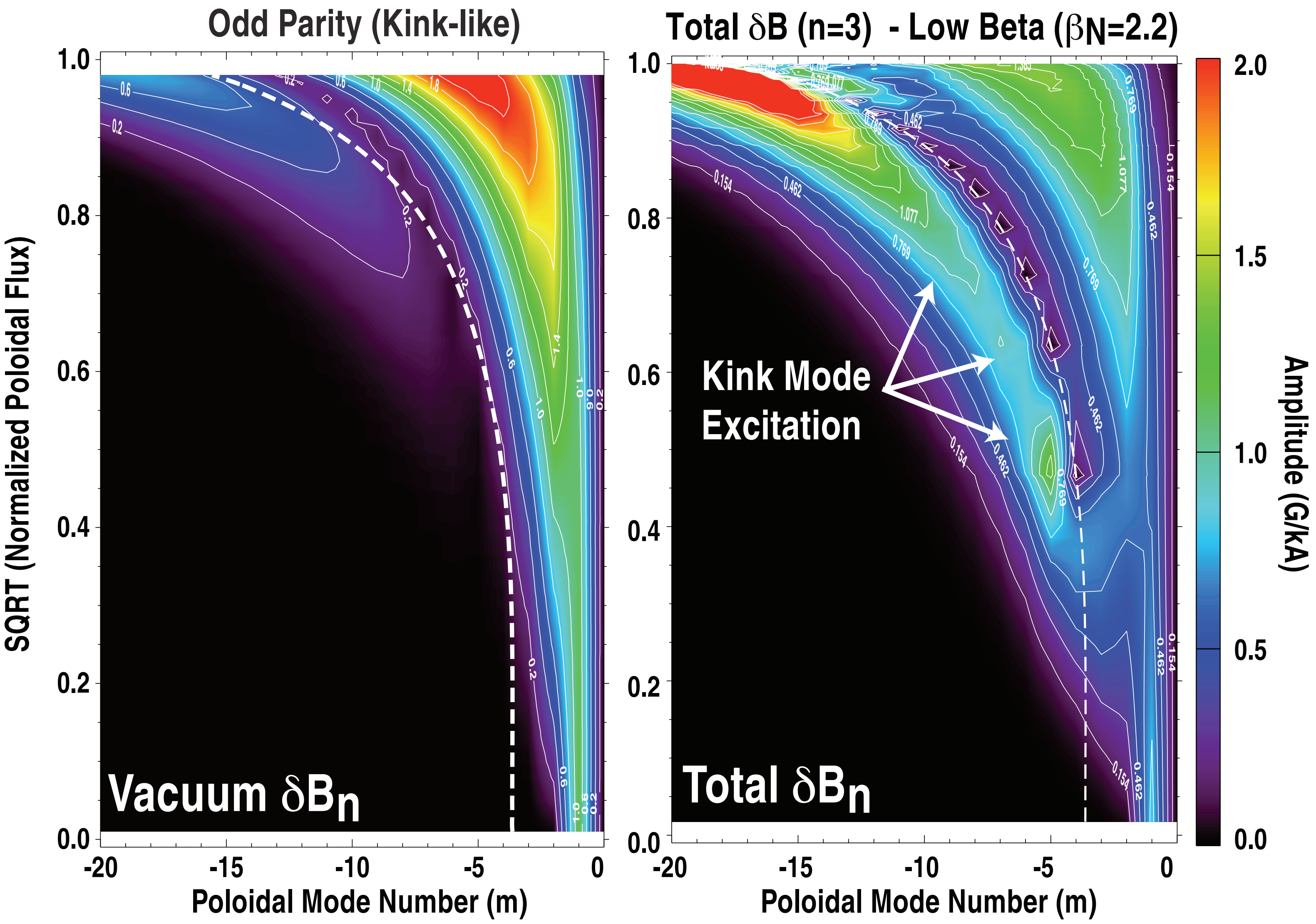


Linear Ideal MHD Calculations (MARS-F) Show Odd Parity Field Primarily Excites the Kink Mode



Kink Mode Excitation Increases with Beta



Odd Parity (Kink-like)

Total δB (n=3) - Low Beta (β_N=2.2)

Kink Mode Excitation

DIII-D NATIONAL FUSION FACILITY SAN DIEGO

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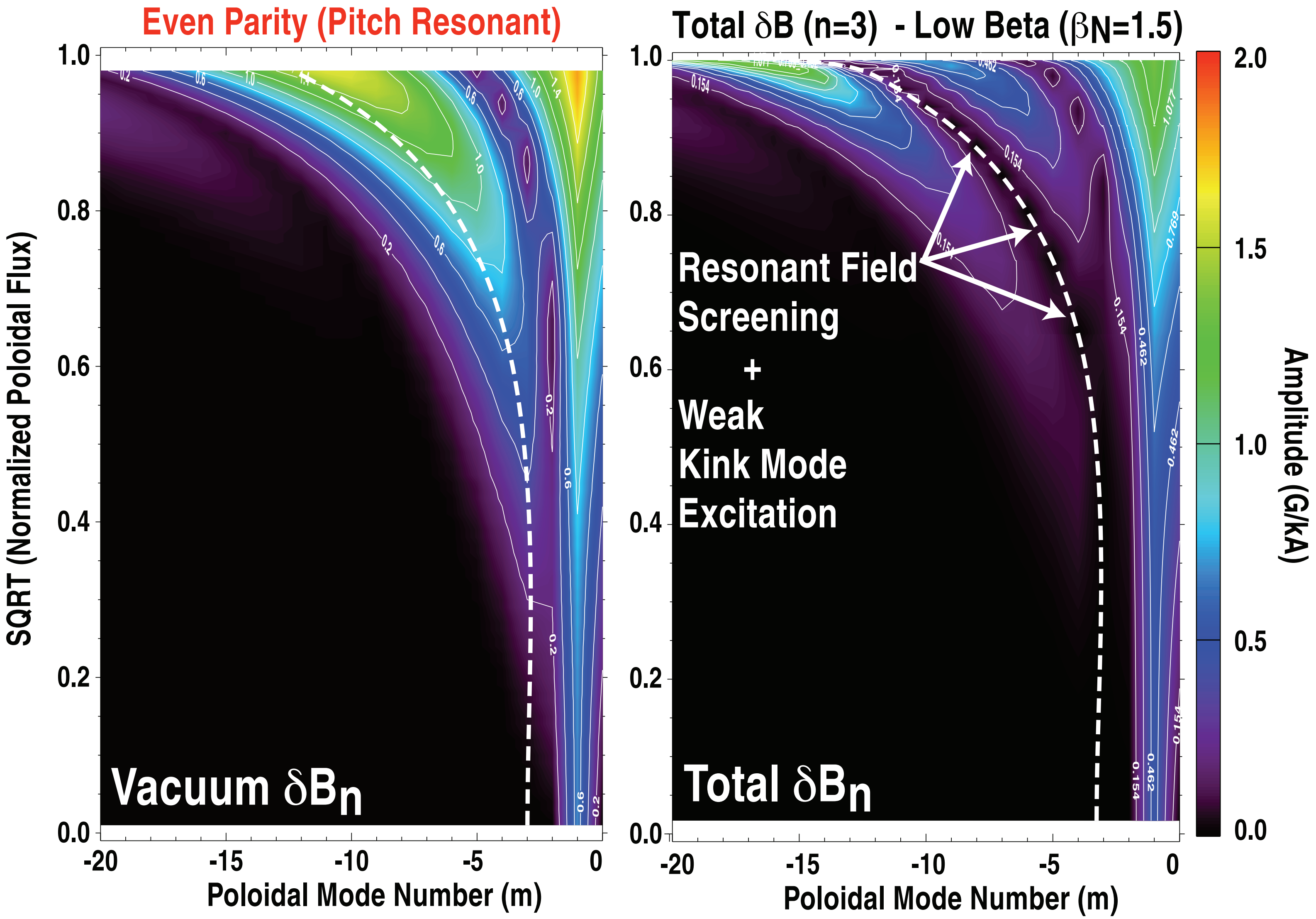
Plasma Response Screens The Resonant Components of The Even Parity Field



Even Parity (Pitch Resonant)

Total δB (n=3) - Low Beta (βN=1.5)

Resonant Field Screening
+
Weak Kink Mode Excitation

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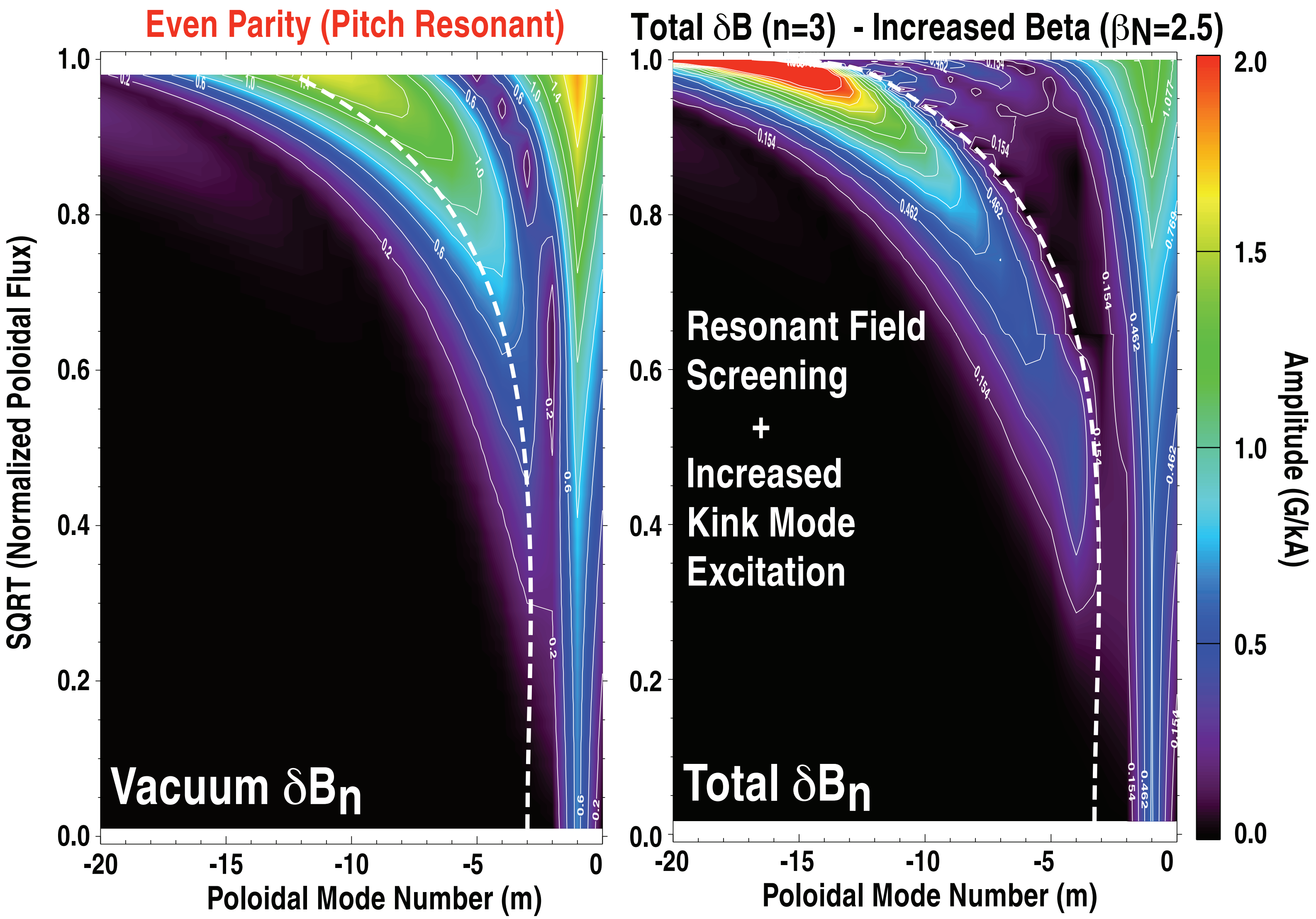
Even Parity Field Also Excites The Kink Mode As Beta Increases



Even Parity (Pitch Resonant)

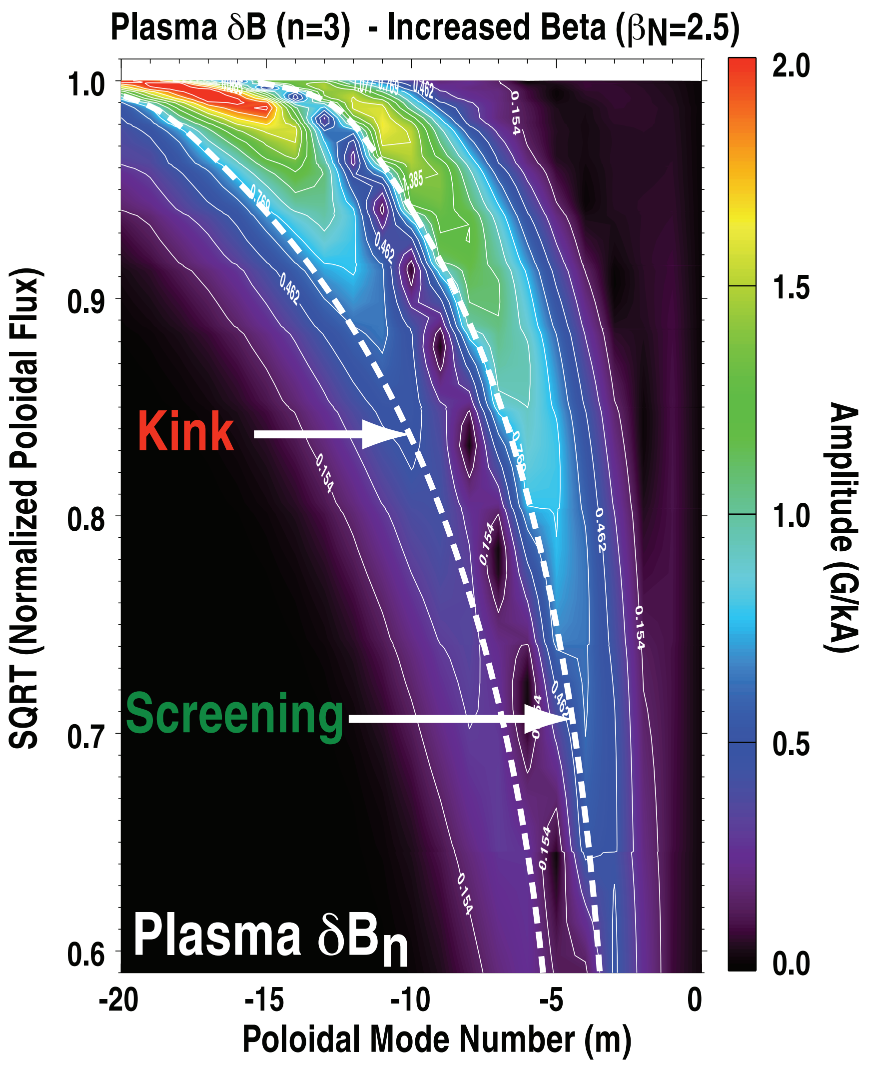
Total δB (n=3) - Increased Beta (β_N=2.5)

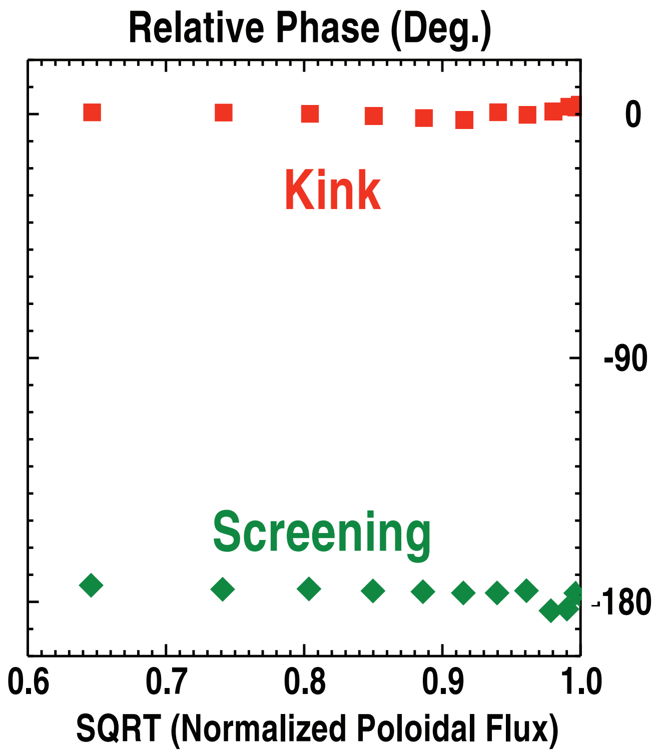
Resonant Field Screening

+

Increased Kink Mode Excitation

Phase of the Plasma δB Shows Screening Field Decreases the Local Field While Kink Mode Excitation Increases δB

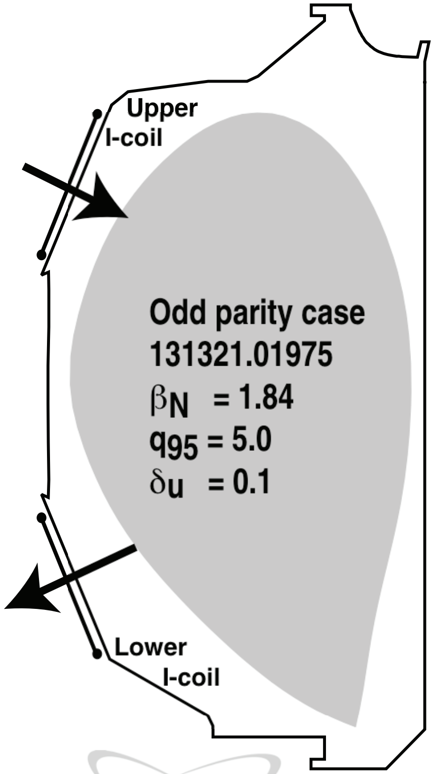


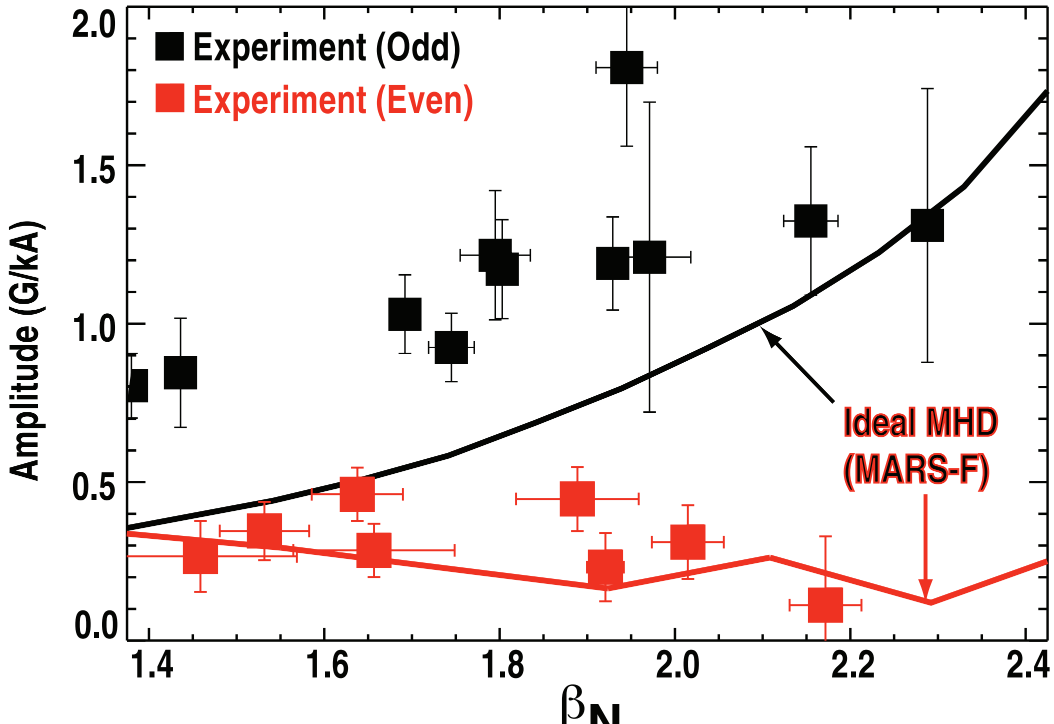


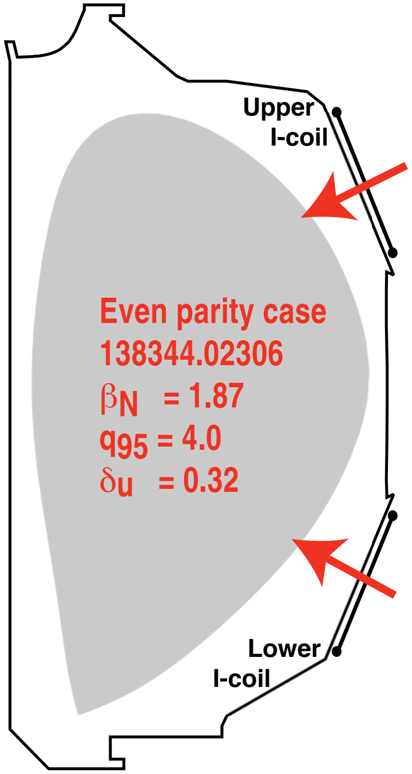
➜ **Results In A Competition Between the Two Effects**

Measured Beta Dependence of n=3 Plasma Response Is Consistent Linear Ideal MHD Model

- **Probe plasma with <u>static</u> n=3 fields using odd and even parity**
  - Cannot rotate n=3 external field with I-coil
- **Measurements and modeling (MARS-F) show plasma δB at midplane**
  - Increases with β_N for odd parity field
  - Decreases with β_N for even parity field



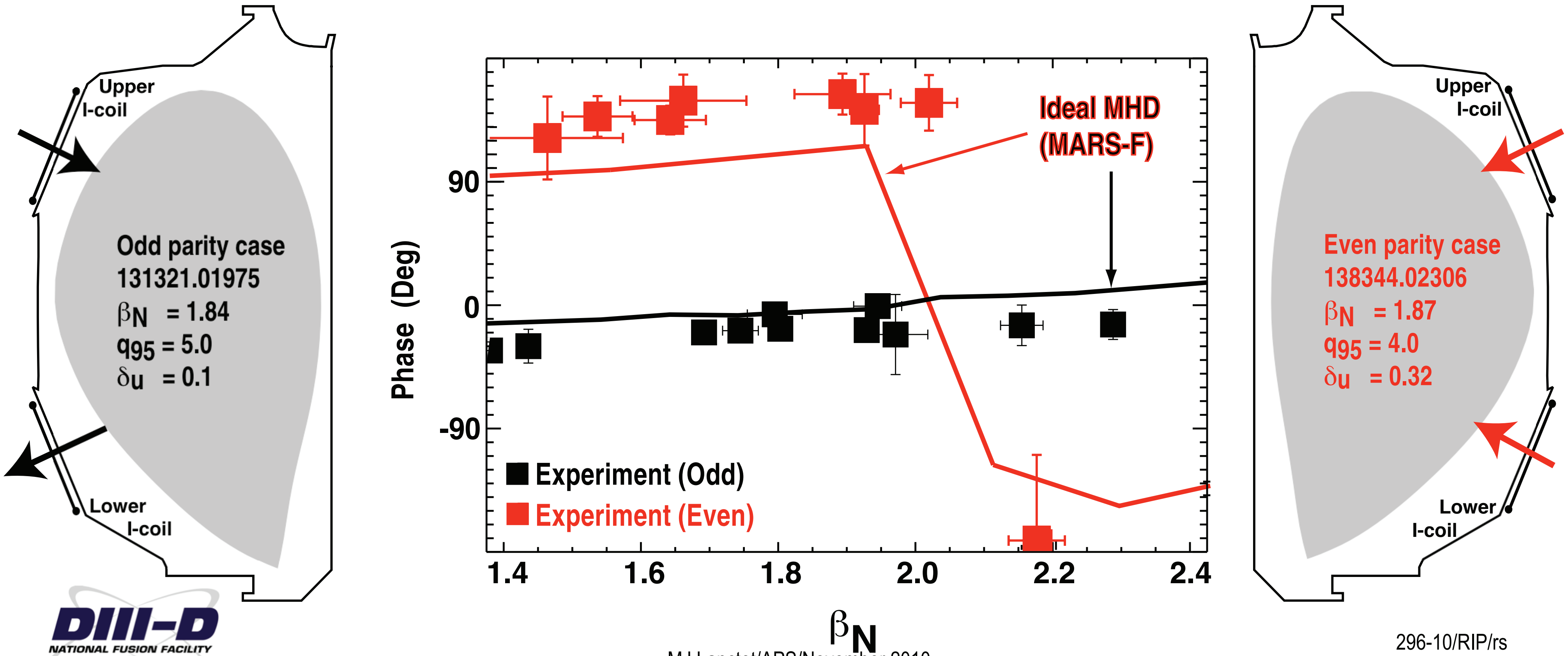




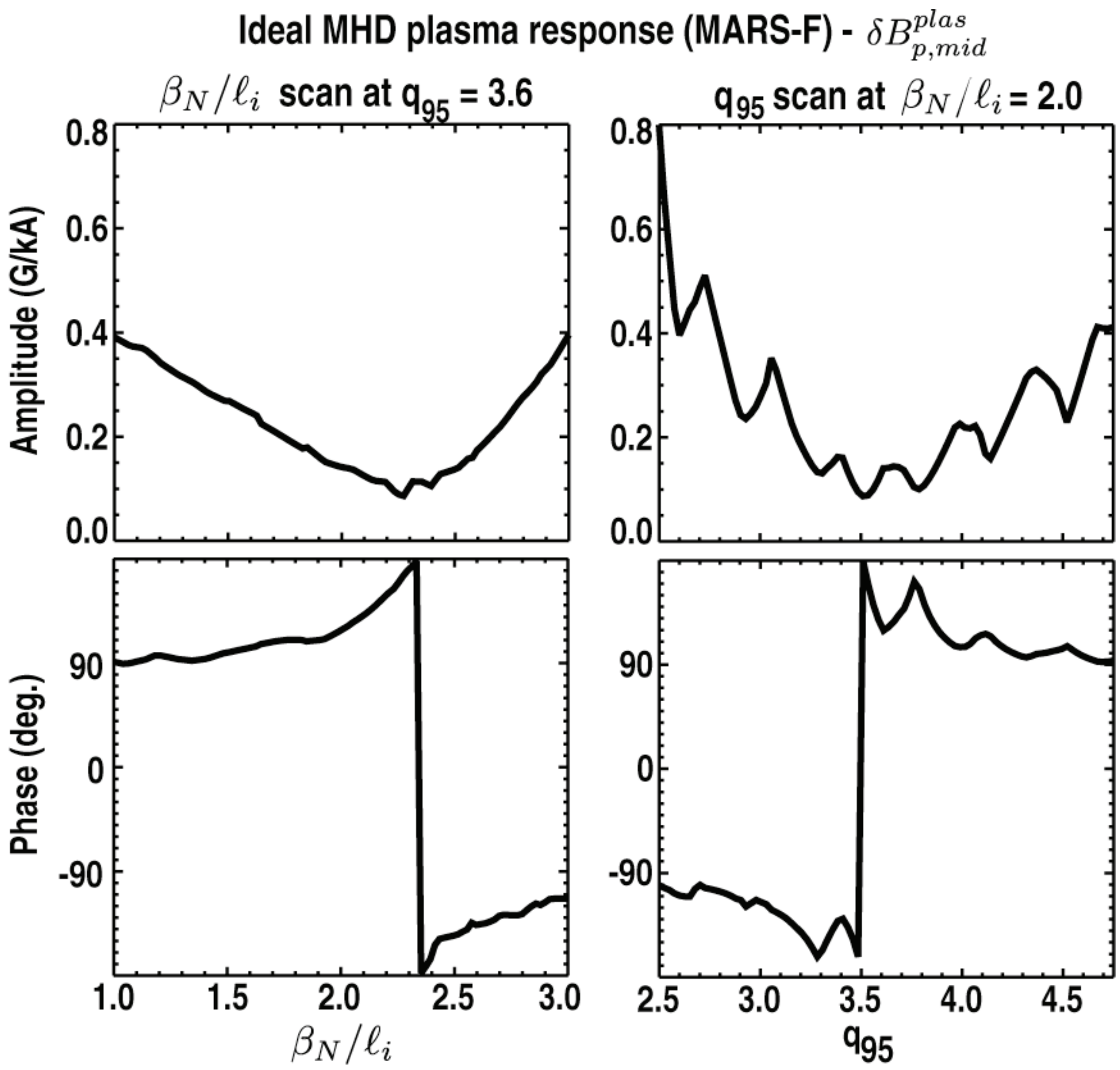
Total δB (even parity) is Dominated By Kink Mode Excitation At High Beta

- **Measured phase of even parity response field drifts by 60° with β_N**
  - Odd parity phase is relatively constant

- **Phase shift occurs when Kink Mode Excitation is equal to the Resonant Screening Field at the probe location**



Numerical Study Shows Even Parity Field Excites The Kink Mode Also At Sufficiently High Plasma Current



- **Use equilibrium solver in CORSICA* code to vary plasma pressure and plasma current in ITER-shaped DIII-D discharge**
  [*J.A. Crotinger et al., LLNL Report UCRL-ID-126284]
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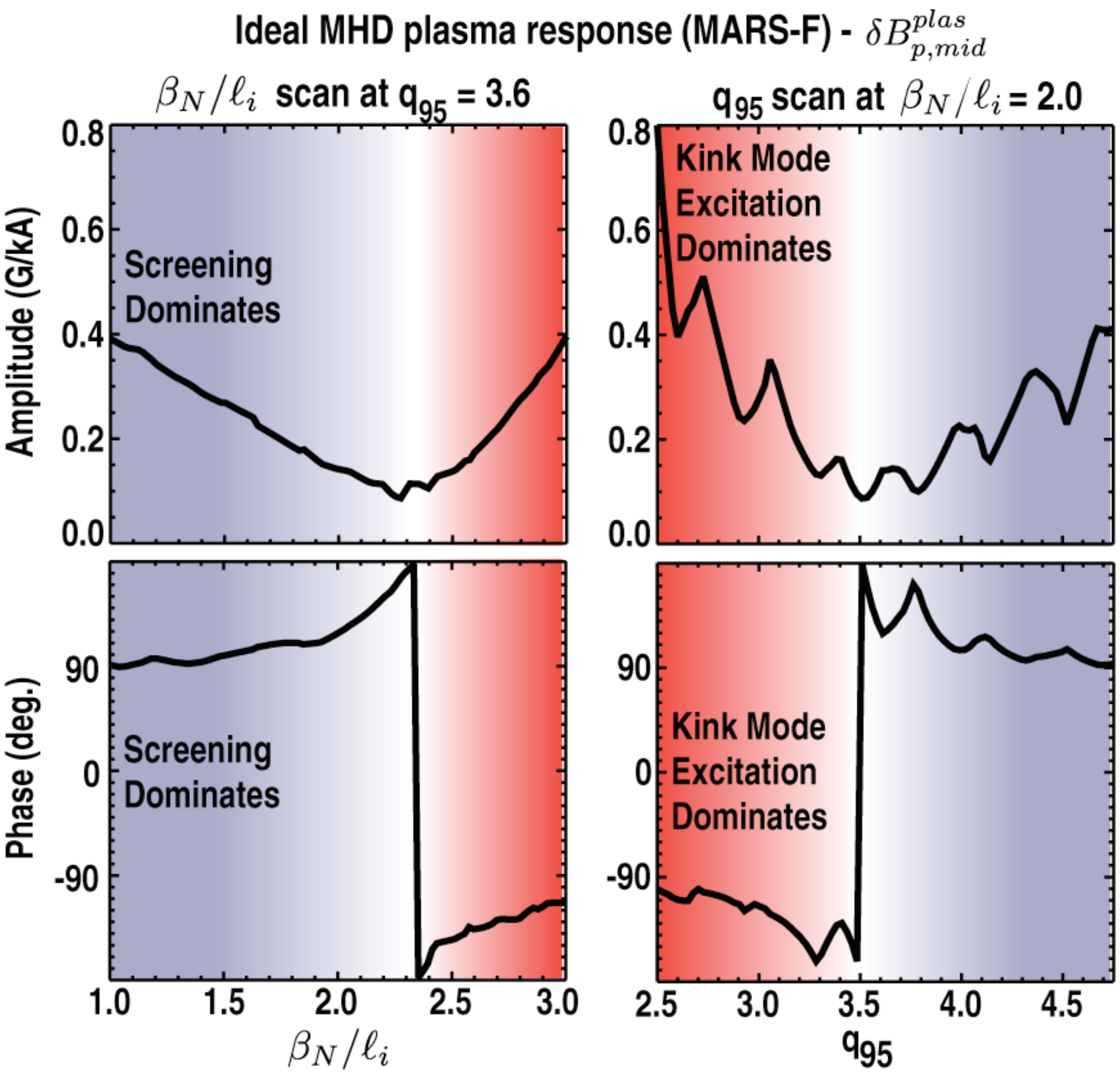

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- **Calculate plasma response to even parity n=3 I-coil field**
- **Kink Mode Excitation becomes dominant when**
  - Kink mode becomes less stable at higher β_N
  - Largest harmonics of δB^{ext} exceed nq and couple to the kink mode

Linear Ideal MHD Can Describe 3D Equilibria Over a Wide Range of Plasma Beta

I: Linear ideal MHD model is adequate where the measured $\delta B^{plas} \propto \beta_N$

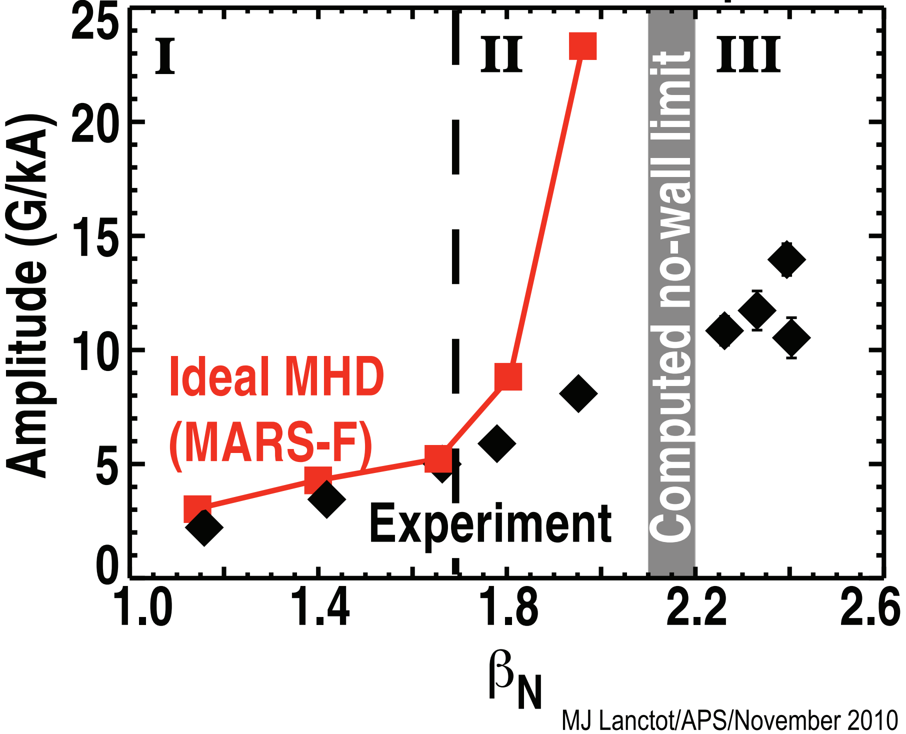
- See Hanson (UP9.00065) for real-time stability control using δB^{plas}

II: Model overestimates δB^{plas} above ~ 80% of the no-wall limit

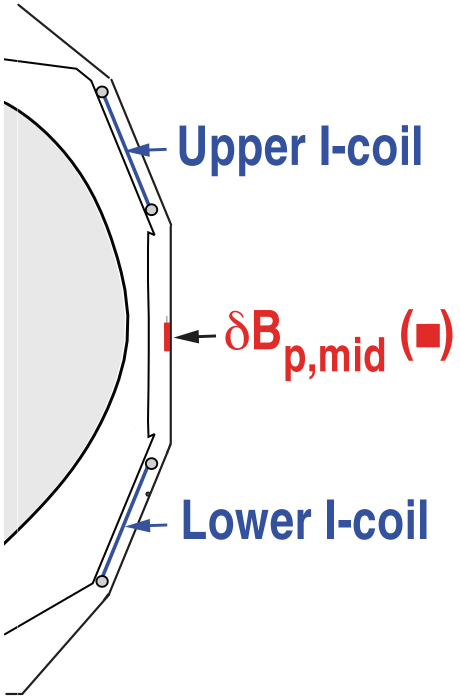
- Implies non-ideal effects are important below the no-wall limit

III: Ideal MHD predicts kink mode is unstable above the no-wall limit but experiment is stable

Plasma response (n=1) $\delta B_p/I_{I\text{-coil}}$



Magnetic sensor location



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DIII-D Discharges Exceed the No-wall Limit With a Wide Range of Rotation Profiles

- **Above the no-wall limit, ideal MHD predicts an unstable RWM**
  - RWM growth rate (γ) given by energy principle [Haney and Freidberg, *Phys. Fluids B* 1989]

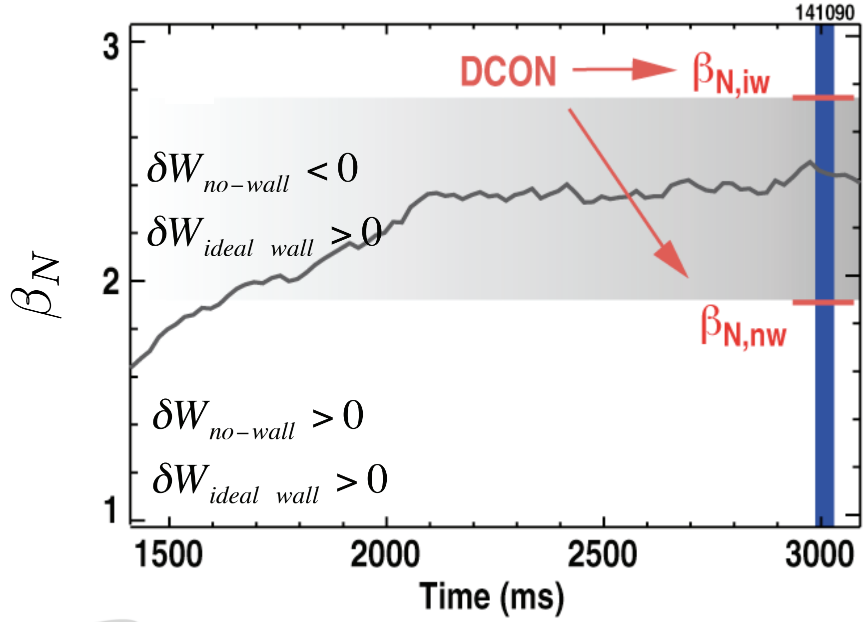
Growth rate normalized to inverse wall time →

$$\gamma\tau_W = -\frac{\delta W_{no-wall}}{\delta W_{ideal\ \ wall}}$$

← Perturbed energy assuming no wall

← Perturbed energy assuming ideal wall

Low rotation discharge above the no-wall limit



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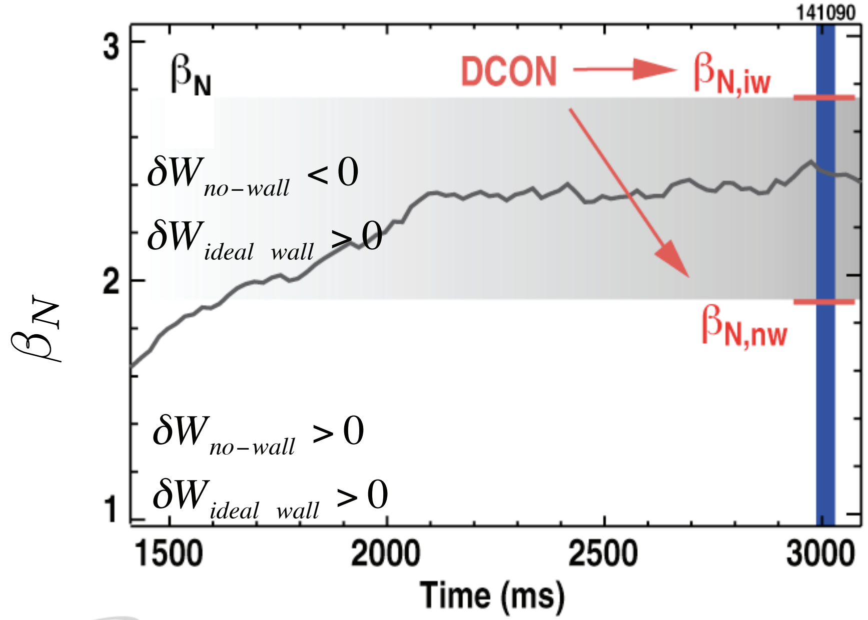
← Perturbed energy assuming no wall

← Perturbed energy assuming ideal wall

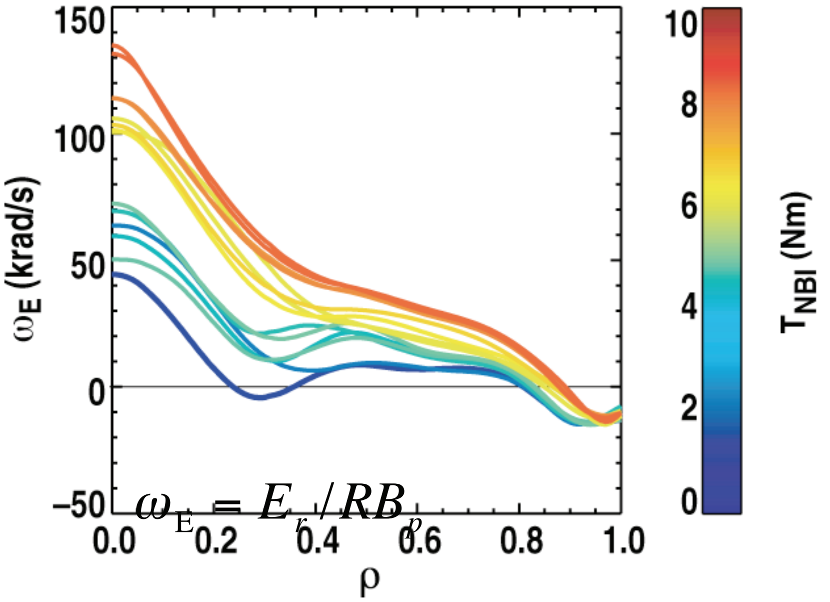
- **Rapid plasma rotation originally thought to stabilize RWM**
  - But RWM stable even for low plasma rotation

Low rotation discharge above the no-wall limit



Stable rotation profiles

β_N = 2.3-2.4



Wave-Particle Interactions Can Lead to an Exchange of Energy Between the RWM and Plasma

- **RWM rotates at ω_E in the plasma frame**

$$\omega_{\mathrm{E}} = \frac{E_r}{RB_p}$$

- **Important particle frequencies for RWM**
  - Transit frequency of passing particles:
    [Bondeson, Chu, *Phys. Plasmas* 1996]

$$\omega_{\mathrm{t}} \sim \frac{V_{th}}{qR}$$

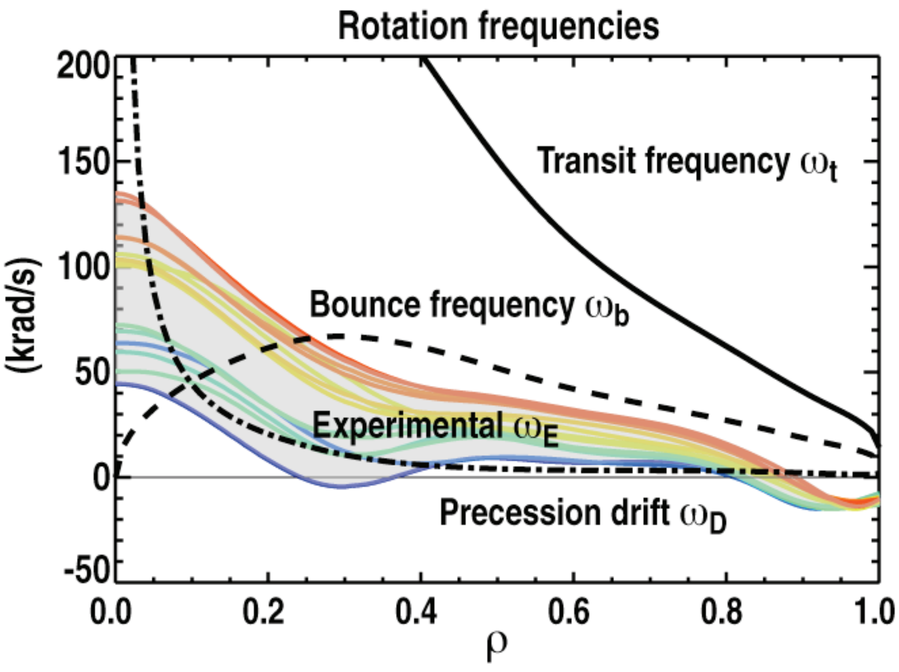
  - Bounce frequency of trapped particles:
    [Bondeson, Chu, *Phys. Plasmas* 1996]

$$\omega_{\mathrm{b}} \sim \sqrt{\frac{r}{2R}}\frac{V_{th}}{qR} \quad < \quad \omega_{\mathrm{t}}$$

  - Precession drift frequency of trapped particles:
    [Hu, Betti, *Phys. Rev. Lett.* 2004]

$$\omega_{\mathrm{D}} \sim \frac{qr_L}{r}\frac{V_{th}}{R} \quad << \quad \omega_{\mathrm{b}}$$

- **DIII-D ω_E rotation typically between precession and bounce frequencies**



Perturbed Kinetic Energy Can Be Calculated With the MISK Code*

*[Berkery, et al., *Phys. Plasmas* 2010]

- **Energy principle has been extended to include kinetic effects**
  [Hu, Betti, *Phys. Rev. Lett.* 2004]

$$\gamma\tau_{\mathrm{W}} = -\frac{\delta W_{\mathrm{nw}} + \delta W_{\mathrm{K}}}{\delta W_{\mathrm{iw}} + \delta W_{\mathrm{K}}}$$

- **The perturbed energy due to wave-particle resonances δW_K (for trapped particles) has the form**

$$\delta W_{\mathrm{K}}^{\mathrm{T}} \propto \sum_{l=-\infty}^{+\infty} \frac{\omega_{*N} + \left(\widehat{\varepsilon} - 3/2\right)\omega_{*T} + \omega_{\mathrm{E}} - \omega_{\mathrm{RWM}}}{\left\langle \omega_{\mathrm{D}} \right\rangle + l\omega_{\mathrm{b}} + \omega_{\mathrm{E}} - \omega_{\mathrm{RWM}}}$$

Precession drift

Bounce frequency

∝ Plasma rotation

Mode rotation

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$\omega_{\mathrm{RWM}} \approx 0$

Resonances when $\omega_E = -\langle\omega_D\rangle$ or $\omega_E = -l\omega_b$

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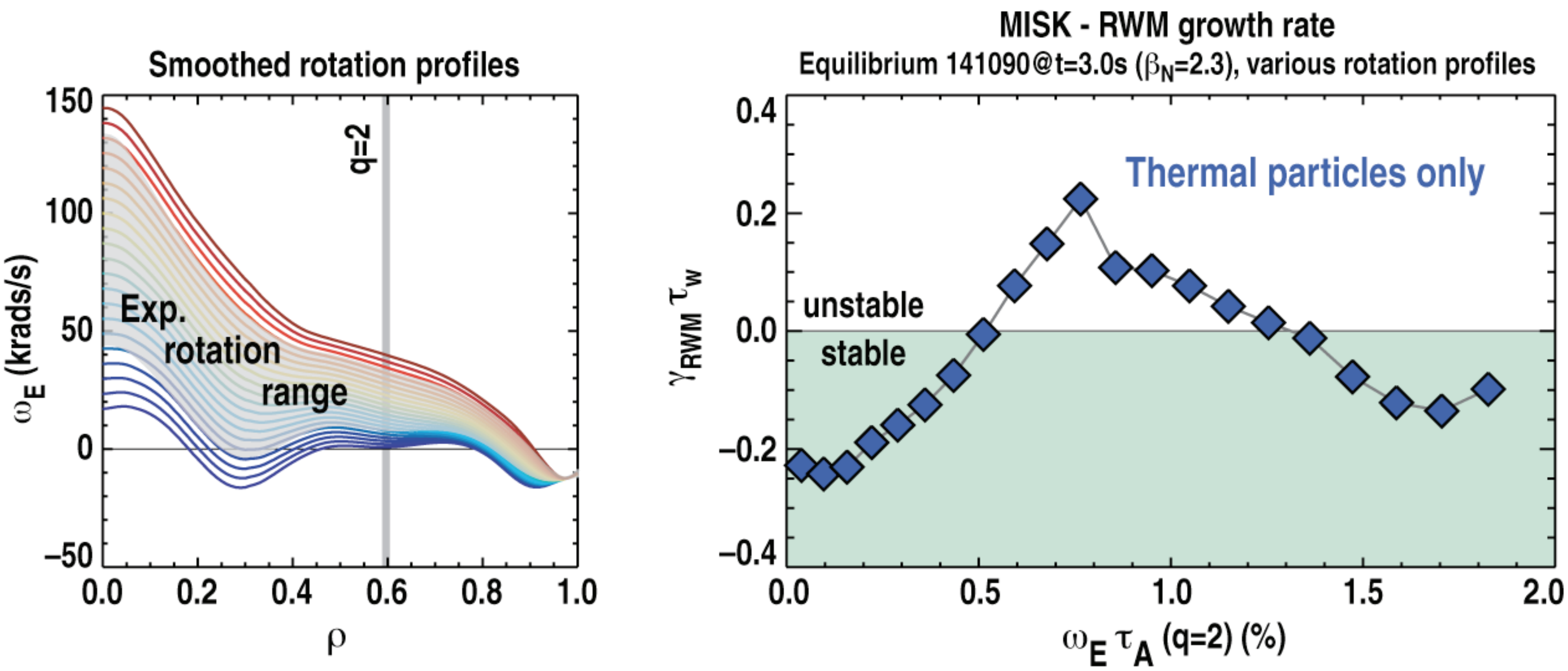
Resonances when $\omega_E = -\langle\omega_D\rangle$ or $\omega_E = -l\omega_b$

- **Perturbative approach implemented in MISK code**
  - Assume the ideal MHD mode structure of a marginal stable RWM
  - Neglect RWM growth rate and mode rotation frequency

MISK Model Can Explain the Stability Over the Entire Range of Rotation Profiles



- **Thermal particles alone are not sufficient to explain observed stability**

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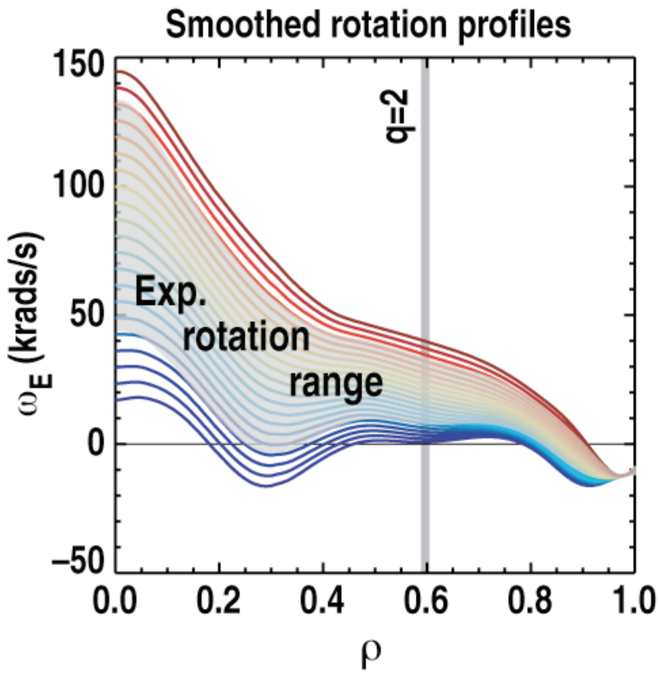
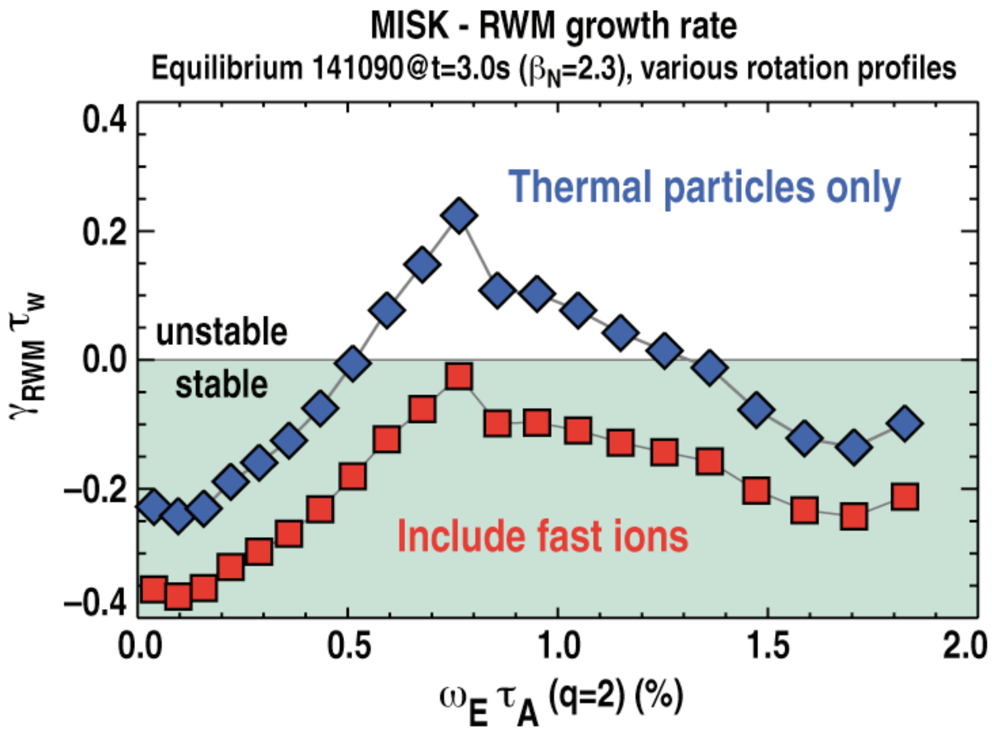


Smoothed rotation profiles



MISK - RWM growth rate

Equilibrium 141090@t=3.0s (β_N=2.3), various rotation profiles

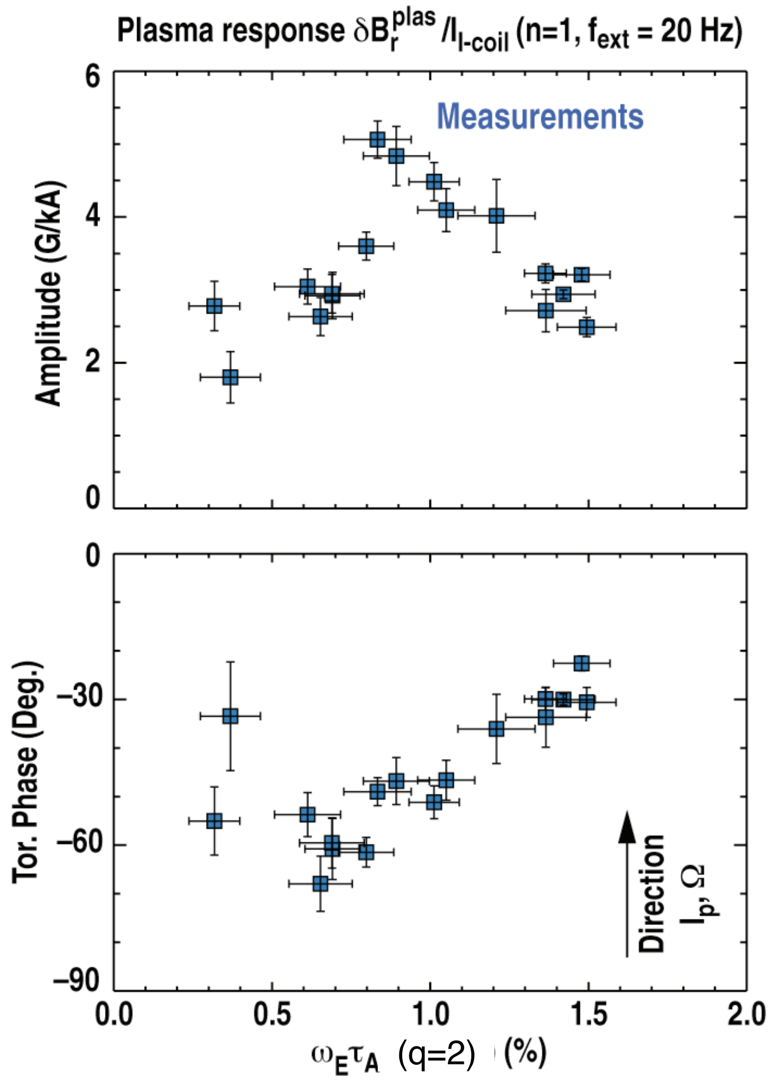
- **Thermal particles alone are not sufficient to explain observed stability**
- **Kinetic model has to include the stabilizing effect of fast ions from the NBI heating to be consistent with the experiment**
  - Fast ions constitute ~20% of the kinetic energy
  - MISK presently assumes only an isotropic slowing down distribution

Probe the RWM Damping Rate Using a Rotating n=1 Magnetic Perturbation

- **Vary neutral beam torque T_{NBI} from 1.5 to 8.0 Nm while keeping** $\beta_N \approx 2.3 \left(> \beta_N^{no-wall} \right)$

- **Plasma response largest at finite plasma rotation:** $\omega_E \tau_A$ (q=2) ~0.9%

- **Phase shift of plasma response with respect to external field largest at somewhat lower rotation:** $\omega_E \tau_A$ (q=2) ~0.6%

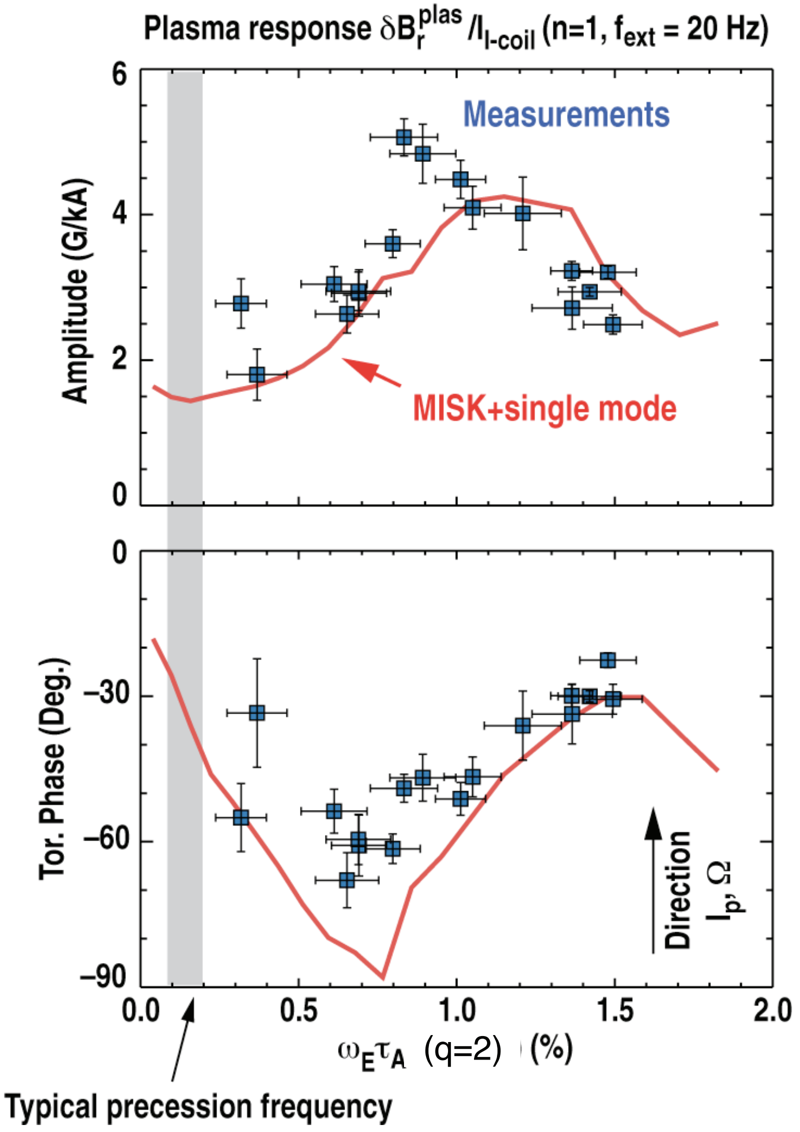


Measured Amplitude and Phase of the Plasma Response Reveals the Characteristics of Kinetic RWM Stabilization

- **RWM growth rate can be mapped to the plasma response using a single mode model**
  [H. Reimerdes, et al., *Phys. Rev. Lett.* (2004)]

- **MISK modeling reproduces all the characteristics of the measured plasma rotation dependence of δB^{plas}**

➔Stability at low rotation direct effect of resonance with the precession drift of trapped ions



Summary

- **Below the no-wall limit, the linear ideal MHD perturbed equilibrium model**
  - Describes the linear plasma response in rotating plasmas
  - Predicts two types of response fields: "Resonant field screening" and "Kink mode excitation"
  - Global Kink Mode is easily excited at high β_N and low q_{95}

- **Above the no-wall limit, the measured rotation dependence of the plasma response reveals direct evidence of kinetic RWM stabilization**